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PERSISTENT APPRECIATIONS AND OVERSHOOTING: A NORMATIVE ANALYSIS

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ABSTRACT

Most economies experience episodes of persistent real exchange rate appreciations, when the question arises whether there is a need for intervention to protect the export sector. In this paper we present a model of irreversible destruction where exchange rate intervention may be justified if the export sector is financially constrained. However the criterion for intervention is not whether there are bankruptcies or not, but whether these can cause a large exchange rate overshooting once the factors behind the appreciation subside. The optimal policy includes ex-ante and ex-post interventions. Ex-ante (i.e., during the appreciation phase) interventions have limited effects if the financial resources in the export sector are relatively abundant. In this case the bulk of the intervention takes place ex-post, and is concentrated in the first period of the depreciation phase. In contrast, if the financial constraint in the export sector is tight, the policy is shifted toward ex-ante intervention and it is optimal to lean against the appreciation. On the methodological front, we develop a framework to study optimal dynamic interventions in economies with financially constrained agents.

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1 Introduction

Most economies experience episodes of large real exchange rate appreciations. There are many factors with the potential to fuel these appreciations. For example, they can stem from domestic policies aimed at taming a stubborn inflationary episode, from the absorption of large capital inflows caused by domestic and external factors, from exchange rate interventions in trading partners, from domestic consumption booms, from a sharp rise in terms of trade in commodity producing economies or, in its most extreme form, from the discovery of large natural resources wealth (the so-called Dutch disease).

While there are idiosyncrasies in each of these instances, the common policy element is that, when the appreciation is persistent enough, the question arises whether there is a need for intervention to protect the export sector (often referred as "competitiveness" policies). This widespread concern goes beyond the purely distributional aspects associated to real appreciations. The fear is that somehow the medium and long run health of the economy is compromised by these episodes. If this concern is justified, should policymakers intervene and stabilize the exchange rate before it is too late? More generally, what does the optimal policy look like?

In this paper we propose a framework to address this common policy element. We present a dynamic model of entry and exit in the export sector where entrepreneurs face financial constraints and exchange rate stabilization may be justified. In our model, when financial constraints damage the export sector's ability to recover, the economy experiences a large exchange rate overshooting once the factors behind the appreciation subside and nontradable demand contracts. Although not always present, overshooting episodes are pervasive, especially when financial frictions are widespread. Figure 1 illustrates three recent examples: The Finnish, Mexican and Asian episodes of the early 1990s, mid 1990s, and late 1990s, respectively. In each of them, the pattern is one where the appreciation is followed by a depreciation of the real exchange rate that significantly overshoots its new medium term level.[1]

In our model, the overshooting results from the export sector's inability to absorb the resources (labor) freed from the contraction in nontradable demand. This inability leads to an amplified fall in real wages, which is costly to consumer-workers.[2] There is scope for policy intervention because

[1] The figure shows real exchange rate indices normalized to 100 at date 0. The latter corresponds to June 1997 for the Asian Crisis, November 1994 for Mexico, and October 1991 for Finland. The Asian crisis real exchange rate is a simple average of the indices for Indonesia, Korea, Malaysia, Philippines, and Thailand. Data Source: IFS (Effective Real Exchange Rate) for Finland, Indonesia, Malaysia and Philippines. Real Exchange Rate from Hausmann et al (2006) for Mexico, Korea and Thailand.

[2] In practice, the drop in the relative price of nontradables and real wages often takes the form of a sharp nominal depreciation which is not matched by a rise in the nominal price of nontradables and wages. See, e.g., Goldfajn and Valdes (1999) and Burnstein et al (2005).

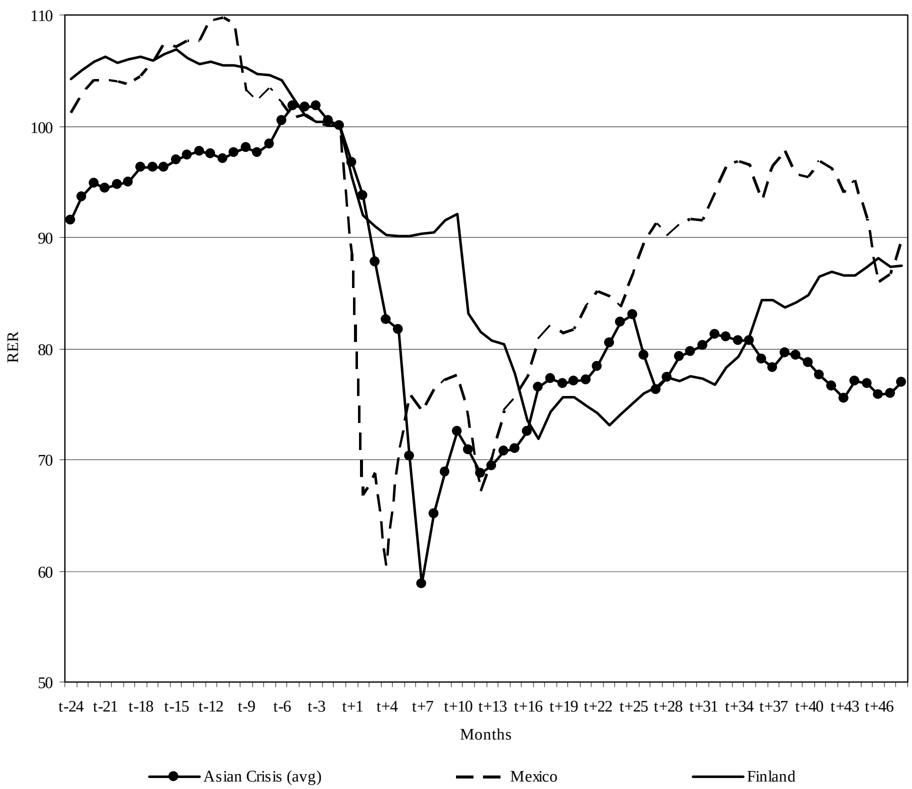

Figure 1: 1990's Overshooting in Finland, Mexico and Asia

there is a connection between the severity of the overshooting and the extent of the contraction in the export sector during the preceding appreciation phase. If consumers were to reduce their demand for nontradables in this phase, then there would be less destruction ex-ante and a faster recovery ex-post. However, rational atomistic consumers ignore the effect of their individual decisions during the appreciation phase on the extent of the overshooting during the depreciation phase. It is this pecuniary externality that justifies and informs policy intervention in our framework.

Our analysis has two parts, a positive one and a normative one. The former consists of a dynamic model of factor reallocation in the presence of financial constraints. Our model economy starts by transiting into an appreciation phase, which it exits into a depreciation phase with a Poisson probability. There are several regions of interest, indexed by the financial resources of the export sector at the onset of the appreciation. When financial resources are plentiful, the economy reaches the first best as real exchange rates (and real wages) are pinned down by purely technological free entry and exit conditions, and hence are orthogonal to consumers' actions. At lower levels of financial resources, financial constraints may become binding during the appreciation phase, the depreciation phase, or both. If they are only binding during the appreciation phase, then the economy experiences bankruptcies but the recovery of the export sector is swift once the depreciation phase starts and the exchange rate is again pinned down by purely technological factors. In contrast, if the financial constraint is binding during the depreciation phase, the recovery of the export sector is slow and the initial real depreciation overshoots the long run depreciation.

On the normative side, we consider the optimal intervention of a benevolent planner that seeks to maximize consumers' welfare, subject to not worsening entrepreneur' welfare. The planner faces the same financial constraint present in the private economy. This limits the planner's instruments, as it rules out direct transfers across groups. Instead, we focus on market-mediated transfers implemented through interventions that influence the real exchange rate. That is, interventions that affect consumers' choices and, thus, the entrepreneurial sector through their effect on equilibrium prices. From this perspective, we show that consumers gain from stabilizing the appreciation whenever this leads to a faster recovery of the export sector once the appreciation subsides. The gain derives from the increase in real wages associated to a faster reconstruction of the export sector.

Importantly, even when overshooting is expected, intertemporal consumption allocation considerations put limits on how much intervention is desirable during the appreciation phase. This connects our analysis to a central consideration for policymakers when dealing with an asset appreciation, be it the currency, real estate or any other asset with potential macroeconomic implications: If there is a need for intervention, how much should be done as prevention (ex-ante) and how much should be left for after "the crash" (ex-post)? In our framework, the answer to this question depends primarily on the extent of the financial constraint in the export sector. On one end, when the financial constraint is severe, ex-ante intervention is most effective. On the other end, when the constraint is loose, ex-post intervention is most desirable and effective. In general, the optimal policy has elements of both, ex-ante and ex-post intervention.

The approach to optimal policy proposed in this paper resembles that of the literature on dynamic optimal taxation. In this dimension, the main innovation of the paper is to apply this methodology to an environment where a subset of agents are financially constrained, imposing restrictions on the ability of policy to reallocate resources between these agents and the rest of the economy. This approach and the solution method developed should prove useful outside our particular application.

Our paper belongs to an extensive literature on consumption and investment booms in open economies, as well as on the role of financial factors in generating inefficiencies in these booms (see, e.g. Gourinchas et al 2001, Caballero and Krishnamurthy 2001, Aghion et al 2004). There is a growing theoretical and empirical literature on the effect of financial frictions on the behavior of exporters (e.g. Chaney, 2005, Manova, 2006), and on the slow response of exports after devaluations (Fitzgerald and Manova, 2007). Our paper emphasizes the general equilibrium implications of this behavior (overshooting). The idea that excessive exchange rate fluctuations can hurt financially constrained export firms is also present in Aghion et al (2006), who explore its effects on investment in innovation and growth.

The pecuniary externality that justifies intervention in our framework is related to those identified in Geanakoplos and Polemarchakis (1996), Caballero and Krishnamurthy (2001, 2004), Lorenzoni (2006) and Farhi et al (2006). Aside from its specific context, the main novelty of our paper is to embed this externality in a tractable model of optimal policy, which allows us to fully characterize the economy's dynamics and to analyze the trade-off between ex ante and ex post interventions.

In terms of its mechanism, the paper also belongs to the literature on Dutch disease. There, intervention is justified by the presence of dynamic technological externalities through learning-by-doing (see, e.g. van Wijnbergen 1984, Corden 1984, and Krugman 1987). In contrast, our paper highlights financial frictions and the pecuniary externalities that stem from these. The policy implications of these two approaches are different: While learning-by-doing offers a justification for industrial policies as a development strategy, the financial frictions we highlight have intertemporal reallocation implications of the sort that matter for business cycle policies.

Section 2 presents a stylized model of creative destruction over appreciation and depreciation cycles. Section 3 characterizes optimal exchange rate intervention in such setup. Section 4 discusses different extensions and their impact on optimal policy. Section 5 concludes and is followed by an extensive appendix, containing all the proofs.

2 A Simple Model of a Destructive Appreciation and Overshooting

In this section we present a model of an economy experiencing a temporary, but persistent, real appreciation. The export sector faces large sunk costs of investment, which limit the extent of its desired contraction, in order to keep capital operational and preserve the option to produce once the appreciation is over. However, this waiting strategy generates losses that require financing. If this financing is limited, the export sector experiences a larger contraction than desired. From the point of view of the economy as a whole, these excessive contractions may compromise the recovery of the export sector once the appreciation is over, leading to a prolonged period of deep real depreciation and low wages.

2.1 The Environment

There is a unit mass of each of two groups of agents within the domestic economy: consumers and entrepreneurs (exporters). There are two consumption goods: a tradable and a nontradable good. The consumer supplies inelastically one unit of labor each period, which can be used as an input for the production of tradables or nontradables. In both cases one unit of labor is needed to produce one unit of output. In addition, the production of tradables requires one unit of capital, or an "export unit," i.e., the technology of the tradable sector is Leontief in labor and capital. Creating an export unit requires investing f units of tradable goods. After an export unit has been set up, it needs to be maintained in operation, otherwise it is irreversibly shut down.[3] Entrepreneurs are the only agents that have access to the technology to run and maintain export units. At date 0 they begin with n_{-1} open export units. The markets for tradables, nontradables and labor are competitive.

Entrepreneurs are risk neutral and only consume tradable goods. Their preferences are given by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t c_t^{T,e},$$

where $c_t^{T,e} \geq 0$ denotes entrepreneurs' consumption of tradable goods. Consumers have log-separable instantaneous utility on the consumption of tradables and nontradables, c_t^T and c_t^N. Their preferences are given by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\theta_t\left(u\left(c_t^T\right)+u\left(c_t^N\right)\right),$$

[3] These assumptions capture the fact that export oriented firms often have more specific (sunk) capital and operations than firms producing primarily for domestic markets. Of course there are important exceptions to this generalization. Later in the paper we discuss the effect of introducing adjustment costs in the nontradables sector.

where $u(c) = \log c$ and θ_t is a taste shock.

The taste shock is the *only* source of uncertainty and is determined by the state s_t, which can take two values in $S = \{A, D\}$ and follows a Markov process with transition probability $\pi(s_{t+1}|s_t)$. The economy begins with a transition into the "appreciation" state $s_t = A$, with $\theta_t = \theta_A$. Each period, with probability $\pi(D|A) = \delta$, the economy switches to the "depreciation" state $s_t = D$, with $\theta_t = \theta_D$. Once the latter transition takes place, D is an absorbing state, i.e., $\pi(A|D) = 0$. We assume that:

$$\theta_A > \theta_D = 1.$$

In the appreciation state the taste shock drives up consumers' demand for both tradable and nontradables, putting upward pressure on the real exchange rate (since the supply of tradables is fully elastic while that of tradables is not – see below). In reality, increases in consumption demand are usually due to positive wealth shocks, e.g., an improvement in the terms of trade for a commodity producing country, or to external shocks which generate positive capital inflows. The taste shock is a convenient device to introduce a shift in consumption demand with minimal added complications. We will return to this issue later in the paper, once we have developed our main points.

Both groups have access to the international capital market, where they can trade a full set of state contingent securities. On each date t, agents trade one-period state-contingent securities that pay one unit of tradable good in period $t+1$ if state s_{t+1} is realized. The entrepreneurs holdings of securities are denoted by $a\left(s^t\right)$ where $s^t = \langle s_0, s_1, ..., s_t\rangle$ denotes the history of the economy up to date t. Note that our simple Markov chain yields histories that are limited to a block of periods in A, followed by D's (there are no alternations).

All entrepreneurs begin with an initial financial positions equal to a_0. For consumers, we set it to zero without loss of generality. Consumers face no financial constraints, while entrepreneurs face the financial constraint

$$a\left(s^t\right) \geq 0. \tag{1}$$

That is, entrepreneurs cannot commit to make any positive repayment at future dates. This is a simple form of financial markets imperfection, which captures the idea that entrepreneurs have limited access to external finance. This is the *only* friction we introduce in the model.

The rest of the world is captured by a representative consumer with a large endowment of tradable goods and linear preferences represented by $\mathrm{E}\sum_{t=0}^{\infty}\beta^t c_t^{T,*}$. Therefore asset pricing is risk neutral: at date t, the price of a security paying one unit of tradable in state s_{t+1} is $\beta\pi(s_{t+1}|s_t)$.

2.2 Decisions and Equilibrium

Let $p\left(s^t\right)$ denote the price of the nontradable good in terms of units of tradable (the numeraire), or the real exchange rate (defined à la IMF). Given the linear technology in the nontradable sector, the equilibrium wage in terms of tradables must be equal to this price. Consumers and entrepreneurs take the real exchange rate as given. Equilibrium prices and quantities are functions of the whole history s^t. To save on notation, whenever confusion is not possible we only use the time subindex t, e.g., p_t is shorthand for $p\left(s^t\right)$.

2.2.1 Consumers

Since wages are equal to p_t, markets are complete, and intertemporal prices are pinned down by the world capital market, consumers face the single intertemporal budget constraint

$$\sum_{t,s^t}\beta^t\pi\left(s^t\right)\left(c^T\left(s^t\right)+p\left(s^t\right)c^N\left(s^t\right)\right)\leq\sum_{t,s^t}\beta^t\pi\left(s^t\right)p\left(s^t\right), \tag{2}$$

where $\pi\left(s^t\right)$ denotes the ex ante probability of history s^t. Thus, consumers' demand for tradables and nontradables take the simple form:

$$\begin{aligned} c_t^T &= \kappa\theta_t, \\ c_t^N &= \frac{\kappa\theta_t}{p_t}, \end{aligned}$$

where the constant κ is

$$\kappa=\frac{1}{2}\frac{\sum_{t,s^t}\beta^t\pi\left(s^t\right)p\left(s^t\right)}{\sum_{t,s^t}\beta^t\pi\left(s^t\right)\theta\left(s_t\right)}. \tag{3}$$

There are two important features from the consumption block. First, during A periods the demand curve for nontradables shifts upward. This is the source of the appreciation. Second, κ is endogenous and is increasing in the value of the exchange rate at any future date. The latter feature will be essential in the analysis of optimal policy.

2.2.2 Exporters and Equilibrium

Even though consumption volatility is not the result of any friction, it may create problems for *both* firms and consumers, if the export sector has limited financial resources. Before discussing this issue in detail, we need to understand exporters' decisions.

It is useful to separate the entrepreneurs' decisions regarding consumption and investment from the problem of creating new units. To this end, we assume that there is a competitive adjustment sector that creates and destroys export units and makes zero profits. Let q_t denote the price of an export unit. Equilibrium in the adjustment sector requires that

$$q_t \in [0, f], \tag{4}$$

$$q_t = f \text{ if } n_t > n_{t-1}, \tag{5}$$

$$q_t = 0 \text{ if } n_t < n_{t-1}. \tag{6}$$

That is, if units are being created their price must be equal to the creation cost f, while if they are being destroyed their price must be zero.

The entrepreneur's flow of funds constraint can then written as

$$c^{T,e}\left(s^t\right) + q\left(s^t\right)\left(n\left(s^t\right) - n\left(s^{t-1}\right)\right) + \beta \sum_{s_{t+1} \in S} \pi(s_{t+1}|s_t) a\left(\left\langle s^t, s_{t+1}, \right\rangle\right) \leq \left(1 - p\left(s^t\right)\right) n\left(s^t\right) + a\left(s^t\right), \tag{7}$$

Each period, the entrepreneur uses his current profits, $(1 - p_t)\, n_t$, and his financial wealth, a_t, to finance consumption, investment in new export units, and investment in state contingent securities. Notice that our timing assumption is that production units created at date t are immediately productive, i.e., they immediately generate unitary profits of $1 - p_t$.

The entrepreneur chooses sequences for $c^{T,e}\left(s^t\right)$, $n\left(s^t\right)$, and $a\left(s^t\right)$ to maximize his expected utility, subject to the flow of funds constraint (7) and the financial constraint (1) for each history s^t. Let $V\left(a\left(s^t\right), n\left(s^{t-1}\right); s^t\right)$ denote the expected utility of an entrepreneur in state s^t who is holding $a\left(s^t\right)$ units of cash and $n\left(s^{t-1}\right)$ production units. Then we can write the Bellman equation:

$$V\left(a\left(s^t\right), n\left(s^{t-1}\right); s^t\right) = \max_{\substack{c^{T,e}\left(s^t\right), n\left(s^t\right), \\ \left\{a\left(s^t, s_{t+1}\right)\right\}_{s_{t+1} \in S}}} c^{T,e}\left(s^t\right) + \beta \sum_{s_{t+1} \in S} \pi\left(s_{t+1}|s_t\right) V\left(a\left(\left\langle s^t, s_{t+1}\right\rangle\right), n\left(s^t\right); \left\langle s^t, s_{t+1}\right\rangle\right)$$

s.t.

$$\begin{aligned} (7), \quad c^{T,e}\left(s^t\right) &\geq 0, \quad n\left(s^t\right) \geq 0, \\ a\left(\left\langle s^t, s_{t+1}\right\rangle\right) &\geq 0 \text{ for } s_{t+1} \in S. \end{aligned}$$

It is straightforward to setup the entrepreneur's problem in sequential form and argue directly that the value function is linear in $a\left(s^t\right)$ and $n\left(s^{t-1}\right)$, given that both the objective function and the constraints are linear.[4] Moreover, $a\left(s^t\right)$ and $n\left(s^{t-1}\right)$ only appear in the flow of funds constraint (7), in the form $a\left(s^t\right) + q\left(s^t\right) n\left(s^{t-1}\right)$. It follows that the value function takes the form

$$V\left(a\left(s^t\right), n\left(s^{t-1}\right); s^t\right) = \psi\left(s^t\right) + \phi\left(s^t\right) \cdot \left(a\left(s^t\right) + q\left(s^t\right) n\left(s^{t-1}\right)\right), \tag{8}$$

where $\phi\left(s^t\right)$ represents the marginal return on entrepreneurs' wealth.

[4] For the moment, we suppose that the prices $\left\{p\left(s^t\right), q\left(s^t\right)\right\}$ are such that the entrepreneur's expected utility is finite. We will check later, case by case, that this condition is satisfied in equilibrium.

In each period, the entrepreneur chooses how many production units to operate, how much to consume, and how many contingent claims to purchase. The first order conditions (and complementary slackness conditions) with respect to these choice variables are[5]

$$-\left(q\left(s^t\right)-\left(1-p\left(s^t\right)\right)\right)\phi\left(s^t\right)+\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}|s_t\right)\phi\left(\left\langle s^t,s_{t+1}\right\rangle\right)q\left(\left\langle s^t,s_{t+1}\right\rangle\right)\leq 0,\ n\left(s^t\right)\geq 0,\ (9)$$

$$\begin{aligned} 1-\phi\left(s^t\right) &\leq 0, \qquad c^{T,e}\left(s^t\right)\geq 0, &(10)\\ -\phi\left(s^t\right)+\phi\left(\left\langle s^t,s_{t+1}\right\rangle\right) &\leq 0, \qquad a\left(\left\langle s^t,s_{t+1}\right\rangle\right)\geq 0,\ \text{for all } s_{t+1}\in S. &(11)\end{aligned}$$

The first condition states that the opportunity cost of the resources used in keeping the marginal unit in operation must be equal to the expected value of that unit tomorrow. The cost of keeping a unit in operation corresponds to the price of acquiring that unit, q_t, minus the current profits, $1-p_t$. Remember that an open unit must remain active, so if $p_t>1$ the firm is making current losses and these losses add to the cost of keeping the unit open. The second condition states that if the entrepreneur's consumption is positive, the marginal value of wealth must be equal to one. Otherwise, it can exceed one. The third condition says that the marginal value of wealth must be non-increasing between two consecutive histories. Holdings of financial assets can only be positive between two histories where the marginal value of wealth is constant.

Finally, we are in a position to define a competitive equilibrium.

Definition 1 *A competitive equilibrium is given by a sequence of prices* $\left\{p\left(s^t\right),q\left(s^t\right)\right\}$ *and quantities* $\left\{c^T\left(s^t\right),c^N\left(s^t\right),c^{T,e}\left(s^t\right),n\left(s^t\right),a\left(s^t\right)\right\}$ *such that: (i) the consumer's decisions* $\left\{c^T\left(s^t\right),c^N\left(s^t\right)\right\}$ *are optimal; (ii) the entrepreneur's decisions* $\left\{c^{T,e}\left(s^t\right),n\left(s^t\right),a\left(s^t\right)\right\}$ *are optimal; (iii) the sequences* $\left\{n\left(s^t\right)\right\}$ *and* $\left\{q\left(s^t\right)\right\}$ *satisfy (4)-(6); (iv) the labor market clears for each* s^t,

$$n\left(s^t\right)+c^N\left(s^t\right)=1.$$

2.3 The Appreciation and Depreciation Phases

Recall that our economy starts with a stock of export units, n_{-1}, and has just entered state A. The situation that concerns us is one in which there is destruction of units during the appreciation, and creation during the depreciation. Moreover, we also wish to focus on a scenario where the option to wait is sufficiently positive that it is not optimal to destroy all export units during the appreciation.

[5] The envelope theorem implies that the Lagrange multiplier on the budget constraint is equal to $\phi\left(s^t\right)$.

2.3.1 An Efficient Benchmark

The export sector has financial resources a_0 to finance the losses during the appreciation phase. As a benchmark, let us first study a case where a_0 is sufficiently large that financial constraints are never binding. In this benchmark, there is no need to keep track of the history s^t except for the current state s_t, since, as we will see, equilibrium prices and quantities are constant both in the A and in the D phase.[6] Therefore, with a slight abuse of notation, we will simply index variables using A or D.

In the absence of financial constraints, $\phi\left(s^t\right)$ is constant and equal to one in both phases. We show later that in equilibrium there is destruction when the economy enters phase A, and creation when it switches to D. Correspondingly, $q_A = 0$ and $q_D = f$. It follows that the first order conditions for n in the A and D phases, respectively, reduce to:

$$\begin{aligned} (1-p_A)+\delta\beta f &= 0, \\ -f+(1-p_D)+\beta f &= 0, \end{aligned}$$

which fully determine the real exchange rate in each phase:

$$p_A^{fb} = 1+\delta\beta f, \tag{12}$$

$$p_D^{fb} = 1-(1-\beta)f. \tag{13}$$

We assume that

$$(1-\beta)\,f < 1, \tag{A1}$$

ensuring that creation is profitable in the D phase and that $p_D^{fb} > 0$.

Given these prices we can find the consumption of tradables and nontradables in each state:

$$\begin{aligned} c_A^{T,fb} &= \kappa^{fb}\theta_A, \qquad c_A^{N,fb} = \frac{\kappa^{fb}\theta_A}{1+\delta\beta f}, \\ c_D^{T,fb} &= \kappa^{fb}, \qquad c_D^{N,fb} = \frac{\kappa^{fb}}{1-(1-\beta)f}, \end{aligned}$$

where κ^{fb} is equal to:

$$\kappa^{fb} = \frac{1-\beta(1-\delta)}{2\left((1-\beta)\theta_A+\delta\beta\right)}.$$

Market clearing yields the number of units open in each state:

$$n_A^{fb} = 1-\frac{\kappa^{fb}\theta_A}{1+\delta\beta f}, \qquad n_D^{fb} = 1-\frac{\kappa^{fb}}{1-(1-\beta)f}. \tag{14}$$

[6] By "A phase" we mean all the histories of the form $s^t = \langle A,...,A\rangle$. By "$D$ phase" all those of the form $s^t = \langle A,...,A,D,...,D\rangle$.

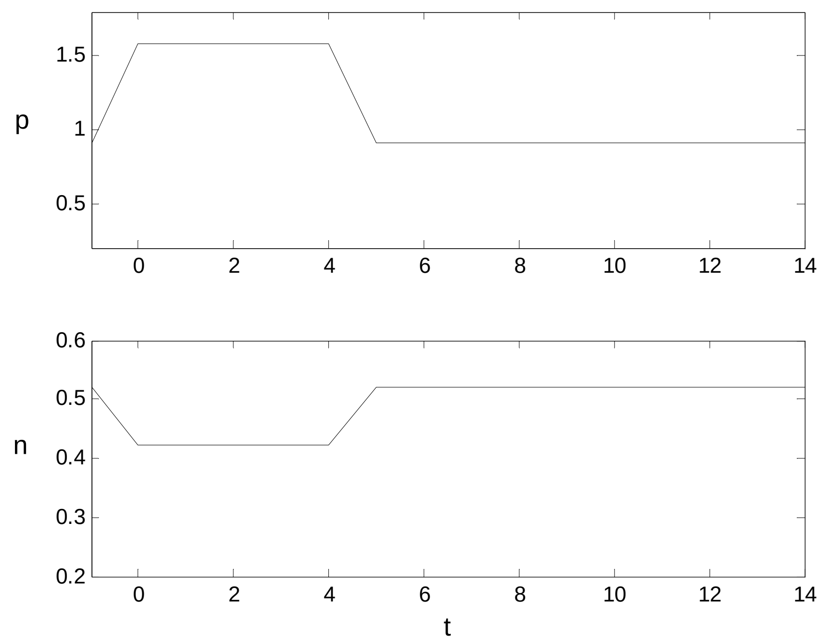

Figure 2: First Best

It is now easy to see that the following two assumptions guarantee that there is destruction when the economy enters state A at date 0, and that there is positive creation when the economy shifts from A to D:

$$n_{-1} > 1 - \frac{\kappa^{fb}\theta_A}{1+\delta\beta f}, \tag{A2}$$

$$\theta_A > \frac{1+\delta\beta f}{1-(1-\beta)f}. \tag{A3}$$

Notice that, as long as the preference shock θ_A is sufficiently large, the equilibrium prices are fully determined on the supply side of the model, by (12) and (13). In particular, the price during the appreciation is such that the current losses, $p_A - 1$, equal the opportunity cost of creating a unit when the switch to the D phase occurs, $\delta\beta f$ in expected value.

Figure 2 summarizes the benchmark economy, plotting the equilibrium dynamics of the real exchange rate and of the number of export units in the event the appreciation lasts five periods.[7] During the A phase, it is optimal for the economy to accommodate the increased demand for

[7] The parameters to generate this figure are $\beta = 0.97$, $\delta = 0.2$, $f = 3$, $\theta_A = 2.1$. We choose $n_{-1} = n_D^{fb}$ and $p_{-1} = p_D^{fb}$ as conventional initial conditions. These initial conditions arise if the economy makes an unexpected transition from the D state to the A state in period 0. We plot an appreciation lasting 5 periods, since that is its expected duration when $\delta = 0.2$.

nontradables by contracting the export sector temporarily. However, since shutting down units wastes creation costs, it is also optimal for the export sector to keep $n_A^{fb} > 0$ units in operation, with each of them incurring flow losses of $p_A^{fb} - 1$.

The following Proposition summarizes the case of high entrepreneurial wealth. The cutoff $\hat{a}^{fb}$ is derived explicitly in the Appendix.

Proposition 1 *(First best) There is a cutoff $\hat{a}^{fb}$ such that if the entrepreneurs' initial wealth satisfies $a_0 \geq \hat{a}^{fb}$, then the equilibrium real exchange rate and the number of firms are constant within the A and D phases, and are given by (12), (13), and (14). The marginal value of entrepreneurial wealth, $\phi\left(s^t\right)$, is constant and equal to 1.*

2.3.2 The Constrained Economy and Overshooting

Suppose now that a_0 is not large enough to implement the first best path (i.e., $a_0 < \hat{a}^{fb}$). There are two margins through which this deficit can materialize. First, the export sector may not have enough resources to finance the flow of losses $(p_A^{fb} - 1)n_A^{fb}$ during the appreciation. Second, even if it can, it may not have enough resources left to finance the investment $f\left(n_D^{fb} - n_A^{fb}\right)$ when the appreciation phase ends.

Relative to the benchmark case, we now need to keep track not only of the current exogenous state s_t, but also of the number of periods since the D phase started. The reason for this is that in this case there is a gradual transition in the D phase where the export sector rebuilds and is constrained by limited financial resources. At the same time, due to complete markets the A phase is still stationary, with constant prices and quantities. The next proposition summarizes the properties of equilibrium prices and quantities that will be useful in the following characterization.

Proposition 2 *If $a_0 < \hat{a}^{fb}$ then $p\left(s^t\right), n\left(s^t\right)$, and $a\left(s^t\right)$ are constant in the A phase. In the D phase, $p\left(s^t\right)$, $n\left(s^t\right)$, and $a\left(s^t\right)$ only depend on the number of periods that the economy has spent in D. The price $q\left(s^t\right)$ is equal to zero in the A phase and to f in the D phase.*

This proposition allows us to write $p\left(s^t\right) = p_A$ in the A phase and $p\left(s^t\right) = p_{D,j}$ in the D phase, where j is the number of periods the economy has spent in D. Analogous notation is used for $n\left(s^t\right)$, $a\left(s^t\right)$ and $\phi\left(s^t\right)$.

Let us focus on the A phase. The relevant optimality conditions for the entrepreneur are

$$(1 - p_A)\,\phi_A + \delta\beta f \phi_{D,0} = 0, \tag{15}$$

and

$$\phi_A \geq \phi_{D,0} \qquad a_{D,0} \geq 0. \tag{16}$$

The stationarity of entrepreneurial wealth in A implies that $a_A = a_0$ and the budget constraint (7) can be written as[8]

$$(1-(1-\delta)\beta)a_0 = (p_A - 1)n_A + \beta\delta a_{D,0}. \tag{17}$$

The flow generated by the initial resources a_0 can be used to finance the operational losses of the export units that remain open during the appreciation, and to transfer financial resources to the recovery phase in D. Going back to the first order conditions, the complementary inequalities in (16) distinguish the case where the financial resources are used for both purposes ($\phi_A = \phi_{D,0}$ and $a_{D,0} > 0$) and the case where they are only used to cover operational losses in A ($\phi_A > \phi_{D,0}$ and $a_{D,0} = 0$).

The first order condition for n_A (equation (15)) yields an expression for the real exchange rate in the A region:

$$p_A = 1 + \beta\delta f \frac{\phi_{D,0}}{\phi_A} \leq p_A^{fb},$$

where the inequality comes from (16). As in the benchmark case, the appreciation is such that production units incur losses, as $p_A > 1$. When $a_{D,0} > 0$ the real exchange rate is equal to that in the first-best, and entrepreneurs are indifferent between holding state contingent securities or holding production units. This indifference means that these two assets have the same expected return, which pins down the equilibrium exchange rate as in the unconstrained economy. Instead, when $a_{D,0} = 0$, the expected return on export units is larger than that on state contingent securities. This wedge is possible because only entrepreneurs can purchase export units, and they are financially constrained. This depresses the exchange rate to a p_A smaller than p_A^{fb}. Far from being good news, this smaller appreciation reflects the fact that financially constrained firms are unable to keep open as many production units as they would like and hence are forced to reduce production and labor demand.

For given parameters, we can show that the initial level of a_0 determines which of the two cases discussed in the previous paragraph arises in equilibrium.

Proposition 3 *(Constrained appreciation phase) There is a cutoff $\hat{a}^A < \hat{a}^{fb}$ such that if $a_0 > \hat{a}^A$ the real exchange rate in the A phase is p_A^{fb} and $a_{D,0} > 0$, while if $a_0 < \hat{a}^A$ the real exchange rate in the A phase is $p_A < p_A^{fb}$ and $a_{D,0} = 0$.*

Let us focus now on the case where $a_0 < \hat{a}^A$. To determine n_A, note that from the consumption side and labor market equilibrium, we have

$$n_A + \frac{\kappa\theta_A}{p_A} = 1.$$

[8] As long as $a_0 < \hat{a}^{fb}$ we have $\phi_A > 1$ and so c_A is set to zero (see the proof of Proposition 2).

Solving for p_A and substituting into the budget constraint (17) pins down the number of production units that are kept active during the appreciation:

$$a_0(1-(1-\delta)\beta) = \left(\frac{\kappa\theta_A}{1-n_A} - 1\right) n_A$$

For a given κ, lower financial resources a_0 reduce the number of production units that are kept open during the appreciation (the right-hand side is increasing in n_A) and lead to more destruction. Notice, however, that in general equilibrium κ falls as well (see the argument below in 2.3.3) so, in extreme cases, an economy with lower a_0 may end up with a higher level of n_A.

Let us now turn to the D region. Starting backwards, once the recovery phase is completed, entrepreneurs consume and $\phi_{D,j}$ is equal to 1. Thus, from the first order conditions, we have that in the stationary phase of D:

$$p_{D,j} = 1-(1-\beta)f = p_D^{fb}$$

Eventually, the real exchange rate converges to the benchmark level.

From the equilibrium condition in the labor market and the fact that consumers' demand is lower in the constrained than in the benchmark case (as we show below), it follows that the long run size of the export sector, after the recovery is completed, is:

$$\bar{n}_D = 1 - \frac{\kappa}{p_D^{fb}} > 1 - \frac{\kappa^{fb}}{p_D^{fb}} = n_D^{fb}.$$

Since in the constrained economy not only entrepreneurs but also consumers are poorer than in the benchmark economy, demand is depressed and hence the export sector eventually expands to absorb the labor freed by the smaller nontradable sector. However, unlike in the benchmark case, this stationary state is not reached instantly since financial constraints also hamper the recovery phase. The flow of funds constraint and the first order conditions for the transition are:

$$f(n_{D,j} - n_{D,j-1}) = (1-p_{D,j})n_{D,j}, \tag{18}$$

$$\phi_{D,j} > 1, \ \phi_{D,j} > \phi_{D,j+1}, \tag{19}$$

$$(-f+1-p_{D,j})\,\phi_{D,j} + \beta f \phi_{D,j+1} = 0, \tag{20}$$

for $j = 0, ..., J$, where J is the last period of the transition phase in D and where $n_{D,-1}$ is equivalent notation for n_A.

Equation (18) states that during the recovery phase, firms use all their profits to rebuild the sector. The inequalities in (19) reflect the fact that the financial constraint is tightest early on in the recovery and gradually declines, and hence there is no reason to accumulate "cash" or to consume. Reorganizing (20), we obtain an expression for the real exchange rate during the transition:

$$p_{D,j} = 1 - f\left(1 - \beta\frac{\phi_{D,j+1}}{\phi_{D,j}}\right) < 1 - f(1-\beta) = p_D^{fb}.$$

That is, during the recovery phase the depreciation is deeper when the economy is constrained. We refer to this deeper depreciation as the *overshooting* implication of financial constraints.

The presence of overshooting means that wages are not only lower than in the benchmark case during the appreciation phase, but also during the transition phase of D. This observation closes our argument, as it explains why consumption levels are lower in the constrained case, given that κ reflects the consumers' lifetime income (see (3)), and that, history by history, the wages p_t are smaller than in the first best.

Figure 3 depicts the constrained economy, assuming for simplicity that $n_{-1} = \bar{n}_D$ and $p_{-1} = p_D^{fb}$.[9] The exchange rate appreciates in the A phase, as in the benchmark economy (represented with dashes in each panel), and in the depreciation phase it experiences a large and protracted overshooting. The export sector contracts during the A phase and, unlike in the benchmark economy, the recovery is only gradual during the D phase. The bottom panel shows the path of the marginal value of a unit of wealth, which is highest in the A region, drops sharply upon the transition into D, and gradually declines within the D region.

Let us conclude with a summary proposition:

Proposition 4 *(Constrained depreciation phase and overshooting) There is a cutoff* $\hat{a}^D < \hat{a}^{fb}$ *such that if* $a_0 \geq \hat{a}^D$ *the real exchange rate throughout the D phase is* p_D^{fb}*, while if* $a_0 < \hat{a}^D$ *the real exchange rate in the D phase overshoots its long run value early on in the transition. That is,* $p_{D,j} < p_D^{fb}$ *for* $j = 0, 1, ..., J$ *and* $p_{D,j} = p_D^{fb}$ *for* $j = J + 1, ...$ *for some* $J \geq 0$*. (Note that the cutoff* $\hat{a}^D$ *may be greater or smaller than* $\hat{a}^A$*, depending on the model's parameters).*

2.3.3 General Equilibrium Feedback

Our discussion above highlights the export firms' problem for a given consumption demand. However, firms' actions affect households' income through labor demand. The tighter is the financial constraint on firms, the lower is labor demand and income. This feedback is captured by the relation between a_0 and κ. Figure 4 plots this relation and shows that κ is increasing in a_0 until it reaches its maximum for $a_0 \geq \hat{a}^{fb}$ (see Lemma 2 in the Appendix).

Note that this general equilibrium feedback generates some counterintuitive results. For example, the model has a sort of sclerosis as a_0 declines. Even though export firms are more financially constrained when financial resources are low, in the long run they absorb a larger share of n. To see this, recall that $\bar{n}_D = 1 - \kappa / p_D^{fb}$ which rises as κ drops. This simply says that an economy with poorer consumers allocates a larger share of its resources to satisfy foreign than domestic demand.

Up to now, we have developed a model of equilibrium destruction and overshooting. The next section turns to the other main concern in this paper. In particular, it shows that when $a_0 < \hat{a}^{fb}$,

[9] The parameters are the same as those used for Figure 2, plus $a_0 = 0.5$.

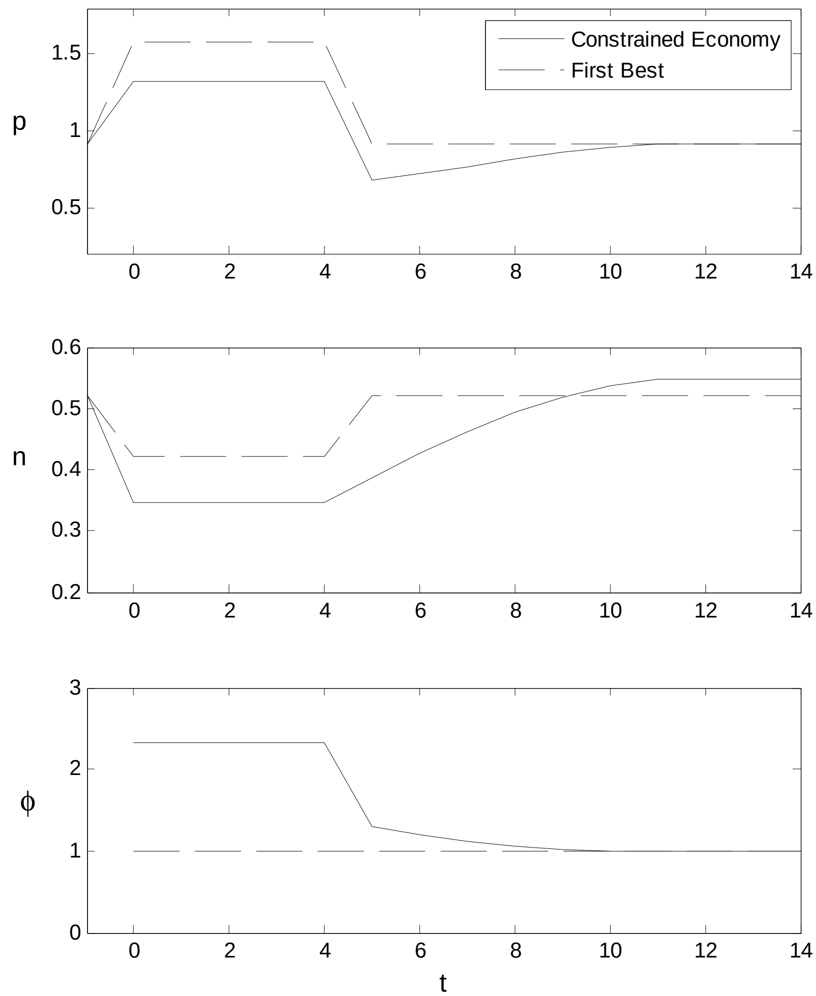

Figure 3: Constrained Economy

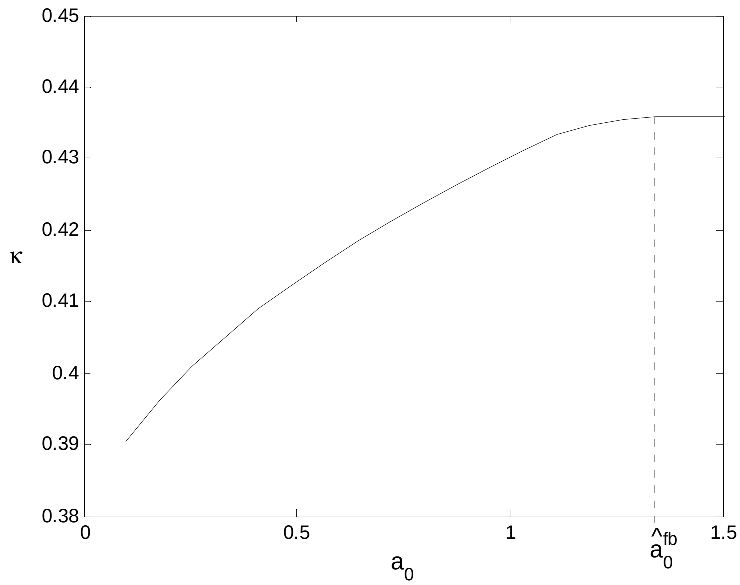

Figure 4: Consumers' Income and Financial Constraint

the social planner may be able to raise κ by inducing consumers to choose a different path for c_t^N (and hence n_t).

3 Optimal Ex-ante and Ex-post Intervention

In the previous section we showed that when the export sector has limited financial resources, the depreciation phase following a persistent appreciation may come with a protracted exchange rate overshooting (a sharp real wage decline) while the export sector rebuilds. Either explicitly or implicitly, in practice it is this overshooting phase that primarily concerns policymakers and leads to a debate on whether intervention should take place during the appreciation phase. In particular, the concern is whether by overly stressing the export sector during the appreciation, the economy may be exposing itself to a costly recovery phase once the factors behind the appreciation subside. In this section we study this policy problem and conclude that if an overshooting is expected, there is indeed scope for policy intervention. The reason for such intervention is that the competitive equilibrium is not constrained efficient, as consumers ignore the effect of their individual decisions on the severity and duration of the overshooting during the depreciation phase.

The optimal policy includes *ex-ante* and *ex-post* interventions. There are instances when the focus of intervention is ex-ante, and the bulk of it consists in stabilizing the exchange rate during the appreciation phase. There are others where the scope for appreciation stabilization is limited and the policy intervention is concentrated in the first period of the depreciation phase (ex-post intervention).

3.1 A Fiscal Intervention

We consider a government that uses a set of fiscal instruments to affect the time profile of consumers' demand. In particular, the government can impose a sequence of linear taxes $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ on consumers' spending on tradables and non-tradables (a negative tax rate corresponds to a subsidy). Any tax revenue is returned to the consumers as a lump-sum transfer at date 0, T_0, so consumers face the budget constraint

$$\sum_{t,s^t} \beta^t \pi\left(s^t\right)\left(\left(1+\tau^T\left(s^t\right)\right) c^T\left(s^t\right)+\left(1+\tau^N\left(s^t\right)\right) p\left(s^t\right) c^N\left(s^t\right)\right) \leq \sum_{t,s^t} \beta^t \pi\left(s^t\right) p\left(s^t\right)+T_0. \quad (21)$$

For a given tax sequence $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ we can define a competitive equilibrium as we did in the economy with no taxes (see Definition 1), replacing the consumers' budget constraint with (21) and adding the condition that the lump-sum transfer T_0 satisfies the government budget balance condition

$$\sum_{t,s^t} \beta^t \pi\left(s^t\right)\left(\tau^T\left(s^t\right) c^T\left(s^t\right)+\tau^N\left(s^t\right) p\left(s^t\right) c^N\left(s^t\right)\right)=T_0.$$

We study a benevolent government that chooses $\left\{\tau^T\left(s^t\right), \tau^N\left(s^t\right)\right\}$ so as to maximize the utility of the representative consumer subject to the constraint of not making entrepreneurs worse off:

$$\sum_t \sum_{s^t} \beta^t \pi\left(s^t\right) c^{T,e}\left(s^t\right) \geq U, \quad (22)$$

where U is the entrepreneurs' expected utility in the competitive equilibrium.

Notice that in this context the choice of linear taxes is not restrictive. The crucial constraint on the set of feasible policies is the condition that no direct transfers can be made between consumers and entrepreneurs. We choose to focus on this exercise for two reasons. First, if the government could implement direct transfers, it would be simple in this economy to undo the effects of the financial constraint (1). The government would transfer resources to the entrepreneurs in the A phase and in the early stage of the D phase, and then transfer resources back to consumers later in the D phase, hence replicating the equilibrium of a frictionless economy. In practice, targeted transfers of this kind take place but are limited by a host of informational and institutional impediments which we do not model here. In this sense, the set of policies that we consider respect the spirit of the financial constraint (1).[10] Second, the policy described is more flexible than the specific forms of macroeconomic interventions that are contemplated in the policy debate. Namely,

[10] A "fundamental" view of constraint (1) is that it is impossible to extract payments from entrepreneurs, whether in the form of financial payments or in the form of taxes.

most proposed interventions are geared towards increasing domestic savings, thus reducing the pressure on the real exchange rate by reducing the demand of *both* tradables and nontradables. As we will see, our policy-maker will choose not to distort tradable consumption decisions and will only intervene on non-tradable consumption.

In summary, we consider a general form of intervention but stay within the boundaries of a constrained efficiency exercise by ruling out direct transfers between consumers and entrepreneurs. This choice allows us to identify a basic pecuniary externality, which should play a relevant role in all practical policy proposals that attempt to curb persistent appreciations.

3.2 Policy Perturbation and Pecuniary Externality

Before characterizing the optimal policy, let us identify the pecuniary externality by studying the impact of small policy interventions around the competitive equilibrium. Consider a planner that maximizes the consumer's utility

$$\theta_A \left(u \left(c_A^T \right) + u \left(c_A^N \right) \right) + \delta\beta \left(\frac{1}{1-\beta} u \left(c_D^T \right) + \sum_{j=0}^{\infty} \beta^j u \left(c_{D,j}^N \right) \right),$$

where we have normalized expected utility by the factor $(1 - \beta (1 - \delta))$.[11] As usual in optimal taxation problems, it is easier to characterize the problem directly in terms of equilibrium quantities, rather than in terms of the underlying tax rates. Thus, we let the planner choose directly the consumption paths for tradables and nontradables. Substituting the government budget balance, the consumers' budget constraint is (also multiplying through by $(1 - \beta (1 - \delta))$):

$$c_A^T + p_A c_A^N + \delta\beta \left(\frac{1}{1-\beta} c_D^T + \sum_{j=0}^{\infty} \beta^j p_{D,j} c_{D,j}^N \right) \leq p_A + \delta\beta \sum_{j=0}^{\infty} \beta^j p_{D,j}. \tag{23}$$

Relative to the problem of an individual consumer, the planner's problem is different in that it takes into account the effect of consumers' decisions on p_A and $\{p_{D,j}\}$. Consumers' decisions affect the equilibrium prices by changing the demand for nontradables and, thus, equilibrium wages. The entrepreneurs' optimality condition and market clearing in the labor market give an equilibrium relation between the quantities chosen by the planner and the prices p_A and $\{p_{D,j}\}$. Namely, the planner chooses the c_A^N and $c_{D,j}^N$, market clearing gives $n_A = 1 - c_A^N$ and $n_{D,j} = 1 - c_{D,j}^N$,

[11] See the Appendix for a detailed setup of the planner problem. In the Appendix we show that the second best allocation shares the following features with the competitive equilibrium: the consumption of tradables is constant and equal to c_A^T and c_D^T, respectively, in the A phase and in the D phase, and the consumption of non-tradables is constant and equal to c_A^N in the A phase and only depends on j in the D phase. Therefore, also in the perturbation argument we focus directly on allocations with these features.

and entrepreneur's optimality gives the associated equilibrium prices. Finally, the constraint that entrepreneurs cannot be made worse off is (also multiplying through by $(1-\beta(1-\delta))$):

$$c_A^{T,e} + \delta\beta \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T,e} \geq (1-\beta(1-\delta))\, U. \tag{24}$$

Let us study the effect of stabilizing the appreciation phase, starting from the competitive equilibrium studied in Section 2. Specifically, consider the effect of reducing c_A^N or, equivalently, increasing n_A, while keeping the $n_{D,j}$'s unchanged. This change will affect equilibrium prices and thus the net present value of the consumer's income. Suppose the planner adjusts c_A^T so that the consumer's budget constraint (23) is satisfied.

The following expression captures the marginal effect of a change in n_A on the consumer's utility:

$$\begin{aligned} -\theta_A u'(1-n_A) + p_A\lambda + \\ +\lambda\left(\frac{\partial p_A}{\partial n_A} n_A + \beta\delta \frac{\partial p_{D,0}}{\partial n_A} n_{D,0}\right), \end{aligned} \tag{25}$$

where $\lambda = u'\left(c_A^T\right)$ is the Lagrange multiplier on the consumer's budget constraint in the competitive equilibrium. The first row of (25) captures the direct effect of the policy. It is equivalent to the consumer's first order condition in the competitive economy and so it is equal to zero at the equilibrium allocation. The second row captures the net income effect for the consumer.[12] Since we keep all the $n_{D,j}$'s constant, this policy only affects the prices p_A and $p_{D,0}$, and the entrepreneurs' consumption at date $t_{D,0}$.

We consider two cases. First, suppose the competitive equilibrium displays $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ (overshooting).

Let us start with the effect of a unit increase in n_A on p_A. If the planner wants entrepreneurs to carry an extra unit of n_A, then p_A must drop for the firm to be able to finance the extra losses of that unit. Recall that the firm's budget constraint in phase A is

$$(1-(1-\delta)\beta)\, a_0 = (p_A - 1)\, n_A,$$

from which we obtain:

$$\frac{\partial p_A}{\partial n_A} = -\frac{p_A - 1}{n_A}. \tag{26}$$

We now turn to the effects of n_A on $p_{D,0}$. Since $p_{D,0} < p_D^{fb}$, i.e., there is equilibrium overshooting, the entrepreneur budget constraint at date $t_{D,0}$ is

$$f(n_{D,0} - n_A) = (1 - p_{D,0})\, n_{D,0}.$$

[12] Note that the consumer's labor income is $p\left(s^t\right)$ while its expenditure on nontradables is $p\left(s^t\right)\left(1-n\left(s^t\right)\right)$. Thus, net income is $p\left(s^t\right) n\left(s^t\right)$.

The entrepreneur's financial constraint is binding and he uses all his current profits to invest in new units. In this case, a unit increase in n_A affects $p_{D,0}$ since it reduces by f the investment required to rebuild to $n_{D,0}$. Wages must rise to compensate for this fall in investment expenditure, so as to keep the financial constraint exactly binding at $n_{D,0}$. Thus,

$$\frac{\partial p_{D,0}}{\partial n_A} = \frac{f}{n_{D,0}}. \tag{27}$$

Finally, notice that in the competitive equilibrium $c_A^{T,e} = c_{D,0}^{T,e} = 0$, and the planner is keeping the sequence $n_{D,0}, n_{D,1}, ...$ unchanged. Therefore, the consumption of the entrepreneurs is unaffected by a marginal change in n_A and (24) is satisfied.

Consumers are hurt by the decline in their wage (real exchange rate) during the A phase, but gain from the rise in their wage in the first period of the D phase. Which effect dominates? Replacing (26) and (27) in the second row of (25) we have:

$$\frac{\partial p_A}{\partial n_A} n_A + \delta\beta \frac{\partial p_{D,0}}{\partial n_A} n_{D,0} = 1 - p_A + \beta\delta f > 0.$$

The inequality follows from $p_A < p_A^{fb} = 1 + \beta\delta f$. That is, in the planner's problem there is an extra term capturing the marginal benefit of increasing n_A on the expected present value of wages. The planner has an incentive to reduce nontradables consumption, so as to reduce the appreciation (i.e. reduce p_A) and allow firms to keep a larger number of units open, which in turn raises wages at $t_{D,0}$. Because $\phi_A > \phi_{D,0}$, reducing wages in A generates an excess return in export firms that is transferred back to workers in the form of higher wages at $t_{D,0}$. Summing up, when $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ the expression in (25), computed at the competitive equilibrium, is positive, so consumers are strictly better off, while entrepreneurs are indifferent to the change.

Consider now a second case, where $p_A < p_A^{fb}$ and $p_{D,0} = p_D^{fb}$ (no overshooting). In this case, entrepreneurs are unconstrained at $t_{D,0}$ and hence $c_{D,0}^{T,e} > 0$. This implies both

$$\frac{\partial p_{D,0}}{\partial n_A} = 0$$

and

$$\frac{\partial c_{D,0}^{T,e}}{\partial n_A} = f.$$

Replacing these terms in (25) we obtain that the effect on the consumer's expected utility is negative

$$-\theta_A u'(1 - n_A) + \lambda p_A + \lambda(1 - p_A) < 0,$$

given that $\lambda < \lambda p_A$. The consumer would benefit from increasing c_A^N and *reducing* n_A. The reason is that it makes no sense for consumers to cut their wage today if this action does not raise wages in the future, which it will not when there is no overshooting to remedy and $p_{D,0}$ is pinned down by the technological free entry condition. Instead, the planner, representing the consumers, would like to exercise its "monopoly" power during the appreciation phase and raise wages by increasing their demand for nontradables. However this increase would reduce the consumption of entrepreneurs, given that $\partial c_{D,0}^{T,e}/\partial n_A = f$, and violate their participation constraint (24). In fact, when there is no expected overshooting, it is optimal not to intervene. In this case, there exists a Lagrange multiplier μ such that the planner's first order condition for n_A takes the form

$$-\theta_A u'(1-n_A) + \lambda p_A + \lambda(1-p_A) + \mu\beta f = 0,$$

where n_A and p_A are at their competitive equilibrium values. The following proposition shows that no other feasible intervention can lead to a Pareto improvement.

Proposition 5 *(Constrained efficiency) If $a_0 > \hat{a}^D$ (no overshooting), then the competitive equilibrium with no taxes solves the planner's problem. It is optimal not to stabilize the appreciation, even if firms are financially constrained and the export sector contracts more than in the first best (i.e., even if $a_0 < \hat{a}^A$).*

Put differently, if there is no overshooting, there is no intertemporal pecuniary externality for consumers, so they cannot trade-off a wage reduction today for a wage increase in the recovery phase. The flip side of this argument is that it is the presence of overshooting that makes individual consumers underestimate the social cost of their increased demand during the appreciation phase.

It is useful to show that the argument just made for n_A can also be made for $n_{D,j}$ during the depreciation phase. That is, suppose the planner can only intervene in period j of the D phase and change $n_{D,j}$ by a small amount. Suppose the entrepreneurial sector has not fully recovered in periods $t_{D,j}$ and $t_{D,j+1}$, i.e., we are in the middle of the overshooting phase with $p_{D,j} < p_{D,j+1} < p_D^{fb}$. Then, it is optimal to reduce $c_{D,j}^N$ further and *exacerbate* the depreciation in period j. By doing so, the consumers accelerate the recovery of the export sector and of real wages. The first order effects of a small intervention are similar to those derived for n_A. In particular, since future n's are taken as given, a change in $n_{D,j}$ only affects the current and next period's prices. As before, the financial constraint is still binding after a small intervention, so $c_{D,j+1}^{T,e} = c_{D,j}^{T,e} = 0$. Therefore, the marginal effect of an increase in $n_{D,j}$ is given by

$$-u'(1-n_{D,j}) + p_{D,j}\lambda + \lambda\left(\frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1}\right). \tag{28}$$

Proceeding as we did above for (26) and (26), we get

$$\frac{\partial p_{D,j}}{\partial n_{D,j}} n_{D,j} + \beta \frac{\partial p_{D,j+1}}{\partial n_{D,j}} n_{D,j+1} = -\left(f - (1-p_{D,j})\right) + \beta f > 0,$$

The inequality follows from the fact that $p_{D,j} < p_D^{fb} = 1 - (1-\beta) f$. This, together with the consumer's first order condition, implies that a reduction in $p_{D,j}$ leads to a marginal welfare gain.

In the competitive equilibrium, the firms' financial constraint depresses labor demand, making non-tradables cheaper and inducing consumers to demand more of them. The social planner offsets the consumers' reaction to the overshooting and reduces $c_{D,j}^N$ (it taxes nontradable consumption). Note that as a result of this reduction in $c_{D,j}^N$ the overshooting is exacerbated, but this is precisely what increases profits and allows financially constrained firms to accelerate investment. The trade-off is between a deeper overshooting and lower wages today in exchange for a faster recovery in wages. Because $\phi_{D,j} > \phi_{D,j+1}$, reducing wages at $t_{D,j}$ generates an excess return in export firms that is transferred back to workers in the form of higher wages at $t_{D,j+1}$.

Once we allow the planner to set policy optimally in both phases, A and D, some of the incentive to exacerbate the overshooting in the D phase goes away, because the planner can already achieve higher levels of investment by protecting entrepreneurial wealth in the appreciation phase. This interaction between preventive intervention and intervention in the depreciation phase is a central aspect of the optimal policy discussion that follows.

3.3 Optimal Policy

We learned from the perturbation argument above that if there is an expected overshooting, then the competitive equilibrium is constrained inefficient and there is scope for policy. We now turn to characterizing the economy's dynamics under the optimal policy.

Figure 5 plots the real exchange rate and the number of export units in the competitive equilibrium and in the "second best" allocation, in a baseline scenario.[13] The first feature of the optimal policy is that the planner attenuates the exchange rate appreciation. This attenuation has the effect of keeping more export units open in the appreciation phase, and makes the recovery of the export sector faster during the depreciation phase. In turn, the faster recovery increases the demand of non-tradables by entrepreneurs and reduces the extent and duration of the real exchange rate overshooting during the depreciation phase. The final effect of the intervention is a form of real exchange rate stabilization.

Two robust features of the optimal price path are that p_A is lower than the competitive equilibrium level, and that the increase in the expected present value of the $p_{D,j}$'s more than offsets the decline in p_A. The price path depicted in Figure 5 also displays a reduction in the initial overshooting at $t_{D,0}$. However, this feature is less robust and depends on the economy's initial conditions. We will discuss other possible cases below.

[13] The parameters are the same as those used for Figures 2 and 3. As in those figures, we plot the realized path when the appreciation lasts five periods. The optimal paths are computed using the characterization result in the Appendix (proof of Proposition 6).

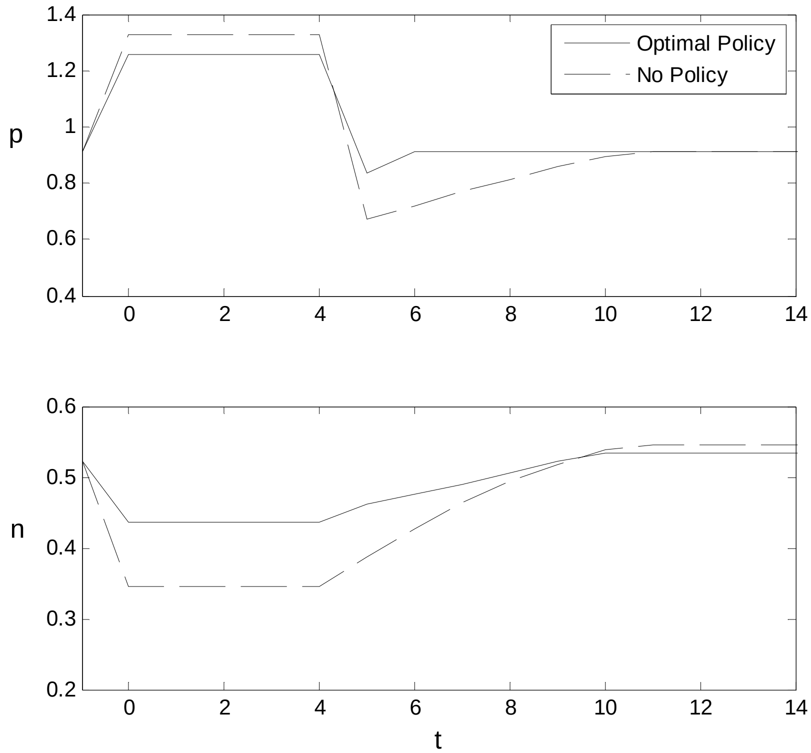

Figure 5: Optimal Policy

Let us explore the optimal paths in more detail, by analyzing first the intervention in the A phase (i.e., the choice of n_A) and then the intervention in the D phase (i.e., the $n_{D,j}$'s).

There is an optimal degree of exchange rate stabilization during the A phase. Given that $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$ we can substitute (26) and (27) in (25), simplify, and obtain the planner's first order condition:

$$\theta_A u'(1 - n_A) = \lambda p_A^{fb} = \lambda (1 + \beta \delta f). \tag{29}$$

The social planner allocates n_A *as if* prices were at first best. In the competitive equilibrium, consumers increase their demand for nontradables in response to the taste shock, which leads to an appreciation of the real exchange rate. However, due to the firms' financial constraint, the appreciation is smaller than it would be in the first best. This price gap implies that consumers further increase their consumption of nontradables, at the expense of export units. The planner taxes consumption of nontradables enough to offset this additional effect, and in so doing lowers the real exchange rate and allows firms to maintain a larger number of production units open.

Turning to the D phase, notice that along the recovery path the entrepreneurs' financial constraint is exactly binding:

$$f(n_{D,j} - n_{D,j-1}) = (1 - p_{D,j}) n_{D,j},$$

until the point where $n_{D,j}$ reaches its "nondistorted" level, i.e., the value $\bar{n}_D$ that satisfies

$$u'(1 - \bar{n}_D) = \lambda p_D^{fb}.$$

Entrepreneurs use all their profits for investment and delay their consumption until they have reached $\bar{n}_D$ (this happens at $t = 11$ in Figure 5). Notice also that some amount of overshooting is still present in the second best, i.e., $p_{D,0} < p_D^{fb}$. Recall the argument made above on the social benefits of the overshooting, which makes the recovery faster and increases future values of $p_{D,j}$. At the optimum, the argument is subtler since $p_{D,1}$ is equal to p_D^{fb}. If the planner were to increase $p_{D,0}$ he would have to reduce $n_{D,0}$. But then, since the entrepreneurs are exactly constrained at $t_{D,1}$, this would imply a wage loss at that date, i.e., $p_{D,1} < p_D^{fb}$. For consumers, the net effect of a reduction in $n_{D,0}$ would be, rearranging (28),

$$\begin{aligned} &u'(1 - n_{D,0}) - \lambda p_{D,0} - \lambda (1 - f - p_{D,0} + \beta f) = \\ &u'(1 - n_{D,0}) - \lambda p_D^{fb} < 0. \end{aligned} \tag{30}$$

On the other hand, if the planner tried to increase $n_{D,0}$, the current wage would drop but there would be no gain in terms of future wages, given that the entrepreneurs' would be unconstrained and would employ the extra funds for consumption. In this case, there would be a benefit in terms of relaxing the entrepreneur's participation constraint and the marginal effect would be

$$-u'(1 - n_{D,0}) + \lambda p_{D,0} + \lambda (1 - f - p_{D,0}) + \mu \beta f \leq 0,$$

where μ is the Lagrange multiplier on the entrepreneur's participation constraint.[14] The two conditions just derived show that the planner is at a 'kink,' where neither decreasing nor increasing $n_{D,0}$ leads to an improvement. Notice that the reasoning behind these conditions applies only because $p_{D,j} = p_D^{fb}$ for all the periods following $t_{D,0}$. This is key since it shows that under the optimal policy, the overshooting can only happen in the first period of the recovery. If we had $p_{D,j} < p_D^{fb}$ in some other period, then in the previous period it would be optimal to increase $n_{D,j-1}$ and accelerate the adjustment toward $\bar{n}_D$. Essentially, the optimal path requires that if the planner wants to allow for some depreciation in the D phase to speed up the recovery, it completely frontloads this depreciation.

In terms of consumption of non-tradables, a distortion is also concentrated in the early periods of the D phase (although, not only in the first period). In these periods the following inequality holds,

$$u'(1 - n_{D,j}) < \lambda p_D^{fb},$$

which can be derived in same way as (30). An individual consumer would like to decrease his consumption of nontradables (i.e., increase $n_{D,j}$). However, since the entrepreneurs financial constraint is exactly binding, increasing $n_{D,j}$ in any of these periods would reduce the current wages, $p_{D,j}$, below their first best level. This has no advantages in terms of future wages, given that $p_{D,j+1}$ is already at its maximum level p_D^{fb}. The potential cost in terms of current wages (plus the shadow cost of the entrepreneurs' participation constraint) exactly compensates for the distortion in nontradable consumption.

Let us summarize the main results of this section with a proposition. The superscript *ce* is used to denote prices and quantities in the competitive equilibrium with no intervention, while starred variables denote the second best.

Proposition 6 *(Optimal policy) If $a_0 < \hat{a}^D$, then the competitive equilibrium is constrained inefficient. Suppose $\phi_A^{ce} \leq (f-1)/(\beta f)$ and $n_A^{ce} \leq n_A^{fb}$. Then, the optimal policy has $p_A^* \leq p_A^{ce}$ and $p_{D,0}^* \leq p_D^{fb}$. Depending on parameters, the optimal policy involves some depreciation of the exchange rate in A (relative to the competitive equilibrium), $p_A^* < p_A^{ce}$, some overshooting in the first period of the D phase, $p_{D,0}^* < p_D^{fb}$, or some combination of both. The overshooting phase lasts at most one period. If $p_{D,0}^* < p_D^{fb}$ the optimal tax on non-tradables in A is such that $p_A^*(1+\tau_A^*) = p_A^{fb}$.*

Let us briefly discuss the role of the assumptions on ϕ_A^{ce} and n_A^{ce}. The first assumption is

[14] It is possible to complete this argument by deriving the optimal value of μ at the optimum. The proof of Proposition 6 in the Appendix reaches the same conclusion using a different approach. The reason for the different approach is that, given the nonconcavity of the problem, the perturbation arguments derived here only yield necessary conditions for an optimum, while the argument in the Appendix can be used to show sufficiency as well.

made for simplicity, as it ensures that the planner can disregard the constraint $p_{D,j} \geq 0$.[15] The second assumption is made to rule out the extreme case discussed in 2.3.2, where the wealth effect on consumers is so large that the constrained equilibrium displays less destruction than the first best.[16]

For completeness, note that in terms of quantities the optimal policy reduces the fluctuations in n and the long run size of the export sector. The former result is just the counterpart of exchange rate stabilization. The reason for the latter result is that consumers are richer in the "second best," and hence use a larger share of their labor resources to produce nontradables. To illustrate this wealth effect, Figure 6 compares the value of κ in the competitive equilibrium and the "second best," for different levels of financial resources in the export sector, a_0. As expected, for high values of a_0 the competitive equilibrium is close to the second best (and they coincide for $a_0 \geq \hat{a}^D$), which in turn is closer to the first best. However, for low levels of a_0, the pecuniary externality is significant and the second best income is substantially higher than that of the competitive equilibrium. Of course, if the government had effective direct transfer instruments, then by increasing a_0 it would be able not only to narrow the wedge between the competitive equilibrium and the second best, but also that between the latter and the first best.

3.4 Ex-ante versus Ex-post Intervention

In our discussion of the optimal policy, we touched on a pervasive policy concern in the presence of an appreciation in the value of the currency or of any other asset with potential macroeconomic consequences (e.g., real estate or stocks). Should the intervention take place *ex-ante* (i.e., during the appreciation phase) or *ex-post* (i.e., after the "crash" or depreciation takes place)?

In the example we used in Figure 5, the optimal policy involved a combination of both. The planner stabilized the exchange during the appreciation, but preserved some of the overshooting in the first period of the depreciation phase as well, both helping export companies accelerate the recovery.

In contrast, panel (a) of Figure 7 represents a case in which the intervention to offset the pecuniary externality is entirely done ex-ante.[17] An attenuation of the appreciation in A, by increasing n_A, increases $p_{D,0}$. Therefore, it is possible that, before reaching the level of n_A that satisfies (29), $p_{D,0}$ reaches its first best level p_D^{fb}. At this point there is no gain for the consumer

[15] The characterization can be easily extended to the case $\phi_A^{ce} > (f-1)/(\beta f)$. In that case, the optimal path may involve a real exchange rate equal to zero for the first period(s) of the D phase. The overshooting is still frontloaded to the early periods of D, but may last more than one period.

[16] See the discussion on page 14. The condition holds in all the examples presented (and in all reasonable parametrizations we have looked at).

[17] Parameters are the same as those used for Figures 5 except for $a_0 = 0.15$ in panel (a), $a_0 = 0.5$ in panel (b), $a_0 = 1.1$ in panel (c).

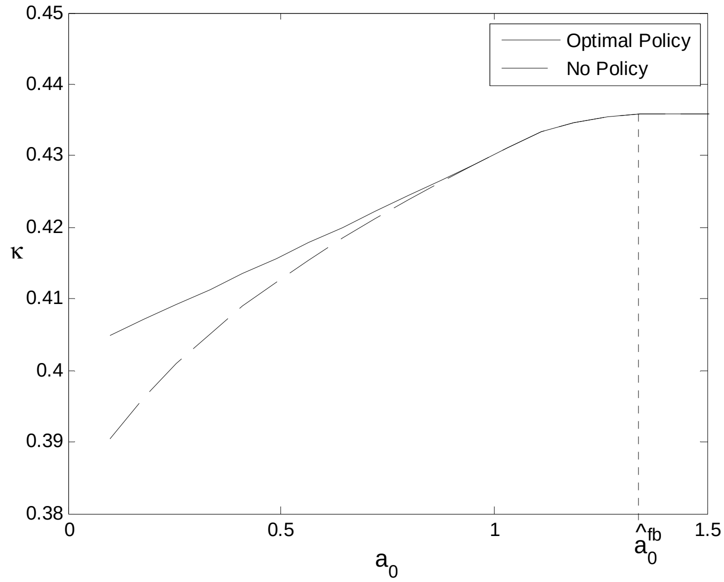

Figure 6: Consumers' Income and Optimal Policy

from cutting wages further during the A phase, since this has no effect on wages in the D phase. Remember that (29) was derived under the assumption that $p_A < p_A^{fb}$ and $p_{D,0} < p_D^{fb}$. Once n_A reaches the level such that $p_{D,0} = p_D^{fb}$, the Lagrangian for the planner problem has a kink similar to the one discussed above in the D phase. A marginal reduction in n_A now gives

$$\begin{aligned} \theta_A u'(1-n_A) - \lambda p_A - \lambda(1 - p_A + \beta\delta f) &= \\ u'(1-n_{D,0}) - \lambda p_D^{fb} &< 0, \end{aligned} \tag{31}$$

while an increase in n_A gives[18]

$$-\theta_A u'(1-n_A) + \lambda p_A + \lambda(1-p_A) + \mu\delta\beta f \leq 0.$$

In this case, the optimal values of n_A and p_A are determined by the entrepreneurs' participation constraint.

The polar opposite happens in panel (c) of Figure 7 (panel (b) simply reproduces Figure 5), where the policy is all ex-post. This takes place if the competitive equilibrium price during the appreciation is equal to p_A^{fb}, for in such case there is no scope for intervention during this phase. The only difference between this scenario and that in panel (a), is the level of financial resources a_0, which is relatively low in panel (a) and high in (c) (while it is at an intermediate level in panel

[18] See footnote 14.

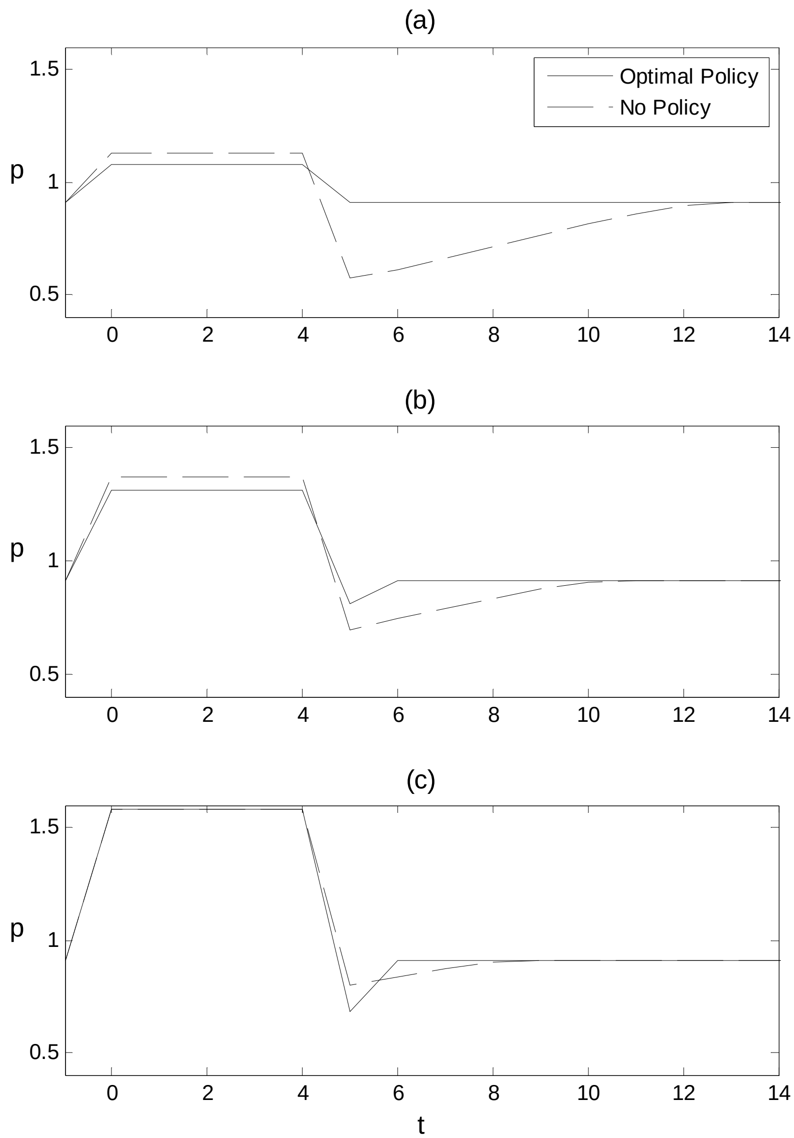

Figure 7: Ex-ante and Ex-post Optimal Intervention

(b)). When financial resources are relatively abundant, the price-distortion during the appreciation phase is small and hence the cost of distorting intertemporal consumption by taxing consumption of nontradables is high. Thus the social planner opts for postponing the intervention.[19]

Figure 8 generalizes the message of the previous figure and shows the level of p_A and $p_{D,0}$ in the competitive equilibrium (solid) and optimal policy (dashes) for a wide range of financial resources a_0. At low levels of a_0 the intervention during the appreciation phase has a large impact on allocations and there is no need to exacerbate the initial overshooting in the depreciation phase. In fact, the latter is significantly reduced in this case. However, as a_0 rises there is less scope for ex-ante intervention and a larger share of the adjustment is deferred to the depreciation phase. Eventually, when a_0 is sufficiently large, the policy is mostly concentrated ex-post, even to the point of causing an over-overshooting (the region where the dashed line is below the solid line in the bottom panel). Finally, as a_0 is sufficiently high that there is no overshooting in the competitive equilibrium, there is no longer scope for policy.

In terms of implementation of the optimal policy in each of these scenarios, Figure 9 reports the paths of nontradable consumption taxes, τ^N, corresponding to the three panels in Figure 7. The pure ex-ante policy in panel (a) requires strictly positive taxes during the appreciation phase and a subsidy during the depreciation phase. That is, the ex-ante aspect of the policy refers to the fact that the pecuniary externality is entirely resolved during the appreciation phase. The subsidy component of the policy is simply a mechanism by which, through higher wages consumers take back any surplus transferred to the entrepreneurs during the intervention in the appreciation phase. Panel (b) shows the intermediate case, where the exchange rate is allowed to depreciate in the first period of the D phase. In this case the path for the subsidy is kept lower in $t_{D,0}$, increases in $t_{D,1}$ and then converges to zero. Panel (c), the pure ex-post policy case has no taxes during the appreciation phase, and instead the tax is concentrated in the first period of the depreciation phase.

4 Further Considerations for Intervention

In this section we briefly discuss three important considerations for intervention: The persistence of the appreciation, distortions in consumers' perceptions, and frictions in the nontradables sector.

[19]Note also that when export firms have abundant financial resources, there is a sort of Ricardian equivalence, in that any (at least small) intervention can be undone by the private sector (this is an exact result whenever $\phi_A = \phi_{D,0}$).

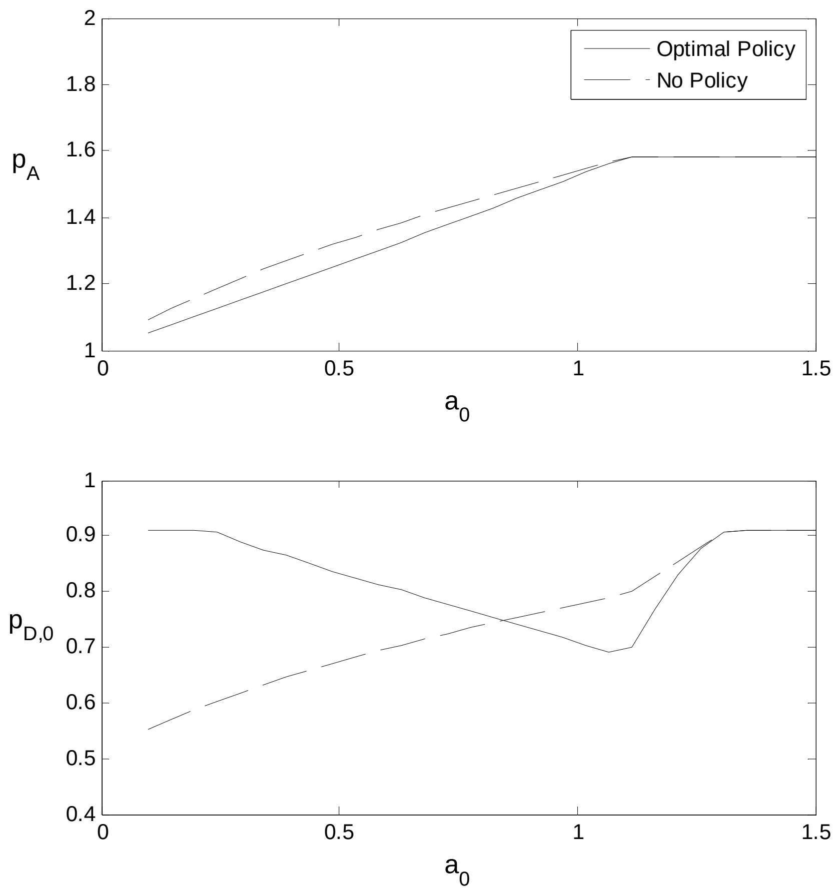

Figure 8: Entrepreneurs' Wealth and Optimal Intervention

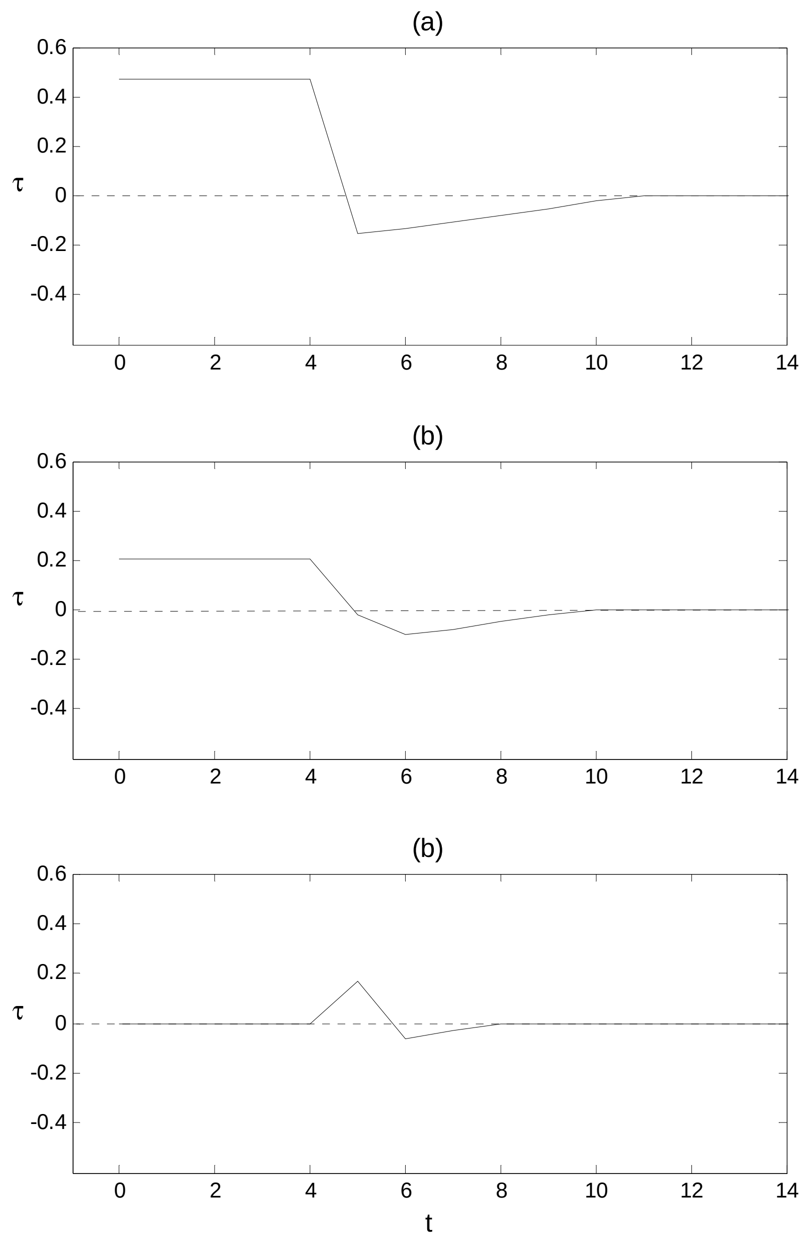

Figure 9: Implementation: Optimal Tax on Nontradables

4.1 Appreciation Persistence

In our complete markets context, persistence matters only in an ex-ante sense and is captured by the parameter δ. On one extreme, if δ is close to one (very short lived appreciations) then the losses to be financed are not much and entrepreneurs' internal resources may suffice. On the other extreme, if δ is very close to zero (very persistent appreciations), then the option value of keeping units is low, and there is no reason to protect the export sector either. It is for intermediate δ's that policy intervention may be needed.

Figure 10 illustrates this non-monotonicity by showing the region where policy intervention is called for in the $(1/\delta, a_0)$ space. The shaded region corresponds to the case where the equilibrium is constrained inefficient and exchange rate intervention is warranted. Note that there are many general equilibrium effects hidden in this figure. For example, as δ changes, so does κ. Also, when δ rises, firms reluctance to destroy during the appreciation rises. This reluctance exacerbates the (now shorter lived) appreciation, and hence the resources required to survive each period of appreciation. However, none of these additional effects is strong enough to change the qualitative shape of the figure and the conclusion that follows from it. Medium run appreciations are most likely to justify intervention.[20]

4.2 Consumers' Overoptimism and Incomplete Insurance

In reality, consumption binges rarely occur by themselves. In the international context, they often come as a response to a rise in national income due to a positive terms of trade shock in commodity producing countries, or due to a large increase in the supply of capital flows to the country. Adding external income shocks to our complete markets, rational representative agent setup, would have no effect on the path of consumption, given that consumers fully insure against these shocks. We need to add some "friction" on the consumption side as well.

One extension along these lines is to replace the taste shocks with income (terms of trade) shocks, but assume that either foreigners charge an insurance premium to consumers, or that the

[20] In an earlier draft we relaxed the complete markets assumption and studied the polar opposite case, where export firms only have access to a riskless bond. In this context, the export sector resources dwindle as the appreciation progresses. The main policy implication that follows from this modification is the timing of the exchange rate stabilization in the appreciation phase. Early on in the appreciation, the optimal policy is to postpone much of the intervention to the D phase. However, as the appreciation continues and the export sector's resources dwindle, the optimal policy shifts a larger share of the intervention to the appreciation phase (essentially, this amounts to a gradual leftward movement in Figure 8 .)

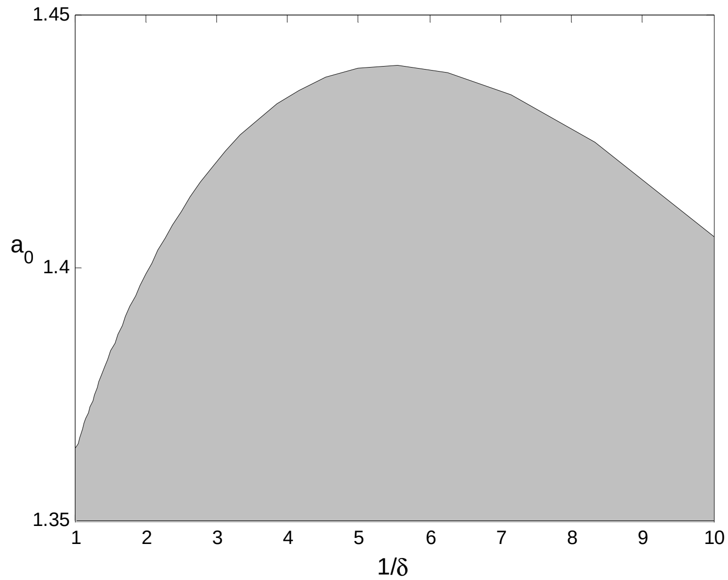

Figure 10: Expected Duration and Policy Intervention

latter are overoptimistic with respect to the expected duration of the high income phase A:[21]

$$\delta^{cons} < \delta.$$

In either case, consumers find D-insurance too expensive and choose not to insure fully. The analysis is more complex in this case since incomplete insurance introduces wealth and price dynamics within the appreciation phase, however the important point for us is that the basic structure of our environment is preserved. In particular, nontradables demand drops at the time of the switch and consumers still ignore the pecuniary externality associated to their high expenditure during the appreciation. Of course, if the social planner does not share in the consumers' optimism, then it would be justified to implement some sort of saving policy, with the goal of reducing not only c_A^N but also c_A^T. More importantly, even if the planner *does share* consumers' view on the expected duration of the appreciation, there is a role for intervention to offset the pecuniary externality, as in our main case.

[21] Alternatively, we could introduce procyclical consumption (or short horizons) through non-representative agents. The extreme version of this formulation is one where consumers live for only one period and must consume their income in that period. The social planner Pareto-weighs a generation t periods from the current one by β^t. If no intergenerational transfer mechanism other than through the real exchange rate is available, then we are again in the situation just described. The constrained goal of the social planner is to reallocate consumption away from nontradables during the appreciation phase. Relative to the pure taste shock scenario, a larger share of the adjustment is done in the A phase, in order to reduce the burden of the adjustment on the first generation in the D phase.

4.3 Rigidities in the Nontradables Sector

Frictions in the nontradable sector generally shift a share of the intervention toward the A phase. For example, this is typically the case in the sudden stops literature, particularly when liabilities are dollarized. The latter limits the possibility and desirability of implementing a large overshooting in D, even if short lived.

Another example is the presence of a real wage rigidity, either as the result of a distortion or of a reservation wage.[22] Yet another is that some of the inputs of production in the nontradables sector are tradables.

Let us develop the simplest of these examples and assume that workers have a reservation wage of w units of tradable goods, which is not binding except, possibly, during the overshooting phase. Suppose that this reservation wage is binding for the optimal policy but not for the competitive equilibrium. That is, in the over-overshooting scenario the social planner would like to bring $p_{D,0}^{sb}$ below w, but it cannot. What is the impact of this binding constraint on the optimal policy? In particular, how much of the intervention is reallocated to the appreciation phase? Let us return to the complete markets environment to answer the latter question. We know that in this context the social planner's first order condition in the A phase is:

$$\theta_A u'(1 - n_A) = \lambda p_A^{fb} = \lambda (1 + \beta \delta f).$$

It follows immediately that $p_A^{sb,w} < p_A^{sb}$, where $p_A^{sb,w}$ and p_A^{sb} stand for the second best real exchange rate during the appreciation with and without a reservation wages w, respectively. The reason for the inequality is that the binding constraint must necessarily lower κ relative to the unconstrained case, and this implies that $\lambda = u'(\theta_A \kappa)$ rises with the constraint. In turn, the latter implies that n_A increases, which given the firms financial constraint can only be achieved with a larger intervention that drops the real exchange rate below that of the unconstrained case.

5 Final Remarks

This paper shows how financial frictions lend support to the view that persistent appreciations may justify intervention, even if agents are fully rational and forward looking. The reason for the intervention is not to improve the health of the export sector per se, as our social planner is primarily concerned about consumers (workers), but a pecuniary externality within consumers. By putting excessive cost pressure on financially constrained export firms during the appreciation phase, consumers reduce these firms' ability to recover once the factors behind the appreciation

[22] See Blanchard (2006b) for a more thorough discussion of wage rigidities and appreciations; and Blanchard (2006a) for an application to the case of Portugal.

subside. The result is a severe overshooting and real wage collapse at that stage, which hurts consumers more than they gain from the extra consumption during the appreciation.

Our normative framework sheds light on the perennial policy problem of the timing of intervention. We show that, other things equal, if financial constraints on the export sector are tight during the appreciation phase, then it is optimal to intervene ex-ante. Conversely, if the export sector has substantial financial resources (although not enough to fully finance the recovery), then ex-ante intervention is either costly or ineffective, and it is optimal to postpone intervention until after the "crash."

In abstract, the optimal policy can be implemented through an appropriate sequence of taxes and subsidies on nontradable consumption. In reality, the flexibility of such policies is limited, leaving to expenditure policy and central bank's reserves management most of the burden. While these are not perfect substitutes for taxes and subsidies, much of our insights still carry over to them.

Let us conclude with a few clarifying remarks and extensions. When thinking about policy, it is worth noting the distinction between an "appreciation" and an "overvaluation." The latter is an elusive concept in practice but it has a well defined meaning in ours: an overvaluation refers to a situation where the exchange rate is higher than it is socially optimal. However, this gap is not limited to an appreciation episode as it can also take place during a depreciation phase. The over-overshooting result is an example of an overvaluation during the depreciation phase. A wage rigidity is an example of when such overvaluation cannot be cured fully with intervention within the depreciation phase. The latter example also illustrates the role of early intervention in limiting *future* overvaluations.

We note that our model uses a single reason —a financial friction— for constrained production in the appreciation and depreciation phases. However, some of our conclusions extend to other scenarios as well. In particular, we could replace the financial constraint in the depreciation phase for a technological time to build assumption. In such case, the overshooting is also directly linked to excessive export destruction in the appreciation phase and there is a reason for intervention. The main difference in this instance is that the optimal policy does not prolong the intervention into the depreciation phase.

Finally, while our analysis focuses on the real exchange rate, it seems suitable for other important relative prices within an economy. For example, a real estate boom can have important cost consequences for sectors that compete with the construction sector for inputs and factors of production. More broadly, ours is a model of the optimal management of sectoral reallocation in the presence of temporary (but persistent) shocks, when some sectors have limited financial and technological flexibility.

6 Appendix

6.1 Proof of Proposition 1

The cutoff is given by

$$\hat{a}^{fb} = \frac{\left(p_A^{fb} - 1\right) n_A^{fb} + \beta\delta f\left(n_D^{fb} - n_A^{fb}\right) - \beta\delta\left(1 - p_D^{fb}\right) n_D^{fb}}{1 - \beta\left(1 - \delta\right)},$$

where $p_A^{fb}, p_D^{fb}, n_A^{fb}$, and n_D^{fb} are defined in the text. Let us conjecture and verify that if $a_0 \geq \hat{a}^{fb}$ these prices and quantities form an equilibrium. Given the conjectured prices it is possible to show (by guessing and verifying) that $V\left(a, n^-; s^t\right) = a + q\left(s^t\right) n^-$ (i.e., $\psi\left(s^t\right) = 0$ and $\phi\left(s^t\right) = 1$). Then, inspecting the entrepreneur's optimality conditions (9)-(11) shows that the entrepreneur is, at each s^t, indifferent among all feasible choices of $c^{T,e}\left(s^t\right), n\left(s^t\right)$, and $\left\{a\left(\langle s^t, s_{t+1}\rangle\right)\right\}_{s_{t+1} \in S}$ If the entrepreneur begins with a_0, he can consume the difference $a_0 - \hat{a}^{fb}$ and then adopt the following rule: set $n\left(s^t\right) = n_A^{fb}$, $a\left(\langle s^t, D\rangle\right) = f(n_D^{fb} - n_A^{fb}) - (1 - p_D^{fb})n_D^{fb}$ and $a\left(\langle s^t, A\rangle\right) = \hat{a}^{fb}$, for each history $s^t = \{A, A, ..., A\}$; set $n\left(s^t\right) = n_D^{fb}, a\left(\langle s^t, D\rangle\right) = 0$ for each history $s^t = \{A, ...A, D, ..., D\}$. These decisions are consistent with labor market clearing. One final check, which we left aside in the main text, is that $n_A^{fb} > 0$. This follows from substituting κ^{fb} in the definition of n_A^{fb} and using the inequalities $1 - \beta(1-\delta) < 1$, $\theta_A / \left((1-\beta)\theta_A + \delta\beta\right) < 1$, and $1/\left(1 + \delta\beta f\right) < 1$.

6.2 Proof of Proposition 2

First, we establish a preliminary lemma.

Lemma 1 *Define the function*

$$H\left(n\right) \equiv fn - \left(1 - \frac{\kappa}{1-n}\right) n - x,$$

the equation $H\left(n\right) = 0$ has a unique solution $n^* \in (0, 1)$, for each $\kappa > 0$ and $x > 0$. Moreover, $H\left(n\right) > 0$ for each $n > n^*$. The solution n^* is increasing in x. If $x = 0$ the equation can have one or two solutions, one of which is 0. In this case, the properties above apply to the largest solution.

Proof. A solution exists because H is continuous in $[0, 1)$, $H\left(0\right) = -x$ and $\lim_{n \to 1} H\left(n\right) = \infty$. Consider the case $x > 0$. Let n^* be a solution, then $f - \left(1 - \kappa/\left(1 - n^*\right)\right) > 0$ must hold. If $n > n^*$, $H'\left(n\right) = f - \left(1 - \kappa/\left(1-n\right)\right) + \kappa n/\left(1-n\right)^2 > 0$ follows from $f - \left(1 - \kappa/\left(1-n\right)\right) > f - \left(1 - \kappa/\left(1-n^*\right)\right) > 0$. This implies that $H\left(n\right) > 0$ for each $n > n^*$, and the solution is unique. The comparative statics result with respect to x follows from the implicit function theorem. When $x = 0$ the solution $n^* = 0$ is trivial. If there is another solution $n^* > 0$, the properties stated can be proved following the steps of the case $x > 0$. ■

The proof will proceed in three steps. First, we define a map T for the coefficient κ. Second, we derive some properties of this map. Finally, we show that this map has a unique fixed point. From this fixed point we can construct an equilibrium with the desired properties.

Step 1. Fix a value for $\kappa \in \left[0, \kappa^{fb}\right]$ and construct an equilibrium as follows.

Phase A. If

$$\left(1-\beta\left(1-\delta\right)\right)a_0 > \left(p_A^{fb}-1\right)\left(1-\frac{\kappa\theta_A}{p_A^{fb}}\right), \tag{32}$$

then set p_A equal to p_A^{fb}, set $n_A = 1-\kappa\theta_A/p_A^{fb}$ and

$$a_{D,0} = \frac{1}{\beta\delta}\left[\left(1-\beta\left(1-\delta\right)\right)a_0 - \left(p_A^{fb}-1\right)n_A\right] > 0. \tag{33}$$

Notice that $n_A > 0$. Since $\kappa \leq \kappa^{fb}$ we have $1-\kappa\theta_A/p_A^{fb} \geq 1-\kappa^{fb}\theta_A/p_A^{fb} > 0$, where the last inequality follows from assumption (A1).

If (32) does not hold, then set p_A equal to the solution of

$$\left(1-\beta\left(1-\delta\right)\right)a_0 = \left(p_A-1\right)\left(1-\frac{\kappa\theta_A}{p_A}\right), \tag{34}$$

(which has a unique solution in $[1, p_A^{fb}]$), set $n_A = 1-\kappa\theta_A/p_A$ and $a_{D,0}=0$. Notice that when $p_A = \kappa\theta_A$, the right-hand side of (34) is zero, therefore $p_A \in \left[\kappa\theta_A, p_A^{fb}\right]$ and $n_A \geq 0$.

Phase D. Define

$$\bar{n}_D = 1-\frac{\kappa}{p_D^{fb}}.$$

Construct the sequence $\{n_{D,j}\}$ that satisfies:

$$f\left(n_{D,0}-n_A\right) = \left(1-\frac{\kappa}{1-n_{D,0}}\right)n_{D,0}+a_{D,0} \tag{35}$$

$$f\left(n_{D,j}-n_{D,j-1}\right) = \left(1-\frac{\kappa}{1-n_{D,j}}\right)n_{D,j} \text{ for } j=1,2,...,J \tag{36}$$

until $n_{D,J+1}$ is larger than $\bar{n}_D$. From then on set

$$n_{D,j} = \bar{n}_D \text{ for all } j > J.$$

Letting $x = a_{D,0}+fn_A$, Lemma 1 ensures that (35) has a solution for $n_{D,0}$ (if $a_{D,0}+fn_A=0$, pick the solution with the largest $n_{D,0}$). To show that $n_{D,0} \geq n_A$ consider the following: Either $H\left(\bar{n}_D\right) \leq 0$, and the solution will be larger than $\bar{n}_D$. In this case the economy converges to $\bar{n}_D$ immediately and

$$n_A \leq 1-\kappa\frac{\theta_A}{p_A^{fb}} \leq 1-\kappa\frac{1}{p_D^{fb}} = \bar{n}_D,$$

where the inequality in the middle follows from assumption (A3). If, instead $H\left(\bar{n}_D\right) > 0$ then $H\left(n_{D,0}\right)=0$. Notice that

$$H\left(n_A\right) = \left(\frac{\kappa}{1-n_A}-1\right)n_A - a_{D,0} \leq \left(p_D^{fb}-1\right)n_A - a_{D,0} < 0$$

where the inequality in the middle follows from

$$\frac{\kappa}{1-n_A} = \frac{\kappa\theta_A}{1-n_A}\frac{1}{\theta_A} < p_A^{fb}\frac{1}{\theta_A} < p_D^{fb} < 1,$$

(the second inequality follows from (A3)). Therefore, Lemma 1 implies that $n_{D,0} > n_A$. In a similar way, it is possible to prove that (36) implies $n_{D,j} \geq n_{D,j-1}$ for each j.

From these two steps we obtain a sequence $p_A, \{p_{D,j}\}$, which can then be substituted in expression (3), to obtain κ'. This defines a map $T : [0, \kappa^{fb}] \to [0, \kappa^{fb}]$.

Step 2. It can be shown that the map T is continuous. Furthermore, let us prove that

$$\kappa' = T(\kappa(1+\Delta)) < (1+\Delta) T(\kappa).$$

In the construction in Step 1, an increase in κ leads to a (weak) reduction in n_A and $n_{D,j}$ for all j (for the initial conditions of phase D notice that if (32) is satisfied, then, using the definition of p_A^{fb}, it is possible to show that $a_{D,0} + fn_A$ is independent of κ; if (32) is not satisfied, then an increase in κ leads to a decrease in n_A). But since $n_A = 1 - \theta_A \kappa / p_A$, $n_{D,j} = 1 - \kappa / p_{D,j}$, this implies that the prices p_A and $p_{D,j}$ must increase less than proportionally than κ. Therefore, κ' increases less than proportionally.

Step 3. Define the following map for $z \equiv \log(\kappa)$:

$$z' = \tilde{T}(z) \equiv \log(T(\exp^z)).$$

Step 2 shows that this map is continuous and has slope smaller than 1. Therefore this map has a unique fixed point (uniqueness is not needed for the statement of this proposition, but will be useful for the following results). Let κ be the fixed point and consider the prices and quantities constructed in Step 1. To ensure that they are an equilibrium, it remains to check that the sequence of prices and quantities are optimal for the entrepreneur. Derive the marginal utility of money at $t_{D,0}$ from the recursion:

$$\phi_{D,j} = \beta \frac{f}{f - (1 - p_{D,j})} \phi_{D,j+1}. \tag{37}$$

By construction we have $p_{D,j} \leq p_D^{fb}$, which implies that $\phi_{D,j} \geq 1$. Moreover, entrepreneurs' consumption and cash savings, $a_{D,j+1}$, are zero until the point where $\phi_{D,j} = 1$. To check optimality in phase A, notice that

$$\phi_A = \frac{\beta \delta f}{p_A - 1} \phi_{D,0}$$

and $\phi_A > \phi_{D,0}$ iff $p_A < p_A^{fb}$, and, by construction $a_{D,0}$ is zero iff $p_A < p_A^{fb}$. Notice that as long as $a_0 < \hat{a}^{fb}$ either p_A or some $p_{D,j}$ will be strictly below their first best value. Therefore, $\phi_A > 1$.

6.3 Proof of Propositions 3 and 4

The following lemma provides a useful preliminary result.

Lemma 2 *The equilibrium value of κ is non-decreasing in a_0.*

Proof. Let $T(\kappa; a_0)$ be the mapping $[0, \kappa^{fb}] \to [0, \kappa^{fb}]$ defined in the proof of Proposition 2, indexed by the initial wealth a_0. Choose two values $a_0' < a_0''$. Let κ' and κ'' be the corresponding equilibrium values of κ. Now, fixing κ' we want to show that T is monotone in a_0, i.e., $T(\kappa'; a_0'') \geq T(\kappa'; a_0')$. If (32) holds at a_0', then an increase in a_0 leaves p_A unchanged, and it increases $a_{D,0}$ (from (33)) and leaves n_A unchanged. If (32) does not hold, an increase in $a_{D,0}$ leads to an increase in p_A, and an increase in $a_{D,0} + fn_A$, since

$a_{D,0}$ either remains zero or becomes positive and n_A increases. In both cases, $a_{D,0} + fn_A$ increases. This means that, in phase D, there will be a (weak) increase in $n_{D,j}$ for all j, and, thus, a (weak) increase in $p_{D,j}$ for all j. Therefore, $T(\kappa'; a_0'') \geq T(\kappa'; a_0') = \kappa'$. This implies that $T(\kappa; a_0'')$ has a fixed point in $[\kappa', \kappa^{fb}]$. Since T has a unique fixed point and $T(\kappa''; a_0'') = \kappa''$, by construction, this implies $\kappa'' \geq \kappa'$. ■

Now we can prove the two propositions. Consider first Proposition 3. Suppose that at a_0' we have $p_A = p_A^{fb}$ in equilibrium. This means that (32) holds at a_0'. Since $\kappa'' \geq \kappa'$, (32) holds *a fortiori* for $a_{D,0}'', \kappa''$, it follows that at the new equilibrium $p_A = p_A^{fb}$ and $a_{D,0} > 0$.

Consider next Proposition 4. Suppose that at a_0' we have $p_{D,0} = p_D^{fb}$ in equilibrium. This means that the following inequality holds

$$f\bar{n}_D' \leq \left(1 - \frac{\kappa'}{1 - \bar{n}_D'}\right)\bar{n}_D' + fn_A' + a_{D,0}' \tag{38}$$

where $\bar{n}_D' = 1 - \kappa'/p_D^{fb}$. Now we want to prove the following inequality

$$fn_A'' + a_{D,0}'' \geq fn_A' + a_{D,0}'. \tag{39}$$

If (32) holds at a_0' then some algebra (using the definition of p_A^{fb}) shows that

$$fn_A'' + a_{D,0}'' = fn_A' + a_{D,0}' = \frac{1}{\beta\delta}\left[(1 - \beta(1-\delta))\, a_0\right].$$

If (32) does not hold at a_0', then we have $a_{D,0}'' \geq 0 = a_{D,0}'$. Furthermore, we can show that $n_A'' \geq n_A'$. Notice that $\kappa'' \geq \kappa'$ holds because, on average, equilibrium prices are larger. However, $p_{D,j}' = p_D^{fb}$ at a_0', and $p_{D,j}'' \leq p_D^{fb}$. This implies that

$$\frac{p_A''}{p_A'} \geq \frac{\kappa''}{\kappa'},$$

and since $n_A = 1 - \kappa/p_A$ this implies $n_A'' \geq n_A'$. Therefore, (39) holds in all cases. This implies that (38) also holds at a_0'', and therefore $p_{D,0} = p_D^{fb}$. Notice that if (32) holds at a_0' then, we can proceed as in the proof of Proposition 3 and show that $a_{D,0}'' > a_{D,0}'$ and $n_A'' = n_A'$.

6.4 Setup of the Optimal Policy Problem

The set of feasible allocations is defined as follows.

Definition 2 *(Feasibility) The allocation $\{c^T(s^t), c^N(s^t), c^{T,e}(s^t), n(s^t)\}$ is feasible iff there exists a sequence of tax rates $\{\tau^T(s^t), \tau^N(s^t)\}$, wealth levels $\{a(s^t)\}$, and prices $\{p(s^t), q(s^t)\}$ such that the prices and quantities $\{p(s^t), q(s^t)\}$ and $\{c^T(s^t), c^N(s^t), c^{T,e}(s^t), n(s^t), a(s^t)\}$ constitute a competitive equilibrium under the tax rates $\{\tau^T(s^t), \tau^N(s^t)\}$.*

We now derive three necessary conditions, (40) to (42) below, that any feasible allocation must satisfy. Then, we define the problem of a planner that chooses an allocation subject only to the consumer's budget constraint, the entrepreneur's budget constraint (7), and conditions (40)-(42). This is a relaxed version of the original planning problem, given that this set of constraints is necessary but, in general, not sufficient for feasibility. We also perform a change of variables that makes the relaxed planning problem a concave problem and we derive first-order conditions which are sufficient for an optimum. In the proof of Proposition 6 we make use of these first-order conditions to find allocations that solve the relaxed planning problem.

First, notice that the entrepreneur's optimality implies that a feasible allocation must satisfy the condition

$$\left(q\left(s^t\right)-\left(1-p\left(s^t\right)\right)\right)n\left(s^t\right)\geq\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}|s_t\right)q\left(\left\langle s^t,s_{t+1}\right\rangle\right)n\left(s^t\right). \tag{40}$$

To prove this inequality, multiply by $n\left(s^t\right)$ both sides of (9), and use the complementarity condition to obtain

$$\left(q\left(s^t\right)-\left(1-p\left(s^t\right)\right)\right)\phi\left(s^t\right)n\left(s^t\right)=\beta\sum_{s_{t+1}\in S}\pi\left(s_{t+1}|s_t\right)\phi\left(\left\langle s^t,s_{t+1}\right\rangle\right)q\left(\left\langle s^t,s_{t+1}\right\rangle\right)n\left(s^t\right).$$

Moreover, the entrepreneur's optimality condition for $a\left(s^t\right)$ implies that $\phi\left(\left\langle s^t,s_{t+1}\right\rangle\right)\geq\phi\left(s^t\right)$ for all s_{t+1}. Substituting in the equation above, gives (40).

Second, recall that $q\left(\left\langle s^t,s_{t+1}\right\rangle\right)\leq f$ which implies

$$q\left(\left\langle s^t,s_{t+1}\right\rangle\right)n\left(s^t\right)\leq fn\left(s^t\right). \tag{41}$$

Finally, define the function

$$G\left(x\right)\equiv f\max\left\{x,0\right\},$$

for a generic variable x, and notice that equilibrium in the adjustment sector implies that, for all s^t and s_{t+1},

$$q\left(\left\langle s^t,s_{t+1}\right\rangle\right)\left(n\left(\left\langle s^t,s_{t+1}\right\rangle\right)-n\left(s^t\right)\right)=G\left(n\left(\left\langle s^t,s_{t+1}\right\rangle\right)-n\left(s^t\right)\right). \tag{42}$$

We perform a change in variables, defining

$$\begin{aligned} z\left(s^t\right) &\equiv \left(p\left(s^t\right)-1\right)n\left(s^t\right)+q\left(s^t\right)n\left(s^t\right), \\ y\left(s^t\right) &\equiv q\left(s^t\right)n\left(s^{t-1}\right), \end{aligned}$$

for all consecutive histories s^{t-1} and s^t.

Substituting the government budget balance in the consumer's budget constraint (21), and using the market clearing condition $c^N\left(s^t\right)=1-n\left(s^t\right)$, we obtain the budget constraint

$$\sum_{t,s^t}\beta^t\pi\left(s^t\right)c^T\left(s^t\right)\leq\sum_{t,s^t}\beta^t\pi\left(s^t\right)p\left(s^t\right)n\left(s^t\right).$$

Substituting z and y and using (42) on the right-hand side we get

$$\sum_{t,s^t}\beta^t\pi\left(s^t\right)c^T\left(s^t\right)=\sum_{t,s^t}\beta^t\pi\left(s^t\right)\left[z\left(s^t\right)-y\left(s^t\right)+n\left(s^t\right)-G\left(n\left(s^t\right)-n\left(s^{t-1}\right)\right)\right]. \tag{43}$$

Substituting z and y in the entrepreneur's flow of funds constraint gives

$$c^{T,e}\left(s^t\right)+z\left(s^t\right)-y\left(s^t\right)+\beta\sum_{s_{t+1}\in S}\pi(s_{t+1}|s_t)a\left(\left\langle s^t,s_{t+1},\right\rangle\right)\leq a\left(s^t\right). \tag{44}$$

The relaxed planner problem is to choose sequences $\{c^T(s^t), c^{T,e}(s^t), n(s^t), a(s^t)\}$ and $\{z(s^t), y(s^t)\}$, to maximize the consumer's expected utility subject to (22), (40), (41), (43), and (44). The relaxed problem is a concave problem, so the first order conditions are sufficient for an optimum. It is possible to show that the solution is stationary in phase A and that, in phase D, quantities and prices only depend on the number of periods since the transition. To save space and help the interpretation, we write the problem directly in terms of variables indexed by A and (D, j). Moreover, we normalize the utility of the consumer, the budget constraint, and the entrepreneur's participation constraint by the constant $(1-\beta(1-\delta))$. Then, the planner maximizes

$$\theta_A \left[u(1-n_A) + u\left(c_A^T\right)\right] + \delta\beta \sum_{j=0}^{\infty} \beta^j \theta_D \left[u(1-n_{D,j}) + u(c_{D,j}^T)\right],$$

subject to

$$c_A^T + \sum_{j=0}^{\infty} \beta^j c_{D,j}^T + G(n_A - n_{-1}) + \delta\beta G(n_{D,0} - n_A) + \delta\beta \sum_{j=1}^{\infty} \beta^j G(n_{D,j} - n_{D,j-1})$$
$$\leq (z_A - y_A + n_A) + \delta\beta \sum_{j=0}^{\infty} \beta^j (z_{D,j} - y_{D,j} + n_{D,j}), \qquad (\lambda)$$

$$c_A^{T,e} + \delta\beta \sum_{j=0}^{\infty} \beta^j c_{D,j}^{T,e} \geq (1-\beta(1-\delta))\, U, \qquad (\mu)$$

$$(1-\beta(1-\delta))\, a_0 - \beta\delta a_{D,0} + y_A - z_A - c_A^{T,e} \geq 0, \qquad (\nu_A) \qquad (45)$$
$$a_{D,0} - \beta a_{D,1} + y_{D,0} - z_{D,0} - c_{D,0}^{T,e} \geq 0, \qquad (\delta\beta v_{D,0}) \qquad (46)$$
$$a_{D,j} - \beta a_{D,j+1} + y_{D,j} - z_{D,j} - c_{D,j}^{T,e} \geq 0 \quad \text{for } j \geq 1, \qquad (\delta\beta^{j+1} v_{D,j}) \qquad (47)$$

and conditions (40)-(41), which take the form

$$\begin{aligned} y_A &\leq f n_A, && (\gamma_A) \\ y_{D,j} &\leq f n_{D,j} \quad \text{for all } j, && (\delta\beta^{j+1}\gamma_{D,j}) \\ z_A &\geq \delta\beta y_{D,0}, && (\eta_{D,0}) \\ z_{D,j} &\geq \beta y_{D,j+1} \quad \text{for all } j. && (\delta\beta^{j+1}\eta_{D,j+1}) \end{aligned}$$

Next to each constraint we write the respective Lagrange multiplier.

Let us take first-order conditions with respect to n

$$\begin{aligned} -\theta_A u'(1-n_A) - \delta\beta f\lambda + \lambda + f\gamma_A &= 0, \\ -\delta\beta^{j+1}\theta_D u'(1-n_{D,j}) - \delta\beta^{j+1} f\lambda + \delta\beta\beta^{j+1} f\lambda + \delta\beta^{j+1}\lambda + \delta\beta^{j+1} f\gamma_{D,j} &= 0, \end{aligned}$$

with respect to y and z

$$\begin{aligned} -\lambda + \nu_A - \gamma_A &= 0, \\ -\lambda\delta\beta^{j+1} + \delta\beta^{j+1}\nu_{D,j} - \delta\beta^{j+1}\gamma_{D,j} - \beta\delta\beta^j\eta_{D,j} &= 0, \\ \lambda - \nu_A + \eta_{D,0} &= 0, \\ \lambda\delta\beta^{j+1} - \delta\beta^{j+1}\nu_{D,j} + \delta\beta^{j+1}\eta_{D,j+1} &= 0, \end{aligned}$$

and with respect to a and $c^{T,e}$

$$-\nu_A + \nu_{D,0} \geq 0 \; (a_{D,0} \geq 0), \tag{48}$$

$$-\nu_{D,j} + \nu_{D,j+1} \geq 0 \; (a_{D,j} \geq 0), \tag{49}$$

$$\mu \geq \nu_A \quad (c_A^{T,e} \geq 0), \tag{50}$$

$$\mu \geq \nu_{D,j} \quad (c_{D,j}^{T,e} \geq 0). \tag{51}$$

Rearranging these conditions shows that a sufficient condition for an optimum is that there exist Lagrange multipliers $\lambda \leq \nu_A \leq \nu_{D,0} \leq \nu_{D,1} \leq ... \leq \mu$, such that

$$-\theta_A u'(1-n_A) + \lambda(1+\delta\beta f) + f(\nu_A - \lambda) = 0, \tag{52}$$

$$-\theta_D u'(1-n_{D,0}) + \lambda(1-(1-\beta) f) + f(\nu_{D,0} - \nu_A) = 0, \tag{53}$$

$$-\theta_D u'(1-n_{D,j}) + \lambda(1-(1-\beta) f) + f(\nu_{D,j} - \nu_{D,j-1}) = 0, \tag{54}$$

and conditions (48)-(51) are satisfied.

6.5 Proof of Propositions 5 and 6

We prove Proposition 6 by giving a complete characterization of the optimal allocation. The proof is split in three steps. In the first step, we define two maps J and $\tilde{J}$. In the second step, we use these maps to construct a candidate optimal allocation and we show that this allocation is indeed optimal. The proof of 5 is a side product of this step. In the third step, we derive the implications for the optimal path of the exchange rate and for the optimal tax in A.

Step 1. We define the two maps J and $\tilde{J}$: Define the map $J : [1+\delta\beta f/\phi_A^{ce}, p_A^{fb}] \to \mathbb{R}$ as follows (notice that $1+\delta\beta f/\phi_A^{ce} < 1+\delta\beta f = p_A^{fb}$ since $\phi_A^{ce} > 1$). For any $p_A \in [1+\delta\beta f/\phi_A^{ce}, p_A^{fb}]$ find the unique ξ that solves

$$\xi = \frac{1-\beta}{(1-\beta)\theta_A + \delta\beta\theta_D}\left(p_A n_A + \delta\beta\left(p_{D,0} n_{D,0} + \sum_{j=1}^{\infty}\beta^j p_D^{fb} n_{D,j}\right)\right), \tag{55}$$

where $p_{D,0} = 1 - f + \delta\beta^2 f^2/(\phi_A^{ce}(p_A - 1))$, and the sequence $\{n_A, \{n_{D,j}\}\}$ is given by

$$n_A = \frac{1}{p_A - 1}(1-(1-\delta)\beta) a_0, \tag{56}$$

$$n_{D,0} = \frac{1}{\delta\beta^2 f}\phi_A^{ce}(1-(1-\delta)\beta) a_0, \tag{57}$$

$$n_{D,j} = \min\left\{\beta^{-j} n_{D,0}, \bar{n}_D\right\} \text{ for } j \geq 1, \tag{58}$$

where

$$\bar{n}_D = 1 - \xi\theta_D/p_D^{fb}. \tag{59}$$

To show that such a ξ exists and is unique, notice that the right-hand side of (55) is a continuous non-increasing function of ξ, and ranges between a positive value, at $\xi = 0$, and $-\infty$ for $\xi \to \infty$. Set $J(p_A) = \xi$

(the function J is allowed to take negative values but we will see below that at the relevant values of p_A, $J(p_A) > 0$). Combining the terms containing p_A on the right-hand side of (55) we obtain the expression

$$\frac{p_A}{p_A - 1}(1 - (1-\delta)\beta) a_0 + \delta\beta \left(1 - f + \frac{1}{p_A - 1}\frac{\delta\beta^2 f^2}{\overline{\phi}_A}\right)\frac{1}{\delta\beta^2 f}\phi_A^{ce}(1-(1-\delta)\beta) a_0,$$

which is monotone decreasing in p_A. Applying the implicit function theorem, it follows that $J'(p_A) < 0$.

Define the map $\tilde{J} : [0, (1-(1-\delta)\beta) a_0/(p_A^{fb} - 1)]$ as follows: For any $n_A \in [0, (1-(1-\delta)\beta) a_0/(p_A^{fb} - 1)]$ find the unique positive ξ that solves

$$\xi = \frac{1-\beta}{(1-\beta)\theta_A + \delta\beta\theta_D}\left(p_A^{fb} n_A + \delta\beta\left(p_{D,0} n_{D,0} + \sum_{j=1}^{\infty}\beta^j p_D^{fb} n_{D,j}\right)\right),$$

where $p_{D,0} = 1 - f + \beta f/\phi_A^{ce}$, and the sequence $\{n_{D,j}\}$ is given by (57)-(58). Again, it is easy to show that such a ξ exists and is unique. Set $\tilde{J}(n_A) = \xi$. It is immediate to show that $\tilde{J}'(n_A) > 0$.

Step 2. Define the function

$$L(p_A) \equiv 1 - \frac{\theta_A J(p_A)}{p_A^{fb}} - \frac{1}{p_A - 1}(1-(1-\delta)\beta) a_0.$$

From step 1, we know that $L(p_A)$ is an increasing function of p_A. Therefore, three mutually exclusive cases are possible. Either there exists a unique $p_A \in [1 + \delta\beta f/\phi_A^{ce}, p_A^{fb}]$ that solves the equation $L(p_A) = 0$, or $L(1 + \delta\beta f/\phi_A^{ce}) > 0$, or $L\left(p_A^{fb}\right) < 0$. We can construct an optimum for each of these cases. We will analyze in detail the first case, which correspond to the case depicted in Figure 5. Let p_A^* be such that $L(p_A^*) = 0$. Set

$$p_{D,0}^* = 1 - f + \frac{1}{\phi_A^{ce}}\frac{\delta(\beta f)^2}{p_A^* - 1},$$

the assumption $\phi_A^{ce} \leq (f-1)/(\beta f)$ ensures that $p_{D,0}^* \geq 0$, given that $p_A^* \leq p_A^{fb}$. In the case $\phi_A^{ce} > (f-1)/(\beta f)$ the argument needs to be amended to allow for a number of periods in which $p_{D,j} = 0$, this requires a slightly more involved definition of the function J, but otherwise the argument is analogous to the one for the case analyzed here. Set $p_{D,j}^* = p_D^{fb}$ for all $j \geq 1$, $q_A^* = 0$, and $q_{D,j}^* = f$ for all $j \geq 0$. Let $\xi^* = J(p_A^*)$ and set $c_A^T = \theta_A\xi^*$ and $c_{D,j}^T = \theta_D\xi^*$. Set the sequence $\left\{n_A^*, \left\{n_{D,j}^*\right\}\right\}$ according to (56)-(58). Finally, the values for the entrepreneur's consumption are set as

$$\begin{aligned} c_A^{T,e*} &= (1-\beta(1-\delta)) a_0 + (1-p_A) n_A, \\ c_{D,0}^{T,e*} &= (1-p_{D,0}) n_{D,0} - f(n_{D,0} - n_A), \\ c_{D,j}^{T,e*} &= (1-p_{D,j}) n_{D,j} - f(n_{D,j} - n_{D,j-1}) \text{ for } j \geq 1. \end{aligned}$$

Given the construction of the sequence $\left\{n_A^*, \left\{n_{D,j}^*\right\}\right\}$, entrepreneur's consumption is always non-negative.

Having defined a candidate optimal allocation, we can define the corresponding sequences for y_A^*, $\left\{y_{D,j}^*\right\}$ and z_A^*, $\left\{z_{D,j}^*\right\}$, and show that we have found an optimum for the relaxed problem defined in 6.4. To do so, we need to find Lagrange multipliers $\lambda^*, \nu_A^*, \{\nu_{D,j}^*\}$, and μ^* such that conditions (48)-(54) are satisfied. Set $\lambda^* = 1/\xi^*$. Notice that the condition $L(p_A^*) = 0$ can be re-arranged to give

$$\lambda^* p_A^* = \theta_A u'(1 - n_A^*). \tag{60}$$

Moreover, by construction

$$n^*_{D,j} \leq 1 - \xi^* \theta_D / p^{fb}_D \text{ for all } j,$$

with equality for j greater or equal than some J^*. This implies that

$$\lambda^* p^{fb}_D \leq \theta_A u' \left(1 - n^*_{D,j}\right) \text{ for all } j,$$

with equality for $j \geq J^*$. Then we can set $\nu^*_A = \lambda$ and

$$\begin{aligned} \nu^*_{D,0} &= \nu_A + \frac{1}{f}\left(\theta_D u' \left(1 - n^*_{D,0}\right) - \lambda^* p^{fb}_D\right), \\ \nu^*_{D,j} &= \nu_{D,j-1} + \frac{1}{f}\left(\theta_D u' \left(1 - n^*_{D,j}\right) - \lambda^* p^{fb}_D\right). \end{aligned}$$

By construction $\nu^*_{D,j}$ will be constant for $j \geq J^*$ and we can set $\mu^* = \nu^*_{D,J^*}$. This confirms that (51) is satisfied for all j and we can check that $c^{T,e*}_{D,j} > 0$ only for $j \geq J^*$, i.e., when $\nu^*_{D,j} = \mu^*$.

Furthermore, we can check that the proposed allocation satisfies the consumer's budget constraint and the entrepreneur's participation constraint. The consumer's budget constraint can be rewritten as

$$c^{T*}_A + \sum_{j=0}^{\infty} \beta^j c^{T*}_{D,j} \leq p^*_A n^*_A + \delta\beta \sum_{j=0}^{\infty} \beta^j p^*_{D,j} n^*_{D,j},$$

the construction of the functions L and J (in particular equation (55)) guarantees that this condition holds as an equality. Some lengthy but straightforward algebra, using the flow of funds constraints, shows that

$$\frac{1}{1-\beta(1-\delta)}\left(c^{T,e*}_A + \delta\beta \sum_{j=0}^{\infty} \beta^j c^{T,e*}_{D,j}\right) = \frac{\beta\delta f}{p^*_A - 1} \prod_{j=0}^{\infty} \frac{\beta f}{f - \left(1 - p^*_{D,j}\right)} a_0.$$

Given the prices p^*_A and $\left\{p^*_{D,j}\right\}$, the right-hand side of this equation is equal to $\phi^{ce}_A a_0$, which is equal to the entrepreneur's expected utility U in the competitive equilibrium, since

$$U = \frac{\beta\delta f}{p^{ce}_A - 1} \prod_{j=0}^{\infty} \frac{\beta f}{f - \left(1 - p^{ce}_{D,j}\right)} a_0 = \phi^{ce}_A a_0. \tag{61}$$

This completes the argument that the candidate allocation solves the relaxed planning problem. It remains to show that this allocation is feasible. To do so, we first derive values for the ϕ^*_A and $\phi^*_{D,j}$. We set $\phi^*_A = \phi^{ce}_A$,

$$\phi^*_{D,0} = \prod_{j=0}^{\infty} \frac{\beta f}{f - \left(1 - p^*_{D,j}\right)},$$

and $\phi^*_{D,j} = 1$ for all $j \geq 1$. These, can be used to check that entrepreneur's behavior is optimal, i.e. that (10) and (11) are satisfied. To check these conditions notice that $\phi^*_A \geq \phi^*_{D,0} \geq \phi^*_{D,1}$ and $\phi^*_{D,j} = 1$ for all j, while $c^{T,e*}_A = c^{T,e*}_{D,0} = 0$ and $a^*_{D,j} = 0$ for all j. Finally, the tax rates are set as follows: $\tau^{T*}_A = \tau^{T*}_{D,j} = 0$ for all j and τ^{N*}_A and $\left\{\tau^{N*}_{D,j}\right\}$ are such that

$$\theta_A u' \left(1 - n^*_A\right) = \lambda^* p^*_A \left(1 + \tau^*_A\right), \tag{62}$$

$$\theta_D u' \left(1 - n^*_{D,j}\right) = \lambda^* p^*_{D,j} \left(1 + \tau^*_{D,j}\right). \tag{63}$$

Let us discuss briefly the cases where $L\left(1+\delta\beta f/\phi_A^{ce}\right)>0$ and $L\left(p_A^{fb}\right)<0$. In the first case, we have that condition (60) now holds as an inequality, and we have $\nu_A^*>\lambda^*$. The rest of the construction is analogous to the one derived above. In the second case, we make use of the function $\tilde{J}$ to find the value of ξ^*. In particular, define the function

$$\tilde{L}\left(n_A\right)\equiv 1-\frac{\theta_A\tilde{J}\left(n_A\right)}{p_A^{fb}}-n_A,$$

and find an $n_A^*\in[0,\left(1-\left(1-\delta\right)\beta\right)a_0/(p_A^{fb}-1)]$ such that $\tilde{L}\left(n_A^*\right)=0$ (notice that $\tilde{L}\left(\left(1-\left(1-\delta\right)\beta\right)a_0/(p_A^{fb}-1)\right)=L\left(p_A^{fb}\right)<0$ and $\tilde{L}\left(0\right)>0$). Then, set $\xi^*=\tilde{J}\left(n_A^*\right)$ and $\lambda^*=1/\xi^*$. In this case, (60) holds as an equality. but we have $n_A^*<\left(1-\left(1-\delta\right)\beta\right)a_0/(p_A^{fb}-1)$, so the entrepreneurs have positive financial savings when they enter phase D,

$$a_{D,0}^*=\left(1-\left(1-\delta\right)\beta\right)a_0-(p_A^{fb}-1)n_A^*.$$

This is consistent with feasibility, given that $p_A=p_A^{fb}$, so that $\phi_A^*=\phi_{D,0}^*$ (which implies that (11) is satisfied with $a_{D,0}^*>0$). The rest of the proof proceeds as in the baseline case.

Step 3. That $p_A^*\leq p_A^{fb}$ and $p_{D,j}^*\leq p_D^{fb}$ follows immediately from the construction of the optimal allocations. We want to show that $p_A^*\leq p_A^{ce}$. Consider first the case where $p_A^*=1+\delta\beta f/\phi_A^{ce}$. In this case, it follows from (61) and $p_{D,j}^{ce}\leq p_D^{fb}$ that $\left(p_A^{ce}-1\right)/\left(\delta\beta f\right)\phi_A^{ce}=\prod_{j=0}^{\infty}\beta f/\left(f-\left(1-p_{D,j}^{ce}\right)\right)\geq 1$. This implies that $p_A^{ce}\geq p_A^*$. Next, consider the case where $p_A\geq 1+\delta\beta f/\phi_A^{ce}$. In this case, we have, by construction $L\left(p_A^*\right)\leq 0$, which implies

$$n_A^{fb}=1-\theta\frac{\kappa^{fb}}{p_A^{fb}}\leq 1-\frac{\theta_A J\left(p_A^*\right)}{p_A^{fb}}\leq\frac{1}{p_A^*-1}\left(1-\left(1-\delta\right)\beta\right)a_0, \tag{64}$$

where the first inequality follows because it is possible to show that $J\left(p_A^*\right)\leq\kappa^{fb}$. If $p_A^{ce}=p_A^{fb}$ then it immediately follows that $p_A^{ce}\geq p_A^*$. Therefore, consider the case $p_A^{ce}<p_A^{fb}$, where $n_A^{ce}=\left(1-\left(1-\delta\right)\beta\right)a_0/\left(p_A-1\right)$ (recall the construction of the equilibrium in Proposition 2). The assumption $n_A^{ce}\leq n_A^{fb}$ and inequality (64) imply

$$\frac{\left(1-\left(1-\delta\right)\beta\right)a_0}{p_A-1}\leq\frac{\left(1-\left(1-\delta\right)\beta\right)a_0}{p_A^*-1},$$

giving the desired inequality.

Let us derive the optimal tax τ_A when $p_{D,0}^*<p_D^{fb}$. By construction, $p_{D,0}^*<p_D^{fb}$ implies $p_A^*>1+\delta\beta f/\phi_A^{ce}$. Also by construction, whenever $p_A^*>1+\delta\beta f/\phi_A^{ce}$ the following condition holds as an equality $1-\theta_A\xi^*/p_A^{fb}=n_A^*$, which, together with (62), implies that $p_A^*\left(1+\tau_A^*\right)=p_A^{fb}$.

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