Persistent spin current in mesoscopic ferrimagnetic spin ring

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(Received 16 July 2005; revised manuscript received 16 September 2005; published 8 November 2005)

Using a semiclassical approach, we study the persistent magnetization current of a mesoscopic ferrimagnetic ring in a nonuniform magnetic field. At zero temperature, there exists persistent spin current because of the quantum fluctuation of magnons, similar to the case of an antiferromagnetic spin ring. At a low temperature, the current shows activation behavior because of the field-induced gap. At a higher temperature, the magnitude of the spin current is proportional to temperature *T*, similar to the reported result of a ferromagnetic spin ring.

DOI: 10.1103/PhysRevB.72.172405 PACS number(s): 75.10.Jm, 75.10.Pq, 75.30.Ds, 73.23.Ra

Persistent charge current in a mesoscopic metal ring was predicted[1] and observed[2] a decade ago. In such a ring threaded by a magnetic flux, if the phase coherence length of electrons is larger than the size of the ring, then the electrons can pick up an Aharonov-Bohm phase after circling the ring once. Such a phase lag (or advance) would lead to a persistent current, which is a periodic function of the threaded magnetic flux,[3] and can be detected via the magnetic response of the (isolated) ring. This phase difference depends only on the magnetic flux passing through the ring, but not on the ring's geometric shape, and can be seen as a particular example of the (topological) Berry phase.[4] It has been proposed that another Berry phase can appear for an electron moving around the metal ring that is subject to a textured magnetic field (or magnetization).[5] This (geometrical) Berry phase, which depends upon the solid angle associated with the textured magnetic field, can lead to persistent charge and spin currents.[5] A similar topological Berry phase appears due to the spin-orbit interaction in one-dimensional rings,[6,7] which is a manifestation of the Aharonov-Casher effect.[8] More studies on the persistent current related to the Aharonov-Casher effect can be found in Refs. 9–11.

Understanding the fundamental properties and behaviors of spin current is very crucial for the progress of spintronics.[12] Among these investigations, spin transport in pure spin systems plays a special role since there is no complication from charge degrees of freedom. In a recent paper, using a semiclassical spin wave analysis, Schütz *et al.* predicted the existence of persistent spin current in a mesoscopic ferromagnetic (FM) spin ring in a *nonuniform* magnetic field.[13] The FM spin ring being considered is a charge insulator with Heisenberg spin interaction, and the spin current is carried by magnon excitations. Similar to the case of charge transport in a metal ring subject to a textured magnetic field,[5] the magnon in a mesoscopic FM spin ring acquires a geometric phase from the (nonuniform) spin texture of the classical ground state. The persistent current is found to be zero at temperature $T=0$ and proportional to T when k_BT is larger than the field-induced energy gap of the magnons.

A similar method has been applied to an antiferromagnetic (AFM) spin ring with a Haldane gap.[14] As compared with the FM case, there are some subtleties in using the semiclassical method in the AFM case. Due to the problem of infrared-diverging magnetization, the spin-wave approach is not valid for AFM spin chains with half-integer spins.[14] That is the reason why only the integer-spin cases are considered in Ref. 14. Nonetheless, in the integer-spin case, an additional staggered field in the direction of the classical magnetization vectors still has to be introduced. Its value needs to be determined self-consistently before quantitative predictions can be made. The authors of Ref. 14 find that, unlike the case of the FM spin ring, the persistent spin current in an AFM spin ring can be nonzero at $T=0$ due to quantum fluctuations. When the spin correlation length is much longer than the size of the ring, the magnitude of the spin current exhibits sawtooth variation with respect to the geometric phase, similar to the behavior of persistent charge current in a metal ring. Recently, the investigation has been extended to an anisotropic FM spin ring,[15] a spin-1/2 AFM spin ring,[16,17] and an anisotropic AFM spin ring.[18]

In this report, we study the persistent spin current in a ferrimagnetic (FIM) spin ring with alternating spins S^A and S^B under a textured magnetic field. Contrary to the AFM case, the problem of infrared-diverging magnetization does not exist in the present FIM case, no matter whether the

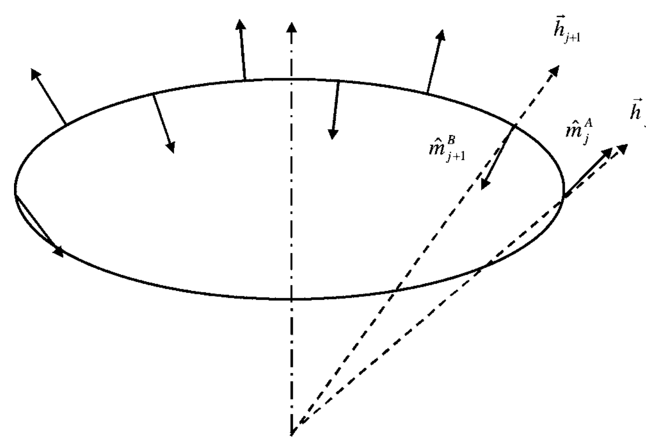

FIG. 1. Classical spin configuration of a FIM spin ring in a crown-shaped magnetic field.

constituent spins are integer or half-integer.[19–22] Thus the self-consistently determined staggered field needs not be introduced, and physical quantities can be calculated directly as long as system parameters are known. We find that the FIM spin ring can have either FM or AFM characteristics. For example, a quantity proportional to $|S^A - S^B|$ plays a role similar to the Haldane gap in the AFM spin ring. Moreover, a nonzero spin current exists at $T=0$, again similar to the case of the AFM spin ring.[14] On the other hand, when the thermal energy is higher than the field-induced gap, the magnitude of the spin current is proportional to temperature T, similar to the case in the FM spin ring.[13]

The Hamiltonian of the ferrimagnetic Heisenberg spin ring in a nonuniform magnetic field $\vec{h}_j \equiv g\mu_B \vec{B}(\vec{r}_j)$ is

$$H = J\sum_{j\in A\cup B} \vec{S}_j\cdot\vec{S}_{j+1} - \sum_{j_1\in A, j_2\in B} (\vec{h}_{j_1}\cdot\vec{S}^A_{j_1} + \vec{h}_{j_2}\cdot\vec{S}^B_{j_2}), \quad (1)$$

where $J>0$, and the index j refers to one of the alternating j_1, j_2 sites, which are located at sublattice A and sublattice B, respectively. The lattice spacing $a=L/N$, where L is the length of the ring and N (an even integer) is the number of lattice sites. On the classical level, $\vec{S}_j$ are replaced by classical vectors $S\hat{m}_j$. The classical ground state $\{\hat{m}_j\}$ can be determined from angular variations with respect to each $\hat{m}_j$, which give

$$JS^B(\hat{m}_{j_2-1} + \hat{m}_{j_2}) - \vec{h}_{j_1} + \lambda^A_{j_1}\hat{m}_{j_1} = 0,$$

$$JS^A(\hat{m}_{j_1} + \hat{m}_{j_1+1}) - \vec{h}_{j_2} + \lambda^B_{j_2}\hat{m}_{j_2} = 0, \quad (2)$$

where λ_j are Lagrange multipliers. It shows that the magnetization aligns parallel to the sum of the external and exchange field, as expected. If the Zeeman energy is much smaller than the exchange energy between spins, then $\hat{m}^A_j$ and $\hat{m}^B_{j+1}$ would be nearly antiparallel to each other, as shown in Fig. 1. Moreover, due to nonzero magnetization in the present FIM model, $\hat{m}^A_j$ would lie nearly along the direction of $\vec{h}_j$, instead of nearly perpendicular to $\vec{h}_j$ as in the AFM case[14] (we take $S^A > S^B$ in this report).

When quantum fluctuations are considered, each spin vector operator is decomposed using a local orthogonal triad, in which $\hat{m}_j$ is one of the basis vector. The other two basis vectors can rotate around $\hat{m}_j$, but the spin current can be shown to be independent of such a gauge freedom. To simplify the Hamiltonian, we choose local triads that are related to adjacent ones via a rule of parallel transport.[13,14] Following the same approach introduced by Schütz *et al.*,[13,14] we focus on systems with large spins (which could be integer or half-integer spins), introduce the Holstein-Primakoff (HP) transformation,[23] and neglect the interactions between spin waves. The Hamiltonian finally becomes

$$H = H_c + \sum_k (2JS^B + h^A_q)a^\dagger_{k+q}a_k + (2JS^A + h^B_q)b^\dagger_{k+q}b_k$$
$$- 2J\sqrt{S^AS^B}\sum_k (a_kb_k + a^\dagger_kb^\dagger_k)\cos(ka), \quad (3)$$

where H_c is the classical Hamiltonian, a_k and b_k are HP bosons, and h^A_q and h^B_q are Fourier components of $h^A_{j_1} \equiv \vec{h}_{j_1}\cdot\hat{m}_{j_1}$ and $h^B_{j_2} \equiv \vec{h}_{j_2}\cdot\hat{m}_{j_2}$. We have assumed that the applied magnetic field (as well as $\{\hat{m}_j\}$) has only one Fourier component with momentum q to simplify the expression. A crown-shaped magnetic field with azimuthal symmetry has the $q=0$ component only. The wave vectors take discrete values,

$$k_n = \frac{2\pi}{L}\left(n + \frac{\Omega}{2\pi}\right), \quad n = 0,1,2,\ldots,N/2-1, \quad (4)$$

where Ω is the Berry phase acquired by the magnons after circling around the ring once.[13,14] The Berry phase is the holonomy angle of the parallel transport [see the discussion above Eq. (3)] and equals the solid angle extended by the classical spin texture $\{\hat{m}_j\}$.[24]

For convenience, we consider the crown-shaped magnetic field below. For a large FIM spin ring in a *weak* magnetic field, we also have $h^B_0 \cong -h^A_0 \equiv -h_0$ (see Fig. 1) with h_0 being positive. With the help of the Bogoliubov transformation, the Hamiltonian is diagonalized as

$$H = \sum_k \left[\epsilon^-_k\left(\alpha^\dagger_k\alpha_k + \frac{1}{2}\right) + \epsilon^+_k\left(\beta^\dagger_k\beta_k + \frac{1}{2}\right)\right] - \frac{NJS^A}{2}(1+\gamma), \quad (5)$$

where α_k and β_k are magnon operators, which are linear combinations of the HP bosons. The two energy branches are

$$\epsilon^\pm_k = JS^A[\sqrt{(1-\gamma)^2 + 4\gamma\sin^2(ka)} \pm (1-\gamma)] \mp h_0, \quad (6)$$

with $\gamma = S^B/S^A < 1$. As mentioned before, in contrast to the AFM case,[14] the staggered field needs not be introduced in the present FIM case. Thus physical quantities can be calculated directly as long as the system parameters are known.

Similar to the case of a FIM spin chain under a *uniform* magnetic field,[25] the magnons with energy ϵ^-_k (ϵ^+_k) in the present case correspond to the ferromagnetic (antiferromagnetic) excitations. The energy gaps of these two branches are $\epsilon^-_0 = h_0$ and $\epsilon^+_0 = 2JS^A(1-\gamma) - h_0$, respectively. That is, a gap is induced by the applied field for the ferromagnetic excitations, while the gap of the antiferromagnetic excitations is reduced by the applied field. In the absence of external magnetic field, the ferromagnetic branch ϵ^-_k becomes gapless with quadratic k dispersion at small k, which corresponds to the Goldstone mode due to the spontaneously broken rotational symmetry. Calculations using the quantum Monte Carlo method yield nearly the same curve for ϵ^-_k, but ϵ^+_k is separated from ϵ^-_k with a larger (k-independent) gap.[19,22] Such a discrepancy is reduced when the spins are larger and the semiclassical formalism works better.[22,26]

It can be explicitly shown that the longitudinal (gauge-invariant) spin current

$$I_s \equiv \langle \hat{m}_j \cdot \vec{I}_{j\to j+1}\rangle = -\frac{\partial F(\Omega)}{\partial\Omega}, \quad (7)$$

where $\vec{I}_{j\to j+1} \equiv J\vec{S}^\perp_j \times \vec{S}^\perp_{j+1}$, $\vec{S}^\perp_j$ is the spin component that is perpendicular to the local magnetization $\hat{m}_j$, and F is the free energy of the spin system.[13,14,27] This is similar to the rela-

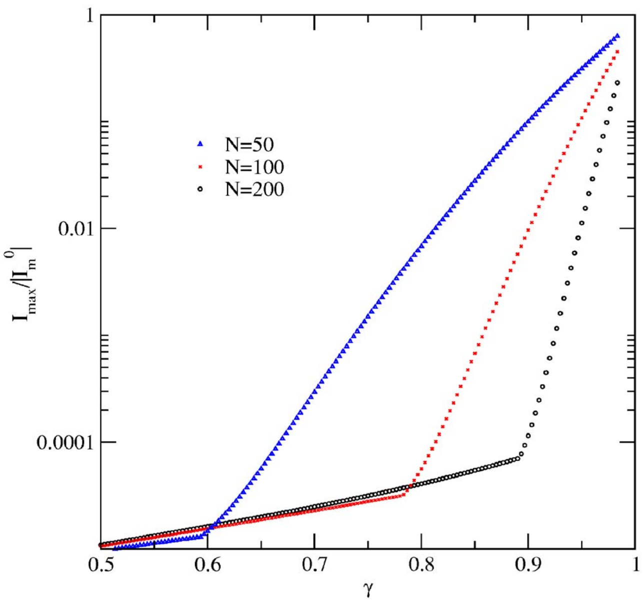

FIG. 2. (Color online) Amplitude of the magnetization current at zero temperature as a function of γ, plotted for three different ring sizes.

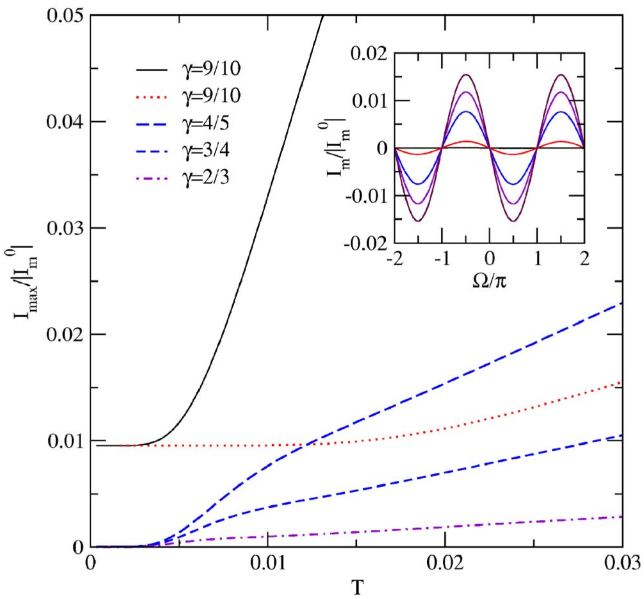

FIG. 3. (Color online) Amplitude of the magnetization current as a function of temperature T. Solid (dotted) line corresponds to a system ($N=100$, $\gamma=0.9$) in a magnetic field $h_0/2JS^A=0.01$ (0.05). Other curves with different γ all have $h_0/2JS^A=0.01$. Inset: The variation of the magnetization current with respect to the change of Ω and T ($N=100$, $\gamma=0.8$). The temperatures for the curves with the smallest amplitude (indiscernible from the horizontal axis) to the largest amplitude are $T/JS^A=0$, 0.005, 0.010, 0.015, 0.020, and 0.025.

tion for persistent charge current in a normal metal ring, but with Ω replacing magnetic flux ϕ. From this relation, we obtain the magnetization current,

$$I_m = \frac{g\mu_B}{\hbar} I_s = -\frac{g\mu_B}{L} \sum_{k,\alpha=\pm} v_k^\alpha \left(n_k^\alpha + \frac{1}{2} \right), \tag{8}$$

where

$$v_k^\alpha = \frac{1}{\hbar}\frac{\partial \epsilon_k^\alpha}{\partial k} = \frac{2JS^A a}{\hbar} \frac{\gamma \sin(2ka)}{\sqrt{(1-\gamma)^2 + 4\gamma \sin^2(ka)}} \tag{9}$$

are the velocities of the magnons, and $n_k^\alpha = 1/[\exp(\epsilon_k^\alpha/T) - 1]$ are the Bose occupation numbers.

For the FIM case, the magnetization current at $T=0$ is nonzero even for vanishing magnon numbers [see Eq. (8)], similar to the AFM case,

$$I_m = I_m^0 \sum_k \frac{\gamma \sin(2ka)}{\sqrt{(1-\gamma)^2 + 4\gamma \sin^2(ka)}}, \tag{10}$$

where $I_m^0 \equiv -(2g\mu_B/\hbar)(JS^A/N)$. The magnon velocity within the summation is a periodic function of k. Therefore, after summing over the first Brillouin zone, the current is zero if the k points are distributed symmetrically ($\Omega=\pi$). For other values of Ω, the summation is nonzero but the magnitude of the magnetization current decreases rapidly as $1-\gamma$ becomes larger. Comparing with the AFM case,[14] it can be seen that $\Delta \equiv \sqrt{(1-\gamma)^2/4\gamma}$ plays a role similar to the Haldane gap, and its inverse determines the scale of the spin correlation length ξ. Therefore, when γ is small enough such that $\Delta \gg 2\pi/N$, the spin correlation length $\xi \ll L$; while if $\gamma \simeq 1$, such that $\Delta \ll 2\pi/N$, we have $\xi \gg L$. Therefore, by varying the ratio of the two different spins, qualitatively different regimes, $\xi \gg L$ and $\xi \ll L$, can be reached. In Fig. 2, the maximum amplitude of the magnetization current I_{max} is plotted as a function of γ. The functional form of $I_{max}(\gamma)$ shows a very clear crossover between these two different regimes.

Thermal energy could excite magnons and generate a larger magnetization current. Typical influence of the temperature on the magnitude of the magnetization current can be seen in the inset of Fig. 3. We have also studied the dependence of I_{max} on the parameters T, h_0, and γ. From the magnon dispersion relations in Eq. (6), we expect activation behavior for $I_{max}(T)$ at low temperature $T < \epsilon_0^- = h_0$. When the thermal energy is larger than the field-induced energy gap h_0, $I_{max}(T)$ should be proportional to T, similar to the behavior of the persistent spin current in a FM spin ring.[13] These behaviors can be seen in Fig. 3. For the upper two curves with $\gamma=0.9$, there is a significant amount of spin current at zero temperature and the activation behavior is implicit. Also, the persistent magnetization current at very low T is independent of the value of h_0 [see Eq. (10)].

In conclusion, we extend the work of Schütz *et al.*[13,14] and studied the persistent spin current in a FIM spin ring. At $T=0$, the functional form of the magnetization current $I_m(\gamma)$ shows distinctive behaviors above/below a threshold value of γ (Fig. 2), which depends on the size of the ring. When thermal excitation can overcome the field-induced energy gap $\epsilon_0^- = h_0$, the magnitude of the spin current grows linearly with temperature T (Fig. 3).

As in the case of the FM spin ring, the induced electric voltage would be of the order of nV, which poses a stringent experimental challenge. Recently, several types of FIM spin-chain compounds have been synthesized.[28,29] In general, a

FIM ring with a larger ratio of γ tends to have larger spin current (see Fig. 3). For a FIM spin chain composed of transition metal elements, the largest possible on-site spin is 5/2 from a half-filled *d* shell. Therefore, the largest possible γ smaller than 1 is $2/(5/2)=0.8$. To obtain a FIM spin chain with $\gamma>0.8$, one might need to introduce rare-earth elements[30] or fabricate a ring composed of magnetic molecules with large spins.[31]

On the theoretical side, even though a lot of progress related to the persistent spin current has been achieved,[13–18] many questions still remain open, such as the generalizations to quasi-one-dimensional spin rings with multiple chains or spin rings with itinerant electrons. The realistic effect of disorder, interaction, or contact leads also remains to be explored in the future.

J. N. W. and M. C. C. acknowledge the support from the National Science Council of Taiwan under Contract Nos. NSC 93-2112-M-003-009 and NSC 94-2119-M-002-001. M. F. Y. acknowledges the support by the National Science Council of Taiwan under Contract No. NSC 93-2112-M-029-006.

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