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ABSTRACT

This article first gives the authors' perspectives on how the field of plasma-based acceleration (PBA) developed and how the current experiments, theory, and simulations are motivated by long term applications of PBA to a future linear collider and an x-ray free electron laser. We then focus on some early applications that will likely emerge from PBA research such as electron beam radiotherapy, directional but incoherent x-ray beams for science and technology, near single cycle continuously tunable infrared pulses for spectroscopy, and non-perturbative quantum electrodynamics enabled by PBA electron beams. In our opinion, these near term applications could be developed within the next decade with a concerted effort by the community.

I. INTRODUCTION

High-energy particle accelerators based on radio frequency (RF) technology, used in synchrotron light sources, x-ray free electron lasers (X-FEL), and colliders by tens of thousands of scientists and engineers, have become too gargantuan and expensive. Scientists are, therefore, searching for a new paradigm for making these critical instruments of discovery[1] more compact and affordable. It would seem that the footprint of an accelerator can be made considerably smaller by increasing the accelerating gradient by two or more orders of magnitude and it could be made more affordable by increasing its wall plug efficiency.[2] The allure of plasma-based (wakefield) acceleration (PBA) schemes is that it has the potential to deliver both the high gradient and the efficiency.[3] After nearly four decades of research, while even the most ardent critics of the field have become believers that PBAs can deliver on the needed increase in the "fully loaded" average accelerating gradient[4] and probably reach >10% wall plug efficiency needed for future colliders, plasma accelerators continue to face many challenges. In this paper, we give our perspective on the status of the field followed by applications that are emerging from this research.

While PBA is an entirely new paradigm for constructing high-energy accelerators, many basic and technological problems still remain to be solved. For instance, the figure of merit for a particle collider is the luminosity L. A simplified expression that neglects the beam–beam disruption effects gives $L\ (\mathrm{cm^{-2}\ s^{-1}}) = f_{rep}N^2/(4\pi\sigma_x\sigma_y)$. Here, f_{rep} is the repetition rate of collisions, N is the number of particles in the colliding bunch, and σ_x and σ_y are the r.m.s. beam sizes at the collision point in the two transverse directions, respectively. The desired L for a 1 TeV center of mass (CM) electron–positron (e−e+) linear collider (LC) is $10^{34}\ \mathrm{cm^{-2}\ s^{-1}}$ within 1% of the center of mass (CM) energy. Achieving this luminosity would require the colliding beams to have an average power of 20 MW, 10^{10} particles per bunch at a repetition rate of 10 kHz and $\sigma_x\sigma_y < 500\ \mathrm{nm}^2$. Even though there has been spectacular experimental progress in PBA research, the beam parameters achieved todate are at least one or, in most cases, many orders of magnitude away from those needed for just the electron arm of a future e^-e^+ collider.

Future colliders will likely be electron–positron (e−e+) linear colliders (LC): first, e−e+ instead of proton-antiproton (P^+P^-) because unlike protons e^- and e^+ are not composite particles and their collisions are, therefore, "clean" and, second, linear instead of circular because electrons being far lighter than protons will radiate away far more of their energy by emitting synchrotron radiation as they are bent around in a circular path of a given radius than P^+ or P^-.

While substantial progress on the electron arm of a conceptual plasma-based linear collider (PB-LC) has been made,[5] the positron arm situation is still uncertain. While some impressive experimental work has been done to demonstrate high-gradient and high-efficiency acceleration of positrons in a plasma wake generated by a single positron beam,[6] the concepts developed for multi-stage acceleration of an electron bunch do not work for positrons. The work on positron acceleration in electron beam produced wakes is in its infancy. A decade of concerted basic science research is likely needed to bring positrons at the same point as where we are with electrons.

Accelerator-based synchrotron and free-electron-laser facilities[7] have enabled scientists to image sub-cellular structures with spatial (nanometer) resolution better than on a molecular scale. PBA has the promise of generating extremely high brightness, micron length gigaelectronvolt energy beams. Therefore, a second, long term application of PBA is the realization of a compact free-electron laser[8–10] in the x-ray domain (X-FEL), the so-called fifth-generation light source that would considerably reduce the size and cost of these machines. The key figure of merit here is the beam brightness is defined as $B = I/\varepsilon_n^2$ where I is the peak current and ε_n is the normalized transverse emittance of the bunch. A fifth generation light source (e.g., a fully coherent hard x-ray FEL) will require electron beam brightness that is orders of magnitude greater than what can be achieved today from a PBA. On the other hand, plasma accelerators are getting reliable enough and are close to generating the charge, energy and energy spread, and the transverse emittance needed to demonstrate a working vacuum ultraviolet (VUV)-extreme ultraviolet (XUV) FEL in the near future.[11]

So why has this field caught on? The answer is simple. Plasma-based accelerators have arguably been the most successful discovery science-area of plasma-physics, in general, and high-energy density science, in particular, in the past two decades. This, in turn, has attracted high quality students to the field, and funding for research and facilities has followed. The linear collider and a fifth generation light source applications are long-term scientific/engineering grand challenges of the 21st century. So it is worthwhile thinking about some novel applications that will be enabled by electron beams produced by PBA in the near term. In this article, we will discuss what these applications are likely to be and how close the community is to realizing them.

II. PLASMA-BASED ACCELERATION OF CHARGED PARTICLES: LASER AND PARTICLE BEAM DRIVER

The development of PBA is a tale of the synergy between theory, simulation, and experiment. It is also tale of the synergy between separate physics disciplines, and between basic and applied science. At its core PBA combines plasma physics, accelerator technology, ultra-fast laser science, relativistic beam physics, and nonlinear optics. Although the history of particle acceleration by collective fields began with independent proposals from Vekslar[12] and Budker[13] to use fields of a high current, relativistic electron beam to accelerate ions, it was not until 1979 when Tajima and Dawson of UCLA proposed using a relativistic plasma wave or a wake driven by a short laser to accelerate electrons at ultra-high gradients.[14] Such waves could be excited by either resonant Raman scattering (plasma beat wave accelerator, PBWA),[15] stimulated Raman scattering instability[16] (self-modulated laser-wakefield accelerator, SMLWFA), or impulse Raman scattering (laser wakefield accelerator, LWFA[14,17]). In this sense, all laser-driven plasma based accelerators are "Raman" accelerators that rely on relativistic wave-particle interactions to increase the energy of the charged particles. Of these three methods, the LWFA is the easiest to understand and yet was not pursued for the first two decades because short and intense enough [<50 fs (FWHM), $a_0 > 1$] laser pulses needed to excite ultra-high gradient wakes in dense plasmas only became routinely available in the late 1990s. Here $a_0 = 0.85 \times 10^{-19}\sqrt{I\left(\frac{W}{cm^2}\right)}\lambda\,(\mu m)$ is the normalized vector potential of the laser with intensity I and wavelength λ. Furthermore, 100 TW class lasers necessary to properly make a parameter scaling of LWFA have been around for just over a decade. A few years later, Chen and Dawson *et al.* also of UCLA suggested that instead of a laser pulse one could use a high current, tightly focused particle bunch to excite a wake—this scheme came to be known as plasma wakefield accelerator (PWFA).[18] Coincidentally, the electron beams needed to excite high-gradient (>10 GeV/m) wakes in a plasma were also not available until the mid-2000s.

In both cases, the drive pulse excites a wake, while a second appropriately placed charged particle trailing bunch (sometimes called the witness bunch) experiences an accelerating electric field and gains energy from the wake. The energy extraction efficiency from the wake to the trailing bunch can be similar in both cases as can other physical effects such as emittance growth due to energy spread, transverse misalignment, hosing instability, plasma ion motion, etc. The ultimate energy gain in a plasma accelerator is limited by betatron radiation loss where particles that are off the propagation axis oscillate in the wake because of the transverse focusing force of the wake.

There are many similarities between LWFA and PWFA but there are some critical differences as well. The interested reader is referred to several excellent review articles on the topic.[19–22]

III. A HISTORICAL PERSPECTIVE ON PBA EXPERIMENTS

For a history of how the experimental PBA field started, the reader is referred to Refs. 23 and 24. Here we summarize the key advances. The first decade of PBA research was focused on using a two-frequency laser beat wave to resonantly excite a quasi-linear ($n_1/n_p < 1$) plasma wave. Here n_1 is the density perturbation associated with the relativistic (phase velocity close to c) plasma wave excited in a plasma density of n_p. Pre-accelerated electrons were externally injected in this wave and accelerated from 2 MeV to eventually 50 MeV in just over 1 cm or at an average gradient of 5 GeV/m.[25–30] This work is significant because these experiments heralded the dawn of relativistic wave-particle interactions and overcame tremendous skepticism in both the beam physics and plasma physics communities that relativistic plasma waves could be excited in a centimeter-scale plasma and be used to accelerate electrons at greater than several gigaelectronvolt per meter gradients.

Soon after the invention of the chirped pulse amplification technique, ten terawatt (10 TW), picosecond (ps) class lasers became available. These lasers were not short enough to excite a strong wakefield in a $10^{19}\,cm^{-3}$ plasma to self-trap plasma electrons, so the SMLWFA regime was explored using high density (lower phase velocity) plasmas. These experiments confirmed the acceleration of self-injected electrons[31] as well as the generation of a high charge beam upon wave breaking of the wake.[32]

By middle of the second decade (mid 1990s), there was sufficient experimental proof of ultra-high gradient electron acceleration by relativistic plasma waves. Fortunately, for this field, two major opportunities presented themselves that changed the landscape of the PBA field.

The first was the arrival of titanium-sapphire laser that provided 10 TW-class but <50 femtosecond (50 fs) laser pulses that were small enough to fit in a university-scale laboratory. In the early 2000s, simulations showed that it was possible for 100 TW class lasers to trap electrons and produce quasi-mono-energetic beams in moderate to high density plasmas without external guiding in what is now referred to as the bubble regime.[33] Very soon thereafter, three groups in three different countries demonstrated experimentally that 10–20 TW lasers could generate that quasi-monoenergetic electron beams with energies on the order of 100 MeV using wakes generated in either self-ionized plasmas or a preformed plasma channel.[34–36] Each of these experimental results was supported by simulation result, and there were also additional supporting simulation results that were published at the same time.[37] These breakthrough experimental results lowered the "barrier to entry" eventually allowing several dozen groups to enter and contribute to the field.

Around the same time as the arrival of table-top multi-terawatt lasers, considerable effort was devoted by the laser-plasma community to develop longer plasma channels using laser driven shocks,[38] discharge capillaries,[39] and ablative capillary discharges.[40] Such channels increased the length of LWFA experiments from mm scales to several centimeters and made possible milestone experiments at the lower plasma densities needed to obtain higher energy gains. At the same time, simulations showed that it was possible to self-guide 100 TW-class lasers over many vacuum Rayleigh lengths in the so-called blowout/bubble regimes.[41] This synergy between experiments and simulations, which continues to this day, has been indispensable to the rapid progress of the plasma-based accelerator field.

As for beam-driven plasma wakefield acceleration, a proof of principle experiment had already been done at the argonne wakefield acceleration (AWA) facility[42,43] to map out the wakefield structure in plasma excited by a relatively low energy electron beam. In that experiment, the energy changes to a witness beam were measured as the delay between the drive and the witness beam was varied. The acceleration gradients were modest because the peak current of the drive electron beam and plasma density were low. However, a breakthrough came in this method of plasma acceleration with the approval of a "1 GeV in 1 m PWFA experiment (E157)" on the Stanford Linear Accelerator Center's (SLAC) Final Focus Test Beam (FFTB) facility line[44] that provided both high energy (20–40+ GeV) electron and positron bunches to the experimenters. Soon thereafter SLAC physicists compressed the beams from 4 ps to 50 fs that allowed particle beam produced wakes to be excited in the high gradient blow out regime[45–48] in a meter scale plasma. The experiments at SLAC on PWFA have permitted the exploration of fully blown out plasma wake cavities for electron acceleration. The experiments have demonstrated up to 40+ GeV energy gain in less than 1 m of plasma,[49] using electrons in the tail of the drive bunch to gain energy from the wake. This experiment attracted the attention of high-energy physicists because it produced energies of interest to them at gradients almost three orders of magnitude greater than in conventional RF-driven accelerator cavities as promoted by the PBA advocates.

IV. CURRENT STATUS OF PBA EXPERIMENTS

On the pure energy gain front, a maximum acceleration of 8 GeV in a low-density preformed plasma channel has been shown,[50] while many groups have shown acceleration of 1+ to 4 GeV beams using the self-guided blowout regime.[51–53] On the reliability front, a LWFA in the self-guided regime has been shown to produce 200 MeV beams continuously for 10^4 shots. On the energy spread front, energy spreads of 1% have been achieved.[54] On the emittance front, LWFA generated electron bunches have been shown to have less than one millimeter–milliradian (sometimes simply called micrometers) transverse emittance compared to the current photocathode-driven RF guns.[55] Other important physical effects such as beam loading,[56] betatron radiation,[57] photon frequency downshift all the way down to the plasma frequency,[58] and novel injection schemes such as colliding pulse injection,[59] ionization injection,[60] and downramp injection[61,62] have been demonstrated.

Relativistic plasma wakes are extremely transient [lifetime O (picosecond)], microscopic (diameter and wavelength $< 100\,\mu$m) structures that propagate at c. Even so, clever techniques to visualize the wakes using spectral holographic interferometry[63] and the deflection of a few femtosecond duration probe electron beam that "freezes" the wake because of its short duration have been developed.[64] These techniques have enabled a comparison between theory and experiments regarding the wake shape, lifetime, and longitudinal and transverse electric field profiles; thus, they have helped validate the particle-in-cell (PIC) codes.

The PWFA community has similarly made tremendous progress using the ultra-relativistic beam facilities at SLAC. A few years later, UCLA/SLAC collaboration working at the FACET facility at SLAC showed up to 9 GeV energy gain of a distinct trailing bunch containing up to 90 pC charge with an energy spread of 5% and an energy extraction efficiency from the wake of 20%.[4,65] These experiments used typically meter long alkali vapor columns[66] that were ionized by the electric field of the drive beam[67] in a reproducible manner for over a million shots at a time. In fact, the plasma source was so robust that it was possible to interpret variations in the experimental outcomes in terms of the details of the drive and trailing beams rather than that of the plasma. In addition, a new regime for positron acceleration[6] was discovered where under the right conditions, the front half of a single positron bunch lost energy in creating a wake, while the rear half of the same bunch loaded this wake and extracted energy at high gradient to yield a 5% energy spread positron bunch.

In addition to acceleration, other physical phenomena such as directional x-ray emission from betatron motion[68] and a new method of synchronized injection of electrons into ultra-relativistic plasma waves[69] were discovered in the PWFA experiments carried out at the FFFB. Furthermore, plasma accelerator generated electrons were used to obtain very high energy ($\gg$megaelectronvolt) betatron x-rays that in turn generated copious number of e−e+ pairs[70] and ultra-relativistic positron bunches were propagated through meter long plasmas.[71]

V. PERSPECTIVE ON THEORY

Rather than giving a detailed history of the development of theory and simulations, we discuss the development and current status of the key concepts and current research directions and opportunities. This leads naturally to a perspective on where the field is likely headed.

No matter the application, the theoretical description of wake excitation relies on equations that describe how the laser or the beam driver evolves as it gives energy to the wake, how the wake depends on the driver parameters, and how the trailing beam evolves in the wake. These equations are inherently nonlinear but are often linearized.

The fully nonlinear description is also complicated by the complete blowout of plasma electrons. The driver is described in terms of the laser field or the particle beam density and the frequency chirp of the laser or energy chirp of the beam. In some cases, the peak laser amplitude or particle density, spot size, and centroid are used. Under the assumption that the phase velocity of the wake is very close to the speed of light, the forces on a trailing beam in the wake are completely described in terms of the pseudo or wake potential $\psi = (\phi\text{-}A_z)$, where ϕ is the scalar potential and A_z is the z component of the vector potential in the direction the driver . Although the wake potential is scalar, it can fully characterize the three dimensional and nonlinear wakefields. The trailing beam is described in terms of its emittance, energy spread (in each longitudinal slice), and spot size (or sometimes in terms of the Courant–Snyder or Twiss parameters).[72]

Fundamental to the subject is the use of the "speed of light" variables and the quasi-static assumption (QSA),[73] and the use of key properties of the wake including that it has zero group velocity and that the forces on a particle moving "at" the speed of light in the $\hat{z}$ direction can simply be obtained as gradients of the wake potential (this follows from the Panofsky Wenzel theorem,[74] see below). These assumptions are useful and valid for 1D, linear, 3D, and nonlinear regimes. In PBA, both the phase velocity and the velocity of the driver (phase and group velocity of the laser and velocity of the particle beam) are very close to the speed of light. Therefore, it is useful to use the speed of light variables ($\xi = ct - z,\ y,\ z; s = z$) instead of (z, x, y; t). Physically, ξ and s represent the distance with respect to the leading edge (head) for the driver defined as at $\xi = 0$ and the distance that the head of the driver has propagated into the plasma, respectively. If the shape of the driver in all directions and its frequency chirp (laser) or energy chirp (particle beam) do not change as it propagates, then in terms of the variables ($\xi,\ y,\ z$) the wake will look identical at each value of s. The QSA is based on the disparate spatial scales between the distance in s that it takes the driver to evolve and the wavelength of the wake. Under the QSA for a given value of s, the wake is calculated assuming the driver is non-evolving. The wakefields are then used to advance the driver to a new value of s. In the initial work on the QSA for laser drivers, the slowly varying variable was set to s = ct and not s = z. This can lead to issues of causality when interpreting the results and when developing simulation methods based on the QSA (discussed later).

The relevant longitudinal (axial) and transverse forces from the wakefields on a particle moving at the speed of light are given by

$$
\begin{aligned}
F_z &= qE_z = q\left(-\frac{\partial}{\partial z}\phi - \frac{1}{c}\frac{\partial}{\partial t}A_z\right) \approx q\frac{\partial}{\partial \xi}(\phi - A_z),\\
\vec{F_\perp} &= q\left(\vec{E_\perp} + \hat{z}\times\vec{B}\right) \approx q\left(-\left(\vec{\nabla}_\perp(\phi - A_z)\right)\right),
\end{aligned}
\tag{1}
$$

respectively, where $\frac{\partial}{\partial s}$ terms are neglected consistent with the QSA, i.e., $\frac{\partial}{\partial s} \ll \frac{\partial}{\partial \xi}$. Therefore, the axial and transverse forces on a particle being accelerated are the charge times the gradients of the wake potential in the corresponding direction. It, therefore, follows that

$$
\vec{\nabla}_\perp F_z = -\frac{\partial}{\partial \xi}\vec{F_\perp}. \tag{2}
$$

This relationship is referred to as the Panofsky–Wenzel theorem[74] for plasma wakefields.

In addition, there are many analogies between how lasers and/or particle beams (drive and trailing beams) respond to the wakefields. These are best seen by viewing the laser as a collection of photons whose number does not change as it gives (or takes) energy to (from) a wakefield. This implies that the action of the laser is locally conserved.[63] The energy exchange is via a change in the frequency of the photon (called a "dressed" photon while in the plasma). The velocity of the photon is the group velocity at the local plasma density and the associated Lorentz factor γ is, therefore, $\omega_0/\omega_{p.}$[14] The wakefield provides a force (a time rate of change of the relativistic factor times the group velocity) on each photon. Interestingly, in the linear limit, this force is proportional (has the same sign but is ω_p/ω_0 times smaller) to that for relativistic electrons. As the frequency of a photon decreases/increases, it is referred to as photon deceleration/acceleration.[75,76] Lasers can also be focused by wakefields. This can be viewed as the dressed photon being accelerated (deflected) transversely. One important difference between LWFA and PWFA is that the Lorentz factor for charged particles is typically much higher than for lasers (photons). As a result, the laser pulse can distort due to axial motion as it depletes. This is the physics underlying the stimulated forward Raman instability.[75,77]

If the goal is to extract as much energy from the driver as possible, then each particle/photon in the drive beam should slow down together. This requires that the gradient of the wakefield be constant inside the driver which requires the use of shaped pulses.[78,79] In the beam driver case, pump depletion occurs when the particles within the drive bunch come to rest (for a laser, this occurs when the photon frequency is downshifted to the plasma frequency).

This leads to the concept of the transformer ratio.[80] If we consider the particle driver "stopped" when the particle decelerating at the fastest rate comes to rest, then the pump depletion (or acceleration) length is obtained from $qE_+ L_{pd} = \gamma_b mc^2$ where E_+ is the largest decelerating field. A particle in the trailing beam will then gain a maximum energy of qEL_{pd} over this distance where E_- is the peak acceleration field within the beam (often called the loaded field as we discuss later). Therefore, the trailing beam will gain an energy of $\Delta W = (E_-/E_+)\gamma_b mc^2$, where $R \equiv E_-/E_+$ is called the transformer ratio. If each particle is gaining R times the initial energy, then from energy conservation there can only be at most 1/R particles in the trailing beam. So the process increases the "voltage" at the expense of the current except that the wake provides a capacitive coupling rather than the usual inductive coupling as in an electric transformer.

From the lasers point of view, the arguments are similar except that $L_{pd} = \frac{(\omega_0/\omega_p)^2}{E_+}$ and the energy gain is, therefore, $R\left(\frac{\omega_0}{\omega_p}\right)^2 mc^2$. For typical lasers and plasmas used, $(\omega_0/\omega_p)^2 \sim 10^3\text{–}10^4$, while for a beam driver $\gamma_b \sim 2 \times 10^4$ to 10^5. Therefore, particle beam drivers using existing technology can typically lead to more energy gain per stage. In addition, in the afterburner concept where the output beam from a future collider ($\gamma_b \sim 10^5$) is used to as a driver, the energy of the trailing bunch that initially has the same energy as the drive bunch can be doubled.[81]

The acceleration length of a single stage of multi-stage PBA can be limited to a distance less than the pump depletion length due to dephasing (and instabilities to be discussed later). The phase velocity of the wake is often estimated to be the velocity of the driver which is the group velocity for the laser. This assumption is not exact.[82–84] An accelerated particle eventually moves closer to the speed of light than the wake, so the dephasing length can be estimated to be the length it takes a particle moving at c to slip a fraction of the wake's wavelength. For simplicity, assume that half the wavelength is useful (the useful part also must focus the particles and in nonlinear wakes the fraction of the wake that has accelerating and focusing fields for electrons can be significantly larger (or smaller for positrons) than one half. Under these assumptions, the dephasing length can be obtained by setting $\left(1-\frac{v_\phi}{c}\right)L_{dp}=\lambda/2$ or $L_{dp}=\gamma_\phi^2\,\lambda/4$. For the particle beam case, $L_{dp} \gg L_{pd}$, while for the laser case, $L_{dp} \leq L_{pd}$, so unless dephasing is addressed, the efficiency of a laser driver will be less than that of a particle beam driver for a single acceleration stage. Fortunately, dephasing is not a fundamental limitation such as conservation of energy so it may be possible to engineer it away. For example, it is possible to use density gradients so that the wavelength accordions toward (or away from) the driver or perhaps by using the concept of a moving or "flying" focus[85–87] where the group velocity of the laser pulse is continuously increased synchronously with the accelerated particles by feeding energy into the wake at small angles.

Another issue is that the drive beam can diffract away before it propagates a pump depletion distance. The diffraction length of a laser in vacuum is the Rayleigh length $z_R = \pi\, w_0{}^2/\lambda$, while for a particle beam, the diffraction length in vacuum is $\beta^* \equiv \sigma_0{}^2/\epsilon$, where w_0 is the $2^{1/2}$ of the RMS of the laser amplitude (not intensity) and σ_0 is the RMS of the beam density (or $\sigma_x \equiv \langle x^2\rangle^{1/2}$) where x is one of the transverse coordinates and $\epsilon \equiv \left[\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2\right]^{\frac{1}{2}}$ is the geometrical emittance of the beam.[60] It is clear that there is an analogy between λ and ϵ. For typical parameters, $\beta^* \gg z_R$; therefore, while guide is not necessary for a particle beam driver, it is essential for lasers.

It was believed early on that relativistic guiding, where the relativistic mass increase in electrons due to their oscillation in the laser field increases the index of refraction on axis,[17] would be an easy solution to self-guide laser drivers. However, one of the first consequences of the QSA[73] was that the modifications to the index of refraction from the laser could be described completely from the wake potential which does not respond for time scales on the order of the plasma period. Viewed another way, the increase in the index of refraction from the relativistic mass was balanced by a decrease from the density compression from the radiation pressure. The consequence of this is that self-guiding a short laser seemed to not be straightforward. As a result, engineering solutions such as the use of plasma channels[88] or capillaries have been actively investigated. As is often the case, nonlinear effects change the conclusions, and as we discuss later, the prognosis for self-guiding of lasers is more sanguine than originally thought.[89] Externally produced plasma channel may generate the same beam energy using a smaller laser power but at the cost of additional complexity. Choosing between self- and externally guided regimes will depend on the application and on expected advances in channel and laser technology. Although diffraction is not as severe for typical particle beams, they still need to be self-guided over pump depletion distances. Just as for lasers, the focusing forces take time to develop from the head of the beam. In the nonlinear regime, however, the focusing gradients are sufficiently large in the rising edge (head) of the beam, so that even a 10 GeV particle beam is easily guided until it is pump depleted. For an application such as compact XFEL, PBA operating in the nonlinear self-guided regime might be advantageous due its simplicity, while for the linear collider application where efficiency is paramount an externally produced plasma channel may be required.

It is illustrative to examine the linear wakefield theory to understand some differences between laser and beam driven wakes and to discuss accelerating positrons and beam loading. The wakefield is excited by the ponderomotive force of the laser driver and/or the space charge force of the particle beam. Linear theory permits the use of Green's functions, and the analysis is simplified by the assumption of no group velocity. The equation for the wake potential driven by lasers and/or particle beams is

$$\left[\frac{\partial^2}{\partial\xi^2}+k_p^2\right]\left[\nabla_\perp^2-k_p^2\right]\frac{e\psi}{mc^2}=k_p^2\frac{q}{e}\frac{n_b}{n_0}+\left[\nabla_\perp^2-k_p^2\right]\phi_p, \qquad (3)$$

where ϕ_p is the ponderomotive potential $\langle (a0)^2\rangle/2$. The solution for ψ can be obtained using Green's functions

$$\frac{e\psi}{mc^2}=\int_{-\infty}^{\infty}dk_p\xi'\eta\left(k_p(\xi-\xi')\right)\sin k_p(\xi-\xi')\times\left[\phi_p\left(\xi',\vec{x}_\perp\right)\right]$$
$$=\int_0^{2\pi}\left[\frac{d\phi'}{2\pi}\int_0^{\infty}dk_pr'k_pr'K_0\left(k_p\left|\vec{x}_\perp-\vec{x'}_\perp\right|\right)\frac{q}{e}\frac{n_b\left(\xi',\vec{x'}_\perp\right)}{n_0}\right]. \qquad (4)$$

Several points are worth noting. For a laser driver, the transverse profile of the wake is identical to that of the ponderomotive potential. So if the spot size of the laser is much less than c/ω_p, the spot size of the wake is also very narrow. On the other hand, for a particle beam driver, the width of the wake is still $\sim c/\omega_p$ even for very narrow beam drivers. One can still tailor the transverse profile of the wake using a particle beam driver, but this requires more care for the narrow beam. In the linear case, there is also "no" difference between using electron and positron drivers (trailing beams), except for π phase shifts in the wake response (or where one needs to place the trailing beam for it to be accelerated—Fig. 1). For hollow channels, the use of modified Green's functions still works, and for weak axial density gradients (under the QSA) solutions can be obtained by letting k_p be a function of ξ after evaluating the integrals.

Obtaining the expressions for the wake response from a laser or a particle beam driver naturally leads to the concept of beam loading which refers to how to most efficiently extract the energy of the wake while at the same time minimizing its energy spread and emittance growth. Within linear QSA theory, the current theoretical formalism used to study beam loading is straightforward[41] and has not changed since the original work. There are two points of view for determining the efficiency. First is a local (or particle) point of view where one examines the rate of energy loss (decelerating field) from a particle in the drive beam and compare it to the rate of energy gain (accelerating field) of a particle in the trailing beam. This is similar to the analysis used to define the transformer ratio.

Second is a global (or field) point of view. One simply calculates the wake response for the driver and then calculates the total wake response for the driver and trailing beams. For example, for a laser driver, n_b only refers to the trailing bunch, while for a particle beam driver, ϕ_p is set to zero and n_b includes the driver and trailing bunches. To estimate the efficiency, one calculates the energy in the wake per unit length without the trailing beam ($\mathcal{E}_0$). Then the energy in the wake left behind driver and the trailing beam ($\mathcal{E}_1$) is calculated, i.e., we use superposition. The efficiency from the driver to the trailing bunch is simply $\eta \equiv 1-\frac{\mathcal{E}_1}{\mathcal{E}_0}$. This will give an identical result obtained from the local or particle point of view. Implicit in the calculations is the

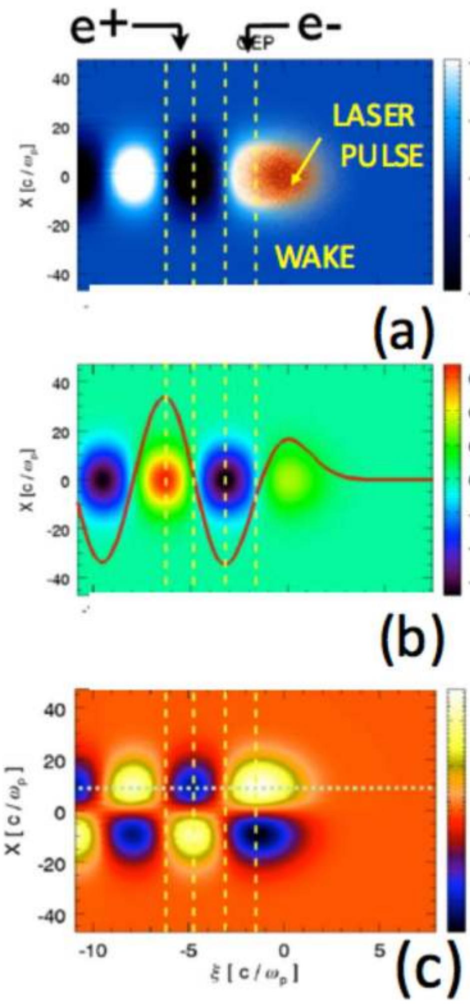

FIG. 1. Linear wake excited in an underdense $(n_e/n_c) \ll 1$ plasma by laser pulse with $a_0 = 0.1$. The contours of (a) the density disturbance, (b) the longitudinal electric field and the on axis magnitude of this field (black curve) and (c) the transverse focusing force. The two vertical dotted lines show the range of phases of the wake where the wakefield is both accelerating and focusing for electrons (e−) and positrons (e+), respectively. There is a π radians phase difference between the phases available for electron and positron acceleration.

assumption of no dephasing. If dephasing occurs, then the efficiency is a function of the propagation distance.

In addition, in many applications, it is desirable to minimize the energy spread and the emittance growth. These can be calculated using the accelerating fields and focusing fields within the trailing beam obtained from superposition. In order to reduce the energy spread by making the accelerating field inside the trailing beam flat in both ξ and (x,y), the trailing beam must be shaped longitudinally and transversely. It is clear there is a trade-off between efficiency, acceleration gradient, and energy spread. High efficiency means the wake amplitude is small at the rear of the trailing bunch. Low energy spread means that the wake amplitude is constant within the bunch; therefore, if it is small at the rear, then it is small throughout the beam. Therefore, in order to simultaneously achieve high efficiency and low energy spread, the acceleration gradient must be low.

In multi-dimensions, one needs to consider how much of the energy density in the cross sectional area of the wake is absorbed by the trailing beam, how the accelerating field varies in the transverse directions, and the properties of the focusing force. If the spot size of the trailing beam is on the order of or larger than the wavelength of the wake, then the cross section of the wake is the same as that of the beam (laser or particle beam). In this case, the problem appears nearly one dimensional and high efficiency can be obtained by using trailing beams with spot sizes comparable to that of the driver. However, such a one dimensional regime is challenging for the PWFA because for typical beam currents (tens of kAs) n_b/n_0 becomes small when the spot size exceeds a plasma wake wavelength.

As discussed earlier, a particle beam diffracts analogously to a laser due to its initial spread in transverse momentum or emittance. The diffraction angle is determined by the beam's geometrical emittance. In order that the accelerated beam can be focused to desired spot sizes after leaving the plasma, its emittance must be very small. As an ideal beam with zero energy spread is accelerated the beams normalized emittance, $\epsilon_n \equiv \gamma\epsilon$ can be conserved if the spot size is "matched" to the focusing force. The matched spot size is very small for typical parameters even for linear wakes. Therefore, the matched spot size of the trailing beam is much smaller than the spot size of the wake. Fortunately, the transverse extent of the wake of the trailing beam scales as c/ω_p [this can be seen from Eq. (4)] and not the beam's spot size. As a result, reasonable efficiency (cancellation of the wake) can still be obtained for a drive beam with a spot size of c/ω_p and a very narrow trailing beam.[90]

The requirements on the charge in a trailing beam and repetition rate f depend on the application. The most challenging is a future linear collider. If one works backward from the requirements on the luminosity that was discussed earlier, then the trailing beam needs to have ∼0.1 to 1 nC and to operate between 1 and 10 kHz. If the trailing beam has 1 nC, a spot size of <10 nm, and a bunch length $\sim c/\omega_p$ (its length scales with the wavelength of the wake), then the density of the trailing bunch to the background plasma density is $n_b/n_0 > 1$ for typical plasma densities. So there is an issue with the use of linear theory to provide a fully self-consistent beam loading scenario. To circumvent this problem, the use of hollow plasma channels,[91] nearly hollow channels,[92] and flattop drivers (by using a superposition of higher order modes)[93] has been considered. Each of these concepts relies on reducing the focusing force, so that the matched spot size can be comparable to the spot size of the wake. It should be emphasized that in all cases the Panofsky–Wenzel theorem for plasma wakefields holds, so there is a relationship between the accelerating and focusing fields.

However, each of these concepts has issues, some based on fundamental physics and others on engineering issues. The use of linear theory is based on the superposition of wakes. So if the wakes "add" to provide the desired accelerating and focusing fields at a specific spacing between the drive and trailing bunches, then as dephasing occurs these properties will change. If higher order laser modes are used, then they too will dephase from each other, causing the shape to change. The lack of focusing fields in hollow channels is strictly true only when there is perfect azimuthal symmetry. These concepts can also be further complicated by the fact that current linear collider designs that minimize beamstrahlung[94] require that the trailing beam have significantly different transverse emittances in the two planes so that the final focus will have beams with asymmetric spot sizes. As a result, it seems challenging to obtain a fully self-consistent beam loading scenario based on linear theory (i.e., a scenario that shows high efficiency, high gradient, emittance preservation, stability, and low energy spread over pump depletion distances) although several suggestions have been made that may provide a solution to this issue.[95] The advantage of this regime is that it is directly applicable to positron acceleration. However, from our perspective, although these concepts are theoretically possible, it will be very challenging to realize them experimentally in the near future.

As mentioned earlier, the attractiveness of the linear regime is that it is in principle straightforward to modify any beam loading design that works for an electron trailing bunch, so that it works for a positron bunch (Fig. 1). The wake of a positron bunch has the opposite sign as that for an electron bunch. So if the electron bunch is replaced with a positron bunch, then rather than being accelerated and focused, it will be decelerated and defocused; or equivalently, rather than absorbing energy from the wake produced by the drive beam, it will give energy to the wake. However, if the positron beam is delayed by half a wavelength, then it will be accelerated and focused with the same efficiency as the original beam loading design.

Addressing issues that arise with linear theory naturally leads to the nonlinear theory. As noted above, beam loading designs based on linear theory lead to considering narrow electron bunches with n_b/n_0 much larger than unity and a current exceeding the Alfvén limit ($I_A = mc^3/e = 17\,\text{kA}$). However, such bunches do not excite linear wakes.[90,96] Therefore, linear theory cannot be used to analyze how such a beam absorbs energy from the wake, i.e., how it is accelerated. It turns out that the wakes made by high current and narrow electron bunches are ideal for accelerating and focusing high current electron bunches [Fig. 2(a)]. Such wakes are excited by the space charge force of the beam, pushing the plasma electrons sideways as well as forward.[33,44,45] These electrons then form a narrow sheath that surrounds the ions (ion column). The space charge force of the ion column then pulls the electrons in the sheath back toward the axis, thereby creating a nonlinear wake. The field structure inside the wake has both electric and magnetic fields, and it can be completely obtained by determining the trajectory of the edge of the sheath [the blowout radius, $r_b(\xi)$] and from phenomenological models for the sheath.[44,45] Others have analyzed the field structure in the blowout regime under the assumption that the cavity is a sphere.[33] Different models for the sheath can be used so long as they are self-consistent. Just as for TEM modes in a waveguide, it turns out that the wake potential can be determined slice by slice using concepts from two-dimensional electro- and magnetostatics. The accelerating field only depends on the trajectory of the blowout radius, i.e., for large blowout radius, it is $\frac{1}{4}\frac{dr_b^2}{d\xi^2}$. When the ion column has a spherical shape such that $r_b(\xi)$ nearly forms a circle, the functional form for the accelerating field is $\frac{eE_z}{mc\omega_p} = \frac{1}{2}\xi$, where $\xi = 0$ is defined to be where r_b is maximum.[44,45] Interestingly, this is identical to the 1D nonlinear wake form. The focusing force (which is from electric and magnetic fields) is the same as that obtained by using 2D electrostatics for an infinitely long ion column, i.e., for a round ion column, it is proportional to the radial distance from the axis, r, and it points in the radial direction. And as long one considers radial distances smaller than the blowout radius, the focusing force does not depend on the axial position [it is focusing with a force proportional to r for electrons for the entire "wavelength" (all phases)]. This last property combined with the Panofsky–Wenzel theorem implies that the accelerating force does not depend on r. These properties are ideal for beam loading (Fig. 3). Therefore, the blowout regime is very attractive for accelerating electrons and unfortunately not for positrons.

Beam loading for nonlinear wakes has been analyzed[97] by examining how the trailing beam affects the trajectory for $r_b(\xi)$. As stated earlier, the second half of the first bucket (bubble) has an accelerating field for electrons. This is the phase for which $\frac{dr_b}{d\xi} < 0$, i.e., the electrons in the sheath are returning to the axis. If an electron beam is placed in this region, then its space charge field repels the sheath electrons. This reduces the magnitude $\frac{dr_b}{d\xi}$ (reduces—eE_z) and elongates the bubble. As in the linear analysis, the field inside the wake can be flattened by shaping the bunch (Fig. 4). The energy left in the wake scales as r_b^4 (it scales as $E_z^2\, r_b^2$ but E_z scales as r_b) so high efficiency can be obtained even with large accelerating fields. An important result from simulations and theory[98] is that even unshaped trailing bunches

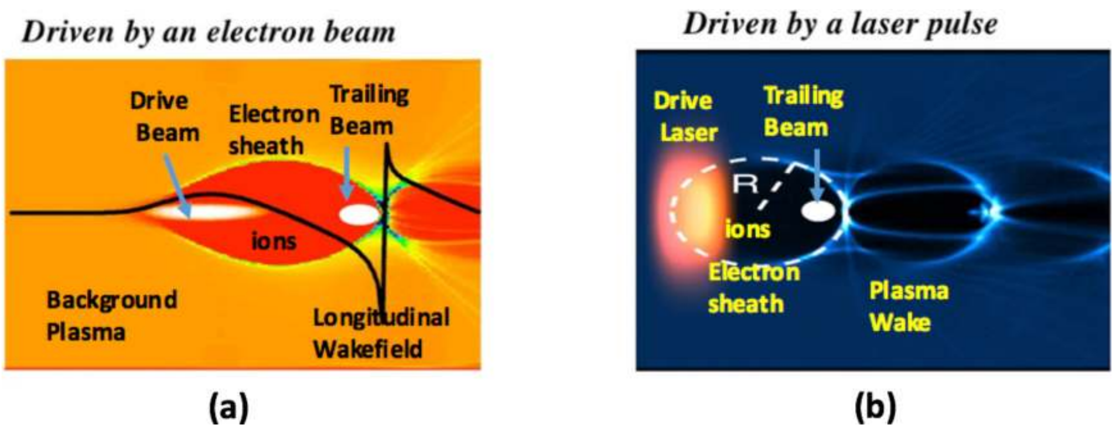

FIG. 2. Nonlinear plasma wakes produced by (a) an electron beam and (b) a laser pulse. In both cases, the drive pulses are traveling from the right to the left. In (a), the black curve shows the on-axis longitudinal electric field of the wake.

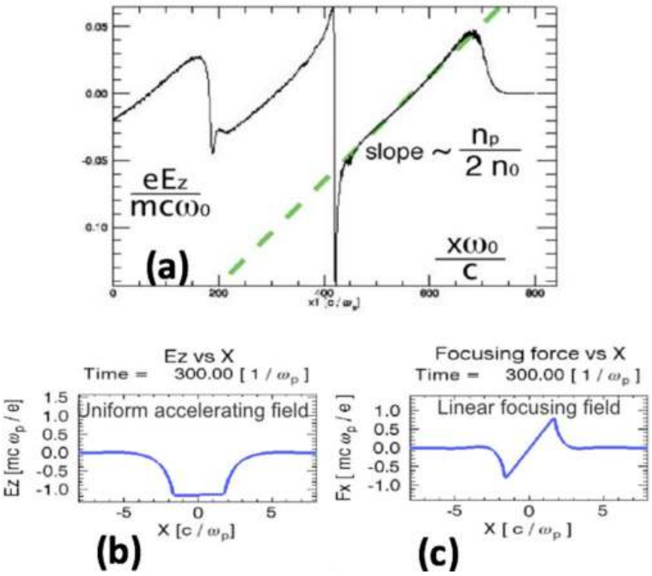

FIG. 3. (a) The on-axis longitudinal electric field E_z of a wake in the "bubble" regime. The slope of the accelerating field in the first bubble is $n_p/2n_0 = \varepsilon/2$. (b) The transverse variation of the longitudinal E_z field and (c) transverse variation of the focusing field $F_x = (E_x\text{-}B_y)$. In the bubble regime, the E_z field is constant, whereas the focusing field is linear in the transverse direction x.

nearly flatten the wake and the wake remains flat as the beam phase slips forward.

Simulations have shown that in the blowout regime nearly 50% energy transfer from a drive beam to a trailing beam can be achieved with beam loaded transformer ratios larger than unity and energy spreads less than 1%.[98] However, these simulations used fixed or immobile ions. For collider emittances and matched trailing beams that have nanocoulomb of charge, the ratio of $\frac{n_b}{n_0} \geq 10^4$, i.e., it can easily exceed the ion to electron mass ratio. Under these conditions, the ions move significantly within the transit time of the beam, i.e., $\frac{\sigma_z}{c}\omega_{pi} \ll 1$.[99] If the ions move, they will modify both the focusing and accelerating fields. The focusing forces will no longer be linear in r, they could increase in strength by orders of magnitude, and they will vary along ξ. It was hypothesized that each of these effects can contribute to catastrophic emittance growth. However, simulations and theory using bunch parameters consistent with designs of a PBA based LC[100] have shown that the emittance growth is limited and can be controlled even for trailing beams with asymmetric spot sizes.[100]

Simulations have confirmed that the nonlinear blowout regime has the potential for accelerating electron beams with high efficiency without significant emittance growth, and low energy spread so long as instabilities such as hosing (discussed below) can be controlled.

An intense laser driver will also excite wakes in the nonlinear blowout regime[101,102] [Fig. 2(b)]. However, in this case, the laser cannot be too narrow because the ponderomotive force only exists where the laser exists and its diffraction length cannot be too short. It is interesting that for a laser driver typically not all the electrons are completely expelled as is the case for a narrow particle beam driver. The ideal shape for a laser driver is, therefore, roughly a sphere (the spot size and pulse length are comparable). For lasers, the blowout radius, $r_{b0,}$ scales as $2a_0^{1/2}(c/\omega_p)$.[41,101] This regime of LWFA is now often referred to as the bubble regime because as noted above for such large blowout radii the shape of the ion column is nearly spherical. In order that the wake remain stable and not varies in propagation distance s, the spot size needs to be "matched" to the blowout radius to prevent oscillation of the laser spot size and hence wake.[41,57,102] In this regime, the wake is produced by the front of the laser, while the bulk

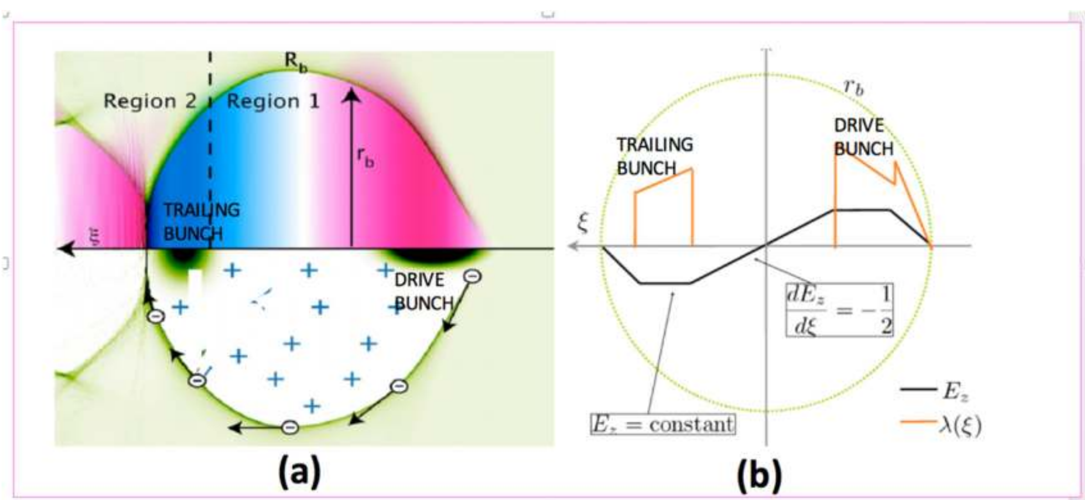

FIG. 4. (a)A near spherical bubble shaped wake (containing positive ions) produced by a shaped drive electron beam as shown in (b). r_b is the local radius of the wake from the axis ξ, and R_b is the maximum radius. The pink region the longitudinal electric field E_z decelerates the drive bunch (green), while in the blue region E_z, it accelerates the trailing bunch. (b) The on-axis variation of the longitudinal electric field produced by a shaped drive bunch and beam loading produced by an inverse trapezoidal trailing bunch. The green circle radius r_b is an approximation of the blowout region produced by the drive bunch. The shaped drive and the trailing bunches lead to flattening of the E_z field (black curve) that leads to a narrow energy spread trailing beam and a high energy transfer efficiency from the drive to the trailing beam. Reproduced with permission from Tzoufras *et al.*, Phys. Rev. Lett. **101**, 145002 (2008). Copyright 2008 American Physical Society.

of the laser resides inside the bubble. As a result, much of the laser can be self-guided.[41] This may appear contradictory to the conclusions from the QSA work.[73] However, in this nonlinear regime, the physics is different. The pump depletion is local and energy can be given to the wake before it diffracts,[103] the diffraction of the head of the beam is still slower than in vacuum, and the bulk if the beam is surrounded and guided by the sheath. Simulations and phenomenological theory indicates that self-guiding can propagate the laser pulse at a distance of hundreds of Rayleigh lengths.[104] This regime of LWFA is of great interest for compact light sources; however, it may not be attractive for a collider unless its efficiency (laser to wake) can be increased.

Plasma wave wakefields have much shorter wavelengths (higher frequencies) than conventional accelerating structures, making it extremely challenging to synchronously inject the trailing beam into the wake. The acceptance volume is small and the injected beam needs to be extremely well aligned with the wake. Fortunately, plasma electrons can be self-injected into the wake. The most obvious to consider is driving a wake to large amplitude such that plasma electrons at rest can be trapped. This is straightforward to do in one-dimension and leads to the concept of wavebreaking. However, in multi-dimensional wakes, while it is clear how to determine a threshold for the wake potential when self-trapping could occur, it is not obvious how to determine a self-consistent form for the wake and the fields when this occurs partly because of structure of the sheath. Although there have been some attempts to investigate self-injection in large amplitude (and non-evolving) wakes.[105–107] this method does not appear to be attractive for producing high quality beams.

Other self-injection schemes are, therefore, being actively investigated. While there have been many proposed schemes, only a few have the ability to generate a beam that has the quality necessary for a linear collider or an X-FEL. The most promising schemes fall under two categories. In the first, electrons are ionized inside the wake, while in the second, the phase velocity of the wake is controlled by varying the wavelength of the wake in propagation distance s. For a wake (linear or nonlinear) with a fixed phase velocity, the trapping condition[56] for an electron is that the change in the wake potential $\Delta\psi < -1 + \frac{1+p_\perp^2}{\gamma_\phi}$ where $\Delta\psi \equiv \psi(\xi) - \psi_i$, and $\psi(\xi)$ and ψ_i are the wake potential at the location of the particle and at initial position of the particle (ψ is normalized to mc^2 and $p_\perp$ is normalized to mc). This condition is valid in one or multiple dimensions. For a background plasma electron, $\psi_i = 0$; therefore, $\psi(\xi)$ must be less than ~ -1 somewhere in the wake which is generally not possible. However, if the electron is created inside the wake (by ionization) near where $E_z = 0$ or equivalently where ψ_i is at a maximum, then the amplitude of the wake needs to only be $\sim$0.5 which is easily satisfied.[60,69,108] Furthermore, this scheme provides well defined mappings between where the particles are born (in ξ and x, y) and where they become trapped, so that very high quality beams can be generated.[109]

Particle trapping can also be induced by controlling the phase velocity of the wake, so that the $(1 + p_\perp^2)/\gamma_\phi$ term lowers the trapping threshold. The speed of the driver is relatively constant (it changes over pump depletion distances); however, the wavelength of the wake depends on density. Therefore, if the driver moves in a density gradient, then the wake minimum or zero can be made to accordion[80] forward or backward if the density is gradually increasing or decreasing. This concept can be used to eliminate dephasing[80] or to permit self-injection. For example, a discontinuous transition can be used.[61,110] In the nonlinear blowout regime, the process for injection is more complicated because the particles are also coming back toward the axis as they get trapped. It turns out that as they get near the axis at the rear of the bubble, they also feel a repelling force, so their transverse momentum is reduced, leading to injection of low emittance beams.[111] There is also a mapping between ξ_i and ξ_f which leads to very low slice energy spread and permits phase space rotation to provide low projected energy spreads after the beam is accelerated.[111] Other ideas to dynamically move the transition point between accelerating an decelerating phase of-in other words to "accordion" the wake are being developed, see, for example, Ref. 112.

For the linear collider application, it is necessary to accelerate positrons. Developing a beam loading scenario for accelerating positrons in the nonlinear regime remains a challenge. A nonlinear fully blown out wake has ideal properties for accelerating electrons including that it provides a constant linear focusing force for all phases, which means that it has defocusing fields for all phases for positrons. In addition, the formation of wakes in the nonlinear regime is fundamentally different.[113] In both linear and nonlinear cases, it is the plasma electrons that respond to the space charge forces of the beam. The positron beam pulls the electrons inward rather than pushing them outward. These electrons then cross the axis continuously and not at one location in ξ. However, if the positron beam is short enough then the electrons cross near the same location and electrons then form a bubble-like wake afterward.

Accelerating positrons in nonlinear wakes and developing beam loading scenarios (where the accelerating field of the wake is flattened) is actively being investigated. Using weakly nonlinear wakes produced by an electron beam or laser produced wakes where there is a limited phase of acceleration/focusing has issues because the positron beam will not flatten the wake. Other ideas have been suggested[114] including using higher order Laguerre laser modes with a hole on axis[115] and experiments and simulations have shown that positrons in the rear of a positron beam can be accelerated and focused with a flattened accelerating field.[6] The key appears to be that there must be plasma electrons trapped on axis that overlap with the positron beam. These electrons can both focus the positrons and flatten the wake. Other recent ideas include the use of an annular drive beam[116] or laser-produced hollow or nearly hollow channels where the focusing fields are zero or small.

This is an appropriate time to mention the use of ultra-high energy (teraelectronvolt-class) proton (or anti proton) beams such as those existing at CERN as wake drivers for accelerating electrons.[117] However, these beams are currently not short enough to excite a wakefield. Therefore, the AWAKE experiment at CERN[118] is relying on a nanosecond-scale proton beam breaking up into short beamlets through a self-modulation instability inside a plasma.[119] The status of this work is very similar to where LWFA research was in the early to mid-1990s as there is a great deal of overlap between how lasers and particle beams self-modulate.[21,77]

This leads us to issues related to how the driver and trailing beams are susceptible to self-modulation like instabilities. Accelerator physicists refer to these as head to tail instabilities while plasma physicists classify them as streaming instabilities in which two harmonic oscillators are coupled together. Besides positron acceleration, which is arguably the biggest obstacle for a PB-LC, the hosing instability of the trailing beam[120,121] is a possible highly detrimental undesirable effect. Current linear collider designs are based on using tens of stages. A new driver must be inserted in front of the trailing bunch in ease stage. Hosing can occur in each stage, and this can be exacerbated by the transverse offsets between the new driver and trailing bunch. Hosing can lead to projected emittance growth as well as difficulty in colliding the beams.

Fortunately, there are mitigation methods. The hosing instability grows from centroid offsets of the two bunches in the PWFA or from centroid offsets of each longitudinal slice of the bunch caused by the coherent synchrotron instability in a conventional beamline and not by unpredictable thermal noise, so feedback techniques might help. Hosing occurs from a coupling between the betatron motion of the beam and the oscillations of the wake. Therefore, if different slices of the beam oscillate differently or the wake frequency changes with propagation distance, then phase mixing can mitigate hosing. This could be achieved through energy chirps,[122] ion motion,[123,124] asymmetric drivers making asymmetric wakes,[123] or density gradients.

For both the X–FEL and LC applications, it is important that beams can be transported out of and into new plasma or conventional accelerator sections. This can lead to large emittance growth if there is energy spread on the beam because of the mismatch in the focusing force (or beta functions) the bunch feels when inside the plasma compared to that due to the external coupling optic. There has been recent progress in understanding this issue and developing concepts to mitigate them. These rely on appropriately chosen density profiles at the exit (and if needed at the entrance) of an acceleration stage or a plasma lens between stages.[125–127] There has been recent progress in understanding this issue and developing concepts to mitigate them.[128] These rely on appropriately chosen density profiles at the exit (and if needed at the entrance) of an acceleration stage or a plasma lens between stages for linear or blowout wakes.[129–131] The matching sections can be short tailored profiles or longer adiabatic matching sections.

As noted earlier, as particles are accelerated (or decelerated) in plasma wakefields they undergo betatron oscillations from the focusing fields. A relativistically moving electron that accelerates transversely will radiate. Therefore, the betatron radiation can generate broadband incoherent radiation near $\omega_r = 2\gamma_b^2 k_\beta / c$ which scales as $\omega_r = \gamma_b^3/2$. It is ultimately the limit to the energy gain in PBA.[132]

From a theoretical perspective, the future of PBA is very bright. New ideas continue to emerge and this together with the expected progress in simulations (see below) will make it possible to soon explore complete linear collider and X-FEL concepts using start to end simulations.

We close this section with some personal perspective on the use of particle beam drivers vs laser drivers. For a collider application where high efficiency from the driver to the output beam is required, it would seem that an electron beam driver (in the blowout regime) offers advantages for accelerating the electrons. It is simpler (uniform plasmas can be used and no customization of the beams is required), and there are already fully self-consistent beam loading scenarios (including ion motion) that would appear to provide 50% energy transfer efficiency from the drive to the witness beam with an acceptable emittance growth and hosing. On the other hand, there are currently no efficient beam loading scenarios for accelerating positrons using nonlinear wakes. Therefore, it may be necessary to operate in the linear or even weakly nonlinear regimes (including the use of hollow channels). In these regimes, lasers have advantages as they are easier to manipulate and customize. It may, therefore, be the case that the cheapest design for a linear collider will use an electron beam to accelerate electrons and laser drivers to accelerate positrons. For the XFEL application where only ultra-high brightness electrons are needed and efficiency is less important, the choice will be dictated by the injection scheme that is used and on cost and size concerns which might favor lasers (perhaps in the blowout regime) as the driver.

VI. PERSPECTIVE ON SIMULATIONS

Simulations have played a decisive role in the development of plasma based acceleration and intense laser and beam plasma interactions. From the very beginning, the particle-in-cell method (PIC) has been the simulation tool of choice for modeling PBA and high field processes. The PIC method is very robust and efficient. The formation of the wake typically involves trajectory crossing and relativistic mass corrections. Therefore, fully nonlinear and kinetic physics needs to be included. This requires either the use of PIC or a relativistic Vlasov approach. The physics is also inherently three dimensional, so 3D models are required.

The PIC method essentially models the Klimontovich equation[133,134] for finite size particles which differs from a Vlasov description which is an ensemble average over many Klimontovich states. The use of finite size particles greatly reduces strong scattering of particles from close encounters for impact parameters less than the particle size. In the absence of collisions (when there are many particles per debye sphere), the Klimontovich and Vlasov descriptions merge. Therefore, with a sufficient number of particles per cell, the PIC method can effectively model the Maxwell–Vlasov system. From a floating point operation count perspective, the PIC method is an efficient method to model the Vlasov equation when the number of particles per cell is less than the number of cells needed in momentum space to represent phase space. In multi-dimensions, the PIC method is, therefore, almost always more efficient. This essentially makes the Vlasov approach impractical unless adaptive meshes are used in momentum space; the distribution function can be expanded into a properly chosen basis and then truncated; or the beam and plasma are modeled with different methods.

Currently, simulations of PBA and intense laser and beam plasma interactions are done using fully explicit (traditional) or QSA PIC codes. In a fully explicit PIC code, Maxwell's equations are solved using cell sizes and time steps that resolve the shortest time and space scales. In PBA, only the physics near the driver is important (both the driver and trailing bunch moves near the speed of light). Therefore, the simulation window needs to only "keep up" with the driver. As a result, nearly all simulations using the traditional PIC method use a moving window.[135] The simulations are done in the lab (plasma) frame but the simulation box (window) acts like a treadmill. Fresh plasma is added to the front of the box, and fields and plasma at the rear of the box are dropped. In this window, the driver is essentially running in place.[82,135]

Developing a PIC algorithm based on the QSA is not straightforward both for conceptual and numerical reasons. In fact, the thought process required to develop a QSA algorithm has led to a deeper understanding of the meaning of the QSA and its strengths and weaknesses. A QSA PIC code separates out the excitation of the wake from the evolution of the driver. The "forces" from the driver are assumed to be fixed at a value of s—recall that in the QSA we use (x, y, $\xi = ct-z$, $s = z$) as the variables. These forces are used to excite a wake that would be static (would not change in s) if the driver did not change. The trajectories of plasma particles are evolved in (x, y; ξ) space assuming that s is fixed, i.e., the trajectory is collapsed onto a 2D space with ξ acting as a time variable. If the driver is evolved in (x, y, ξ; s), then s acts like the time variable. The field equations are a reduced set of Maxwell equations. They are obtained by making a mathematical transformation from (x, y, z; t) to (x, y, ξ *and* s = z) and dropping all $\partial/\partial s$ derivatives, i.e., the equations are solved in (x, y , ξ) space. The first implementation of a QSA PIC code was developed by Whittum[136] to study PWFA and hosing (it was 3D), and relied on the assumption that plasma particles only move transversely (this greatly simplifies the concept but is only valid for narrow and weak drivers). Mora and Antonsen[137] were the first to figure out how to develop a fully nonlinear QSA PIC algorithm. Their code was in 2D r–z geometry and it was initially used to study LWFA. Developing a fully nonlinear 3D QSA algorithm required additional improvements[138–140] including developing strategies for parallelizing the algorithm.

The use of the moving window and the QSA PIC codes can be viewed as making Galilean transformations into a frame moving at the speed of light c. Under certain conditions (where all modes of interest move in the forward direction with phase velocities close to the speed of light), it is also natural to do the calculations in a Lorentz transformed frame[141,142] in which the plasma moves toward the driver. The Lorentz boosted idea is a reduced model as it eliminates physics associated with modes with slower phase or backward moving phase velocities. The basic idea is that in this frame the driver length is Lorentz expanded (if the driver is a laser, then its wavelength is also expanded), while the acceleration length (length of the plasma) is Lorentz contracted. If there is no reflected light and there are no modes with phase velocities "significantly" different than v_ϕ, then in the new frame the smallest scales are all Lorentz expanded, so that the required number of simulation cells required does not change. If all the requirements just mentioned are met and the number of particles per cell does not change, then this technique can lead to savings that scale as γ_ϕ^2 over the use of the moving window.

It turns out that there is a robust numerical instability that arises in multi-dimensions when plasma drifts relativistically across the grid. This is what prevented the boosted frame concept from being successfully used in the 1990s. When the idea of using a Lorentz boosted frame was independently proposed again in 2007, it initiated new research on its use. This led to the identification of the numerical Cerenkov instability (NCI) as the source of the instability[143,144] (the same issue had prevented the successful implementation of the boosted frame in the early 1990s) and to numerical methods to mitigate it or even eliminate it, e.g., Refs. 145–147. Even with this progress, the use of a Lorentz boosted frame remains an active area of research. Challenges still remain for using the boosted frame technique for studying the nonlinear blowout regime, PWFA, self-injection, and instabilities such as hosing. Furthermore, when modeling narrow particle beams, the cell sizes will have very different scales in each direction in the boosted frame. Understanding the consequences of this will require research as well.

Other ideas that that have improved the capability of PIC are the ponderomotive guiding center (PGC) approach and combining the PIC method on an r-z grid with a gridless method in ϕ. The former concept utilizes the idea that the motion of an electron in a laser field can be obtained by averaging over the laser period/wavelength to a high degree of accuracy so long as the a_0 is not too large, and it is not focused to a spot size comparable to its wavelength. The condition for a_0 not being too large is roughly satisfied when particles do not move forward with sufficient speed, so that the Doppler shifted frequency of the motion in the laser field is still much smaller than the plasma frequency which can be roughly quantified as $a_0 < \left(\frac{\omega_0}{\omega_p}\right)^{1/2}$ for particles that started at rest. The PGC method continues to be actively used[148] and developed.[149–151]

Incorporating 2D r-z algorithms in standard PIC codes to study PWFA has been very successful. However, the assumption of azimuthal symmetry is not valid for linearly polarized lasers. A linearly polarized laser with a symmetric spot size only has the m = 1 azimuthal mode. This led to the idea of expanding the fields and currents in r-z space for each azimuthal mode[152,153] and truncating the expansion at a mode number that adequately captures the physics of interest. This is now referred to as a quasi-3D method, and it can also be combined with the boosted frame technique.[154] The use of the quasi-3D method can lead to computational savings of factors of 100 if only a few m modes are needed. This idea has also been implemented into a QSA PIC code.[154]

Despite being over half a century old, the PIC method continues to evolve and improve. The concept of the PIC method is simple and the algorithm can be broken down in the following steps: (1) load particles onto a grid with continuous values for their location and momentum and initialize or launch fields, (2) deposit their charge and current onto the corners (or half-cell offsets) of the grid, (3) solve for the fields on the corners (or half-cell offsets) using either FFTs or finite difference methods, and (4) interpolate the fields to the particle locations and advance their positions and momentum. This is then repeated for a desired number of time steps. In some cases, there is trade-off between accuracy and performance, and this includes choices in numerical parameters, algorithms, and models. At present, there is often not a clear answer as to which software to use; however, with recent advances, there are options for studying most problems of interest.

There have been recent advances in field solvers which in some cases impacts how the current is deposited. The advantages to using FFT based Maxwell solvers has been known since electromagnetic PIC codes were first developed.[133,134] The advantages include less dispersion errors for light waves and that the algorithm converges to the "correct" answer as the time step is reduced if the cell size is kept fixed. An extension of using FFTs is the pseudo-spectral analytic time domain (PSATD) approach.[133,155] This algorithm assumes the currents are constant and then analytically integrates the fields forward in time in wave number space. It eliminates dispersion errors for light waves in vacuum, but it assumes the current is constant within a time step. There is therefore room for improvement in the PSATD. The use of FFTs can lead to issues for scaling the algorithm to many computer nodes. However, it was proposed that when using the PSATD approach the FFTs can be done locally with minimal errors because all modes move exactly at the speed of light.[156] This method combined with using a Galilean frame transformation has been proposed to eliminate the NCI.[147] Others have proposed using FFT solvers along the drifting direction that modify the k operator slightly around where NCI couplings occur.[157] This was extended to using customized higher order finite difference solvers.[158] The goal is to provide a solver with nth order accuracy but to keep more coefficients than are required for this accuracy. The coefficients are then chosen to minimize errors between it and the desired k operator while keeping n-th order accuracy. The use of customized solvers can also be used to minimize errors in the light dispersion as well as to minimize errors in the motion of single electrons as they free stream or interact with intense laser fields. For example, the fields that surround an electron that moves near the speed of light have numerical errors that can lead to self-forces on a single electron or energy spread of a bunch of electrons.[159] These can be mitigated with a customized solver (or a FFT based solver in the propagation direction). It is important that Gauss's law always be satisfied. This can be accomplished by solving Gauss's law directly. It can also be accomplished by advancing the electric field forward in time using Ampère's law so long as the continuity equation is rigorously satisfied. When higher order solvers are used then the current (which is usually obtained using a charge conserving method for second order finite difference solvers) must be modified. This can be done through an extension to charge conserving current deposit,[160] by correcting the second order charge conserving current, or by using an approximate Boris correction to the longitudinal part of the electric field. The choices have trade-offs in speed and accuracy. Very recently, it was proposed to use a semi-implicit finite difference algorithm that uses a grid where the cells are effectively rotated so that one coordinate is aligned along the diagonal of the cell. It was shown that for certain time steps this algorithm seems to have properties similar to the PSATD with a Gallilean frame to eliminate the NCI. There is, thus, no single method that provides high fidelity in all cases, but there are a growing number (we can expect more) of options to choose from.

There are also many new options for the particle pusher. There is the standard relativistic Boris pusher,[161] and several newer pushers[161,162] that are second order accurate in time. The difference between them is how they define the average velocity during a time step for the $\vec{v} \times \vec{B}$ force. In the standard method, the magnetic field also has to be averaged as it is not known at the time step index of the electric field. On the other hand, new field solvers (PSATD or customized[163]) can provide the magnetic field at the appropriate time step with higher accuracy. Others have proposed using subcycling[164] to improve the accuracy of the pusher of particles moving in intense laser fields. Recently, ideas for semi analytical pushers have been developed. The concept is analogous the PSATD method for the fields where analytical solutions are used for constant current. In this case, if the fields (forces) are constant during an interval of time, then the relativistic push can be done analytically. This was shown to be possible when using the proper time.[165] However, recently it has been shown that using the analytic solution one can generate a mapping between the lab frame and proper time for each particle. This mapping can be solved iteratively leading to an analytic pusher which is generally slower than the second order pushers. Thus, as for the field solvers, there are now a variety of options for the pusher with varying degrees of accuracy and speed.

The best choices for the field solver and pusher depend on the problem being studied, and determining the best choices will require experimentation. It is, therefore, important that software provide as many options as possible. The recent advances also make it easier to do convergence tests.

As the applications evolve, new physics must be added to the software. This includes adding radiation reaction (with and without quantum corrections), adding quantum electrodynamics (QED) processes, improving the accuracy of the ionization rates, adding spin as a degree of freedom during ionization, tracking the spin for selected particles as they are accelerated, and developing models for how to handle collisions with ions in the ion column. These models are important for both "standard" and QSA PIC. Recently, radiation reaction (classical and quantum)[166] and QED[167,168] processes have been included in some PIC codes.

It is very challenging to develop complex software that is also computationally efficient. Simulations of PBA are performed on the entire ecosystem of computing resources, including single, many core, and graphics processing units (GPU) servers, and including midscale clusters and leadership class facilities. Leadership class computers consist of millions of cores and thousands of nodes. They are useful if the computational load can be evenly distributed among the cores. This is done by breaking the problem into spatial domains.

Within a given simulation, the number of particles within a given domain can dramatically change in time. If the decomposition of these domains onto nodes and cores is not updated, then the simulation can effectively stall. In addition, the effective use of GPUs requires that data be streamed onto many low level cores and this must also be "load balanced." Developing robust dynamic load balancing routines that also effectively utilize GPUs is daunting. Furthermore, in some problems, the required spatial resolution may vary dramatically in space. Some researchers are investigating how to incorporate adaptive mesh refinement (AMR).[169] For full Maxwell solvers, this can lead to numerical issues with respect to reflection of light and self-forces of particles as they cross into regions with different resolution. AMR could also be useful for QSA based PIC codes. Another issue is determining the appropriate time step to use in each region with difference cell sizes. Thus, AMD brings an additional layer of complexity. Add to this the challenge of adding new physics packages and field solvers while ensuring that the low level parallelization routines continue to work and the computational load is uniformly distributed across 1 000 000 compute cores, and one has a software engineering challenge.

As a result, there are no obvious answers to which choices are the best for algorithms and software development, so there are still several software development efforts. This is not a bad thing, rather it is a requirement for a trustworthy ecosystem of software tools. As the complexity of the software grows it is imperative that more than one software be available to the community and that the software be developed in a collaborative (not necessarily open source) environment that allows new ideas to blossom.

The progress in both new physics modules, improved fidelity in full and reduced models, computational efficiency, and computational power is likely to have a profound impact on research in plasma based acceleration. In some cases, real-time steering of experiments (simulations can be finished in minutes on dedicated clusters) will be possible. Furthermore, end-to-end modeling of some X-FEL designs could become a reality in the not so distant future on leadership class computers. Simulations of multiple stages (or a single afterburner stage) of a plasma accelerator-based LC design including the final focus and disruption/bremsstrahlung at the collision location may also be possible within the next decade on leadership class machines. This will permit a detailed study of the interplay between physics in multi-stage concepts, how design choices at the interaction point affect the required beam parameters at the beginning stages, and tolerances on beam and plasma parameters, perhaps decades before experiments can. The future in simulation capability is very bright.

VII. PERSPECTIVE ON LONG-TERM APPLICATIONS

As we have already discussed, the research on PBA continues to be motivated and largely funded by two long term applications—a LC operating at the energy frontier of particle physics and a fifth generation light source, such as a compact X-FEL. There are "community-driven" studies on the R&D required toward, making the first two goals viable that the readers might find very useful.[170] However, several more near-term applications based on already achieved beam parameters have emerged. At the risk of some repetition, we give our perspective on these two longer-term applications in this section first to give the context and then address the near term applications that might be enabled by PBAs in Sec. VIII.

There is still considerable basic research needed before one can do a technology feasibility study needed for a conceptual design of the two long term applications. In order to comprehend the challenge at hand, the question can be split into key areas where progress is critical: (i) generation of collider-quality e^- and e^+ beams (relatively high charge but ultra-low transverse and longitudinal emittance bunches), (ii) preservation of such bunches while being accelerated inside and transported in-between the plasma stages (e^- and e^+), and (iii) do all the previous with high wall-plug energy efficiency and high repetition rate to enable the needed luminosity.[171]

In principle, the generation of a collider-quality beam can be achieved by injecting polarized electrons or positrons into the damping rings (Fig. 5) to reach the required emittance, as considered for the proposed International linear collider (ILC).[172] Plasmas can play a role here as a new type of photocathode that will enable a new source of ultra-bright beams of electrons by using an auxiliary laser pulse to locally ionize higher-ionization-potential electrons and inject them in a plasma-based accelerator[173–177] or through concepts in which the phase velocity of the wake is controlled. New ideas for positron source have also been proposed,[178] but their viability for a collider remains to be demonstrated.

Once a collider-quality beam has been generated, a plasma-based accelerator, driven by laser pulses (LWFA) or particles beams (PWFA), can be used to bring the particles' energy up to the energy frontier—which is expected to be at 1.5 TeV (3 TeV in the center of mass CM at the collision point) by the time PBA is at a conceptual design report stage for a PB-LC. For electrons, the nonlinear blowout regime is considered as the most promising route as discussed in the theory section, with its ideal field properties[179] that can preserve emittance and absolute slice energy spread because of the linear focusing force and of the radially-independent accelerating field. But for very low emittance, the matched spot size in blowout cavity is at the

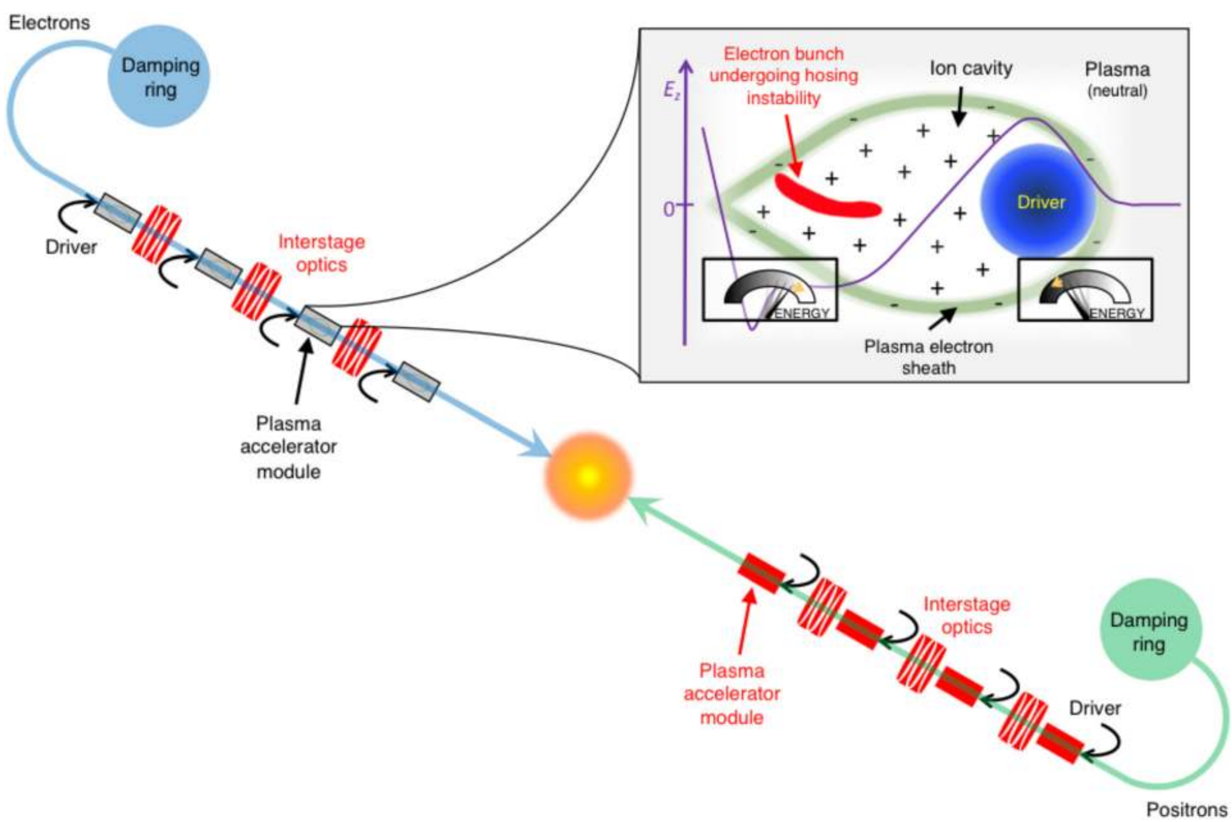

FIG. 5. Schematic of a plasma-based electron-positron collider, highlighting in red some of the key challenges that require basic R&D, namely, hosing instability, staging, and plasma-based positron acceleration (not exhaustive).

nanometer scale, and as a result the electron bunch density greatly exceeds the ion density and leads to ion motion. It was feared that ion motion would induce emittance growth and beam quality degradation; however, recent simulations and theory have shown that the emittance growth can be mitigated for a single plasma accelerator stage.[100] Furthermore, transverse beam instabilities, in particular, the hose instability, can also be a very important limitation for plasma acceleration of high-quality electron beams,[121] as it ultimately leads to beam loss or emittance growth. Several means have been proposed to mitigate this instability[122–124] including a promising route that takes advantage of ion motion to suppress hosing. As a result, it appears that a fully self-consistent beam loading scenario that provides high efficiency and beam quality exists for electrons using an electron beam-driven driver in the nonlinear blowout regime.

While these strategies are very promising for the preservation of collider-quality electron beams inside a single plasma accelerator stage, staging plasma accelerator modules is considered as the main avenue toward high particle energies,[180–182] unless using a driver with extremely high stored energy, such as a teraelectronvolt-class proton bunch,[96] for single-stage acceleration to the designed particle collision energy. When staging plasma accelerator modules, the beam needs to be captured out of each plasma cell to be refocused into the next one while preserving its emittance, which is a significant challenge due to the chromaticity of the focusing elements and tight alignment tolerances into each plasma cell.[183,184] The development of plasma matching sections for in and outcoupling[125–127] is critical to reduce the required length between stages and to mitigate emittance growth.

Solving these critical problems could make plasma-based accelerator a viable technology for the electron arm of an electron–positron plasma-based collider. Yet, positron acceleration in plasma cannot benefit from the field structure within the blowout regime. Although high-field positron acceleration was experimentally demonstrated in plasmas,[6,185] no self-consistent solution for pump depletion distances has been experimentally or computationally demonstrated to-date for quality-preserving positron acceleration necessary for collider parameters. In uniform plasma, the motion of plasma electrons within the positron bunch induces nonlinear focusing and radially-dependent accelerating field that compromises the beam quality, while in hollow plasma channels, strong transverse wakefields[186] may lead to severe transverse instabilities for which a mitigation strategy is yet to be demonstrated. Solving this problem of plasma-based positron acceleration may require to go beyond conventional wisdom, e.g., not aiming for perfectly linear focusing but for a beam equilibrium distribution with acceptable emittance growth from an initially Gaussian distribution, or to consider other means to provide focusing to the positrons, e.g., using electron lensing. Because of these challenges, plasma-based acceleration is also considered for electron–proton or gamma–gamma colliders, for which only electrons need be accelerated in a plasma.

These critical problems are summarized in the schematic of Fig. 5, where some of the main challenges at hand for a plasma-based electron–positron collider are shown in red; e.g., how to deal with the hosing instability in the blowout regime for electron acceleration, how to stage plasma accelerator modules, and how to accelerate positrons in plasmas while maintaining "collider quality."

Finally, for a wall plug power of few hundreds of megawatt at most, reaching collider luminosities exceeding $10^{35}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ is very demanding and in particular, requires high repetition rates and high wall-plug-to-beam energy efficiency.[181] While the energy extraction efficiency from the plasma to the beam can be very competitive,[4] high efficiency might be difficult to achieve simultaneously with instability mitigation[187] and beam quality preservation. Providing drive beams with kilohertz repetition rates or higher and correspondingly high average power is within the capabilities of particle accelerator technology for the PWFA case,[181,188] but considerable advances in laser technology, specifically high-efficiency diode-pump lasers and fiber lasers, are required to fill the technology gap for the LWFA case. This will also require the development of adequate plasma devices and diagnostics. The ultimate repetition rate accessible by plasma-based accelerators will depend on the plasma recovery time, i.e., on the long timescale evolution of the plasma. The full picture of the temporal evolution of the plasma itself, from femtosecond to microsecond time scales, including wakefield excitation, ion motion,[189] diffusion, thermalization and hydrodynamic processes, that can be accompanied by various instabilities, is yet to be investigated in detail.

This panorama of the R&D discovery science that remains to be addressed for the long-term application of a PB-LC showcases the importance of developing early applications that are enabled by the electron beams that are routinely being produced by PBAs. These beams in turn can be used to generate directional incoherent and possibly coherent x-rays beams. FEL is one of two flagship applications of both PWFA[173,190] and LWFA.[191–194] The experimental demonstration of FEL light from a plasma accelerator, most likely in the UV to XUV spectral range as a first step for the LWFA case, will be a game changer for the field and will trigger an exciting and intense development and investment toward plasma-based angstrom-wavelength saturated FEL. Such an achievement will represent a major accomplishment for plasma accelerators—making a transition from accelerator research to accelerator technology for applications in sciences, medicine and industry.

Using electron beams from a plasma-based accelerator (either beam or laser driven) to drive an FEL is, however, extremely challenging due to the very demanding requirements of free-electron lasers and of the specific properties of electron beams produced to-date[195] from laser-plasma accelerators (relatively large energy spread, divergence and shot-to-shot fluctuations). Assuming that the electron beam brightness can be increased close to the value needed for FEL gain, the femtosecond duration beam generated by the plasma-based accelerator has to be transported to a magnetic undulator where the electrons' trajectory is bent in a periodic fashion, leading to the emission of synchrotron radiation (see Fig. 6). In the undulator, the interaction between the electron bunch and the seed radiation at the resonance wavelength (usually produced within the undulator) can lead to a microbunching instability, i.e., lasing, but only if the slice energy spread is narrower than the lasing bandwidth. Although recent simulations have shown that plasma-based acceleration can provide normalized brightnesses in excess of 10^{20} and energy spreads less than 1%, it is unlikely that these parameters will be achieved experimentally within the next five years. To handle the relatively large energy spread of current LWFA electron beams, the use of transverse gradient undulators[196] or of a decompression chicane combined with chromatic matching[196–198] (see Fig. 6) has been proposed and are currently implemented in ongoing LWFA-based FEL projects as a way to effectively reduce the slice energy spread. Because matched electron beams in a plasma wakefield have small beta functions, they typically exit the

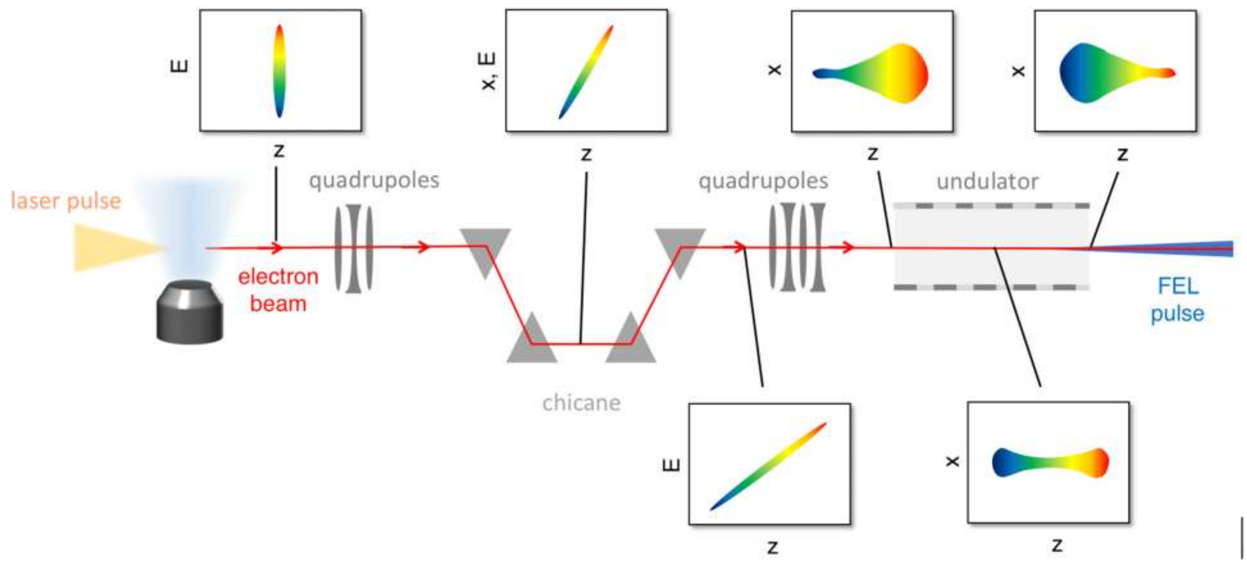

FIG. 6. Schematic of a free electron laser based on a laser-plasma accelerator, using a decompression chicane. At the exit of the laser-plasma accelerator, the electron beam has small bunch length and large energy spread. After the four dipoles of the chicane, the bunch is stretched to a longer bunch length, with high energy electrons at the front, low energy electrons at the rear, and a very small slice energy spread. In the undulator, chromatic matching is used so that low energy electrons are focused at the undulator entrance, electrons at the central energy are focused in the middle, and high energy electrons at the end of the undulator, in a way that the FEL radiation slips along the bunch and stays overlapped with the focused part of the electron bunch.

plasma accelerator with a divergence that is unusually large when compared to conventional accelerators, and this can lead to two potential detrimental effects. First, it induces a large emittance growth due to chromatic aberrations of the focusing elements, which can be mitigated by using very compact permanent quadrupoles[199] with variable strength[200] or plasma lenses[193,201,202] very close to the plasma accelerator and by taking advantage of chromatic matching.[197,198] Second, electrons exiting with large angles cover more path to reach the undulator than those on-axis, resulting in a coupling between angle and longitudinal position along the bunch, which increases the effective bunch length and, when using a decompression chicane, can be the dominant contribution to the slice energy spread. This second effect, where divergence at the plasma accelerator source induces slice energy spread in the undulator, is of paramount concern for the experimental demonstration of first FEL light from a plasma accelerator, and highlights the critical need for plasma matching sections[125] and ultra-compact transport elements.[193,199,200,202]

First steps toward LWFA-based FEL were obtained by observing spontaneous synchrotron radiation emitted by the electrons in the undulator with a rudimentary beam from system from the plasma accelerator to the undulator, at visible[203] and XUV[204] wavelengths. Enormous effort was then invested in the control and optimization of the electron beam transport from the plasma accelerator to the undulator, minimizing the slice energy spread in the undulator using a design such as the one shown in Fig. 6 with a decompression chicane and chromatic matching, and aimed at mitigating the initial weaknesses of the electron source (energy spread, divergence and shot-to-shot fluctuations) and approaching the electron parameters at the undulator entrance necessary for a FEL proof-of-principle demonstration. Such successful transport and control were achieved[191] and allowed observation of high-quality spontaneous synchrotron radiation with its distinctive spatio-spectral purity,[205] only accessible with a properly tuned transport system. Simulations indicate that, coupled with electron divergence of 1 mrad (rms) or less, and electron spectral charge densities of few picocoulomb/megaelectronvolt or more, these results should enable the first experimental demonstration of FEL gain from electrons at the 200 MeV level and FEL in the UV to XUV spectral range.

Finally, as plasmas can be harnessed as injectors and accelerators of ultrabright electron beams, with 6D brightnesses that are beyond the state-of-the-art of conventional accelerators,[206] they hold the promise of not only compact- with much shorter gain length and total undulator length- but also very high performance FEL light sources of unprecedented brightness. From our perspective it is possible that a self- injection scheme could produce electron beams with normalized brightness of 10^{20} A/m^2/rad^2, normalized emittances of 10 nm, energy spreads $<1\%$, peak currents >10 kA, and energies in excess of 1 GeV within the next decade with concerted effort by the community.

VIII. NEAR TERM APPLICATIONS

A. Incoherent directional x-ray and gamma-ray beams

Laser-plasma accelerators are already enabling near term development of a novel class of incoherent and directional x-ray and gamma-ray sources, spanning photon energies from sub-kiloelectronvolt to tens of megaelectronvolt.[10] These light sources can then be used in a broad range of scientific and societal applications, from imaging with absorption and phase contrast tomography in life sciences,[10,207–211] time-resolved x-ray absorption spectroscopy at femtosecond timescale for warm dense matter,[212,213] gamma-ray radiography,[214–218] nuclear medicine, nuclear inspection, non-destructive material inspection and ultrafast probing of high energy density science (HEDS).[219]

Several physical mechanisms can be leveraged to produce x-rays and gamma rays from laser-plasma accelerators. They are summarized in Fig. 7 and differ in the way electrons from the plasma accelerator are forced to wiggle transversely. The basic principle is that a relativistic electron that experiences a transverse acceleration radiates light at very short wavelength, in particular, due to the relativistic Doppler shift.[10] This transverse oscillatory motion occurs naturally in a laser-plasma accelerator due to the strong focusing force of the blow-out cavity (see Fig. 7 left), and leads to the production of the so-called betatron radiation,[68,219–222] whose properties depend on the plasma density, electron energy and transverse amplitude of the betatron oscillation and its spectral range is typically in the 1–100 K eV range. Alternatively, electrons from the laser-plasma accelerator can scatter counter-propagating laser photons (see Fig. 7 center) by inverse-compton (IC) scattering.[223–225] The IC scattered photons form a directional beam, typically ranging from a few kilovolt to tens of megavolt depending on the electron energy. Finally, gamma rays can also be generated efficiently by bremsstrahlung in a high-Z conversion foil[216] (see Fig. 7 right). In this case, electrons from the laser-plasma accelerator are deflected by the electric field of the nuclei when traversing a high Z foil emitting radiation that has a spectrum that extends up to the energy of the highest energy electron. While some of the properties of this LWFA-based bremsstrahlung radiation source (in particular, source size, divergence and spectral bandwidth), are not as competitive as those of betatron and compton radiation sources, the gamma-ray yield and source size (30 μm)[225] make this source very relevant for high-resolution gamma-ray radiography.

To make these x-ray and gamma-ray sources compatible with a wide variety of applications, two approaches are being pursued that may enable transformative applications in the near term. First, a large effort is ongoing toward high repetition rate laser systems, where the high average power has to be properly handled. The availability of LWFA-based betatron, Compton and bremsstrahlung sources at 10 Hz, 100 Hz or even beyond will considerably help in filling the gap between performance of today's sources and average power required by the applications. Second, pushing the efficiency, yield, and brightness of these sources with a better control and optimization of the laser-plasma interaction is also critical. For instance, the use of density tailored plasmas[22,226] or staging a LWFA accelerator into a PWFA radiator[227] can considerably boost the betatron radiation yield and efficiency, and nanocoulomb-class electron bunches produced by direct laser acceleration (DLA)[228–230]—a process analogous to the inverse free electron laser acceleration—can be used to increase the yield of Compton scattering and bremsstrahlung radiation. Recent experimental advances have shown that, by shaping both the longitudinal and the transverse plasma density profiles, betatron radiation can be considerably improved, with critical photon energy and yield boosted by up to an order of magnitude.[231] Combining these brighter plasma-based femtosecond x-ray and gamma-ray sources with high repetition rate lasers will make many near term applications mentioned above a reality.

B. Role of PBA in high-energy density science (HEDS)

In high energy density science, x-ray and gamma-ray beams are valuable tools to probe transient state of matter in extreme conditions of temperature, pressure and density, which is of fundamental importance for inertial confinement fusion, planetary physics and astrophysical systems. X-ray radiography of extremely dense targets is a common tool for diagnosing shock propagation, visualization and quantifying the growth of radiation-driven and hydrodynamic instabilities, estimating the ρ-r product of an inertial confinement fusion target with compressed density ρ and radius r and tomographic imaging of the onset of fatigue, void formation and other changes to materials exposed to hostile environment. We have already discussed above how betatron, IC-scattered or bremsstrahlung radiation generated using PBA generated ultra-short electron bunches is being developed for applications such as phase-contrast microscopy[232] using kiloelectronvolt x-rays on one hand and differential absorption spectroscopy using 100+ keV x-rays generated by IC or bremsstrahlung on the other hand.[233] For HEDS applications, however, much larger x-ray fluxes than what a typical LWFA based source can provide are required because the charge per bunch is typically $\leq$ 100 pC.

To reach the required photon yield and go beyond the capabilities of LWFA-based femtosecond x-ray and gamma-ray sources, laser systems with higher laser energy and picosecond duration can be leveraged to provide electron charge in the tens of nC range and x-ray

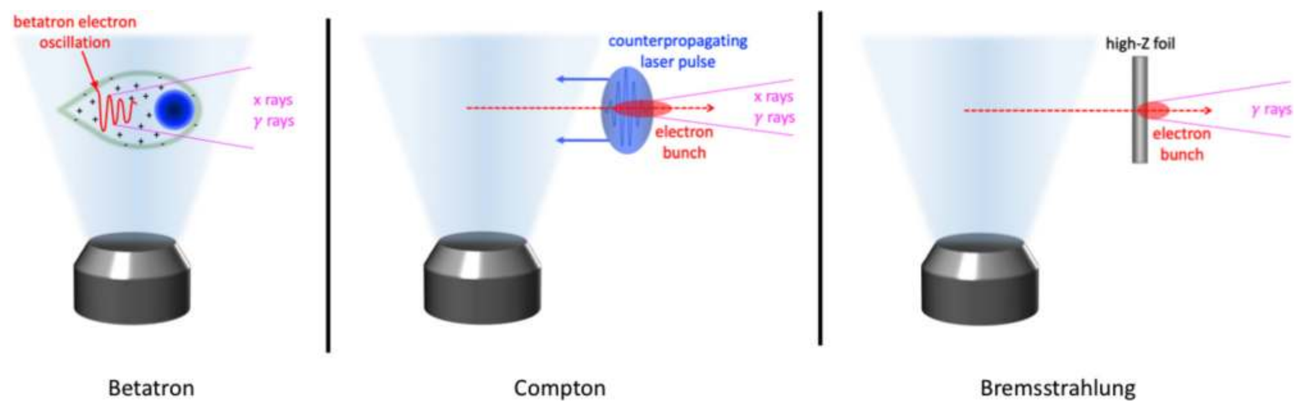

FIG. 7. Principle of betatron, inverse-compton and bremsstrahlung radiation sources. In the first case (left), electrons oscillate in the blow-out cavity of the laser-plasma accelerator and radiate betatron x-rays. For the inverse-compton source (middle), at the exit of the laser-plasma accelerator, electrons radiate IC scattered photons during their oscillations in the field of a counterpropagating laser pulse. Bremsstrahlung radiation (right) can be produced by placing a high-Z foil in the path of the electron beam.

and gamma-ray yield up to two orders of magnitude larger than those obtained using femtosecond laser pulses, and even three orders of magnitude for the Compton source (due to the longer interaction time with the counter-propagating laser pulse), while maintaining sufficient spatial and time resolution for applications to high energy density experiments.[234] Relying on the self-modulation instability and contribution from direct laser acceleration, picosecond 100-J-class laser-driven plasma accelerators have experimentally demonstrated that x-ray and gamma-ray sources can be produced with betatron, with inverse Compton scattering and bremsstrahlung mechanisms. These results open the way to their applications at large-scale laser facilities where laboratory high energy density science is under investigation.

C. Radiotherapy with very high energy electrons (VHEE)

Radiation therapy is a well-established tool for cancer treatment, which aims at killing malignant tumor cells while minimizing the dose received by the surrounding healthy tissue. The dose that quantifies the amount of energy deposited in tissue per unit mass is a critical parameter in radiotherapy and controlling the spatial dose distribution is the key to maximizing the energy deposition in the malignant tumor. Today the most common radiotherapy uses megavolt photon beams (at multiple angles) that overlap at the location of the tumor as the ionizing radiation. Proton beams have favorable ballistic properties as they have a finite penetration range with a characteristic Bragg peak in the dose deposition depth profile [see Fig. 8(a)], yet proton therapy is less common because it requires large scale accelerator facilities and gantries, and is therefore rather expensive. In the last two decades, relativistic laser-plasma interaction has been actively studied for creating for a compact proton therapy tool,[235] but the required proton energy has so far been elusive. The use of laser wakefield accelerated electrons for radiotherapy is now also being considered as a possible application of laser-plasma accelerators.[236–239] For low energy electrons (clinical electron beams with energy ranging from 5 to 20 MeV), the dose is deposited over a short range, and important lateral scattering occurs, increasing the dose deposited in nearby healthy tissue and decreasing the clinical efficacy of the treatment plan. Very high energy electrons (VHEE), corresponding to 100 to 250 MeV in the medical context, can penetrate deeply (see Fig. 8) with sufficiently small lateral scattering and, when used from multiple angles, are a promising choice as a source of ionizing radiation for the treatment of deep-seated (e.g., prostate) tumors.[240–242] Indeed, many studies have shown the clinical advantage of VHEE treatment plans over photon plans. For example, for the case of a treatment plan of the prostate cancer, the comparison between 6 MV photons and 250 MeV electrons shows that with VHEE the dose in healthy tissue is decreased by 20% with respect to 6 MeV photons (see Fig. 8).[243–245] While VHEE is not currently used for cancer treatment, its potential for radiotherapy is seriously considered[246] and, in addition to the laser-plasma accelerator approach discussed here, its realization using conventional accelerator technology to produce very high energy electron beams is also pursued.

In laser-plasma, accelerators readily generate electron beam parameters required for VHEE radiotherapy. For example, an electron bunch with 30 pC of charge with energy spread of a few percent at 10 Hz is sufficient to deliver the required dose in about a minute.[240] To make laser-plasma based VHEE radiotherapy a reality, there are a number of engineering issues that need to be addressed, such as filtering and providing shielding from the unwanted low energy electrons, electron monochromator to choose electrons with a certain energy and energy spread and a magnetic beam transport system[237] that can be integrated in a gantry. But the real challenge lies in the demonstration of an electron pre-clinical beam line that is robust, reliable and can deliver the required beam parameters consistently over 24 h for days of continuous operation, with minimal required maintenance, as well as being cost effective. The laser-plasma community has already made significant progress in this direction, transforming proof-of-principle research experiments into stable and controllable accelerator

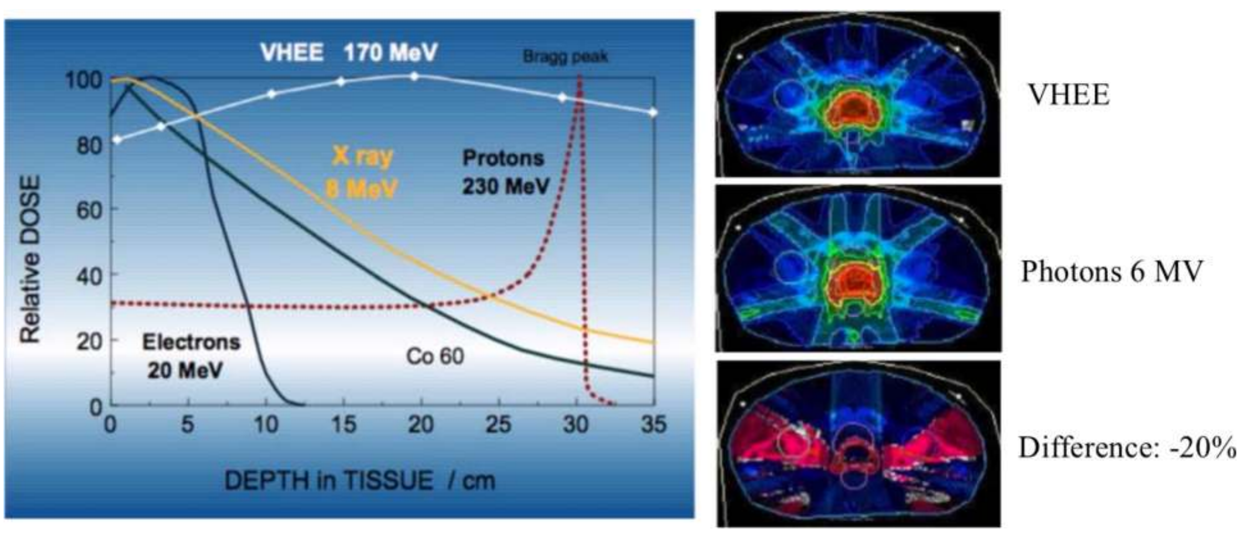

FIG. 8. Radiotherapy with very high energy electrons: (left) depth profiles of dose deposition in tissue for protons, photons, low energy electrons (20 MeV) and very high energy electrons (170 MeV). Comparison between treatment plans with high energy electrons (VHEE) and photons (6 MV). Reproduced with permission from Malka *et al.*, Mutat. Res./Rev. Mutat. Res. **704**, 142–151 (2010). Copyright 2010 Elsevier.

operation, for instance with the LUX facility[192] that has demonstrated stable continuous operation and repetition rate of up to 5 Hz. Furthermore, the adequate control over the spatial distribution of the dose will also represent a major milestone for showing the viability of laser-plasma accelerators for VHEE radiotherapy.

Finally, by using electron beams from a laser-plasma accelerator, it is also possible to take advantage of the temporal distribution of the dose, using fast dose fractionation[247] and high dose rate, as for example in the so-called FLASH effect[248–250] in which there is a reduced toxicity on healthy tissue while the effect on the malignant cells in the tumor is preserved. This temporal control and the extremely high dose rate that comes along with the ultrashort (femtosecond) duration of electron beams from laser-plasma accelerators, could become an additional key benefit of LWFA-based VHEE radiotherapy.

D. Spectroscopy enabled by tunable mid-infrared (IR) radiation pulses

Until now, we have discussed possible applications arising from electrons accelerated by PBA or from the radiation generated by those electrons. However, in the case of wakes excited by a laser pulse, an entirely new type of radiation source is possible. This source can generate continuously tunable, near-single cycle intense coherent radiation pulses in the long-wavelength infrared region (LW-IR) from 5 to 20 μm.[54] The physical process that makes this possible is asymmetric self-phase modulation of the drive laser pulse that.[251–253] The frequency downshifted photons have a slower group velocity than the initial laser photons; therefore, they slip backward. The lowest frequency photons enter the plasma wake cavity that is nearly devoid of any plasma electrons. The plasma cavity acts as a low loss container where these long wavelength components phase lock to form a nearly transform limited pulse.

Recently this concept has been realized in the laboratory where near single cycle, relativistic pulses have been produced in the entire LW-IR region[54] A plasma source with a density upramp is used to first compress the 50 fs, 0.8 μm laser pulse to less than 10 fs using density gradients associated with a plasma wake in the low density plateau region. The self-compressed pulse then traverses a second density (up) ramp and enters a much shorter but higher density region where it now undergoes the asymmetric self-phase modulation followed by frequency dependent group velocity dispersion as described above.

Such pulses have already been used to probe the wake dynamics itself and hold great promise for broadband (or conversely impulse) Raman spectroscopy, attosecond science and pump–probe experiments in the molecular fingerprint region.

E. Role of plasma-based accelerators in strong field-quantum electrodynamics (SF-QED) experiments

There is considerable interest in understanding QED in the non-perturbative regime that becomes accessible when a relativistically intense laser pulse is collided with a highly relativistic electron beam.[167,168,254,255] This regime is reached when $\chi = E/E_{cr} \geq 1$. Here E is the rest frame electric field and $E_{cr} = 1.3 \times 10^{18}$ V/m is the so-called QED critical field. In this regime one can experimentally observe quantum radiation suppression, multiple photon emission and quantum radiation reaction effects.[256] For beam and laser parameters available at FACET II, i.e., 10 GeV beam and 20 TW laser, $\chi \approx 0.6$ $\varepsilon(10\,\text{GeV})\,\sqrt{(I\,[10^{20}\,\text{W/cm}^{-2}])} \approx 0.6$, close to the desired value of ≥ 1. The easiest way to clearly be in the non-perturbative regime is to either increase the laser power to >100 TW level or to use the 20 GeV beam expected from FACET II E300 experiment.[171] The successful use of the latter will be a new research application of PBA.

IX. PERSPECTIVE ON THE FUTURE

So what is likely to happen in the near future in PBA research? In the United States, the research in this field has traditionally been funded by the high energy physics (HEP) branch of the Department of Energy (DOE) at national laboratories such as at Brookhaven, Argonne, SLAC National Accelerator Laboratory and Lawrence Berkeley National Laboratory and at number of universities throughout the U.S. As experiments have become more advanced and complex much of the work is likely to become consolidated at DOE's flagship facilities for advanced acceleration research called, FACET II at SLAC National Accelerator Laboratory and BELLA at Lawrence Berkeley National Laboratory. This consolidation will focus on systematic exploration of science and engineering issues with the goal of a future plasma-based particle collider. For example, the FACET II 10 GeV electron beam facility will address the issues of minimizing the energy spread, maintaining the emittance and throughput of a high charge electron bunch while adding a 10+ GeV energy in a single stage of a PWFA. In addition, PWFA experiments will be aimed at demonstrating a high (>40%) drive to trailing bunch energy transfer efficiency, while energy depleting the pump beam. BELLA has the ability to demonstrate a meaningful staging experiment where the accelerating beam is shown to gain 5+ GeV energy per stage without significant loss of charge and increase in emittance. These are very difficult yet important goals. Several other competing facilities dedicated to advanced acceleration techniques and light sources such as FLASHForward (DESY, Germany) and CoReLS (Korea) have already come online while many others such as EuPRAXIA and ELI Pillars (in Europe) and many others in Asia (China, Japan, and India) will be coming on-line in the next several years. All these facilities are expected to have plasma acceleration as well as the demonstration of FEL action using electron beams produced by plasma accelerators in their research portfolio.

There are many discovery science topics yet to be addressed. The LC application may require the use of spin polarized electron and positron beams. Typically spin polarized positron beams are generated by first producing undulator radiation in the multi-megavolt range that with angular momentum using a 10 GeV class electron beam. These x-rays then decay via pair production in a high Z target. The resulting spin polarized electrons are then collected, cooled in a storage ring and accelerated in a normal RF accelerator. To-date no viable alternative scheme (including by using a PBA) for generating a spin-polarized positron bunch has been put forward. As for generating a spin polarized electron beam using a PBA is concerned several ideas have been put forward. One idea uses injecting spin polarized electrons, generated by ionizing pre-aligned highly polarizable molecules, in a plasma wake using density downramp injection.[257] Another idea proposes to create a spin polarized electron bunch *in situ* by employing ionization injection using spin-dependent ionization rates.[258] The latter scheme is closely related to how polarized electron beams are generated using a GaAs photoinjector in conventional accelerators. These schemes are interesting because they have the potential of generating synchronized (to the wake), high current and low emittance bunches.

Current LC designs also rely on small angle collision of asymmetric emittance flat beams because the synchrotron radiation (beamstrahlung) produced in the interaction region is a factor of 10 less for flat beams than for round beams. Presently asymmetric beams are unintentionally generated as a consequence of asymmetric drivers or different radiation loss in the two transverse planes by betatron radiation, etc., and have rather large emittances. Intentional generation and acceleration of low emittance flat beams is a completely open issue. The parameters of PBA generated beams are unique, and if a paradigm for using round beams can be found, then this would significantly impact the physics within each plasma stage.

Strategies for emittance preservation (including the mitigation of hosing) throughout a multi-stage PBA accelerator are arguably the most pressing current problem. The first challenge is how to generate ultra-low emittance beams and how to inject, accelerate, and extract them from a PBA. One approach is to use the current photo-injector based technology to produce ultra-low emittance and possibly polarized electron bunches, and then to externally inject them in a number of PBA stages each providing an energy gain of 10–20 GeV. The difficulty is how to get both high a charge (0.5–1 nC) in a very short [0 (1 μm σ_z)] bunch and extremely small transverse and longitudinal emittance at the same time? The R&D for the proposed International linear collider (ILC) will go a long way toward providing such bunches. A second approach is to generate such bunches within the PBA itself by a number of wake injection techniques such as colliding pulse injection, downramp-injection and ionization injection. Each has its pros and cons. In the case of a PWFA both the drive and the trailing bunches may develop longitudinal microstructure due to the coherent synchrotron instability. This in turn may seed the hosing instability. Techniques for the suppression of both these instabilities are of paramount importance to PBA research.

New concepts for compact positron beam sources and clever methods for accelerating positrons in plasma structures to give a collider quality beam are also an unsolved problem as has already been stated throughout this paper.

In this paper, we have given our personal perspectives on the status of and challenges for the PBA field. These challenges must be overcome for continuing the march toward the long term grand challenges of a LC at the energy frontier and a fifth generation compact X-FEL. Fortunately, several important basic science and societal applications have emerged that offer near term opportunities for this field. We have discussed these near term applications for the electron beams, radiation generated by the electron beams and tunable radiation generated by the laser pulse. In our opinion these near term applications could be developed within the next decade with a concerted effort by the PBA community.

AUTHORS' CONTRIBUTIONS

All authors contributed equally to this work. All authors reviewed the final manuscript.

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This paper is not intended to be a comprehensive review of this subject. The authors apologize in advance if some important work has unintentionally not been included here. The perspectives are those of the authors only.

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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