The uniqueness of the solution can be proved as follows: Suppose that $\Phi_1(x)$ and $\Phi_2(x)$ are two solutions such that

$$\Phi_1(x) = f(x) + \int_0^\infty K(x, s)\Phi_1(s)ds,$$

$$\Phi_2(x) = f(x) + \int_0^\infty K(x, s)\Phi_2(s)ds. \tag{7}$$

Denote $\Phi_1(x) - \Phi_2(x) = \Psi(x)$; we have by subtraction

$$\Psi(x) = \int_0^\infty K(x, s)\Psi(s)ds. \tag{8}$$

This is a Fredholm integral equation without a free term. Consequently, by (3) the solution is

$$\Psi(x) = 0 \quad \text{or} \quad \Phi_1(x) = \Phi_2(x), \tag{9}$$

so that the equation (1) has a unique solution.

The preceding proofs of convergence and uniqueness can also be extended to the solution of the system of equations for a semi-infinite strip subjected to more general symmetrical loadings [6].

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# journal of applied mechanics book reviews

**Analysis of Turbulent Boundary Layers.** By Tuncer Cebeci, and A. M. O. Smith. Academic Press 1974. 404 Pages. Cost $35.00.

### REVIEWED BY P. A. LIBBY[1]

Turbulent shear flows such as boundary layers, mixing layers, wakes and jets occupy a central position in the spectrum of fluid mechanical phenomena. Because of the essential difficulty of closure, no complete theory, methodology, or phenomenology for the analysis of these flows exists or is to be expected. However, the advent of high-speed computers has permitted more complete and detailed treatments to be developed than was possible in the past; thus the current literature on turbulent shear flows is replete with new phenomenology and applications to complex flow situations beyond consideration a few years ago.

The authors of the present book have developed, over a period of years, one of the new methods of analysis of turbulent boundary layers. Since they have been employed in a research department of an aircraft company, the motivation for their work has been to provide more accurate, yet practical, means of estimating skin-friction, heat transfer, and separation on aircraft components under flight conditions. Their approach has been based on careful, detailed, yet empirical correlations of the several turbulent transport coefficients which can be introduced into the describing equations for the mean quantities in turbulent flows. With these correlations and appropriate numerical techniques, calculations of turbulent boundary layers under a variety of conditions can be carried out. Only two-dimensional and axisymmetric layers are considered.

The authors devote the first half of the book to a review of turbulent phenomena, to the development of the boundary-layer equations, to a discussion of the general features of turbulent boundary layers, and to a brief (23 pages) discussion of various current methods for the treatment of such layers. This material, somewhat expanded, is typical of that given in a one year, senior-masters level course in engineering turbulence. The second half of the book describes the experimental correlations and the numerical techniques developed by the authors for the turbulent boundary layer.

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This book will be useful to practicing engineers, not only for the detailed presentation of the authors' method but for the careful presentation of the available experimental data and for the bibliography. More research-oriented engineers, engineering scientists, and turbulence workers involved with the more advanced methods associated with "second-order-closure" will consider this book a statement of the situation relative to the phenomenology of turbulent shear flows circa 1965, but will still find the data and bibliography useful.

**Perturbation Method in Fluid Mechanics.** By Milton Van Dyke. 1975 Publication. Parabolic Press. P.O. Box 3032, Stanford, Calif. 94305. Cost $7.00. 274 Pages.

### REVIEWED BY S. ROSENBLAT[2]

The first edition of Van Dyke's book *Perturbation Method in Fluid Mechanics,* published in 1964, introduced the techniques of singular perturbation theory to a whole generation of research workers. Since that time, as the author points out in the preface to the Annotated Edition (1975, The Parabolic Press, Stanford, Calif.), these techniques have become "part of the analytical apparatus of anyone interested in research. . . . . The resulting research papers, even if restricted to fluid mechanics, number in the thousands." In this sense Van Dyke's book, which was the first systematic account of the subject, could well be regarded as one of the most influential books of its time.

The Annotated Edition differs from the original mainly through the insertion of 34 pages of Notes on the text. The Notes draw attention to mathematical and procedural difficulties which have come to light during a decade of application of the techniques; and they comment on recent results pertaining to particular problems. In addition the bibliography has been substantially, though selec-

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tively, expanded. Apart from these addenda and some minor corrections the text remains unchanged.

Van Dyke's book is no longer the only one which treats the subject of singular perturbations. Moreover the problems on which the techniques are demonstrated are not quite as central to the mainstream of fluid mechanics as they were in 1964. Nevertheless this book continues to provide a valuable first glimpse of singular perturbation theory for prospective researchers.

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**Linear and Nonlinear Waves.** By G. B. Whitham. Published 1974. Cost $22.50. John Wiley and Sons, Inc., Publishers, 605 3rd Avenue, New York 10016. 636 Pages.

### REVIEWED BY T. C. T. TING[3]

This book is divided into two parts, not into linear and nonlinear waves as one might guess from the title of the book, but into hyperbolic and dispersive waves. Hyperbolic waves are waves whose partial differential equations are of hyperbolic type. In fact, the wave motion in most mathematical books is identified synonymously with hyperbolic equations. Thus hyperbolic waves are defined by their governing equations. Dispersive waves are, on the other hand, defined by their solutions. A linear dispersive system is any system which admits solutions of the form

$$\phi = a \cos (kx - \omega t)$$

in which the frequency $\omega$ is a real function of the wave number $k$ and $d^2\omega/dk^2 \neq 0$. Clearly, the partial differential equations for dispersive waves can be hyperbolic. They can be parabolic as in the case of waves in a beam, or even elliptic as in the ocean waves. The fact that the book is divided almost equally into hyperbolic waves and dispersive waves seems to underscore Professor Whitham's remark which says: "It is probably fair to say that the majority of wave motions fall into the dispersive class."

Introduction and general outline of the book are presented in Chapter 1. Part I on Hyperbolic Waves is presented in Chapter 2–10. Part II on Dispersive Waves occupies Chapters 11–17. The titles of the chapters are as follows: 2 Waves and First-Order Equations; 3 Spccific Problems; 4 Burgers' Equation; 5 Hyperbolic Systems; 6 Gas Dynamics; 7 The Wave Equation; 8 Shock Dynamics; 9 The Propagation of Weak Shocks; 10 Wave Hierarchies; 11 Linear Dispersive Waves; 12 Wave Patterns; 13 Water Waves; 14 Nonlinear Dispersion and the Variational Method; 15 Group Velocities, Instability, and Higher-Order Dispersion; 16 Applications of the Nonlinear Theory; 17 Exact Solutions: Interacting Solitary Waves.

The book covers all the major well-established ideas. The nonlinear theory is emphasized from the outset. Most of the typical techniques for solving problems are presented, but these are not pursued to the point where they cease to give information about the nature of the waves and become exercises in mathematical methods. The mathematical ideas are interspersed with discussion of specific physical fields to illuminate the mathematical arguments. Most physical examples are taken from fluid dynamics, a reflection of Professor Whitham's personal interest and contributions in the area. One of Professor Whitham's many contributions is nonlinear dispersive waves and the variational method. This is covered in Chapter 14.

The book is unique. It will be extremely valuable to anyone who is working in the area of wave propagation. It is a truly applied mathematics book in the sense that the mathematical theories and the physical ideas are well blended. One learns not only the mathematical techniques from this book, but also the physical interpretation of the wave phenomena.

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