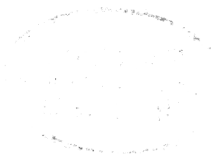


Bhimsen K. Shivamoggi

Perturbation Methods for Differential Equations



Birkhäuser
Boston • Basel • Berlin

Table of Contents

Preface	xi
Chapter 1 Asymptotic Series and Expansions	1
1.1 Introduction	1
1.2 Taylor Series Expansions.....	2
1.3 Gauge Functions.....	2
1.4 Asymptotic Series and Expansions.....	4
1.5 Asymptotic Solutions of Differential Equations	10
1.6 Exercises	11
Chapter 2 Regular Perturbation Methods	13
2.1 Introduction	13
2.2 Algebraic Equations	14
2.3 Ordinary Differential Equations	22
2.4 Partial Differential Equations	29
2.5 Applications to Fluid Dynamics: Decay of a Line Vortex	31
2.6 Exercises	33
2.7 Appendix. Review of Partial Differential Equations	35
A1. Transformation to Canonical Forms	35

Chapter 3 The Method of Strained Coordinates/Parameters	41
3.1 Introduction	41
3.2 Poincaré-Lindstedt-Lighthill Method of Perturbed Eigenvalues.....	42
3.3 Eigenfunction Expansion Method	52
3.4 Lighthill's Method of Shifting Singularities	55
3.5 Pritulo's Method of Renormalization	59
3.6 Wave Propagation in an Inhomogeneous Medium	63
3.7 Applications to Solid Mechanics: Nonlinear Buckling of Elastic Columns.....	65
3.8 Applications to Fluid Dynamics	71
3.8.1 Nonlinear Hyperbolic Waves in a Gas	71
3.8.2 Rayleigh-Taylor Instability of Superposed Fluids.....	75
3.9 Applications to Plasma Physics	82
3.9.1 Nonlinear Waves in an Electron-Plasma.....	82
3.9.2 Rayleigh-Taylor Instability of a Plasma in a Magnetic Field.....	84
3.10 Limitations of the Method of Strained Parameters	91
3.11 Exercises	93
3.12 Appendix 1. Fredholm's Alternative Theorem	95
3.13 Appendix 2. Floquet Theory	97
3.14 Appendix 3. Bifurcation Theory.....	107
 Chapter 4 Method of Averaging	 113
4.1 Introduction	113
4.2 Krylov-Bogoliubov Method of Averaging.....	114
4.3 Krylov-Bogoliubov-Mitropolski Generalized Method of Averaging	118
4.4 Whitham's Averaged Lagrangian Method.....	122
4.5 Hamiltonian Perturbation Method	125
4.5.1 Systems with Constant Parameters.....	125
4.5.2 Systems with Slowly-Varying Parameters.....	130
4.6 Applications to Fluid Dynamics: Nonlinear Evolution of	

	Modulated Gravity Wave Packet on the Surface of a Fluid	135
4.7	Exercises	139
4.8	Appendix 1. Review of Calculus of Variations.....	141
	A1.1 Functionals with Second-Order Derivatives.....	141
	A1.2 Functionals with Higher-Order Derivatives	144
	A1.3 Functionals with Several Independent Variables	145
4.9	Appendix 2. Hamilton-Jacobi Theory	148
	A2.1 Hamilton's Equations	148
	A2.2 Canonical Transformations	150
	A2.3 Hamilton-Jacobi Equation.....	152
	A2.4 Action-Angle Variables.....	153
 Chapter 5 The Method of Matched Asymptotic Expansions.....		155
5.1	Introduction	155
5.2	Physical Motivation.....	155
5.3	The Inner and Outer Expansions	158
5.4	Hyperbolic Equations.....	166
5.5	Elliptic Equations	176
5.6	Parabolic Equations.....	179
5.7	Interior Layers	181
5.8	Latta's Method of Composite Expansions	185
5.9	Turning Point Problems	190
	5.9.1 JWKB Approximation	190
	5.9.2 Solution Near the Turning Point.....	194
	5.9.3 Langer's Method	197
5.10	Applications to Fluid Dynamics: Boundary-Layer Flow Past a Flat Plate	199
	5.10.1 The Outer Expansion	199
	5.10.2 The Inner Expansion	201
	5.10.3 Flow Due to Displacement Thickness.....	206

5.11	Exercises	207
5.12	Appendix 1. Initial-Value Problem for Partial Differential Equations	209
5.13	Appendix 2. Review of Nonlinear Hyperbolic Equations	211
Chapter 6 Method of Multiple Scales		219
6.1	Introduction	219
6.2	Differential Equations with Constant Coefficients	219
6.3	Struble's Method	243
6.4	Differential Equations with Slowly Varying Coefficients	252
6.5	Generalized Multiple-Scale Method	262
6.6	Applications to Solid Mechanics: Dynamic Buckling of a Thin Elastic Plate	265
6.7	Applications to Fluid Dynamics	273
6.7.1	The Problem of Aerodynamically Generated Sound	273
6.7.2	Mathematical Formalization of Lighthill's Theory of Aerodynamically Generated Sound	276
6.7.3	Nonlinear Shallow Water Waves	281
6.7.3(a)	Governing Equations	282
6.7.3(b)	Korteweg-deVries Equation	284
6.7.3(c)	Solitary Waves	286
6.7.3(d)	Stokes Waves	288
6.7.3(e)	Perturbed Solitary Wave Propagation	290
6.7.3(f)	Wave Modulation and Nonlinear Schrödinger Equation	295
6.7.3(g)	Long-Time Evolution of the Modulation	299
6.7.3(h)	Second-Harmonic Resonance	303
6.8	Applications to Plasma Physics	307
6.8.1	Nonlinear Longitudinal Waves in a Hot Electron-Plasma	307
6.8.2	Ion-Acoustic Solitary Waves in an Inhomogeneous Plasma	310
6.9	Exercises	316

Chapter 7 Miscellaneous Perturbation Methods.....319

7.1 A Quantum-Field-Theoretic Perturbative Procedure..... 319

 7.1.1 Blasius Equation..... 320

7.2 A Perturbation Method for Linear Stochastic Differential Equations..... 322

 7.2.1 Application to Wave Propagation in a Random Medium..... 325

 7.2.2 Renormalization Procedure..... 327

 7.2.2(a) Exact Solution 327

 7.2.2(b) Perturbative Solution..... 328

 7.2.2(c) Renormalized Solution..... 329

7.3 Exercises 333

References..... 335

Answers to Selected Problems..... 343

Index 347

Permissions 353