

# Phase-coherent link from optical to microwave frequencies by means of the broadband continuum from a 1-GHz Ti:sapphire femtosecond oscillator

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An optical clockwork is created with a compact 1-GHz repetition-rate laser and three nonlinear crystals. The broadband continuum output of the laser covers sufficient bandwidth to provide direct access to its carrier-envelope offset frequency without the use of a microstructure fiber. We phase lock the femtosecond comb to a Ca optical standard and monitor the stability of the repetition rate,  $f_r$ , at 1 GHz. We demonstrate that the short-term stability of the microwave output of the optical clock is at least as good as that of a high-performance hydrogen maser.

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Frequency metrologists have long sought to employ lasers locked to optical transitions in atoms as frequency standards, because optical standards have the potential to provide orders-of-magnitude greater stability than their microwave counterparts. The challenge lies in dividing the optical frequency down to the microwave domain, where electronics can count the oscillations. Kerr-lens mode-locked femtosecond lasers can now provide this bridge.<sup>1-5</sup> These lasers have a comb of discrete spectral components  $f_n$  that may be expressed in terms of two microwave frequencies: the repetition rate,  $f_r$ , and the carrier-envelope offset frequency,  $f_{\text{CEO}}$ , where  $f_n = nf_r + f_{\text{CEO}}$  and  $n$  is an integer. The parameter  $f_{\text{CEO}}$  can be determined through a self-referencing technique; if a full octave of bandwidth is available, the second harmonic of the low-frequency wing of the comb can be heterodyned with the high-frequency wing.<sup>4</sup> Most commonly, such a broad bandwidth has been attained by coupling the output of a Ti:sapphire laser oscillator into a microstructure fiber.<sup>3,4,6</sup> Although these fibers can easily broaden the femtosecond laser spectrum to greater than one octave, they have limitations with regard to long-term operation, i.e., time scales of the order of several hours. Specifically, it is challenging to maintain efficient coupling into the small core of the fiber, the cleaved faces can be damaged under the high intensities, and broadband amplitude noise is present in the output.<sup>7</sup> A recent breakthrough in this area is the generation of an octave of bandwidth directly from a femtosecond laser.<sup>8</sup> With this laser, Morgner *et al.* directly measured and stabilized  $f_{\text{CEO}}$ .<sup>9</sup>

In this Letter we describe experiments with a new laser that has a much higher repetition rate of 1 GHz and a more compact and simple design.<sup>10</sup> This laser allows a self-referencing scheme that utilizes two-thirds of an octave, heterodyning the third and

second harmonics of portions of the spectrum to provide access to  $f_{\text{CEO}}$ . We phase lock this new laser to a high-performance optical standard, thereby incorporating it into an optical clock. We demonstrate that the short-term instability of the microwave output of this optical clock at  $f_r = 1$  GHz is at least as good as one of the most stable microwave atomic frequency standards.

The laser was described in detail elsewhere,<sup>10</sup> so only a brief description is given here. The five-element laser has a bow-tie ring configuration with a 1-GHz repetition rate. One of the cavity mirrors is slightly convex, with a radius of curvature of 1000 mm. This convex mirror is the key element in the generation of the broadband continuum, which spans an octave from 560 to 1150 nm at  $-50$  dB below the maximum. Pumping the laser with 8 W of power at 532 nm produces the output spectrum displayed in Fig. 1. The

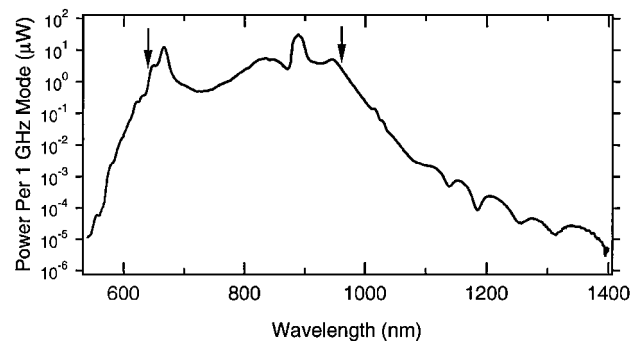


Fig. 1. Output spectrum of a 1-GHz femtosecond laser, showing the power per 1-GHz mode measured in microwatts given 640-mW total average power. We obtain the offset frequency,  $f_{\text{CEO}}$ , by heterodyning the third harmonic of light at 960 nm with the second harmonic of 640 nm (indicated by the two arrows).

low power in the wings makes it difficult to determine  $f_{\text{CEO}}$  by heterodyning the high-frequency wing of the spectrum with the second harmonic of the low-frequency wing. Instead, we observe the beat frequency between the third and second harmonics of portions of the output spectrum separated by two-thirds of an octave. The frequency difference  $3f_n - 2f_m = 3(nf_r + f_{\text{CEO}}) - 2(mf_r + f_{\text{CEO}})$  yields  $f_{\text{CEO}}$  with  $m = 3/2n$ .

The setup for measuring  $f_{\text{CEO}}$  is similar to that described previously.<sup>9</sup> The laser output is split into two arms by a dichroic beam splitter. Light near 640 nm enters arm I, and the spectrum at 960 nm passes through to arm II. In arm I, a 0.3-mm-thick type I  $\beta$ -barium borate (BBO) crystal is angle-tuned to produce the second harmonic at 320 nm. In arm II, 960-nm light is frequency tripled in two steps, which also produces 320 nm. The 960-nm light is first doubled in a 2-mm-thick angle-tuned type I lithium iodate crystal. The 960- and 480-nm outputs are then separated by a dichroic beam splitter. The 480-nm light passes through a half-wave plate and a delay stage before recombining with the 960-nm light. Both beams are then focused into a 0.5-mm-thick BBO crystal for type I sum-frequency generation. The 320-nm beams from arms I and II are subsequently superimposed on a 50% beam splitter.

For one to observe a heterodyne beat note, the two pulsed beams must have the same polarization and must overlap temporally, spectrally, and spatially. Temporal overlap is achieved by use of a delay stage incorporated into arm I. Spectral overlap is obtained by angle tuning of the nonlinear crystals and is verified with a UV spectrometer. Spatial overlap is an especially important issue for the two 320-nm beams. The required tight focusing into nonlinear crystals results in poor spatial quality, especially for the 320-nm light from arm II, which incorporates two cascaded nonlinear crystals. For these reasons, spatial mode matching is accomplished by coupling of both beams into a 2- $\mu\text{m}$ -core single-mode UV optical fiber. Typical powers in the 320-nm beams after the fiber were a few hundred nanowatts and a few tens of nanowatts for arms I and II, respectively.

The output of the fiber is then incident upon a photomultiplier tube. The beat-note frequency at  $f_{\text{CEO}}$  was detected with  $\sim 20$ -dB signal-to-noise ratio (SNR) in a 300-kHz bandwidth. Although the beat frequency could also be observed without the use of the mode-matching fiber, the noise floor increased by 20 dB and the SNR decreased by 3 to 5 dB in a 300-kHz bandwidth. These results testify to the importance of mode matching in the system.

The relatively low SNR of the heterodyne  $f_{\text{CEO}}$  is improved by phase-locking a voltage-controlled oscillator to this beat. The output of the voltage-controlled oscillator, which is the filtered and amplified  $f_{\text{CEO}}$ , is then phase locked to a frequency synthesizer that is referenced to a hydrogen maser. The phase-locked loop (PLL) is implemented by control of the power of the pump laser with an acousto-optic modulator. A second heterodyne beat frequency,  $f_b = f_n - f_{\text{Ca}}$ , is measured between one element of the femtosecond laser

comb and the 456-THz (657-nm) stabilized diode laser of the National Institute of Standards and Technology Ca optical standard.<sup>11</sup> A second PLL serves to lock  $f_b$  by varying the cavity length of the femtosecond laser by means of a piezoelectric transducer behind one of the cavity mirrors. When both  $f_{\text{CEO}}$  and  $f_b$  are phase locked, every tooth in the femtosecond comb is fixed with respect to the Ca optical standard.

It is thus necessary to quantify the stability of this comb as a phase-coherent clockwork linking optical and microwave frequencies. To obtain sufficient measurement resolution, we heterodyne the repetition rate,  $f_r$ , against a low-noise 1-GHz frequency synthesizer that is referenced to a hydrogen maser. Offsetting the synthesizer frequency from  $f_r$  provides a 10-kHz beat frequency that is counted with various gate times  $\tau$ . We then calculate the Allan deviation,  $\sigma(\tau)$ , a measure of fractional frequency instability,<sup>7,12</sup> for each value of  $\tau$ . The result is displayed by the circles in Fig. 2. The error bars represent a statistical uncertainty of  $1/\sqrt{N}$ , where  $N$  is the number of sample points at the respective gate times.<sup>13</sup> The solid curve represents the Allan deviation of the hydrogen maser that references the frequency synthesizer against which we beat  $f_r$ . The Allan deviation of the additional phase noise that is due to the 1-GHz frequency synthesizer used in the measurements described above is recorded as triangles in Fig. 2.

During the counting of the 10-kHz beat frequency, the quality of the two phase locks is ensured by means of synchronously counting the phase-locked beats  $f_{\text{CEO}}$  and  $f_b$ . Any deviations of  $f_{\text{CEO}}$  and  $f_b$  by more than  $1/\tau$  ( $\tau$  in seconds) results in the rejection of the corresponding 10-kHz counter reading. This rejection serves to remove instances when the PLLs may have slipped cycles.<sup>7,14</sup> In a majority of the points, however, the variation of  $f_{\text{CEO}}$  and  $f_b$  is significantly better than  $1/\tau$ . For example, for a 1-s gate time, a typical

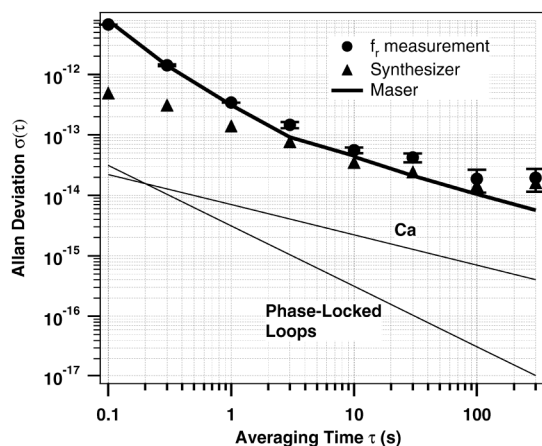


Fig. 2. Allan deviation (circles) for the Ca-referenced optical clock at  $f_r = 1$  GHz measured relative to a low-noise frequency synthesizer referenced to a hydrogen maser. Error bars of  $1/\sqrt{N}$ , where  $N$  is the number of sample points, are shown. The Allan deviation of the synthesizer (triangles) and the maser (solid curve) are also shown. The two thin lines illustrate the upper limits of the instability of  $f_r$  that are due to the Ca standard and the phase locks.

variation would be 0.2 Hz for  $f_{\text{CEO}}$  and 0.02 Hz for  $f_b$ . More specifically, the threshold of  $1/\tau$  ensures that the fluctuations measured in the repetition rate will not be limited by the quality of the PLLs. The repetition rate may be expressed as  $f_r = (f_b - f_{\text{CEO}} + f_{\text{Ca}})/n_0$ , with fluctuations in  $f_r$  coming from instabilities in  $f_b$ ,  $f_{\text{CEO}}$ , and  $f_{\text{Ca}}$ . Here  $n_0$  is the index of the mode that is locked to Ca, and  $n_0 \sim 460,000$ . The  $1/\tau$  criterion sets an upper limit of  $3 \times 10^{-15}/\tau$  on deviations in  $f_r$  because of variations in  $f_b$  or  $f_{\text{CEO}}$ . The fluctuations in  $f_r$  as a result of the Ca standard have an upper limit<sup>15</sup> of  $7 \times 10^{-15}/\sqrt{\tau}$ . These upper limits are also plotted in Fig. 2 and are well below the instability measured in  $f_r$ . Assuming that no unknown noise sources contribute, the figure shows that the Ca system should ultimately limit the stability of the femtosecond optical comb. It is important to note that the contribution of the maser-referenced synthesizers used for the PLLs to the instability of  $f_r$  is negligible. Specifically, it can be expressed as  $\sigma_{\text{maser}}(\tau)f_{\text{CEO}}/(n_0f_r) \approx 10^{-18}$  at 1 s, and the same contribution holds for  $f_b$ . The use of the maser reference in the two PLLs can be eliminated<sup>15</sup> by referencing of all the synthesizers to  $f_r/100 = 10$  MHz. In this Letter, however, our measurements indicate that the stability of  $f_r$  is limited either by the frequency synthesizer used to beat against the repetition rate or by the hydrogen maser to which the synthesizer is referenced. The data show that for  $\tau < 10$  s, the hydrogen maser limits the measurement, whereas for  $\tau > 10$  s the synthesizer becomes the limiting factor. It follows that the next step is to compare the comb with something more stable than the hydrogen maser, such as another optical clock. For example, one tooth of the Ca-stabilized comb could be heterodyned with the local oscillator for the Hg<sup>+</sup> optical standard at 563 nm.<sup>16</sup> Although the power per mode at this wavelength of the femtosecond laser spectrum is less than 100 pW, a SNR of 10 dB in 300-kHz bandwidth was achieved for this beat note in a preliminary trial.

In summary, we have implemented an optical clockwork using a new high-repetition-rate femtosecond laser that delivers a broadband continuum and has a simple and compact design. By direct comparison, we demonstrate that the short-term stability of this optical clock is at least as good as that of a high-performance hydrogen maser, the most stable microwave frequency standard readily available to us. We believe that these initial results represent a significant improvement over a microstructure fiber system in that the power fluctuations of the optical spectrum are significantly reduced. This reduction results in a constant amplitude for the observed beats and makes it possible to acquire cycle-slip-free data over longer times (hours). Future directions of study include measuring the instability of  $f_r$  with respect to another optical clock to definitively test the stability of the femtosecond comb. Furthermore, given the demonstrated control of the carrier-envelope offset,  $f_{\text{CEO}}$ , we control the equivalent in time, the pulse-to-pulse carrier-envelope phase slip for the few-cycle pulses in this laser. This is of particular

interest for experiments involving high-intensity ultrashort pulses.

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*Note added in proof:* We can now produce a phase lock of  $f_{\text{CEO}}$  without the aid of a voltage-controlled oscillator for near-arbitrary durations in excess of 14 h.

## References

1. Th. Udem, J. Reichert, R. Holzwarth, and T. W. Hänsch, *Phys. Rev. Lett.* **82**, 3568 (1999).
2. J. Reichert, M. Niering, R. Holzwarth, M. Weitz, Th. Udem, and T. W. Hänsch, *Phys. Rev. Lett.* **84**, 3232 (2000).
3. S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, R. Holzwarth, Th. Udem, and T. W. Hänsch, *Phys. Rev. Lett.* **84**, 5102 (2000).
4. D. J. Jones, S. A. Diddams, J. K. Ranka, A. Stentz, R. S. Windeler, J. L. Hall, and S. T. Cundiff, *Science* **288**, 635 (2000).
5. H. R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, and U. Keller, *Appl. Phys. B* **69**, 327 (1999).
6. J. K. Ranka, R. S. Windeler, and A. J. Stentz, *Opt. Lett.* **25**, 25 (2000).
7. L. Hollberg, C. W. Oates, E. A. Curtis, E. N. Ivanov, S. A. Diddams, Th. Udem, H. G. Robinson, J. C. Bergquist, R. J. Rafac, W. M. Itano, R. E. Drullinger, and D. J. Wineland, *IEEE J. Quantum Electron.* **37**, 1502 (2001).
8. R. Ell, U. Morgner, F. X. Kärtner, J. G. Fujimoto, E. P. Ippen, V. Scheuer, G. Angelow, T. Tschudi, M. J. Lederer, A. Boiko, and B. Luther-Davies, *Opt. Lett.* **26**, 373 (2001).
9. U. Morgner, R. Ell, G. Metzler, T. R. Schibli, F. X. Kärtner, J. G. Fujimoto, H. A. Haus, and E. P. Ippen, *Phys. Rev. Lett.* **86**, 5462 (2001).
10. A. Bartels and H. Kurz, *Opt. Lett.* **27**, 1839 (2002).
11. C. W. Oates, F. Bondu, R. W. Fox, and L. Hollberg, *Eur. Phys. J. D* **7**, 449 (1999).
12. J. A. Barnes, A. R. Chi, L. S. Cutler, D. J. Healey, D. B. Leeson, T. E. McGunigal, J. A. Mullen, Jr., W. L. Smith, R. L. Sydnor, R. F. C. Vessot, and G. M. R. Winkler, *IEEE Trans. Instrum. Meas.* **IM-20**, 105 (1970).
13. These error bars are included to offer the reader information regarding the number of samples taken, especially at large  $\tau$ . They are not an attempt to account specifically for all possible noise processes. Such an analysis is beyond the scope of this work.
14. S. A. Diddams, Th. Udem, K. R. Vogel, C. W. Oates, E. A. Curtis, R. S. Windeler, A. Bartels, J. C. Bergquist, and L. Hollberg, *Proc. SPIE* **4269**, 77 (2001).
15. S. A. Diddams, Th. Udem, J. C. Bergquist, E. A. Curtis, R. E. Drullinger, L. Hollberg, W. M. Itano, W. D. Lee, C. W. Oates, K. R. Vogel, and D. J. Wineland, *Science* **293**, 825 (2001).
16. R. J. Rafac, B. C. Young, J. A. Beall, W. M. Itano, D. J. Wineland, and J. C. Bergquist, *Phys. Rev. Lett.* **85**, 2462 (2000).