Phase-matchable nonlinear optical interactions in periodic thin films*

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A proposal for a new method of phase matching in nonlinear optical interactions is made. A periodic perturbation of the surface of a thin-film waveguide generates space harmonics with new propagation constants which can be phase matched. An analysis of this proposal shows it to be particularly interesting for a class of thin-film nonlinear devices using the cubic optically isotropic semiconductors (such as GaAs, GaP, etc.) which possess high nonlinear optical coefficients but are not phase matchable by the conventional birefringent techniques.

Optically isotropic materials have, to date, not been used in nonlinear optical applications because the conventional technique of birefringent phase matching cannot be applied. This has, so far, precluded the use of many highly nonlinear materials, such as GaAs, from practical utilization in second-harmonic generation, parametric oscillation, and frequency upconversion.

One approach to phase matching in optically isotropic materials involves the use of dimensional dispersion in thin-film waveguides.[1] Another suggestion[2,3] utilizes periodically laminated structures. The realization of this last approach involves a fractional wavelength control of the lamination period and has not yet been demonstrated.

As an outgrowth of our experiments on light guiding in epitaxial GaAs thin films,[4] we have considered a new spatial modulation approach to phase matching, which involves a periodic corrugation of one (or more) of the boundaries of the thin-film waveguide, as shown in Fig. 1.

A more careful investigation of this idea leads to the conclusion that the expected magnitude of the effect, using presently available materials and techniques, is such as to encourage serious efforts to implement it. The main results of the analysis are presented in what follows.

Consider a single propagating mode, say m, in a thin (uncorrugated) waveguide with a principal transverse field component,

$$E_m^\omega(x,z,t)=C_m^\omega \exp[i(\omega t-\beta^\omega z)]\mathscr{E}_m^\omega(x), \tag{1}$$

in the presence of a sinusoidal boundary perturbation whose period Λ is large enough so as not to couple into the continuum ("leaky") modes[5]; the wave has the Floquet form

$$\begin{aligned}E_m^\omega(x,z,t)&=C_m(z)\exp[i(\omega t-\beta_m^\omega z)]\mathscr{E}_m^\omega(x)\\&=\exp[i(\omega t-\beta_m^\omega z)]\mathscr{E}_m^\omega(x)\sum_{n=-\infty}^{\infty}A_{mn}^\omega \exp[-in(2\pi/\Lambda)z],\end{aligned} \tag{2}$$

where $C_m(z)$ is periodic in Λ. The mode consists of an infinite number of space harmonics, each with its phase constant

$$\beta_{mn}^\omega=\beta_m^\omega+n2\pi/\Lambda, \quad n=\pm1,2,\ldots . \tag{3}$$

Phase-matched interactions are thus no longer limited to the principal value of β but can involve the space harmonics. As an example, second-harmonic generation can be achieved by matching the fundamental $(n=0)$ space harmonic at ω to the first $(n=\pm1)$ space harmonic at 2ω, so that

$$\beta_0^{2\omega}=2\beta_0^\omega\pm(2\pi/\Lambda). \tag{4}$$

The penalty for using the space harmonics is that the conversion efficiency is reduced relative to the phase-matched interaction in the bulk by a factor which in the example just quoted is approximately equal to $|A_{01}^{2\omega}|^2$. A meaningful evaluation of the feasibility of phase matching by periodic surface perturbation prerequires a solution for the amplitudes of the space harmonics A_{mn}.

A solution of Maxwell's equations for a TE mode, to be described elsewhere, gives

$$\frac{\partial^2 C_m}{\partial z^2}-2i\beta_m\frac{\partial C_m}{\partial z}=-k_0^2C_m\int_{-\infty}^{\infty}|\mathscr{E}_m(x)|^2\Delta n^2(x,z)\,dx, \tag{5}$$

where $\Delta n^2(x,z)$ is the deviation, due to surface corrugation, of the actual index (squared) of refraction from that of a planar boundary film. $k_0\equiv 2\pi/\lambda_0$ and $\mathscr{E}_m(x)$ is normalized according to $\int_{-\infty}^{\infty}|\mathscr{E}_m(x)|^2dx=1$. When a, the corrugation amplitude, is small compared to the thickness t, we can replace $\mathscr{E}_m(x)$ in Eq. (5) by $\mathscr{E}_m(0)$, and assuming $\partial^2C_m/\partial z^2\ll\beta_m\partial C_m/\partial z$, we obtain

$$C_m(z)=C_m(0)\exp\left\{i\frac{k_0^2|\mathscr{E}_m(0)|^2(n_2^2-1)a\Lambda}{4\pi\beta_m}\left[\cos\left(\frac{2\pi}{\Lambda}\right)z-1\right]\right\}. \tag{6}$$

The corrugation thus causes a (spatial) phase modulation of the mode, so that, using Eq. (2) and a Bessel-func-

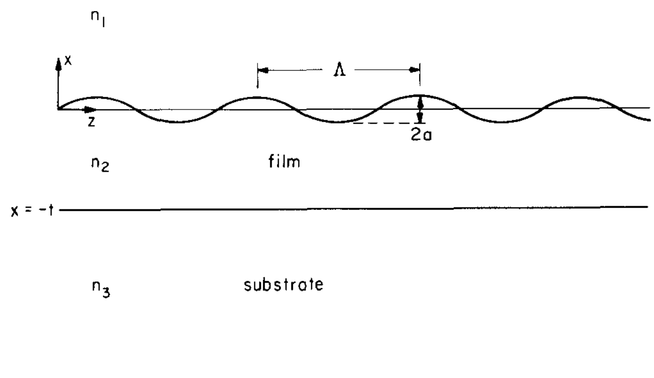

FIG. 1. Model of a thin-film waveguide with a periodic perturbation of one interface.

tion expansion of Eq. (6), we get the following solution for the mode:

$$E_m(x,z,t)=C_m(0)\exp[i(\omega t-\phi)]\mathscr{E}_m^{\omega}(x)\times\sum_{n=-\infty}^{\infty}(i)^n J_n(M_m)\exp\{-i[\beta_m^{\omega}-n(2\pi/\Lambda)]z\},$$

where

$$M_m^{\omega}\equiv\frac{k_0^2|\mathscr{E}_m(0)|^2(n_2^2-1)a\Lambda}{4\pi\beta_m}\ . \qquad (8)$$

Equation (7) gives explicitly the relative amplitudes of the space harmonics generated by the surface corrugation. This result can now be used in analyzing nonlinear interactions in a thin film. To be specific we consider an example of second-harmonic generation in which both the input (ω) and output (2ω) are in the same, say $m=0$, waveguide mode. We thus have

$$E_0^{\omega}(x,z,t)=\sum_n A_n\exp[i(\omega t-\beta_n^{\omega})z]\mathscr{E}_0^{\omega}(x),$$
$$E_0^{2\omega}(x,z,t)=\sum_m B_m\exp[i(2\omega t-\beta_m^{2\omega})z]\mathscr{E}_0^{2\omega}(x), \qquad (9)$$

where $\beta_n^{\omega}=\beta_0^{\omega}-n2\pi/\Lambda$, and A_n and B_m are defined by Eq. (7). The second-harmonic polarization generated by $\mathscr{E}_0(x,z,t)$ is taken as

$$p^{2\omega}(x,z,t)=d\sum_n\sum_l A_nA_l\exp\{i[2\omega t-(\beta_n^{\omega}+\beta_l^{\omega})z]\}[\mathscr{E}_0^{\omega}(x)]^2, \qquad (10)$$

where d is the appropriate bulk nonlinear tensor element. The rate of growth of the average power in the second-harmonic mode is

$$\frac{dP^{2\omega}}{dz}=\omega W\,\mathrm{Im}\left(\int_{-\infty}^{\infty}E^{2\omega}(p^{2\omega})^*\,dx\right)$$
$$=W\omega d\,\mathrm{Im}\left(\sum_n\sum_l\sum_m A_n^*A_l^*B_m\exp[i(\beta_n^{\omega}+\beta_l^{\omega}-\beta_m^{2\omega})z]\times\int_{-\infty}^{\infty}[\mathscr{E}_0(x)]^2\mathscr{E}_0^{2\omega}(x)\,dx\right), \qquad (11)$$

where W is the waveguide width in the y direction. Equation (11) shows immediately that a phase-matched interaction is due to triplets (n,l,m) or space harmonics for which the exponent in Eq. (11) vanishes, or, using Eq. (3), a phase-matched interaction occurs when

$$2\beta_0^{\omega}-\beta_0^{2\omega}+(n+l-m)(2\pi/\Lambda)=0. \qquad (12)$$

Phase matching in *first order* occurs if we choose the period Λ so that $2\beta_0^{\omega}-\beta_0^{2\omega}\pm 2\pi/\Lambda=0$. The synchronous contributions, in this case, arise from the triplets (0,0,1), (0,−1,0), and (−1,0,0). A simple manipulation of Eq. (11) in which all non-phase-matched contributions are ignored, in which β is assumed to be equal to the bulk propagation vector, and where the energy is assumed to be confined mostly within the height t of the guide gives

$$\frac{P^{2\omega}(l)}{P^{\omega}}=\frac{2\omega^2d_{\mathrm{eff}}^2l^2}{(n^{\omega})^2n^{2\omega}}\left(\frac{\mu_0}{\epsilon_0}\right)^{3/2}\frac{P^{\omega}}{Wt}\ , \qquad (13)$$

in the nondepleted pump approximation. This result is of a form identical to the bulk interaction[6] except that here the effective nonlinear coefficient is

$$d_{\mathrm{eff}}\approx dJ_1(M_0^{\omega}). \qquad (14)$$

The approximate equality is used since the exact numerical coefficient, which is of the order of magnitude of unity, involves the transverse overlap integral and the relative magnitude and sign of the three first-order contributions. It will be included in a more complete forthcoming paper.

The conversion efficiency from ω to 2ω is seen to be proportional to the mode power density P^{ω}/Wt. Since W and t can be made comparable to λ, this power density can become very large even for small power input. The *penalty* for using surface corrugation is a reduction of the effective nonlinear coefficient by the factor $\sim J_1(M_0^{\omega})$. Using Eq. (8) in the case of a GaAs epitaxial film with $t=6\ \mu$, $n_2=3.5$, and $n_2-n_3=0.2$, for a fundamental wave at $\lambda_0=10.6\ \mu$ we find that first-order phase matching requires a corrugation with a period of $\Lambda=107\ \mu$ and that, for $a=0.5\ \mu$, $d_{\mathrm{eff}}\approx\frac{1}{25}d_{\mathrm{GaAs}}$. It is interesting to note that, even allowing for the 25× reduction of the effective nonlinearity, using $d_{\mathrm{GaAs}}\simeq 1.2\times 10^{-21}$ MKS, the effective coefficient is comparable to that of $LiNbO_3$, one of the best phase-matchable materials presently used.

Note added in proof: Nonlinear interactions involving a spatial periodic modulation of the nonlinear coefficient d rather than the modulation of the height are also possible and will be discussed separately.

Experimental techniques for fabricating surface corrugations in GaAs with periods as small as 0.28 μ have been developed in cooperation with Dr. H. Garvin of the Hughes Research Laboratories. The availability of such techniques plus the fast evolving technology of GaAlAs epitaxy should make possible the development of tunable optical parametric oscillators, upconverters, and second-harmonic generators using thin films.

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[1]D.B. Anderson and J.T. Boyd, Appl. Phys. Letters 19, 266 (1971).
[2]A. Ashkin and A. Yariv (unpublished).
[3]N. Bloembergen and A.J. Sievers, Appl. Phys. Letters 17, 483 (1970).
[4]D. Hall, A. Yariv, and E. Garmire, Appl. Phys. Letters 17, 127 (1970).
[5]D. Marcuse, Bell System Tech. J. 48, 3187 (1969).
[6]A. Yariv, *Introduction to Optical Electronics* (Holt, Rinehart and Winston, New York, 1971), p. 190.