Phase noise of four-wave mixing in semiconductor lasers

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(Received 13 January 1992; accepted for publication 13 March 1992)

A simple theoretical analysis shows that the linewidth of the conjugate wave produced in four-wave mixing in semiconductor lasers is equal to the linewidth of the probe plus four times the linewidth of the pump. Experimental results in good agreement with the theory are presented. This result implies an enormous enhancement in the phase noise of the conjugate wave and sets a limitation on some practical applications of four-wave mixing.

Four-wave mixing (FWM) has been studied extensively in recent years. Due to its potential in real-time holography, adaptive optics, etc., phase conjugation through degenerate FWM and nearly degenerate four-wave mixing (NDFWM) has attracted considerable attention in nonlinear optics.[1,2] Since the conjugate wave generated through NDFWM interaction is frequency shifted from the signal frequency by two times the beating frequency between signal and pump, frequency conversion using NDFWM has also been proposed in optical communication applications.[3] However, the effect of noise in NDFWM, to the authors' knowledge, has not yet been studied. Since noise has important impacts in all the applications quoted above, a noise analysis in NDFWM is required and this is the purpose of the present letter.

Collinear interactive FWM in semiconductor lasers and laser amplifiers has been extensively studied using the rate equation model.[4-6] The dominant mechanism of NDFWM is explained by the dynamic population pulsation at the beat frequency between probe and pump waves. In semiconductor lasers and laser amplifiers, the amplified probe and conjugate wave output through NDFWM can be expressed formally as

$$A_1(\Omega)=F_1(\Omega,|A_0|^2)\,|A_0|^2A_{\rm in}, \tag{1a}$$

$$A_2^*(\Omega)=F_2(\Omega,|A_0|^2)\,(A_0^*)^2A_{\rm in}, \tag{1b}$$

where Ω is the pump-probe frequency detuning, A_0 is the mean field of the pump inside the active waveguide, and $A_{\rm in}$ is the input probe signal amplitude.

In the case of counterdirectional NDFWM in traveling wave laser amplifiers (TWAs) with equal intensity of the pump waves impinging upon the external facets of the laser, the coefficients F_1 and F_2 are[4]

$$F_1(\Omega,|A_0|^2)=\frac{p\exp(-\bar{\alpha}L)}{p\cos(pL)+\alpha\sin(pL)}, \tag{2a}$$

$$F_2(\Omega,|A_0|^2)=\frac{\kappa_2\sin(pL)}{p\cos(pL)+\alpha\sin(pL)}, \tag{2b}$$

with $p=(\kappa_1\kappa_2^*-\alpha^2/|A_0|^4)^{1/2}$, $\alpha=(\alpha_1+\alpha_2^*)/2$, $\bar{\alpha}=(\alpha_1-\alpha_2^*)/2$, $\kappa_{1,2}=-2iC\alpha_0/(1+|A_0|^2\pm i\Omega\tau_s)$, $\alpha_{1,2}=\alpha_0[1-C|A_0|^2/(1+|A_0|^2\pm i\Omega\tau_s)]$ amd $\alpha_0=-(1-i\beta)(g_0/2)/(1+|A_0|^2)$, where the positive or negative sign is chosen for the subscript 2 and 1, respectively. In Eqs. (2), C is the field overlap factor, L the chip length, τ_s the spontaneous emission lifetime, g_0 the small signal gain, and β the linewidth enhancement factor.

When a semiconductor laser operates above threshold, the pump light is produced by itself[7] and the NDFWM process inside this device has special properties compared to that in TWAs. Nevertheless, the probe and conjugate output can still be expressed in the form of Eqs. (1) except for the different coefficients F_1 and F_2:[8]

$$F_1(\Omega,|A_0|^2)=\frac{-\frac{1}{2}g_0\Gamma(1+i\beta)C-i\Omega(1+|A_0|^2-i\Omega\tau_s)/|A_0|^2}{\tau_p\Omega[i\Omega(1+|A_0|^2)+\tau_s(\Omega^2-\Omega_R^2)]}, \tag{3a}$$

$$F_2(\Omega,|A_0|^2)=\frac{\frac{1}{2}g_0\Gamma(1+i\beta)C}{\tau_p\Omega[i\Omega(1+|A_0|^2)+\tau_s(\Omega^2-\Omega_R^2)]}. \tag{3b}$$

Here, Ω_R is the relaxation oscillation frequency of the pump laser, Γ is the confinement factor of the laser waveguide, and τ_p is the cavity roundtrip time of the pump laser.

Equations (1)–(3) have been successfully used to explain most of the features of NDFWM in TWAs and in lasers operating above threshold. In general, the intracavity transmitted probe and pump lights were treated as single frequency. In practical cases, however, both pump and probe waves are affected by amplitude and phase noise, the latter being responsible for the finite linewidth of the field spectrum. Amplitude noise produces only a broadband background in the field spectrum of semiconductor lasers. This is important only at high detunings from the center frequency, where its coupling with phase fluctuations introduces an asymmetry in the resonance peaks present in the semiconductor laser line shape.[9] This amplitude noise will be neglected in the present analysis. The effect of the phase noise of both the pump and the signal can be easily accounted for by substituting A_0 and $A_{\rm in}$ with $A_0e^{i\phi_0(t)}$ and $A_{\rm in}e^{i\phi_{\rm in}(t)}$, respectively, where $\phi_0(t)$ and $\phi_{\rm in}(t)$ are the pump and signal phase noise. For TWAs in the configuration considered above, this means that we consider two mutually coherent pump waves. We will return on the opposite case of mutually incoherent pump waves later on. The substitution of time-dependent phases is not rigorously

 0003-6951/92/202454-03$03.00

correct, because Eqs. (1) are already in the frequency domain. However, it is a good approximation when, as usual, the separation between pump, signal, and conjugate waves is much larger than the linewidth of the fields. This means that we consider only the effect on NDFWM of the low frequency content of the phase noise, which gives rise to the linewidth of the fields. This component is much slower than the inverse of the beating frequency Ω and hence we can assume that the output fields $A_1(\Omega)$ and $A_2(\Omega)$ adiabatically follow the phase noise of signal and pump.

Introducing phase noise of both pump and signal, the field autoconvolutions write

$$\langle A_1(\Omega,t+T)A_1^*(\Omega,t)\rangle = |F_1(\Omega,|A_0|^2)|^2|A_0|^4|A_{\rm in}|^2 \times\langle\exp\{i[\phi_{\rm in}(t+T)-\phi_{\rm in}(t)\}]\rangle, \quad (4a)$$

$$\langle A_2^*(\Omega,t+T)A_2(\Omega,t)\rangle = |F_2(\Omega,|A_0|^2)|^2|A_0|^4|A_{\rm in}|^2 \times\langle\exp\{-2i[\phi_0(t+T)-\phi_0(t)]\} \times\exp\{i[\phi_{\rm in}(t+T)-\phi_{\rm in}(t)]\}\rangle. \quad (4b)$$

For long times (low frequencies), the phase diffusion is a random walk of variance

$$\langle[\phi_j(t+T)-\phi_j(t)]^2\rangle = D_j|T|, \quad (5)$$

where the diffusion constant D_j is related to the linewidth by the following relationship:[10]

$$\Delta\nu_j = D_j/(4\pi). \quad (6)$$

By using the property of Gaussian processes

$$\langle\exp\{i\lambda[\phi_j(t+T)-\phi_j(t)]\}\rangle = \exp\{-\lambda^2 D_j T/2\}, \quad (7)$$

and from the independence of the phase noise of pump and signal, we get the power spectrum of the probe and the conjugate wave output through the Fourier transformation with respect to the slow time scale T:

$$\langle|A_1(\Omega,\omega)|^2\rangle = |F_1(\Omega,|A_0|^2)|^2|A_0|^4|A_{\rm in}|^2 \frac{4\pi\Delta\nu_1}{\omega^2+(2\pi\Delta\nu_1)^2}, \quad (8a)$$

$$\langle|A_2(\Omega,\omega)|^2\rangle = |F_2(\Omega,|A_0|^2)|^2|A_0|^4|A_{\rm in}|^2 \frac{4\pi\Delta\nu_2}{\omega^2+(2\pi\Delta\nu_2)^2}, \quad (8b)$$

where

$$\Delta\nu_1 = \Delta\nu_{\rm in}, \quad (9)$$

$$\Delta\nu_2 = 4\Delta\nu_0 + \Delta\nu_{\rm in}. \quad (10)$$

$\Delta\nu_2$, $\Delta\nu_0$, and $\Delta\nu_{\rm in}$ are the linewidths of conjugate, pump, and probe waves, respectively, and ω is the angular frequency deviation from the center frequency of the probe and conjugate. The above calculations revealed that the linewidth of the probe output is the same as that of the input, while the linewidth of the conjugate is much broader than those of pump and probe, and is very sensitive on the pump linewidth. A simple relationship between the linewidths of probe, pump, and conjugate waves, as expressed by Eq. (10), is the main result in this letter. Equations (9) and (10) hold, in general, when the pump waves are mutually coherent. This is the case, for example, of the counterdirectional NDFWM with the two counterpropagating pump waves obtained by splitting the same laser beam. If the two pump beams are obtained from independent laser sources, one has to consider that $(A_0^*)^2$ in Eq. (1b) is replaced by $A_{0,f}^*A_{0,b}^*$, where $A_{0,f}^*$ and $A_{0,b}^*$ are the forward and the backward propagating pump waves and they have, now, independent phase noise. The same calculations performed above can be easily repeated in this case, obtaining for the linewidth of the conjugate output

$$\Delta\nu_2 = \Delta\nu_{0,f} + \Delta\nu_{0,b} + \Delta\nu_{\rm in}, \quad (11)$$

where $\Delta\nu_{0,f}$ and $\Delta\nu_{0,b}$ are the linewidth of the forward and backward propagating pump waves. Our results are not only restricted to NDFWM, but hold also for highly degenerate FWM where much larger frequency shifts are achieved through intraband transitions in semiconductors.

The experimental setup is described as follows. Two identical DFB-BH laser diodes with an emission wavelength of 1554 nm are used. One of them is used to generate the probe wave and its output is injected into the other laser which works above threshold providing the pump wave. The third DFB laser also with the wavelength of 1554 nm is used as a local oscillator which beats with the output of the pump laser and downshifts the signals to radio frequency. Each laser used is external reflection isolated with a Faraday optical isolator providing more than 70 dB of isolation. A *p-i-n* photodetector and a spectrum analyzer with the bandwidth of 22 GHz are used for the spectrum measurement. In order to prevent the effect of pump depletion, the injected probe power is kept less than −50 dBm. The spectral linewidth of the local oscillator used in the experiment is measured to be about 20 MHz and both the pump and the probe linewidths are more than 100 MHz.

The measured spectrum is shown in Fig. 1 for two different pump levels while the probe power and the frequency detuning are the same. In both cases the linewidth of the conjugate wave is much wider than the linewidth of both probe and pump waves as expected. By comparing Fig. 1(b) to Fig. 1(a), one finds that, as usual, the pump linewidth increases with the decrease of its injection current, and the increase in the linewidth of the conjugate wave is much larger than the increase of the pump linewidth, while the linewidth of the probe is unchanged. A more systematic measurement on the linewidth of the pump, the probe, and the conjugate wave is performed by varying the bias level of the pump laser. The result is reported in Fig. 2. Since the power spectrum of probe, pump, and conjugate waves are Lorentzian, their actual linewidth can be evaluated by the difference between the linewidth displayed by the spectrum analyzer and the linewidth of the local oscillator. It is evident that the linewidth of the amplified probe wave is insensitive to the linewidth of the pump while the linewidth of the conjugate wave is linearly

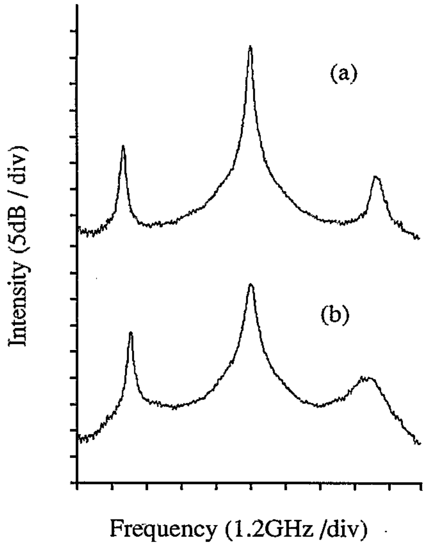

FIG. 1. Measured heterodyne spectrum of NDFWM with the pump laser biased at (a) $I=60$ mA and (b) $I=50$ mA. The pump laser has the threshold current of 37.8 mA.

related to that of the pump. The solid line in Fig. 2 is obtained by using Eq. (10).

It is also interesting to note that in the case of NDFWM in semiconductor lasers operating above threshold, Eqs. (1) and (3) predict a similar optical power for probe and conjugate wave output.[8] However, experiments performed by using Fabry–Perot (FP) interferometers show that the power of the conjugate wave is typically lower than that of its probe counterpart.[7,8] The reason is, up to date, not clear. We attribute this difference to the effect of phase noise. Obviously, the broadening in the linewidth will reduce the peak amplitude in the measured spectrum even if the total optical power remains the same. Our heterodyne measurement system allows us to have a more precise evaluation of both power and linewidth of the probe and conjugate waves. Indeed, our results indicate that $A_1/A_2 \simeq \Delta\nu_2/\Delta\nu_1$, where A_1 and A_2 are the peak values of the probe and conjugate wave power spectrum. This implies that the optical power of the conjugate and the probe waves output are essentially the same within the experimental accuracy. On the other hand, the measurements with FP interferometers[7,8] might suffer from the limitation of their resolution.

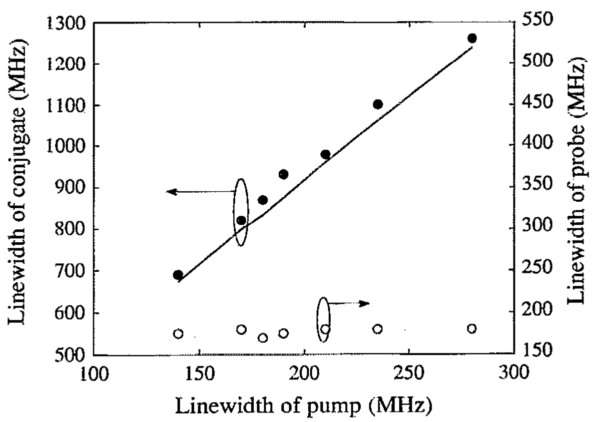

FIG. 2. Measured linewidths of the conjugate wave (solid points) and of the output probe wave (open points) vs the linewidth of the pump wave. The solid line indicates the values of the conjugate linewidth calculated by using Eq. (10).

In conclusion, a simple relationship between the linewidths of probe, pump, and conjugate waves in NDFWM has been found. Experimental results in good agreement with the theory have been reported. The large sensitivity of the phase noise of the conjugate wave on the phase noise of the pump sets a severe limitation on the linewidth of the pump to be used in some practical applications of FWM.

One of the authors (R.H.) thanks N. Caponio, P. Gambini, M. Puleo, and E. Vezzoni for help in the experiment and the CSELT for providing the experimental facilities. The work of R. Hui was supported by CNR and Camera di Commercio di Torino under Research Grant "All-Optical Communication Networks." The work of A. Mecozzi has been carried out in the framework of the agreement between the Fondazione Ugo Bordoni and the Italian PT Administration.

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