Phase-shifting interferometry based on principal component analysis

J. Vargas,[1,*] J. Antonio Quiroga,[2] and T. Belenguer[1]

[1]*Laboratorio de Instrumentación Espacial, Instituto Nacional de Técnica Aeroespacial,*
Carretera de Ajalvir Km 4, 28850 Torrejón de Ardoz, Madrid, Spain

[2]*Optics Department, Universidad Complutense de Madrid, Facultad de CC. Físicas,*
Ciudad Universitaria s/n, 28040 Madrid, Spain
*Corresponding author: jvargas@fis.ucm.es

An asynchronous phase-shifting method based on principal component analysis (PCA) is presented. No restrictions about the background, modulation, and phase shifts are necessary. The presented method is very fast and needs very low computational requirements, so it can be used with very large images and/or very large image sets. The method is based on obtaining two quadrature signals by the PCA algorithm. We have applied the proposed method to simulated and experimental interferograms, obtaining satisfactory results. © 2011 Optical Society of America

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Phase-shifting interferometry (PSI) is a useful technique in optical metrology for measuring the modulating phase of interferograms [1]. In standard PSI, a sequence of N interferograms is obtained having known phase shifts between them so the acquisition is synchronous. Usually a minimum of three phase-shifted interferograms are need to retrieve the phase. Other phase-shifting algorithms, named self-calibrating, allow the determination of the modulating phase without any prior knowledge about the phase steps (asynchronous detection) [2–5]. All these methods are iterative and require considerable computation to converge to a solution. Therefore, these methods are not practical when we use large images or a big number of interferograms. Additionally, these methods require the background and the modulation terms to be approximately spatially constant.

In this Letter, we present an asynchronous phase-shifting method based on the principal component analysis (PCA) algorithm that is very fast and easy to implement. Additionally, it does not use any nonlinear minimization or optimization process, so it is not expensive from a computational point of view. In PSI, an interferogram sequence can be described using the following expression:

$$I_n(x,y) = a(x,y) + b(x,y)\cos[\Phi(x,y) + \delta_n], \quad (1)$$

where $a(x,y)$ is the background illumination, $b(x,y)$ and $\Phi(x,y)$ are the modulation and phase maps, respectively, and δ_n are the phase steps. Expression (1) can be rewritten as

$$I_n = a + b[\cos(\Phi)\cos(\delta_n) - \sin(\Phi)\sin(\delta_n)]. \quad (2)$$

From expression (2) and grouping terms, we obtain

$$\tilde{I}_n = \alpha_n I_c + \beta_n I_s, \quad (3)$$

where we have subtracted the background component, $\alpha_n = \cos[\delta_n]$, $\beta_n = \sin[\delta_n]$, $I_c = b\cos[\Phi]$, $I_s = b\sin[\Phi]$, and the spatial dependence has been omitted for the sake of clarity. From expression (3), we show that any inter-ferogram without background term ($\tilde{I}_n$) can be decomposed into two uncorrelated quadrature signals I_c and I_s that approximately verify the following expression:

$$\sum_{x=1}^{N_x} \sum_{y=1}^{N_y} I_c(x,y) I_s(x,y) \cong 0, \quad (4)$$

where $N_x \times N_y$ is the image size. PCA is a technique from statistics for reducing an image or data set [6]. It involves a mathematical procedure that transforms a number of possibly correlated images into the smallest number of uncorrelated images called the principal components. The principal components are linear combinations of the original variables and are the single best subspace of a given dimension in the least-square sense. In practice, the PCA algorithm is based on three steps. Suppose that we have N images of size $N_x \times N_y$. This image set can be expressed in a matrix form as

$$\mathbf{x} = [x_1, x_2, ..., x_N]^T. \quad (5)$$

In Eq. (5) x_n is a column vector with size $N_x \times N_y$ whose elements are taken columnwise from the nth image. In expression (5), $[\cdot]^T$ denotes the transposing operation, and x has N rows and $N_x \times N_y$ columns. The first step of the PCA algorithm consists in obtaining the covariance matrix C from x as

$$C = (x - m_x)(x - m_x)^T. \quad (6)$$

In expression (6) m_x has the same size as x. All the elements in each column of m_x correspond to the mean value of the respective column of x. Note that, in expression (6), $(x - m_x)$ corresponds to a background suppression operation. Because C is real and symmetric, it is always possible to find a set of real eigenvalues and its corresponding orthonormal eigenvectors. From matrix theory the covariance matrix can be diagonalized as
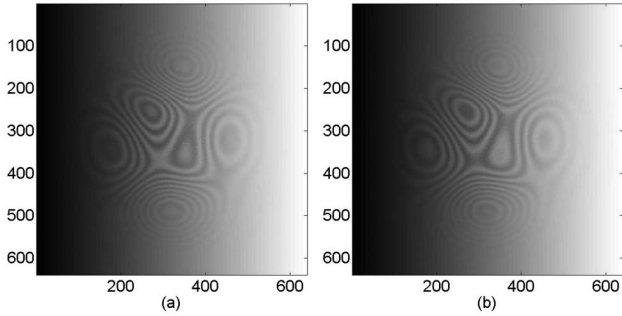
$$D = ACA^T, \quad (7)$$

Fig. 1. Two of the 10 interferograms used in the simulation section.

Fig. 2. Computed basis obtained by the PCA algorithm when 10 phase-shifted patterns with 10% signal-to-noise ratio are processed.

where D is a diagonal matrix, and A is the transformation matrix. This diagonalization process is the second step of the PCA method and is performed in a practical point of view by the singular value decomposition algorithm. The final step consists in obtaining the principal components by the *Hotelling transform* [6] as

$$y = A(x - m_x). \qquad (8)$$

In our case, we only want the first two principal components with the biggest eigenvalues (first and second columns of y), which will correspond to the I_c and I_s signals. Note that the method cannot determine the correct global phase sign as we arbitrarily assign the cosine and sine signals to the first and second principal component, respectively. The phase is obtained as

$$\Phi = \arctan(I_s/I_c). \qquad (9)$$

We have tested the proposed method with simulated fringe patterns in different situations. We have performed simulations with different levels of noise and with different number of phase-shifted fringe patterns. In all cases, we have compared the results obtained by the PCA with the least-squares self-calibrating algorithm presented in [2]. In the first experiment, we have ten phase-shifted fringe patterns affected by different levels of additive white noise. Figure 1 shows two sample fringe patterns with 10% signal-to-noise ratio. The background and modulation maps are $a(x,y) = x/50$ and $b(x,y) = \exp[-0.5(x^2 + y^2)/10^4]$, respectively. The image size is 640×640.

In Table 1 we show the rms errors of the recovered phase and the processing times obtained when ten phase-shifted fringe patterns with different levels of noise are processed by the proposed (PCA) and the self-calibrating (SC) method. Additionally, in Table 1 it is shown the first

Fig. 3. Obtained wrapped phases (a) when the proposed method is used and (b) actual phase map.

four normalized eigenvalues (EVs). As can be seen from Table 1, the proposed method is approximately 2 orders of magnitude more accurate and fast than the SC approach. Figure 2 shows the first and second principal components that are two quadrature signals.

Figure 3 shows the theoretical wrapped phase and the results from the proposed method. In this case, the rms error between the theoretical and the retrieved phase by the PCA algorithm is 0.096 rad. On the other hand, the rms error computed between the theoretical phase and the phase obtained by the SC algorithm [2] is 0.72 rad. The processing times are 0.39 s and 52 s for the proposed and the SC methods, respectively. In all cases the fringe patterns have been processed with a 2.67 GHz laptop and using MATLAB.

In the next experiment, we varied the number of fringe patterns processed while fixing the signal-to-noise ratio to 10%. In Table 2 we show the first four normalized EVs, the rms errors, and the processing times when different numbers of fringe patterns as shown in Fig. 1 are processed. As can be seen from Tables 1 and 2, the rms and processing times obtained by the PCA method are

Table 1.   Results Obtained for Different Levels of Noise

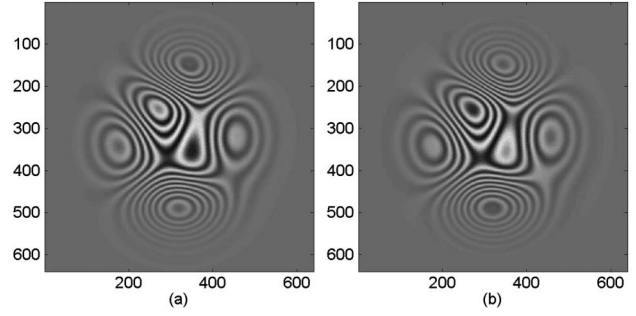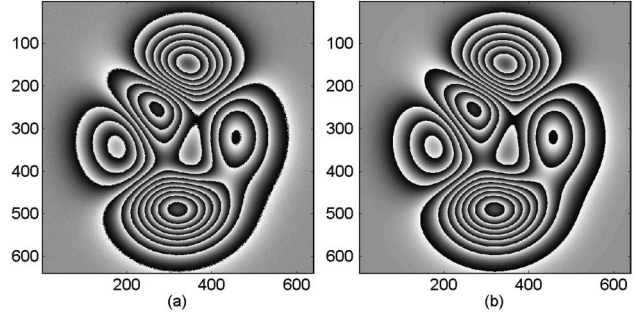| Noise Level	0%	20%	40%	60%	80%
rms PCA (rad)	$1.1e-3$	0.19	0.44	0.81	4.9
Time PCA (s)	0.41	0.46	0.42	0.39	0.42
First EV	$2.5e-2$	$2.6e-2$	$2.5e-2$	$3.3e-2$	$4.0e-2$
Second EV	$4.3e-3$	$1.5e-2$	$1.8e-2$	$2.5e-2$	$2.5e-2$
Third EV	$4.7e-12$	$1.2e-3$	$5e-3$	$1.1e-2$	$1.5e-2$
Fourth EV	$4.2e-12$	$1.2e-3$	$5e-3$	$1.1e-2$	$1.5e-2$
rms SC (rad)	0.94	0.68	8.3	5.06	11.5
Time SC (s)	52	51	52	52	53

Table 2.   Results Obtained for Different Patterns

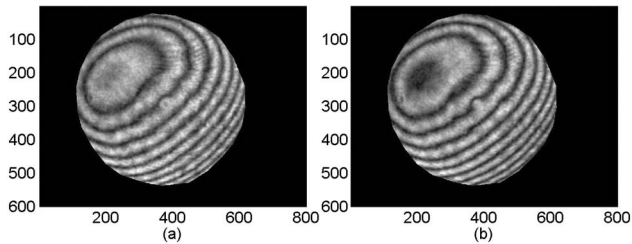| Patterns	5	15	25	35	45
rms PCA (rad)	0.17	0.078	0.072	0.048	0.044
Time PCA (s)	0.22	0.67	1.8	2.3	3.1
First EV	$2.6e-2$	$2.3e-2$	$2.0e-2$	$1.9e-2$	$2.0e-2$
Second EV	$8.0e-3$	$1.4e-2$	$1.0e-2$	$1.9e-2$	$1.9e-2$
Third EV	$6.2e-4$	$2.1e-4$	$1.3e-4$	$9.2e-5$	$7.2e-5$
Fourth EV	$6.2e-4$	$2.1e-4$	$1.3e-4$	$9.2e-5$	$7.2e-5$
rms SC (rad)	0.85	0.77	0.63	0.55	0.55
Time SC (s)	49	56	64	72	77

Fig. 4.   Two sample real interferograms used in the experimental results section.

always significantly lower—approximately 2 orders of magnitude—than when using the SC approach. Additionally, the magnitude of the first and second EVs with respect to the rest is a quantity that weighs the goodness of the retrieved phase. The larger the first and second EVs are, the more accurate the retrieved phase is.

In the simulations shown above, the phase shifts are randomly distributed in the 2π range. If the phase shifts are close to zero with a small number of interferograms, both the PCA and the advanced iterative algorithm methods cannot obtain a reliable phase measurement.

We have applied the proposed algorithm to real interferograms. We have an interferogram sequence formed by 19 fringe patterns. In Fig. 4 we show two phase-shifted interferograms of the sequence. The size of each image is 600×800.

We first process this interferogram sequence with the SC method [2]. We denote the obtained modulating phase as the reference phase. Next, we process the interferogram sequence with the proposed PCA algorithm.

In Fig. 5 are shown the wrapped phases obtained by the PCA method (a) and by the SC algorithm, the reference wrapped phase (b), respectively. As can be seen from Fig. 5, the wrapped phases are similar. The rms error of the difference between both reconstructed phases is 0.056 rad, and the processing times are 0.48 s and 31 s when using the PCA and the SC algorithms, respectively. In order to show the robustness of the proposed algo-
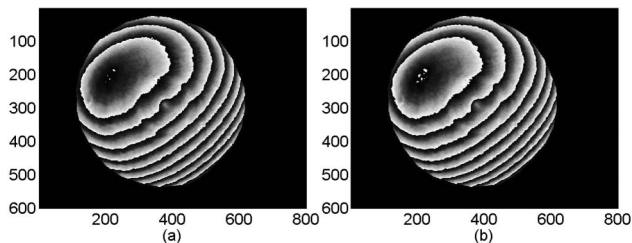

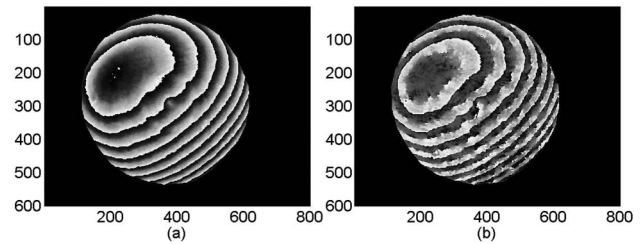Fig. 5.   Wrapped phases obtained by the (a) proposed and (b) SC methods.

Fig. 6.   Obtained wrapped phases when using the (a) proposed method and the (b) SC method to process four real interferograms.

rithm, we have selected the first four interferograms of the sequence, and we have repeated the experiment. The wrapped phases obtained by the PCA and the SC algorithms are shown in Fig. 6. As can be seen from Fig. 6, the retrieved phase by the PCA algorithm is similar than the ones shown in Fig. 5 using 19 interferograms. This is not the case when using the SC approach. The rms error of the difference between the reference and retrieved phases when four interferograms are processed by the PCA and the SC approaches are 0.056 and 6.1 rad, respectively. Finally, the processing times are 0.6 s and 24 s when using the PCA and the SC algorithms, respectively.

In summary, we have presented an asynchronous phase-shifting method based on the PCA algorithm that is very fast and easy to implement, so it is not expensive in a computational point of view. The method does not need any prior guess about the phase steps or any requirements about the background and modulation terms. We have compared the proposed method with a standard SC approach [2], and we have found that the proposed method is approximately 2 orders of magnitude more accurate and faster. All the examples of this work can be reproduced running the MATLAB package that can be found in [7].

References

1. D. Malacara, M. Servín, and Z. Malacara, *Interferogram Analysis for Optical Testing* (Dekker, 1998).
2. Z. Y. Wang and B. T. Han, Opt. Lett. **29**, 1671 (2004).
3. I.-B. Kong and S.-W. Kim, Opt. Eng. **34**, 183 (1995).
4. J. L. Marroquin, M. Servin, and R. R. Vera, Opt. Lett. **23**, 238 (1998).
5. K. G. Larkin, Opt. Express **9**, 236 (2001).
6. R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, 3rd ed. (Prentice-Hall, 2007).
7. http://goo.gl/80Kh6.