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PHASE-SPACE CONSTRATNIS ON SOME HEAVY-ION INERTIAL-FUSION IGNITERS AND EXAMPTE DESIGNS OF 1 MJ RF IITHAC SYSTEMS**

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## ABSIRACT

The design of a high-energy heavy-ion accelerator system for the ignition of inertial-fusion pellet targets starts with the need to satisfy the six-dimensional phase space volume requirement at the target, taking into account dilutions of ion phase space density arising from imperfect beam manipulations throughout the system. Although this need is well known, the phase space condition does not appear to have been widely used as a systematic design guide. Such an approach is presented for systems employing an rf linac either as the main accelerator or as a synchrotron injector; some general properties of final focusing lens systems are included in the analysis. The method is illustrated in some detail, and example designs developed, for one-megajoule linac-accumulator ring systems.

[^0]
## Introduction

In recent months workers at several Laboratories have developed conceptual designs for heavy-ion igniter systems. Although attention has been paid to phase space requirements in these studies it appears that they have been under-used as a primary guide to system design. The main purpose of this paper is to outline a method by which this can be done.

In what follows attention is confined to systems -using an rf linac either as the main accelerator or as an injector into one or more synchrotrons. For an induction linac driver more information from computer codes designed to study transverse and longitudinal particle motions is needed before a corresponding analysis can be carried out.

A second purpose of the paper is to develop a class of conceptual system designs. In the examples presented only a full energy linac with accumulator rings is considered so as to illustrate one method of system parametrization in considerable detail.

Six-Dimensional Phase Space Volume at the Target
We define this quantity as

$$
V_{6 f}=n_{b}\left(e_{\perp f}\right)^{2} e_{11 I}
$$

with $n_{b}$ the number of beams, subscript $f=$ final, $e_{\text {Lf }}$ the normalized transverse emittance (area/ $\pi$ ) per beam, and $e_{\text {uf }}$ the longitudinal emittance (area/J) in eV-.sec. These quantities are given by

$$
e_{\perp f}=(\beta \gamma)_{f} r_{s}\left(R_{p} / R_{v}\right)
$$

with $r_{s}, P_{p}$, and $R_{v}$ the radii of target spot, beam port, and reactor vessel, respectively; and

$$
e_{H \dot{f}}=\left(B^{2} Y\right)_{f} A m_{p} c^{2}(\Delta p / p)_{f}\left(\tau_{f} / 2\right)
$$

with final momentum spread $\pm \Delta p / p$ and $\tau_{f}$ the pulse duration at the target associated with the peak power portion of the pulse shape required. ${ }^{I}$ This shape is imagined to be made up by an appropriately timed sequence of pulses, each of duration $\tau_{f}$.

The irradiated mass is $\pi n_{s} r_{s}{ }^{2} Q$ with $Q$ the ion range in mass/area and $n_{s}$ the number of beam spots, so that the total energy $E$ delivered is

$$
E=\pi n_{s} r_{s}^{2} \&
$$

with $\xi$ the specific energy deposition (energy/mass).
Phase Space Volume from the Linac
This quantity is defined (subscript $L=$ linac) as

$$
v_{6 L}=\left(\varepsilon_{I \mathrm{~L}}\right)^{2} \varepsilon_{\mathrm{iLL}}
$$

with $\varepsilon_{\perp \dot{I}}$ the normalized transverse emittance at the rf linac exit and $\varepsilon_{\| L}$ its longitudinal emittance. The latter is the product of occupied normalized longitudinal emittance $\varepsilon_{\| L b}$ per rf "bucket" and the number of buckets, the number being the product $f_{L b} \Delta t_{L}$ with $f_{\text {Lb }}$ the frequency with which filled buckets emerge from the linac and $\Delta t_{L}$ its on-time. Thie ontime is

$$
\Delta t_{L}=q e E /\left(I_{L} T_{f}\right)
$$

with ion charge $q e$, mean linac electric current $I_{L}$, and final ion kinetic energy $\mathrm{T}_{\mathrm{f}}$. Dilution of phase space density during conversion of the linac bunches to a dc beam by debunching, and dilutions arising from other manipulations downstream from this point, will be introduced later.

We define this factor $\frac{\text { Ideal Available Dilution Factor }}{\mathcal{O}_{\mathrm{i}} \text { as }}$
(actor $\mathcal{O}_{i}$ as

$$
\mathscr{S}_{\mathrm{i}}=\mathrm{v}_{6 \mathrm{f}} / v_{6 \mathrm{~L}}
$$

it is the factor by which phase space density may be diluted by all operations downstream from the linac exit without failing to meet target requirements. Rather than inserting the expressions above for the various factors to evaluate $\mathcal{C}_{i}$ at this point, we next consider certain properties of the final lens systems which focus the beams on the target spots.

## Properties of Final Lens Systems

## First-order Monochromatic

A detailed first-order analysis of a class of final lens systems consisting of quadrupole doublets without a gap between the elements has been presented by A. Garren. ${ }^{2}$ In subsequent work by D. Neuffer ${ }^{3}$ it has been found that oniy small improvements on Garren's systems can be made by reasonable extensions of this class ( optimized individual bores, use of triplets in appropriate cases, etc.). Therefore we adopt Garren's results, leaving open the possibility of shading a numerical coefficient slightly to allow for such potential gains. An important property of Garren's systems (which can be deduced from his paper) is

$$
\left(e_{\perp f}\right)^{2}=(B \gamma)_{f} q B_{Q} r_{s}^{2} X_{\max } /\left(A C_{G}\right)
$$

with $X_{\max }$ the maximum radial beam displacement in the lens system, $B_{Q}$ the quadrupole pole tip field at radius $X_{\max }$, and $C_{G}$ a dimensionless coefficient found from Garren's Table A8-7.1. ${ }^{4}$ With $\varepsilon_{L_{f}}$ in radian-meters, $B_{Q}$ in Tesla, $r_{s}^{\prime}$ and $X_{\max }$ in meters, one finds $C_{G}$ by first evaluating $k=q B_{Q} R_{v}^{2} /\left(3.13 B \gamma A X_{\max }\right)$, then finding $b$ and $x$ corresponding to 1 from the table, and finally forming the product

$$
\mathrm{c}_{\mathrm{G}}=3.13 \mathrm{bx} .
$$

The practical range of cases is stated by Garren as $7 \lesssim b \lesssim 25$, corresponding to values of $k$ varying by a factor $\sim 8$; however, the corresponding range of $C_{G}$ is $100 \approx C_{G} \approx 150$, showing insensitivity to $k$.

## Chromatic Aberration

From the numerical work of Neuffer it has been found that the approximate relation ${ }^{5}(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{f}} \leqslant \mathrm{r}_{\mathrm{s}} / \mathrm{R}_{\mathrm{p}}$ should be replaced by the less optimistic

$$
(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{f}} \leqslant \frac{1}{2} r_{\mathrm{s}} / \mathrm{X}_{\max }
$$

for lenses lacking sextupole elements (such as those in the periodic system proposed by K. Brown ${ }^{6}$ ) arranged to correct chromatic aberration. We represent the effect of such corrections by a sextupole improvement factor $F_{\mathrm{s}} \geqslant 1$; the present hope of those trying to design such systems is $F_{s} \sim 5$ but this may be over-optimistic.

## Third and Higher Order Geometric Aberrations

This subject is not yet fully explored, but numerical work by Neuffer indicates that for beams of present interest these effects restrict $X_{\text {max }}$ to an upper limit of order 30 cm . As will be shown below, a numerical value for $X_{\text {max }}$, although required for lens design, is not needed for the phase space analysis presented here.

Evaluation of $A_{i}$; Discussion
Using the equations above and MKS units,

$$
\vartheta_{i}=\frac{m_{p} c^{2}}{4 e_{G}} \frac{\left(\beta^{3} \gamma^{2}\right)_{f} r_{s}^{3} \tau_{f} T_{f}}{E} \frac{I_{L}}{\left(\varepsilon_{L L}\right)^{2} \varepsilon_{\| L b} f_{L b}} B_{Q} F_{S} r_{b} .
$$

Note that charge state $q$ and final lens bore $X_{\max }$ have cancelled. We express spot size as above and employ
$\beta^{3} \gamma^{2}=\left[(1+\gamma)^{3 / 2} / \gamma\right]\left[T /\left(\operatorname{Am}_{p} c^{2}\right)\right]^{3 / 2}$ so that
$\left(\beta^{3} \gamma^{2}\right)_{\mathrm{f}} \mathrm{r}_{\mathrm{s}}^{3}=\left(\pi m_{\mathrm{p}} \mathrm{c}^{2}\right)^{-3 / 2}\left[\gamma_{\mathrm{f}}{ }^{\left.-1\left(\frac{1+\gamma_{\mathrm{f}}}{n_{\mathrm{s}}}\right)^{3 / 2}\right]\left[\mathrm{T}_{\mathrm{f}} /\left(\mathrm{A}^{2} / \operatorname{Ga}\right)\right]^{3 / 2}\left[\text { AE/E] }{ }^{3 / 2} .\right.}\right.$
We will see that it is desirable to use the minimum number of beam spots; $n_{s}=2$. Then the first square bracket is very close to unity for all $\gamma_{f_{2}}$ of interest. The factors are grouped as shown to display the ratio $T_{f} / A^{2} Z_{l}$ explicitly; in simplest approximation the energy loss rate $\mathrm{dT} / \mathrm{dx} \propto Z^{2}$ $\propto A^{2}$, so the ratio would be constant. In fact it varies by less than a factor of two over the range of interest, as shown in the following table (top of next page):
Thus we have:
$\mathscr{O}_{i} \simeq \frac{1}{4 \pi^{3 / 2}\left(m_{p} c^{2}\right)^{\frac{1}{2}} e_{G}} \cdot\left(\frac{E^{\frac{1}{2}} \tau_{f}}{\zeta^{3 / 2}}\right) \cdot\left(\frac{T_{f}}{A^{2} \not \subset f}\right)^{3 / 2} \cdot\left(\frac{I_{L}}{\left(\varepsilon_{L L}\right)^{2} \varepsilon_{\mu L b} f_{L b}}\right) \cdot \frac{A^{3 / 2} T_{f} B_{Q} F_{s} n_{b}}{\left(n_{s} / 2\right)^{3 / 2}}$.

| $\mathrm{T}(\mathrm{GeV})$ | 5 | 10 | 15 | 20 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A |  |  |  |  |  |
| 131 (Xe) | - | 10 | 7.8 | 6.8 | 5.3 |
| 200 ( Hg ) | 9.6 | 8.9 | 8.0 | 7.1 | 6.4 |
| 238 (U) | 8.8 | 8.0 | 7.4 | 6.5 | 6.1 |

Aside from the constants, the first bracket refers to target requirements, the second is nearly constant, the third constitutes a figure of merit for the rf linac, and the remaining factors are all constrained. Note that in any "funnel-loading" linac scheme ${ }^{8}$ the ratio $I_{L} / f_{L b}$ is ideally a constant; also, it has been commonly assumed that electric current $I_{L}$ may be conserved during stripping to increase charge state at an early stage in the rf linac system. Therefore this figure of merit may be evaluated (with allowance for dilutions in the linac "tree") at its first stage, where $I /\left(e_{\perp}\right)^{2}$ is a direct measure of source brightness.

The system design problem starts with the necessity to choose parameters such that $\theta_{i}$ is as large as the product of all expected transverse and longitudinal dilution factors. (Should it be too large one would reduce $\mathrm{T}_{\mathrm{f}}$ to lower the system size and cost.) We discuss the factors in turn.

The parameters $E=1 \mathrm{MJ}, ' \tau_{f}=6 \mathrm{nsec}, \quad \xi=20 \mathrm{MJ} / \mathrm{g}$ have remained constant during the past two years. For larger $E$ the value of $\tau_{f}$ may not increase ${ }^{9}$ faster than $\mathbb{E}^{1 / 3}$, and there seems to be no change in the requirement $\ell_{\varphi}=30 \mathrm{MJ} / \mathrm{g}$ for $E=10 \mathrm{MJ}$. Therefore the target factor in $\mathcal{A F}_{i}$ increases only by about a factor 3.7 in going from 1 to 10 NJ where $\tau_{\mathrm{f}} \leqslant 13 \mathrm{nsec}$.

The linac figures of merit deduced from the assumptions of several recent studies appear to lie within a range of a factor of four. We quote values from Hearthfire 3 , denoted below by HF3; the 10 MJ design by A. Maschke, ${ }^{11}$ denoted by MJO; and that reported by D. Young, ${ }^{8}$ Case 23, denoted by DY23:

| Parameter | HF3 | MJ1O | DY23 |
| :--- | :---: | :---: | :---: |
| Ion A | $131^{+8}$ | $238^{+2}$ | $200^{+3}$ |
| $I(\operatorname{Amp})$ | 0.03 | 0.16 | 0.40 |
| $e_{L L}(\mathrm{~cm}-\mathrm{mrad})$ | 0.2 | $\sim 0.09$ | 0.2 |
| $e_{11 \mathrm{LD}}(\mathrm{eV}-\mathrm{sec}) \times 10^{3}$ | 0.86 | 8 | $\sim 1.6$ |
| $\mathrm{f}_{\mathrm{Lb}}(\mathrm{MHz})$ | 25 | 16 | 80 |
| Figure of merit | 35 | 154 | 78 |

It is advantageous to use the largest $A(A=238)$, smallest $n_{s}\left(n_{s}=2\right)$, and largest $B_{Q}$ (we use 5 T below). We set the Garren coefficient. $\mathrm{C}_{\mathrm{G}}=100$, expecting optimized design to lower it from the mid-range value 125 . The parameters remaining with which to increase $d \mathcal{F}_{i}$ are $F_{s}, n_{b}$, and $T_{f}$. An upper limit on $F_{s}$ (perinaps $F_{S} \leqslant 5$ ) will soon emerge from studies in progress. Aside from cost of beam lines and magnets, there is a practical upper limit on $n_{b}$ arising from geometrical effects; perhaps $n_{b} \leqslant 24$. For each choice of $A, E$, and $\hat{\zeta}^{\gamma}$ an upper limit is imposed on $T_{f}$ by the minimum spot size requirement $r_{s} \geqslant 1 \mathrm{~mm}$; the limit is higher for larger values of $A$ and $E$. The following table, employing the range-energy relations of Ref. 7, shows a few values:

| Ion | $\begin{aligned} & E=1 \mathrm{MJ} \\ & \theta=20 \mathrm{MJ} / \mathrm{g} \\ & \underline{t} \leqslant 0.8 \mathrm{~g} / \mathrm{cm}^{2} \end{aligned}$ | $\begin{aligned} E & =10 \mathrm{NJ}, \\ \ell & =30 \mathrm{MJ} / \mathrm{g} \\ \mathscr{Q} & \leqslant 5.3 \mathrm{~g} / \mathrm{cm}^{2} \end{aligned}$ |
| :---: | :---: | :---: |
| Xe | 13 | 42 |
| Hg | 22 | 76 |
| U | 27 | 95 |

If the product of all expected dilution factors cannot be accomnodated within the limits indicated the only path open is to start over with larger $\mathbb{E}$.

It is useful to give an expression for $A_{i}$ in the following units: $\mathcal{L}(M J), \mathscr{F}(M J / g), r_{f}(n s e c), T_{f}(\mathrm{CeV}), \mathcal{Q}\left(g / \mathrm{cm}^{2}\right), I(A), B_{Q}(T), e_{\perp}(\mathrm{cm}-\mathrm{mrad})$, $\varepsilon_{\| I b}(\mathrm{eV}-\mathrm{sec}), \mathrm{f}_{\mathrm{Ib}}(\mathrm{MHz})$. Then
$\theta_{i}=\frac{0.46}{C_{G}} \cdot \frac{E^{\frac{3}{2}} \tau_{f}}{\xi^{3 / 2}} \cdot \frac{T_{f}^{5 / 2}}{(A Z Z)^{3 / 2}} \cdot \frac{I_{L}}{\varepsilon_{\perp L}^{2} \varepsilon_{i I L b} f_{L b}} \cdot \frac{B_{Q} F_{S} n_{b}}{\left(n_{S} / 2\right)^{3 / 2}}$.

To illustrate the use of this expression we give numerical values for the factors for $H F 3$ and $M J 10$, using $n_{b}$ (effective) = 32 for the former, $\tau_{f}=13$ and $n_{D}=8$ for the latter, and $n_{s}=2, B_{Q}=5$, and $C_{G}=100$ for both:
HF3: $\quad \mathcal{A}_{i}^{\prime}=\left(4.6 \times 10^{-3}\right)(0.067)(0.54)(35)(5)(32) \mathrm{F}_{\mathrm{s}}=0.93 \mathrm{~F}_{\mathrm{s}}$,
MJ10: $\hat{\vartheta}_{i}^{\prime}=\left(4.6 \times 10^{-3}\right)(0.25)(1.21)(154)(5)(8) \mathrm{F}_{\mathrm{s}}=8.6 \mathrm{~F}_{\mathrm{s}}$.
Decreasing $T_{f}$ in HF 3 to 13 GeV to bring the spot radius upto 1 mm does not appreciably affect $\mathscr{O}_{i}$ but requires $50 \%$ more ions and therefore more or larger synchrontrons and rings. Even so, this design seems to require $F_{s}=10$ to allow the total stated expected dilution factor of $3 \times 3=9$ (transverse only) which does not seem conservative. The value of $\infty$ is larger for MJIO by a factor 8.4 arising from larger $E$ and $A$, and the larger linac figure of merit compensates for smailer $n_{b}$. The MJIO design also requires about $50 \%$ more or larger fings to supply 10 MJ , and other changes to provide the smaller $\tau_{f}$ which appears to be required. The total stated expected dilution factor for this design is $3 \times 3$ (post-linac transverse) $\times 3$ (longitudinal) $=27$, requiring $F_{s}=3.1$.

It must be emphasized that the estimates of $\mathscr{S}_{i}$ presented here should not be expected to agree with those of the workers whose designs are used for illustration because they have made different assumptions regarding final lens systems.

A Class of Linac-Accumulator Designs
In the light of the information above, it was decided to examine the possibility of designing a system similar to MJ10 but with $\mathbb{E}=1 \mathrm{MJ}$ by compensating the smaller $E$ by a larger $n_{b}$. The exercise also provides an example of a design procedure based explicitly on phase space considerations. We have raised the ion ( $A=238$ ) final energy $T_{f}$ to 25 GeV , near the limit set by $r_{s} \geqslant 1 \mathrm{~mm}$, and have assumed a linac figure of merit slightly more conservative than that in MJ10, with $\varepsilon_{11 \mathrm{Lb}}=10^{-3} \mathrm{eV}$-sec and other parameters the same as in DY23, giving the figure of merit 125 in the units used above. The value of $\mathcal{O}_{i}$ is then
$\stackrel{\ddot{Y}_{i}}{ }=\left(4.6 \times 10^{-3}\right)(0.067)(1.4)(125)(5) n_{b} F_{s}=0.27 n_{b} F_{s}=D_{\perp} D_{11}$
with $D_{\perp}, D_{\| l}$ the total transverse and longitudinal dilution factors, respectively. As minimal lower limits we take $D_{1} D_{\|}=16$, a value lying between those of the designs cited above. Then

$$
n_{b} \geqslant 60 / F_{s}=12 \text { for } F_{s}=5
$$

The linac beam is to till $n_{r}$ rings, each of radius $R$ and mean bending field $B$, with $n_{t}$ turns each. To match the transverse phase space

$$
\varepsilon_{\perp R}^{2}=\Gamma_{\perp} n_{t} \varepsilon_{\perp L}^{2}
$$

with $\varepsilon_{\perp \mathrm{R}}$ the ring emittance; we have assumed that all transverse dilution occurs during injection. If every beam bunch emerging from a ring is split transversely into $\sigma^{2}$ beams, each with its own final lens,

$$
\varepsilon_{1 R}=\sigma \varepsilon_{\perp f^{\prime}}
$$

With $B$ in resla, the ring radius in meters is $R=363 /(q B)$ and the total number of turns in all rings is $5.74 q^{2} \mathrm{~B}$. Combining these relations

$$
\frac{\mathrm{qB} \mathrm{D}_{\perp}}{\sigma^{2} n_{r}} \simeq 1.5
$$

The space charge limit in a ring has been calculated in the usual way ${ }^{12}$ except that the allowable number of ions has been taken as one-third of that given by $\Delta v=\frac{1}{4}$ so as to allow adiabatic rebunching with quasiparabolic charge distribution in azimuth to within a phase spread $\pm \pi / 2$ at harmonic $h$ (with $h$ bunches emerging from each ring) just before final implosive compression in the rings. This gives

$$
N_{\mathrm{sc}} \leqslant 2.5 \times 10^{14} \sigma / \mathrm{q}^{3 / 2},
$$

obtained from $\varepsilon_{\text {If }}=0.58 \mathrm{q}^{\frac{1}{2}}$ cm-mrad using $X_{\max }=30 \mathrm{~cm}, R_{v}=5 \mathrm{~m}, \mathrm{C}_{\mathrm{C}} \sim 100$, and the relations above. Because this design requires a total number $N_{o}$ of ions equal to $2.5 \times 10^{14}$, with total charge $4 \times 10^{-5} \mathrm{q}$ coul, we find

$$
n_{r} \geqslant q^{3 / 2} / \sigma
$$

Combining this with the transverse matching requirement,
and

$$
B \geqslant 1.5 \sigma \mathrm{Q}^{\frac{1}{2}} / \mathrm{D}_{1}
$$

$$
n_{t}=8.4 \sigma^{2} \mathrm{q} / \mathrm{D}_{\perp}
$$

To match the longituãinal phase space,

$$
\varepsilon_{\| R}=\varepsilon_{\| L b} n_{L b / c} D_{\|}
$$

with $\varepsilon_{\| R}$ the longitudinal emittance in one ring circumference and $n_{\text {Lb }} / \mathrm{c}$ $=2 \pi R f_{I b} / \beta c=1400 /(q B)$ the number of linac bunches per circumference. Extraction at harmonic $h$ leads to

$$
\varepsilon_{n f}=\varepsilon_{\| R} / h=1.4 D_{\|} /(h q B)=0.25 F_{s} \text {, }
$$

so that

$$
\mathrm{hq} \mathrm{~B}=5.6 \mathrm{D}_{\mu} / \mathrm{F}_{\mathrm{s}} .
$$

The final momentum spread is $(\Delta \mathrm{p} / \mathrm{p})_{f}=1.75 \times 10^{-3} \mathrm{~F}_{\mathrm{S}}$ and the total number of beams is

$$
\mathrm{n}_{\mathrm{b}}=\sigma^{2} \mathrm{~h} \mathrm{n}_{\mathrm{r}} \text {. }
$$

One may then proceed to assume values of $D_{\perp}$ and $\sigma$, and to calculate for each charge state $q$ values of $B_{\min }, n_{r_{\min }}, n_{r}=$ integer $\geqslant n_{r_{\min }}$, $B$, $n_{t}, h_{\max }, h_{\min }, n_{b_{\max }}, n_{D_{\min }}, D_{\| \max }$, and $D_{\| \min }{ }_{\min }$. The maximum and minimum values arise from assuming a maximum value for $n_{b}$ and a minimum value for $D_{11}$; there is also the requirement that $h$ be an integer.
Example designs based on $D_{\perp}=4, \sigma=1, F_{s}=5, n_{b} \leqslant 24, D_{n} \geqslant 4$ are given in the following table:

| $\begin{aligned} & \text { RFL } \\ & \text { Case } \end{aligned}$ | q | $n_{r}$ | $B(T)$ | $\mathrm{n}_{\mathrm{t}}$ | $h_{\text {max }}$ | $h_{\text {min }}$ | $\mathrm{n}_{\mathrm{b}_{\text {max }}}$ | ${ }^{\mathrm{b}_{\text {min }}}$ | DIImax | Dlimin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2/25 | 2 | 3 | 0.55 | 2.1 | 8 | 4 | 24 | 12 | 8 | 4 |
| 3/25 | 3 | 6 | 0.73 | 6.3 | 4 | 2 |  | 12 | 8 | 4 |
| 4/25 | 4 | 8 | 0.73 | 8.4 | 3 | 2 | 24 | 16 | 8 | 5.2 |
| 6/25 | 6 | 16 | 0.98 | 12.6 | 1 |  | 16 |  | 5.2 |  |
| 8/25 | 8 | 24 | 1.09 | 16.8 |  |  | 24 |  | 8 |  |

The larger values of $D_{\|}$are safer because of anticipated dilutions in the funnel-loading linac, in debunching its beam, and perhaps in final compression. Note also that $D_{\|}$is proportional to $F_{S}$ so that if $F_{s}$ is reduced from 5 to the smallest value (2.5) compatible with $n_{b} \leqslant 24$ the values of $D_{\|}$are also reduced by a factor 2. The small value $D_{1}=2 \times 2=4$ for injection is less plausible for the larger values of $n_{t}$

In a detailed development of designs based on such parameters the number of turns per ring should be adjusted to be an integer, and advantage could be taken of the ingenious use in HF3 of longer pulses through some final lenses to increase their effective number.

If one repeats this procedure for $\sigma=2$ with all other values held the same only $q \leqslant 5$ is allowed and larger $n_{t}$ (17-42 turm/ring) is needed. Therefore $D_{1}=4$ is too small; taking it larger reduces $D_{11} / F_{S}$. However, if this problem is ignored the total circumference of all rings is reduced by a factor four from its value ( 6.2 km ) for $\sigma=1$. One pays another price; the peak rf volts per turn required for final sudden compression in the rings ${ }^{13}$ is of order $5 / \mathrm{BMV} /$ turn for $\sigma=1$ and is four times as large for $\sigma=2$. However, the corresponding azimuthally-averaged peak rf electric field for this class of systems depends only on the number of turns injected per ring and is equal to $1.1 \Omega_{t} \mathrm{kV} / \mathrm{m}$ for our initial selection of general parameters. Expressions for total circumference of all rings and peak rf voltage are $2 \pi R n_{r}=1.55 D_{\perp} / \sigma^{2}$ and $V_{o}(M V / t u m)=14 \mathrm{q} / \mathrm{n}_{\mathrm{r}}=20 \sigma^{2} /\left(B \mathrm{D}_{\mathrm{L}}\right)$. Conclusions

The linac on-time per target in the examples above is $q / 10$ msec; it is evident that the system could serve several reactor vessels.

When confronted with the tightness of the phase space constraint, which has been evident to all those who have tried to construct example designs, it is of central importance to emphasize the crucial role played by the linac figure of merit defined above, and the absolute necessity to minimize all phase space dilutions.

## Parameler List for Case RlL 3/25 ( $h_{1}=3$ )

## General

$E=1 \mathrm{MJ}, \quad \underset{f}{ }=20 \mathrm{MJ} / \mathrm{g}, \quad \tau_{f}=6 \mathrm{nsec}$.
$A^{q}=238^{+3}, \quad T_{f}=25 \mathrm{GeV}, \quad \mathrm{R}^{\prime}=0.72 \mathrm{~g} / \mathrm{m}^{2}, \mathrm{n}_{\mathrm{s}}=2, \mathrm{r}_{\mathrm{s}}=1.05 \mathrm{~mm}$.
$N_{0}=2.5 \times 10^{14^{f}}$ ions, $Q=1.2 \times 10^{-4}$ Coul.
$\gamma_{f}=1.113, \quad(B \gamma)_{f}=0.488, \quad \beta_{f}=0.439$.
Iinac
$I_{L}=0.4 \mathrm{~A}, \quad e_{1 L}=0.2 \mathrm{~cm}-\operatorname{mrad}, f_{L b}=80 \mathrm{MHz}$ (DY23)
$\varepsilon_{\| I b}=10^{-3} \mathrm{eV}$-sec, $\Delta \mathrm{t}_{\mathrm{I}}=3 \times 10^{-4} \mathrm{sec}, \quad \varepsilon_{\| I}=24 \mathrm{eV}-\mathrm{sec}$.
Accumulator Rings
$n_{r}=6, n_{t}=6.3$ per ring, $D_{\perp}=4$ (assumed) implies $D_{i l} \leqslant 5.6$.
$B=0.73 \mathrm{~T}, \mathrm{R}=165 \mathrm{~m}$.
$\varepsilon_{1 R}=1 \mathrm{~cm}$-mrad, $f_{r e v}=0.12 \mathrm{MHz}$.
$n_{\mathrm{Lb} / \mathrm{c}}=640, I_{\text {circ }}=2.4 \mathrm{~A}$.

## Rapid Extraction

$h=3(0.36 \mathrm{MHz}), V_{o} \sim 7 \mathrm{MV} /$ turn (peak), $\overline{\boldsymbol{\varepsilon}}_{\mathrm{o}} \sim 7 \mathrm{kV} / \mathrm{m}$ (peak).
fast compression factor $\sim 210$ ( $\sim 30$ inside rings, $\sim 7$ in lines).
$I_{\text {max }}$ at extraction $\sim 150 \mathrm{~A}$.
Number of turns during compression $\sim 20$

## Final Lenses

$B_{Q}=5 \mathrm{~T}, \quad X_{\text {max }}=30 \mathrm{~cm}, \mathrm{R}_{\mathrm{V}}=5 \mathrm{~m}, \mathrm{n}_{\mathrm{b}}=18$.
$C_{G} \simeq 100$ (Garren's $k=3.44, \quad b=10.6, x=3.2$ ).
$e_{L_{f}}=1$ cm-mrad, $(\Delta \mathrm{p} / \mathrm{p})_{\mathrm{f}}=1.75 \times 10^{-3} \mathrm{~F}_{\mathrm{s}}=8.8 \times 10^{-3}$ for $F_{S}=5$ (assumed).
$e_{11 f}=0.25 \mathrm{~F}_{\mathrm{s}} \mathrm{eV}-\mathrm{sec}$, bunch length at target $=0.8 \mathrm{~m}$, current per beam

$$
=1 . I \mathrm{kA} .
$$

All emittances in this paper are area/ $\pi$ and are normalized.

## References

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