

Phase Transitions in the NMSSM

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1. Introduction

the **Baryon Asymmetry of the Universe**

$$\frac{n_B}{s} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq (0.37 - 0.88) \times 10^{-10}$$

← BBN, consistent with WMAP data

scenarios to explain the BAU

- GUTs
- Affleck-Dine
- Leptogenesis
- ★ EW Baryogenesis
 - physics within our reach
- ⋮

Anomalous $(B + L)$ -nonconservation in EW theory

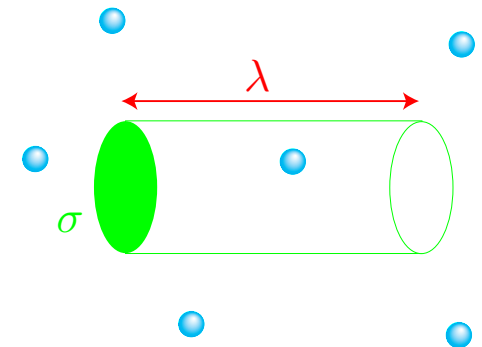
suppressed at $T = 0$ by $e^{-2S_{\text{instanton}}} \simeq 10^{-164}$

— free from proton decay problem

at $T < T_C$; $\Gamma_{\text{sph}}^{(\text{br})} \simeq T e^{-E_{\text{sph}}/T}$

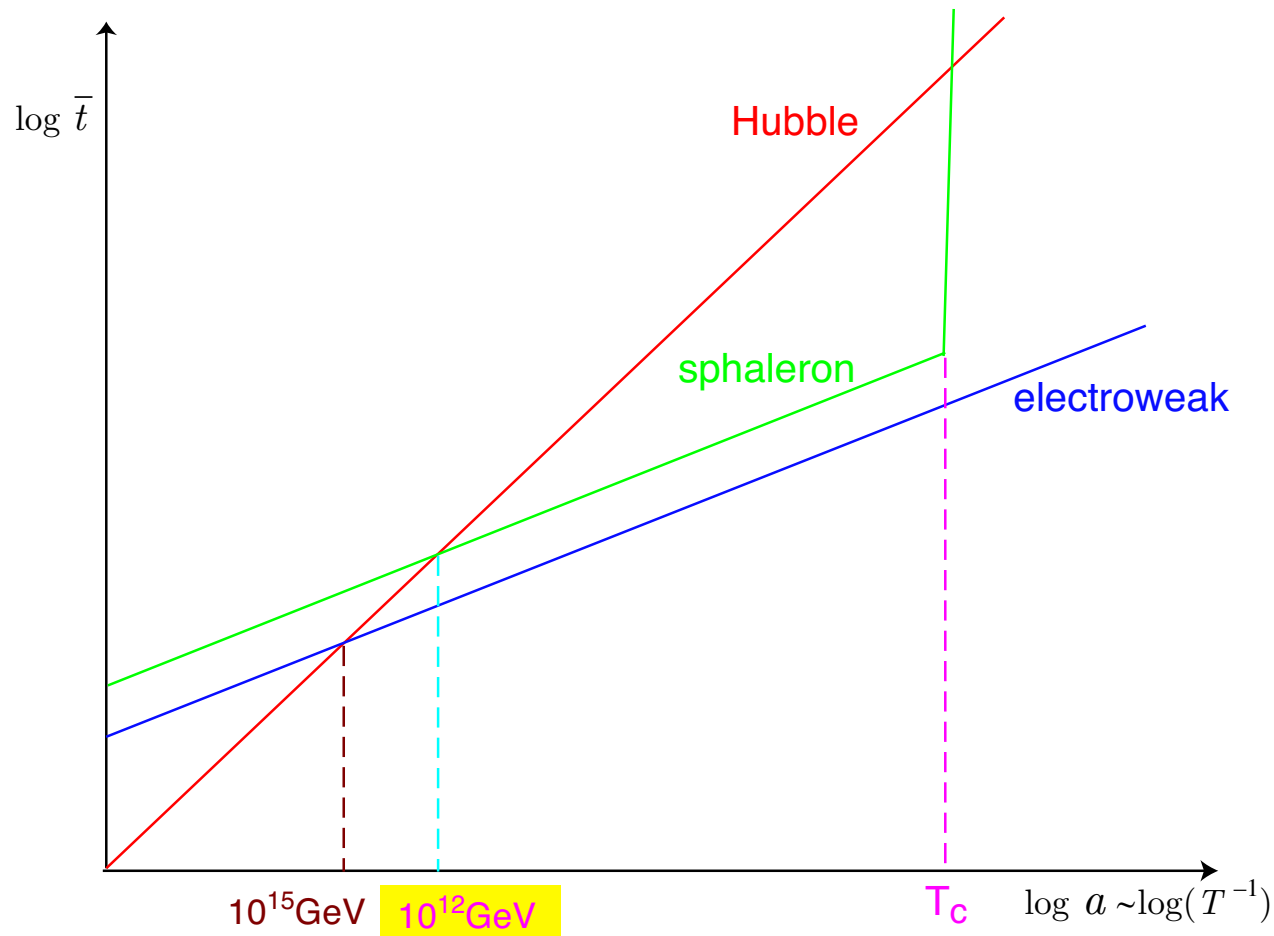
at $T > T_C$; $\Gamma_{\text{sph}}^{(\text{sym})} \simeq \kappa \alpha_W^4 T$ ($\kappa \simeq 1.1$)

mfp (time scale) of elementary processes: $\lambda \cdot \sigma = \frac{1}{n}$
 $m \ll T \Rightarrow \lambda \simeq \bar{t} = \text{mean free time}$



For relativistic particles at T ,

$$\sigma \simeq \frac{\alpha^2}{s} \simeq \frac{\alpha^2}{T^2} \implies \lambda \simeq \frac{10}{gT^3} \left(\frac{\alpha^2}{T^2} \right)^{-1} = \frac{10}{g\alpha^2 T}$$



If $v(T_C) \ll 200 \text{ GeV}$ (eg. 2nd order EWPT), $\exists T_{\text{dec}}$, s.t.

$$T_{\text{dec}} < T < T_C \implies \Gamma_{\text{sph}}^{(b)}(T) > H(T)$$

wash-out of $B + L$ even in the broken phase

To have nonzero BAU,

- (i) we must have $B - L$ before the sphaleron process decouples, *or*
- (ii) $B + L$ must be created at the first-order EWPT, *and* the sphaleron process must decouple immediately after that.

N.B.

$\Delta(B + L) \neq 0$ process is *in equilibrium*, for $T_C \simeq 100\text{GeV} < T < 10^{12}\text{GeV}$.

If $\Delta L \neq 0$ process is in equilibrium in this range of T $\Rightarrow B = L = 0!$

To leave $B \neq 0$, $\Gamma_{\Delta L \neq 0} < H(T)$ for $T \in [T_C, 10^{12}\text{GeV}]$.

\Rightarrow constraints on models with $\Delta L \neq 0$ processes.

e.g., lower bound on m_N in the seesaw model

\rightarrow upper bound on $m_\nu < 0.8\text{GeV}$

$$T \simeq 100\text{GeV} \Rightarrow H^{-1}(T) \simeq 10^{14}\text{GeV}^{-1} \ll \bar{t}_{EW} \simeq \frac{1}{\alpha_W T^2} \sim 10\text{GeV}^{-1}$$

\therefore All the particles of the SM are in *kinetic equilibrium*.

nonequilibrium state \Leftarrow **1st order EW phase transition**

study of the EWPT

★ static properties \Leftarrow effective potential = free energy density

$$V_{\text{eff}}(\mathbf{v}; T) = -\frac{1}{V} T \log Z = -\frac{1}{V} \log \text{Tr} \left[e^{-H/T} \right]_{\langle \phi \rangle = \mathbf{v}}$$

★ dynamics — formation and motion of the bubble wall when 1st order PT

$V_{\text{eff}}(\mathbf{v}; T)$ \Leftarrow parameters of the model \Rightarrow mass, coupling of the Higgs bosons

contents

2. Higgs mass and the EWPT in the MSM

3. EWPT in the MSSM

Higgs mass and couplings — with/without CP violation

4. Phase Transitions in the NMSSM

similarity and difference

between the MSSM and the NMSSM

5. Summary

2. Higgs mass and the EWPT in the MSM

perturbation at the 1-loop level

$$V_{\text{eff}}(\varphi; T) = -\frac{1}{2}\mu^2\varphi^2 + \frac{\lambda}{4}\varphi^4 + 2Bv_0^2\varphi^2 + B\varphi^4 \left[\log\left(\frac{\varphi^2}{v_0^2}\right) - \frac{3}{2} \right] + \bar{V}(\varphi; T)$$

where $B = \frac{3}{64\pi^2 v_0^4} (2m_W^4 + m_Z^4 - 4m_t^4),$

$$\bar{V}(\varphi; T) = \frac{T^4}{2\pi^2} [6I_B(a_W) + 3I_B(a_Z) - 6I_F(a_t)], \quad (a_A = m_A(\varphi)/T)$$

$$I_{B,F}(a^2) \equiv \int_0^\infty dx x^2 \log\left(1 \mp e^{-\sqrt{x^2+a^2}}\right).$$

high-temperature expansion [$m/T \ll 1$]

$$I_B(a^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}a^2 - \frac{\pi}{6}(a^2)^{3/2} - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{4\pi} - \frac{a^4}{16} \left(\gamma_E - \frac{3}{4} \right) \frac{a^4}{2} + O(a^6)$$

$$I_F(a^2) = \frac{7\pi^4}{360} - \frac{\pi^2}{24}a^2 - \frac{a^4}{16} \log \frac{\sqrt{a^2}}{\pi} - \frac{a^4}{16} \left(\gamma_E - \frac{3}{4} \right) + O(a^6)$$

For $T > m_W, m_Z, m_t$,

$$V_{\text{eff}}(\varphi; T) \simeq D(T^2 - T_0^2)\varphi^2 - ET\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

where

$$D = \frac{1}{8v_0^2}(2m_W^2 + m_Z^2 + 2m_t^2), \quad E = \frac{1}{4\pi v_0^3}(2m_W^3 + m_Z^3) \sim 10^{-2}$$

$$\lambda_T = \lambda - \frac{3}{16\pi^2 v_0^4} \left(2m_W^4 \log \frac{m_W^2}{\alpha_B T^2} + m_Z^4 \log \frac{m_Z^2}{\alpha_B T^2} - 4m_t^4 \log \frac{m_t^2}{\alpha_F T^2} \right)$$

$$T_0^2 = \frac{1}{2D}(\mu^2 - 4Bv_0^2), \quad \log \alpha_{F(B)} = 2 \log(4)\pi - 2\gamma_E$$

At T_C , \exists degenerate minima: $\varphi_C = \frac{2ET_C}{\lambda_{T_C}}$

$$\Gamma_{\text{sph}}^{(\text{br})} < H(T_C) \iff \frac{\varphi_C}{T_C} \gtrsim 1 \implies \text{upper bound on } \lambda \quad [m_H = \sqrt{2\lambda}v_0]$$

$$m_H \lesssim 46 \text{ GeV} \implies \text{MSM is excluded}$$

★ Monte Carlo simulations

effective fermion mass : $m_f(T) \sim O(T) \leftarrow$ nonzero modes

\therefore simulation only with the bosons

QFT on the lattice $\left\{ \begin{array}{l} \text{scalar fields: } \phi(x) \text{ on the sites} \\ \text{gauge fields: } U_\mu(x) \text{ on the links} \end{array} \right.$

$$Z = \int [d\phi dU_\mu] \exp \{-S_E[\phi, U_\mu]\}$$

- 3-dim. $SU(2)$ system with a Higgs doublet and a triplet time-component of U_μ
[Laine & Rummukainen, hep-lat/9809045]
- 4-dim. $SU(2)$ system with a Higgs doublet [Csikor, hep-lat/9910354]
EWPT is first order for $m_h < 66.5 \pm 1.4 \text{ GeV}$

Both the simulations found end-point of EWPT at

$$m_h = \left\{ \begin{array}{l} 72.3 \pm 0.7 \text{ GeV} \\ 72.1 \pm 1.4 \text{ GeV} \end{array} \right. \Rightarrow \boxed{\text{no PT (cross-over) in the MSM !}}$$

3. EWPT in the MSSM

superpotential: $W = y_b Q_L B_R^c H_d - y_t Q_L T_R^c H_u - \mu H_d H_u$

2 Higgs doublets: $H_d \ni \Phi_d = \begin{pmatrix} \phi_d^0 \\ \phi_d^- \end{pmatrix}$, $H_u \ni \Phi_u = \begin{pmatrix} \phi_u^+ \\ \phi_d^0 \end{pmatrix}$

Higgs potential

$$V_0 = m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u - (m_3^2 \epsilon_{ij} \Phi_d^i \Phi_u^j + \text{h.c.}) + \frac{g_2^2 + g_1^2}{8} \left(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right)^2 + \frac{g_2^2}{2} \left| \Phi_d^\dagger \Phi_u \right|^2$$

all the parameters are real: no CP violation

$$m_{1,2}^2 = m_{\text{soft}}^2 + |\mu|^2 \leftrightarrow v_0 \text{ and } \tan \beta \quad \text{by} \quad \frac{\partial V_0}{\partial v_d} = \frac{\partial V_0}{\partial v_u} = 0$$

The Higgs mass is not completely a free parameter.

After EWSB $\longrightarrow \phi_d^0 = \frac{1}{\sqrt{2}}(v_d + h_d + i a_d), \quad \phi_u^0 = \frac{1}{\sqrt{2}}e^{i\theta}(v_u + h_u + i a_u)$

vacuum: $v_0 = \sqrt{v_d^2 + v_u^2} = 246\text{GeV}, \quad \tan \beta = v_u/v_d$

1 Nambu-Goldstone mode in (a_d, a_u) and 1 in (ϕ_d^+, ϕ_u^-)

\implies physical modes: 3 neutral (h, H, A) , 1 charged (H^\pm)

tree-level masses

$$m_{h,H}^2 = \frac{1}{2} \left[m_Z^2 + m_A^2 \mp \sqrt{(m_Z^2 + m_A^2)^2 - 4m_Z^2 m_A^2 \cos^2(2\beta)} \right],$$

$$m_A^2 = \frac{\text{Re}(m_3^2 e^{i\theta})}{\sin \beta \cos \beta}, \quad m_{H^\pm}^2 = m_A^2 + m_W^2$$

$$\longrightarrow m_h \leq \min \{m_Z, m_A\}, \quad m_H \geq \max \{m_Z, m_A\}$$

These bounds receive radiative corrections from loops of the top quarks and squarks

$$\longrightarrow m_h \lesssim 135\text{GeV}$$

[Okada, et al. PTP85 ('91) 1]

One-loop Effective potential

 $(T = 0)$

$$V_{\text{eff}} = V_0 + \frac{N_C}{32\pi^2} \sum_{q=t,b} \left[\sum_{j=1,2} \left(\bar{m}_{\tilde{q}_j}^2 \right)^2 \left(\log \frac{\bar{m}_{\tilde{q}_j}^2}{M^2} - \frac{3}{2} \right) - 2 \left(\bar{m}_q^2 \right)^2 \left(\log \frac{\bar{m}_q^2}{M^2} - \frac{3}{2} \right) \right]$$

$\bar{m}^2(v_d, v_u, \theta)$: field-dependent mass

mass² at the one-loop level

$$\mathcal{M}^2 = \begin{pmatrix} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d^2} \right\rangle & \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial h_u} \right\rangle & \frac{1}{\cos \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial a_u} \right\rangle \\ \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial h_u} \right\rangle & \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u^2} \right\rangle & \frac{1}{\sin \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u \partial a_d} \right\rangle \\ \frac{1}{\cos \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_d \partial a_u} \right\rangle & \frac{1}{\sin \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_u \partial a_d} \right\rangle & \frac{1}{\sin \beta \cos \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial a_d \partial a_u} \right\rangle \end{pmatrix}$$

$$m_{H^\pm}^2 = \frac{1}{\sin \beta \cos \beta} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial \phi_d^+ \partial \phi_u^-} \right\rangle \quad \langle \dots \rangle = \text{values at the vacuum}$$

CP-conserving $\Rightarrow \mathcal{M}_{33}^2 = m_A^2$

CP violation in the squark sector $\propto \text{Im}(\mu A_q e^{i\theta}) \Rightarrow \mathcal{M}_{13}, \mathcal{M}_{23} \neq 0$

One-loop Effective potential

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mass eigenstates: (H_1, H_2, H_3)

$$\begin{pmatrix} h_d \\ h_u \\ a \end{pmatrix} = \mathcal{O} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix}, \quad \mathcal{O}^T \mathcal{M}^2 \mathcal{O} = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2)$$

gauge and Yukawa interactions

$$\mathcal{L}_{\text{gauge}} \sim g_2 m_W g_{VVH_i} \left(W_\mu^+ W^{-\mu} + \frac{Z_\mu Z^\mu}{2 \cos^2 \theta_W} \right) H_i + \frac{g_2}{2 \cos \theta_W} g_{ZH_i H_j} Z^\mu \left(H_i \overleftrightarrow{\partial}_\mu H_j \right)$$

$$\mathcal{L}_Y \sim -\frac{g_2 m_b}{2 m_W} \bar{b} (g_{bbH_i}^S + i \gamma_5 g_{bbH_i}^P) b H_i$$

corrections to the couplings

[MSM: $g_{VVH} = 1, g_{ZH H} = 0, g_{bbH} = 1$]

$$g_{VVH_i} = O_{1i} \cos \beta + O_{2i} \sin \beta$$

$$g_{ZH_i H_j} = \frac{1}{2} [(O_{3i} O_{1j} - O_{3j} O_{1i}) \sin \beta + (O_{3i} O_{2j} - O_{3j} O_{2i}) \cos \beta]$$

$$g_{bbH_i}^S = O_{1i} \frac{1}{\cos \beta}, \quad g_{bbH_i}^P = -O_{3i} \tan \beta, \quad g_{bbH_i}^2 = (g_{bbH_i}^S)^2 + (g_{bbH_i}^P)^2$$

★ Electroweak phase transition

$$V_{\text{eff}}(\mathbf{v}; T) = V_{\text{eff}}(\mathbf{v}; T) + 6 \sum_{q=t,b} \sum_{j=1,2} \frac{T^4}{2\pi^2} I_B \left(\frac{\bar{m}_{\tilde{q}_j}}{T} \right) + \dots,$$

where $m_{\tilde{t}_j}^2$ is the eigenvalues of

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + \left(\frac{g_1^2}{24} - \frac{g_2^2}{8} \right) (v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 & \frac{y_t}{\sqrt{2}} (\mu v_d + A(v_2 - i v_3)) \\ * & m_{\tilde{t}_R}^2 - \frac{g_1^2}{6} (v_u^2 - v_d^2) + \frac{y_t^2}{2} v_u^2 \end{pmatrix}$$

light stop scenario

[de Carlos & Espinosa, NPB '97]

$$m_{\tilde{t}_L}^2 = 0 \text{ or } m_{\tilde{t}_R}^2 = 0 \implies \text{smaller eigenvalue: } m_{\tilde{t}_1}^2 \sim O(v^2)$$

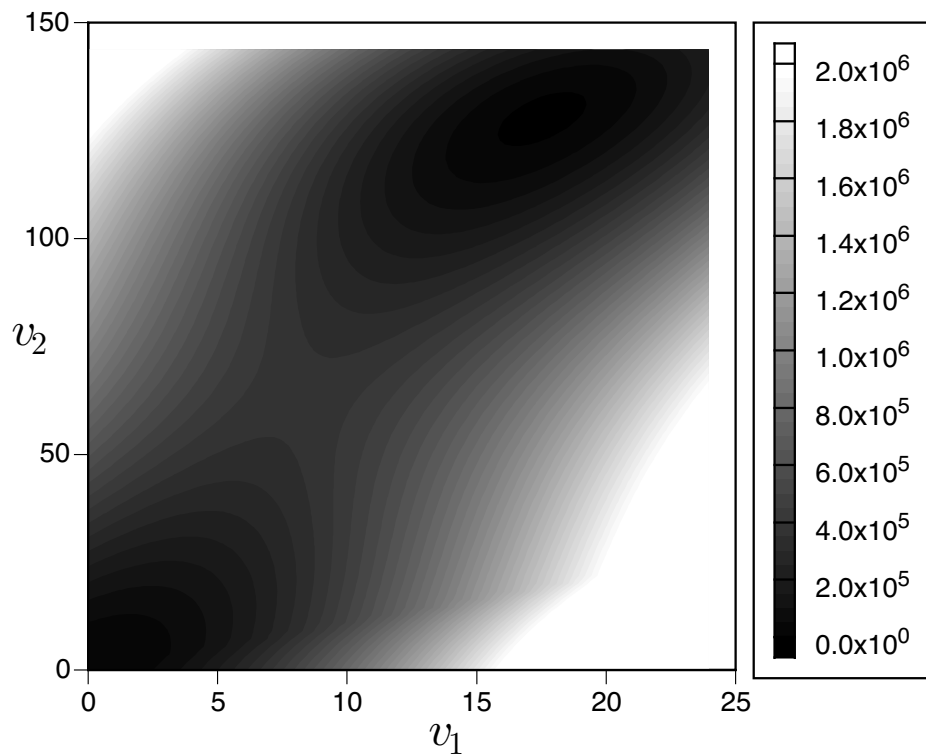
$$\therefore \text{high-}T \text{ expansion: } \Delta_{\tilde{t}} V_{\text{eff}}(\mathbf{v}; T) \implies -3 \frac{T}{6\pi} (m_{\tilde{t}_1}^2)^{3/2} \sim -T v^3 \quad \rightarrow \text{1st order PT}$$

more effective for larger y_t — smaller $\tan \beta$

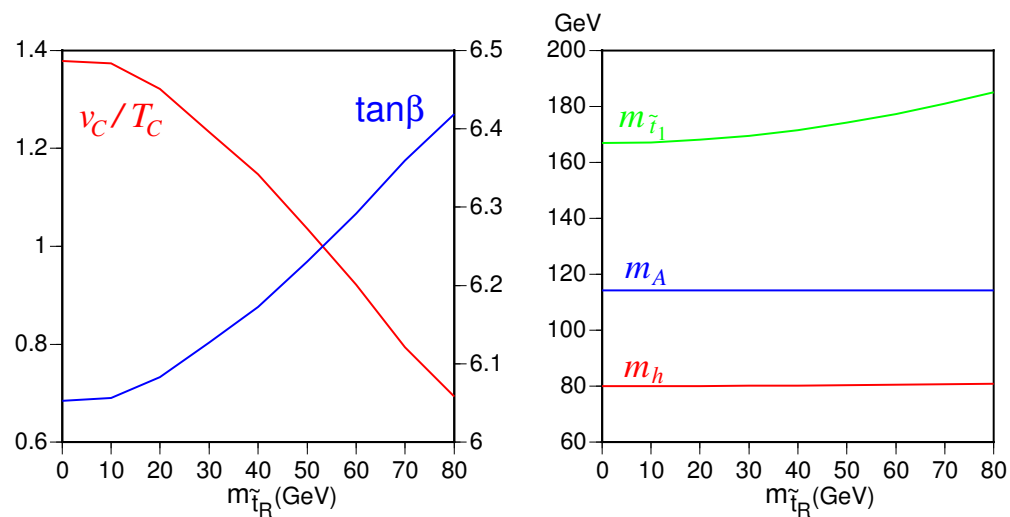
An example: $\tan \beta = 6$, $m_h = 82.3\text{GeV}$, $m_A = 118\text{GeV}$, $m_{\tilde{t}_1} = 168\text{GeV}$

$T_C = 93.4\text{GeV}$, $v_C = 129\text{GeV}$

[KF, PTP101('99)]



$V_{\text{eff}}(v_1, v_2, v_3 = 0; T_C)$



$m_{\tilde{t}_R}$ -dependence ($\tan \beta = 6$)

★ Lattice MC studies

- 3d reduced model

[Laine et al. hep-lat/9809045]

strong 1st order for $m_{\tilde{t}_1} \lesssim m_t$ and $m_h \leq 110\text{GeV}$

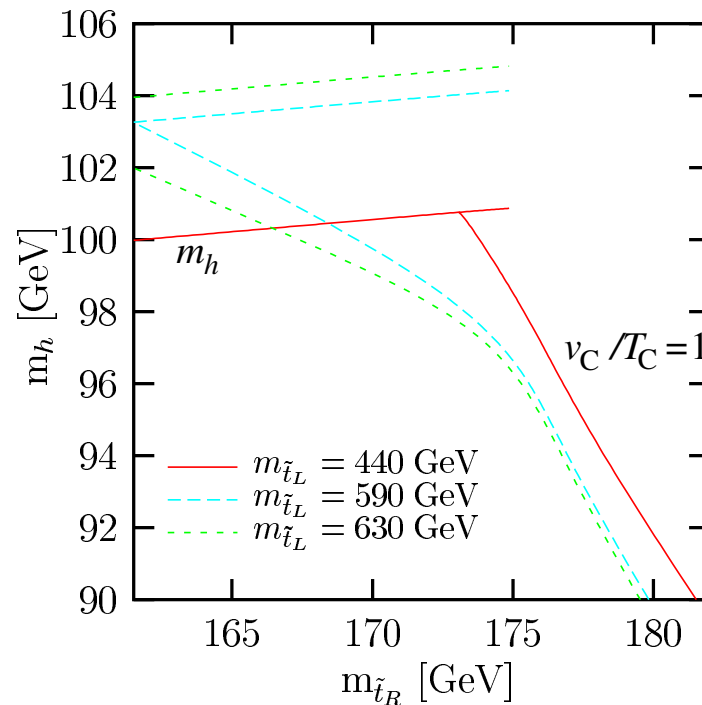
- 4d model

[Csikor, et al. hep-lat/0001087]

with $SU(3)$, $SU(2)$ gauge bosons, 2 Higgs doublets, stops, sbottoms

$A_{t,b} = 0$, $\tan \beta \simeq 6$

→ agreement with the perturbation theory within the errors



$m_A = 500\text{ GeV}$

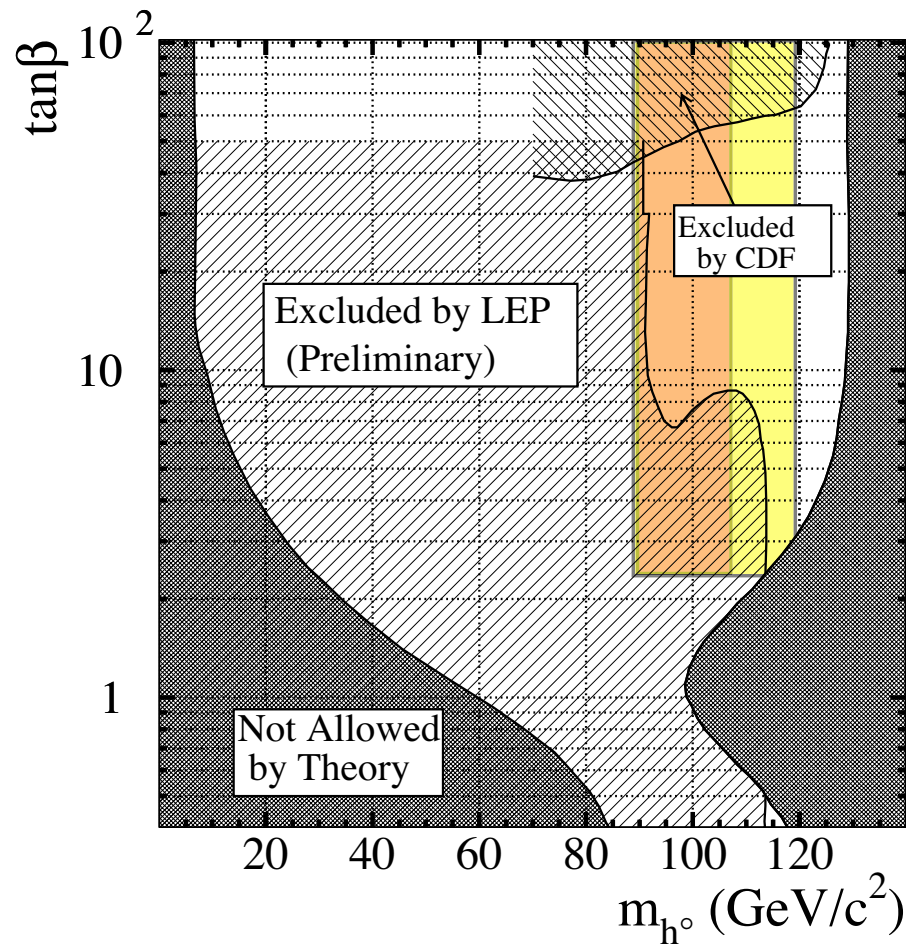
$v_C/T_C > 1$

below the steeper lines



max. $m_h = 103 \pm 4\text{ GeV}$

for $m_{\tilde{t}_L} \simeq 560\text{ GeV}$



[PDG,
<http://ccwww.kek.jp/pdg/>]

light stop: $m_{t_R} = 0$

negative soft mass²: $m_{t_R}^2 > -(65\text{GeV})^2$

[Laine & Rummukainen, NPB597]

★ Effects of CP violation on the EWPT

[KF, Tao & Toyoda, PTP 109('03)]

EWPT in the light-stop scenario [$m_{\tilde{t}_R} = 10\text{GeV}$]

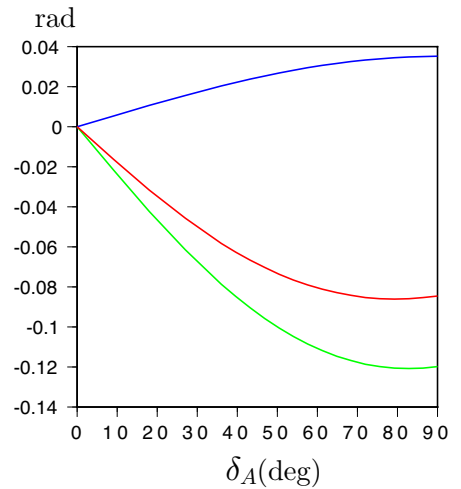
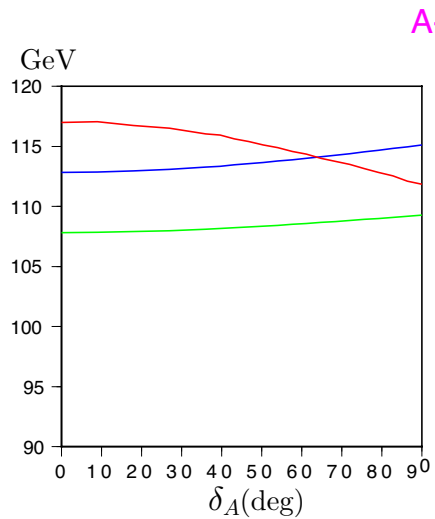
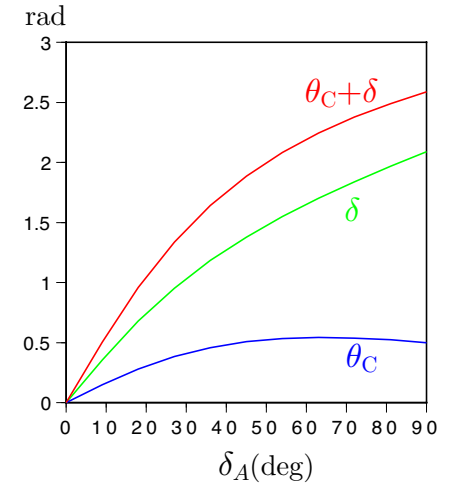
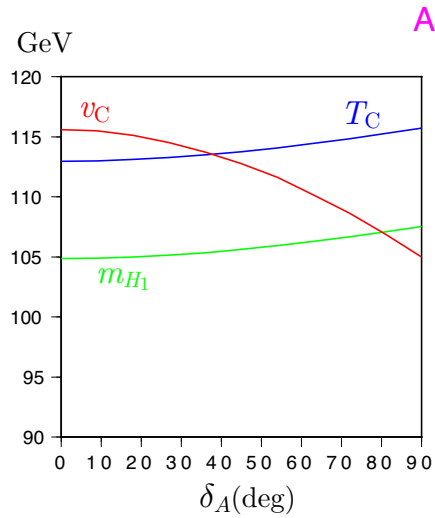
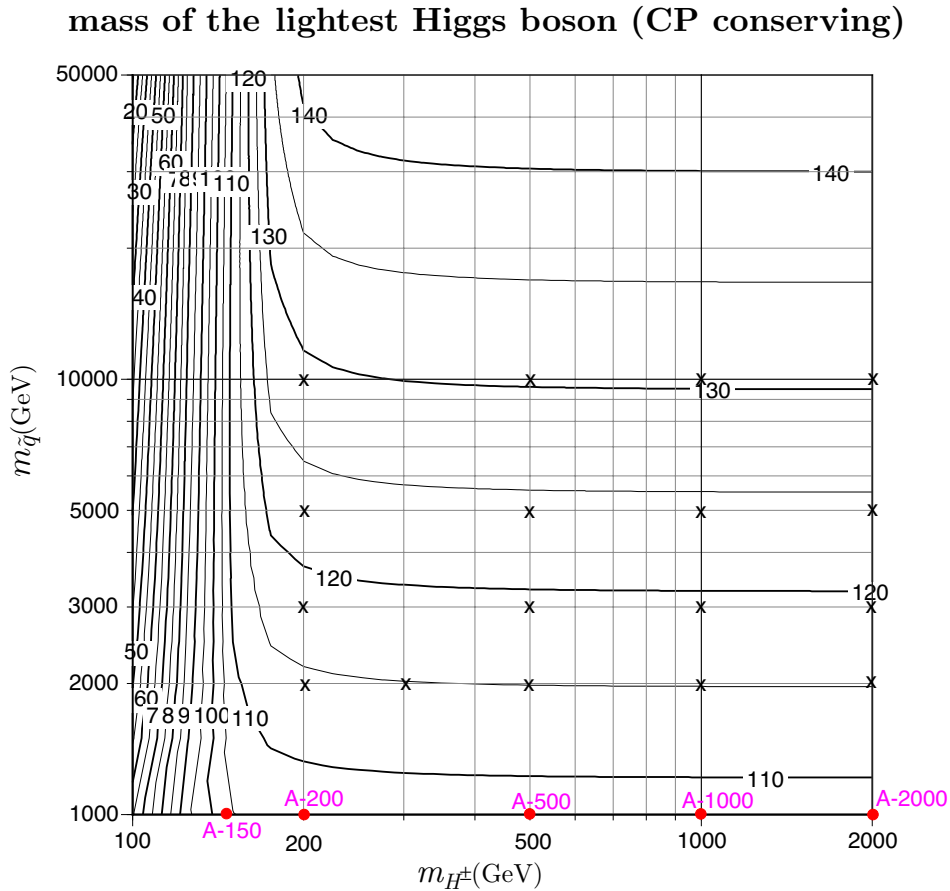
$$\text{Im}(\mu A_t e^{i\theta}) \neq 0 \Rightarrow \begin{cases} \triangleright \text{scalar-pseudoscalar mixing} & [\text{Carena, et al., NPB586}] \\ \triangleright \text{induces } \delta = \text{Arg}(m_3^2) \\ \bullet \text{ weakens the EWPT} \end{cases}$$

field-dependent mass² of the lighter stop:

$$\bar{m}_{\tilde{t}_1}^2 = \frac{1}{2} \left[m_{\tilde{q}}^2 + m_{\tilde{t}_R}^2 + y_t^2 v_u^2 + \frac{g_2^2 + g_1^2}{4} (v_d^2 - v_u^2) - \sqrt{\left(m_{\tilde{q}}^2 - m_{\tilde{t}_R}^2 + \frac{x_t}{2} (v_d^2 - v_u^2) \right)^2 + y_t^2 |\mu v_d - A_t^* e^{-i\theta} v_u|^2} \right]$$

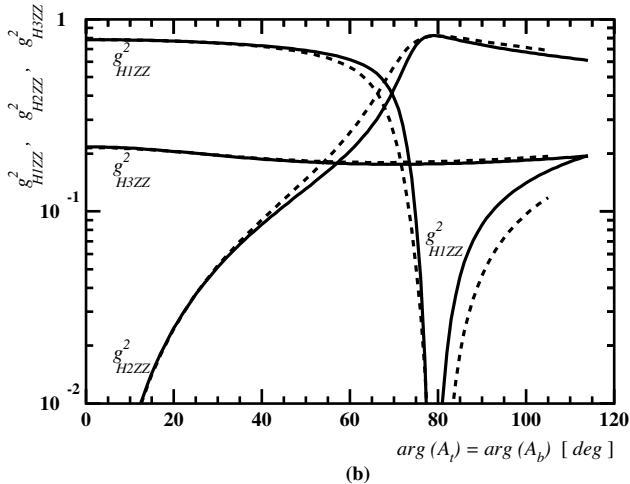
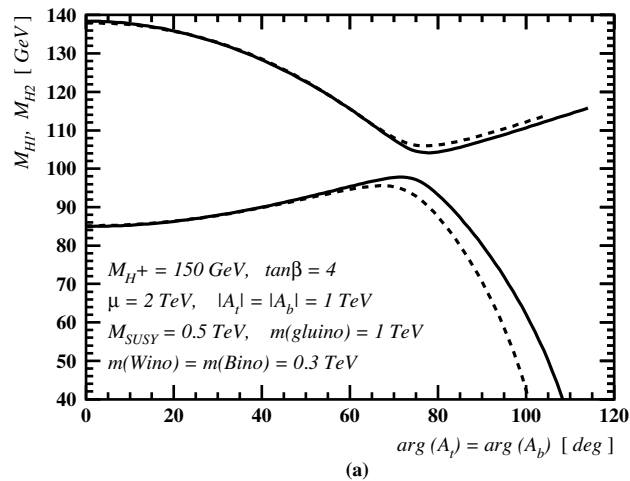
$$\tan \beta = 10, \mu = 1500\text{GeV}, |A| = 150\text{GeV}$$

For parameter sets with $m_{H_1} \geq 105\text{GeV}$, introduce $\delta_A = \text{Arg}A$

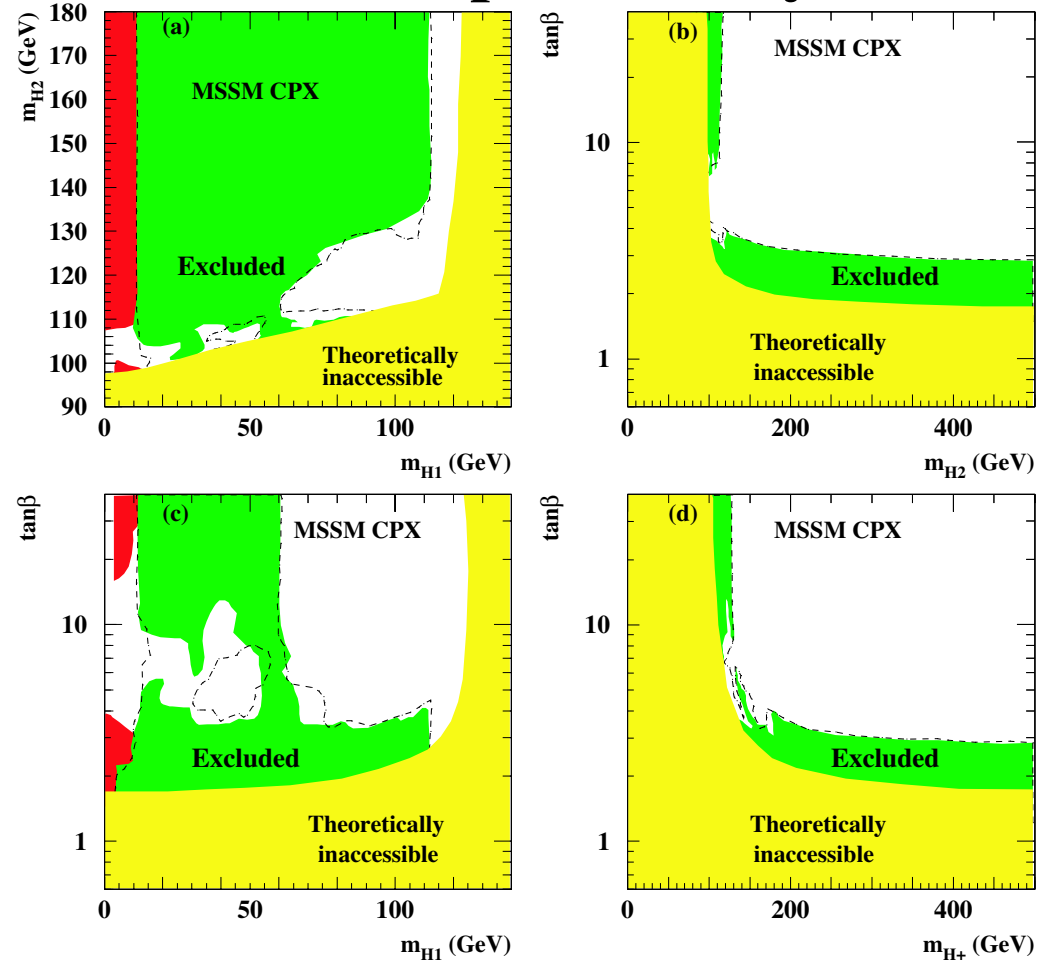


★ Large CP violation and light Higgs

[Carena, et al., NPB586]



OPAL preliminary



OPAL PN524 (July, 2003) CPX scenario

The EWPT in the light-Higgs allowed region has not been investigated.

3. EWPT in the NMSSM

$$W = \epsilon_{ij} \left(y_b H_d^i Q^j B - y_t H_u^i Q^j T + y_l H_d^i L^j E - \lambda N H_d^i H_u^j \right) - \frac{\kappa}{3} N^3$$

$$\lambda \langle N \rangle \sim \mu \text{ in the MSSM}$$

$$\mathcal{L}_{\text{soft}} \ni -m_N^2 n^* n + \left[\lambda A_\lambda n H_d H_u + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right].$$

order parameters: $\langle \Phi_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_{0d} \\ 0 \end{pmatrix}$, $\langle \Phi_u \rangle = \frac{e^{i\theta_0}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{0u} \end{pmatrix}$, $\langle n \rangle = \frac{e^{i\varphi_0}}{\sqrt{2}} v_{0n}$

tree-level Higgs potential:

$$\begin{aligned} V_0 = & m_1^2 \Phi_d^\dagger \Phi_d + m_2^2 \Phi_u^\dagger \Phi_u + m_N^2 n^* n - \left(\lambda A_\lambda \epsilon_{ij} n \Phi_d^i \Phi_u^j + \frac{\kappa}{3} A_\kappa n^3 + \text{h.c.} \right) \\ & + \frac{g_2^2 + g_1^2}{8} \left(\Phi_d^\dagger \Phi_d - \Phi_u^\dagger \Phi_u \right)^2 + \frac{g_2^2}{2} \left| \Phi_d^\dagger \Phi_u \right|^2 \\ & + |\lambda|^2 n^* n \left(\Phi_d^\dagger \Phi_d + \Phi_u^\dagger \Phi_u \right) + \left| \lambda \epsilon_{ij} \Phi_d^i \Phi_u^j + \kappa n^2 \right|^2 \end{aligned}$$

1. 5 neutral and 1 charged scalars (3 scalar and 2 pseudoscalar when CP cons.)

The lightest Higgs scalar can escape from the lower bound 114GeV, because of small coupling to Z, W caused by large mixing among 3 scalars. [Miller, et al. NPB 681]

— “Light Higgs Scenario” —

2. CP violation at the tree level: $\text{Im}(\lambda A_\lambda e^{i(\theta+\varphi)})$, $\text{Im}(\kappa A_\kappa e^{3i\varphi})$, $\text{Im}(\lambda \kappa^* e^{i(\theta-2\varphi)})$

3. $v_n \rightarrow \infty$ with λv_n and κv_n fixed \implies MSSM [Ellis, et al, PRD 39]

\longrightarrow new features expected for $v_n = O(100)\text{GeV}$

-
- ★ study of the Higgs spectrum and couplings without/with CP violation [KF and Tao, hep-ph/0409294]
 - ★ study of the EWPT without/with CP violation [KF, Toyoda and Tao, hep-ph/0501052]
 - ★ sphaleron solution [KF, et al. in preparation]

mass matrix of the neutral Higgs bosons

$$\mathcal{M}^2 \equiv \begin{pmatrix} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_i \partial h_j} \right\rangle & \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial h_i \partial a_j} \right\rangle \\ \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial a_i \partial h_j} \right\rangle & \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial a_i \partial a_j} \right\rangle \end{pmatrix} \xrightarrow{\text{extract NG modes}} \begin{pmatrix} \mathcal{M}_S^2 & \mathcal{M}_{SP}^2 \\ (\mathcal{M}_{SP}^2)^T & \mathcal{M}_P^2 \end{pmatrix}$$

$$\mathcal{M}_S^2 : 3 \times 3, \quad \mathcal{M}_P^2 : 2 \times 2. \quad \mathcal{M}_{SP}^2 : 3 \times 2$$

where the basis is (h_d, h_u, h_n, a, a_n) ,

$$\mathcal{M}_{SP}^2 \propto \begin{cases} \text{Im} (\lambda \kappa^* e^{i(\theta_0 - 2\varphi_0)}) & \text{at the tree level} \\ \text{Im} (\lambda v_n A_{t,b} e^{i(\theta_0 + \varphi_0)}) & \text{at the one-loop level} \end{cases}$$

charged Higgs mass

$$m_{H^\pm}^2 = \frac{1}{\sin \beta_0 \cos \beta_0} \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial \phi_d^+ \partial \phi_u^-} \right\rangle = (m_{H^\pm}^2)_{\mu = \lambda v_n e^{i\varphi_0} / \sqrt{2}}^{\text{MSSM}}$$

At the tree-level,

$$\mathcal{M}_S^2 = \begin{pmatrix} \left(R_\lambda - \frac{\mathcal{R}v_n}{2}\right) v_n \tan \beta + m_Z^2 \cos^2 \beta & -\left(R_\lambda - \frac{\mathcal{R}v_n}{2}\right) v_n - m_Z^2 \sin \beta \cos \beta + |\lambda|^2 v_d v_u & -R_\lambda v_u + \mathcal{R}v_u v_n + |\lambda|^2 v_d v_n \\ \dots & \left(R_\lambda - \frac{\mathcal{R}v_n}{2}\right) v_n \cot \beta + m_Z^2 \sin^2 \beta & -R_\lambda v_d + \mathcal{R}v_d v_n + |\lambda|^2 v_u v_n \\ \dots & \dots & R_\lambda \frac{v_d v_u}{v_n} + 3R_\kappa v_n + 2|\kappa|^2 v_n^2 \end{pmatrix},$$

$$\mathcal{M}_P^2 = \begin{pmatrix} \left(R_\lambda - \frac{1}{2}\mathcal{R}v_n\right) \frac{v_n}{\sin \beta \cos \beta} & (R_\lambda + \mathcal{R}v_n)v_0 \\ (R_\lambda + \mathcal{R}v_n)v_0 & R_\lambda \frac{v_0^2 \sin \beta \cos \beta}{v_n} + 3R_\kappa v_n - 2\mathcal{R}v_d v_u \end{pmatrix},$$

$$\mathcal{M}_{SP}^2 = \begin{pmatrix} 0 & \frac{3}{2}\sin \beta \\ 0 & \frac{3}{2}\cos \beta \\ -\frac{1}{2} & -2\sin \beta \cos \beta \end{pmatrix} \mathcal{I}v_0 v_n.$$

where we have defined

$$R_\lambda = \frac{1}{\sqrt{2}} \operatorname{Re} \left(\lambda A_\lambda e^{i(\theta_0 + \varphi_0)} \right), \quad I_\lambda = \frac{1}{\sqrt{2}} \operatorname{Im} \left(\lambda A_\lambda e^{i(\theta_0 + \varphi_0)} \right),$$

$$R_\kappa = \frac{1}{\sqrt{2}} \operatorname{Re} \left(\kappa A_\kappa e^{3i\varphi_0} \right), \quad I_\kappa = \frac{1}{\sqrt{2}} \operatorname{Im} \left(\kappa A_\kappa e^{3i\varphi_0} \right),$$

$$\mathcal{R} = \operatorname{Re} \left(\lambda \kappa^* e^{i(\theta_0 - 2\varphi_0)} \right), \quad \mathcal{I} = \operatorname{Im} \left(\lambda \kappa^* e^{i(\theta_0 - 2\varphi_0)} \right)$$

— independent of phase convention

We have used the tadpole conditions: $\left\langle \frac{\partial V_{\text{eff}}}{\partial h_i} \right\rangle = \left\langle \frac{\partial V_{\text{eff}}}{\partial a_i} \right\rangle = 0$ ($i = d, u, n$)

$$m_1^2 = \left(R_\lambda - \frac{1}{2} \mathcal{R} v_{0n} \right) v_{0n} \tan \beta_0 - \frac{1}{2} m_Z^2 \cos(2\beta_0) - \frac{|\lambda|^2}{2} (v_{0n}^2 + v_{0u}^2) + \dots$$

$$m_2^2 = \left(R_\lambda - \frac{1}{2} \mathcal{R} v_{0n} \right) v_{0n} \cot \beta_0 + \frac{1}{2} m_Z^2 \cos(2\beta_0) - \frac{|\lambda|^2}{2} (v_{0n}^2 + v_{0d}^2) + \dots$$

$$m_N^2 = (R_\lambda - \mathcal{R} v_{0n}) \frac{v_{0d} v_{0u}}{v_{0n}} + R_\kappa v_{0n} - \frac{|\lambda|^2}{2} (v_{0d}^2 + v_{0u}^2) - |\kappa|^2 v_{0n}^2 + \dots$$

$$I_\lambda = \frac{1}{2} \mathcal{I} v_{0n} + \dots, \quad I_\kappa = -\frac{3}{2} \mathcal{I} \frac{v_{0d} v_{0u}}{v_{0n}}$$

We shall use m_{H^\pm} instead of R_λ :

$$m_{H^\pm}^2 = m_W^2 - \frac{1}{2} |\lambda|^2 v^2 + (2R_\lambda - \mathcal{R} v_{0n}) \frac{v_{0n}}{\sin 2\beta_0} + \dots$$

Definition of the couplings

gauge vs mass eigenstates:
$$\begin{pmatrix} h_d \\ h_u \\ h_n \\ a \\ a_n \end{pmatrix} = \mathcal{O} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{pmatrix}, \quad \mathcal{O}^T \mathcal{M}^2 \mathcal{O} = \text{diag}(m_{H_1}^2, \dots, m_{H_5}^2)$$

$$\mathcal{L}_{\text{gauge}} \ni g_2 m_W g_{VVH_i} \left(W_\mu^+ W^{-\mu} + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right) H_i + \frac{g_2}{2 \cos \theta_W} g_{ZH_iH_j} Z^\mu (H_i \overleftrightarrow{\partial}_\mu H_j)$$

$$\mathcal{L}_{\text{Yukawa}} \ni -\frac{g_2 m_b}{2 m_W} \bar{b} (g_{bbH_i}^S + i \gamma^5 g_{bbH_i}^P) b H_i$$

$$\left\{ \begin{array}{l} g_{VVH_i} = \mathcal{O}_{1i} \cos \beta + \mathcal{O}_{2i} \sin \beta \\ g_{ZH_iH_j} = \frac{1}{2} \{ (\mathcal{O}_{4i} \mathcal{O}_{2j} - \mathcal{O}_{4j} \mathcal{O}_{2i}) \cos \beta - (\mathcal{O}_{4i} \mathcal{O}_{1j} - \mathcal{O}_{4j} \mathcal{O}_{1i}) \sin \beta \} \\ g_{bbH_i}^S = \mathcal{O}_{1i} \frac{1}{\cos \beta}, \quad g_{bbH_i}^P = -\mathcal{O}_{4i} \tan \beta \\ g_{bbH_i}^2 \equiv (g_{bbH_i}^S)^2 + (g_{bbH_i}^P)^2 \end{array} \right.$$

★ MSSM vs NMSSM

tree-level mass relation (CP-conserving)

$$m_h \leq \min\{m_A, m_Z\}$$

$$m_H \geq \max\{m_A, m_Z\}$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2$$

$$m_{A_1} < \hat{m} < m_{A_2}$$

For $\hat{m} \gg v_0, v_n$, $m_{S_1} < m_{S_2} < \hat{m} < m_{S_3}$

$$\hat{m}^2 = m_{H^\pm}^2 - m_W^2 + |\lambda|^2 v_0^2/2$$

tree-level vacuum

The tadpole condition $\left\langle \frac{\partial V_0}{\partial \varphi_i} \right\rangle = 0$ is sufficient for the EW vacuum (v_{0d}, v_{0u}) to be the global minimum of the potential.

Even if the tadpole conditions are satisfied, the prescribe vacuum (v_{0d}, v_{0u}, v_{0n}) is *not always the global minimum*.

Although the NMSSM has **more parameters** than the MSSM, it must satisfy **more constraints** than the MSSM.

$$\lambda, \kappa, A_\lambda, A_\kappa, m_N^2$$

★ Constraints on the parameters

1. vacuum condition

The vacuum $(v_0, v_{0n}, \tan \beta_0, \theta_0, \varphi_0)$ be the global minimum of V_{eff} .

2. spectrum condition

The neutral Higgs boson with $|g_{VVH}| > 0.1$ be heavier than 114GeV.

We scanned the parameter space for (CP-conserving case)

$$\tan \beta_0 = 2 - 10, v_{0n} = 100 - 1000\text{GeV}, m_{H^\pm} = 100 - 5000\text{GeV},$$

$$-1000\text{GeV} \leq A_\kappa \leq 0, 0 \leq \lambda \leq 1, -1 \leq \kappa \leq 1$$

$$(m_{\tilde{q}}, m_{\tilde{t}_R} = m_{\tilde{b}_R}) = \begin{cases} (1000\text{GeV}, 800\text{GeV}) & \text{heavy-squark} \\ (1000\text{GeV}, 10\text{GeV}) & \text{light-squark-1} \\ (500\text{GeV}, 10\text{GeV}) & \text{light-squark-2} \end{cases}$$
$$A_t = A_b = 20\text{GeV}$$

A necessary condition for the vacuum condition: $V_{\text{eff}}(\mathbf{v}_0) < V_{\text{eff}}(\mathbf{0})$



$$m_{H^\pm}^2 < \frac{2|\lambda|^2 v_{0n}^2}{\sin^2 2\beta_0} + \frac{2|\kappa|^2 v_{0n}^4}{v_0^2 \sin^2 2\beta_0} + \frac{\mathcal{R} v_{0n}^2}{\sin 2\beta} - \frac{4R_\kappa v_{0n}^3}{3v_0^2 \sin^2 2\beta_0} + m_Z^2 \cot^2 2\beta_0 + m_W^2$$

for fixed (λ, κ) , upper bound on m_{H^\pm}

this bound becomes irrelevant in the MSSM-limit

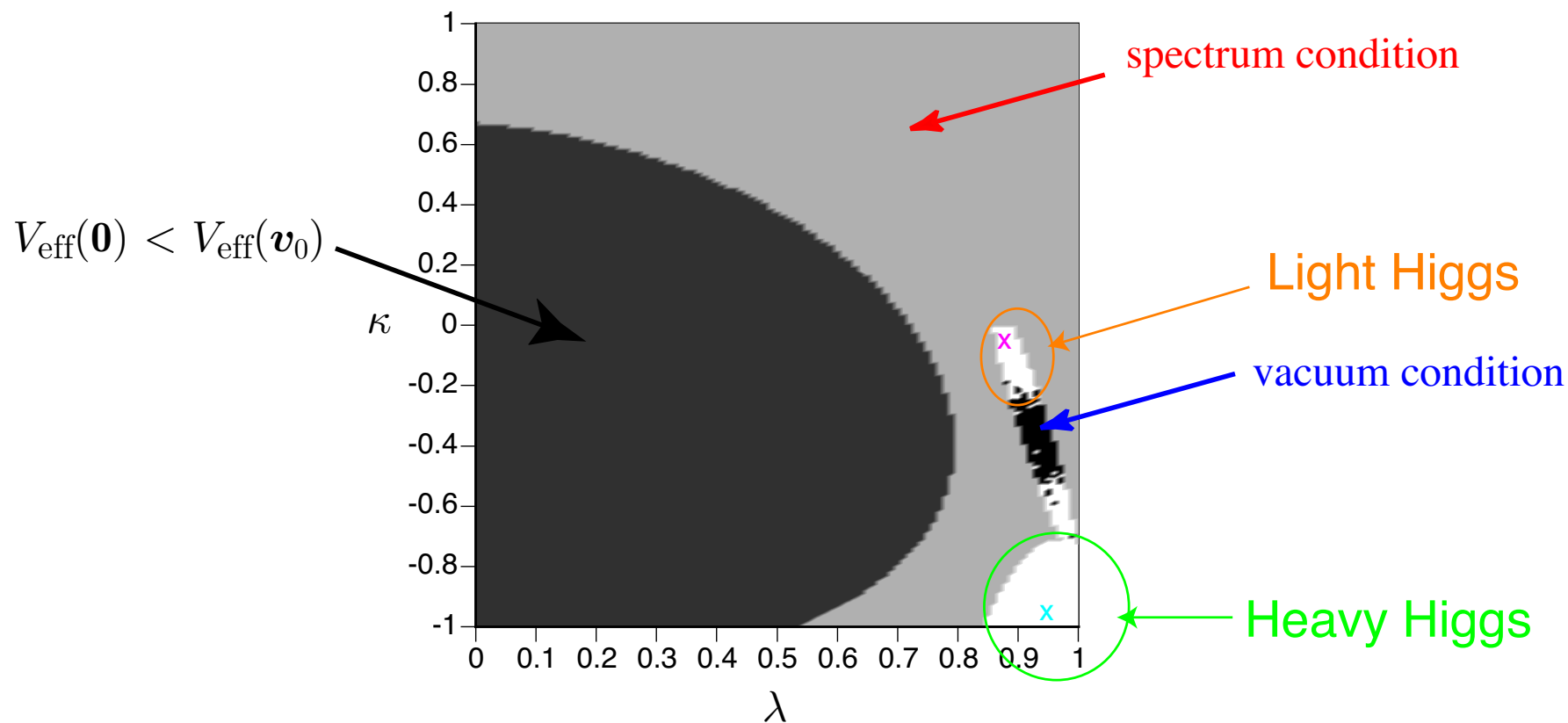
$v_{0n} \rightarrow \infty$ with λv_{0n} and κv_{0n} fixed

for fixed m_{H^\pm} , an elliptic region in (λ, κ) -plane is excluded

the region shrinks to a point for $v_{0n} \rightarrow \infty$

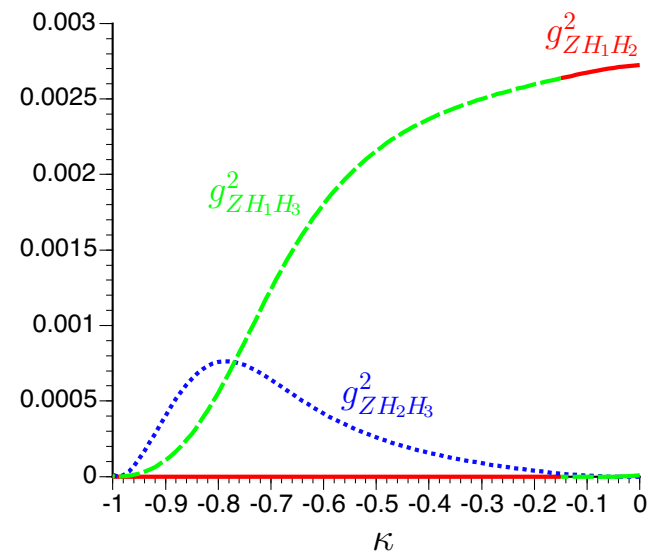
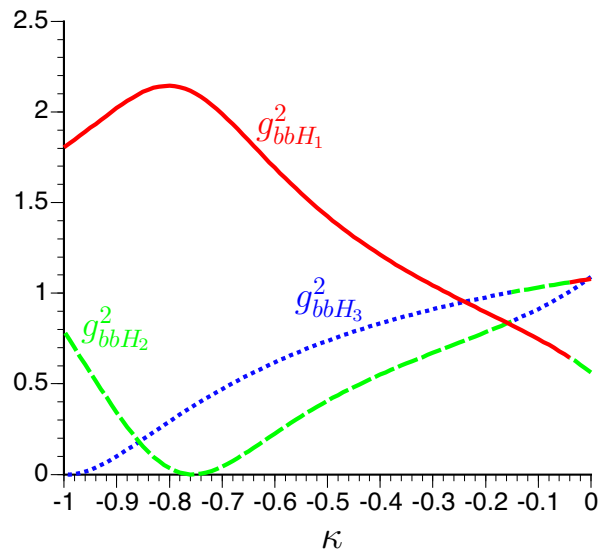
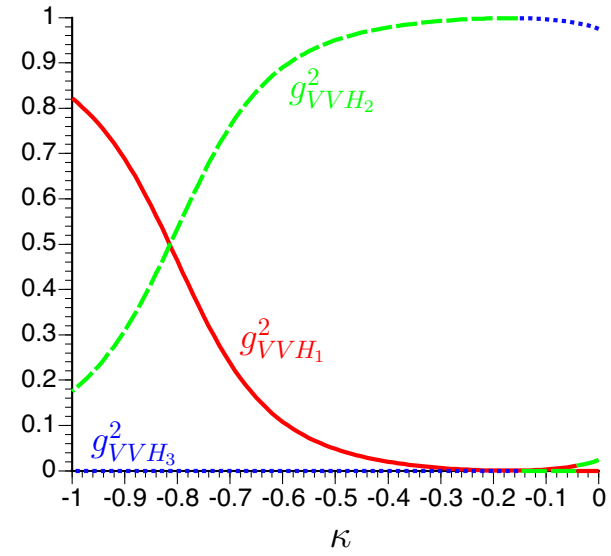
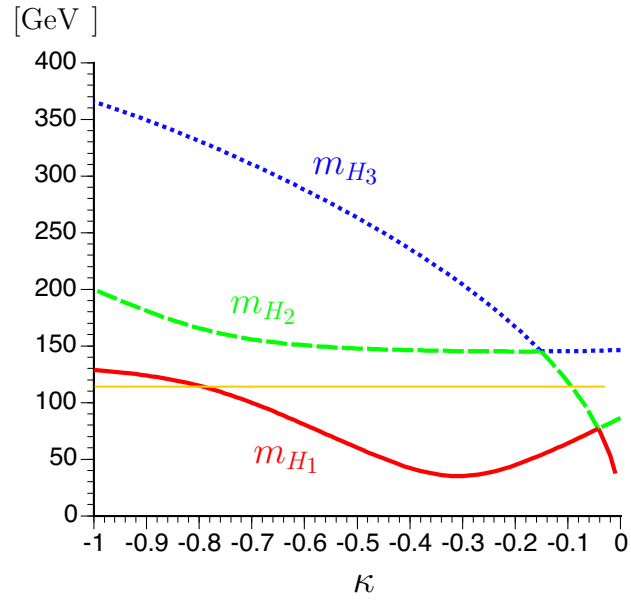
We executed *numerical search* for the global minimum of V_{eff} to ensure the vacuum condition.

e.g., $\tan \beta_0 = 3$, $v_{0n} = 200\text{GeV}$, $m_{H^\pm} = 400\text{GeV}$, $A_\kappa = -200\text{GeV}$, heavy squark



λ	κ	m_{H_1}	m_{H_2}	m_{H_3}	m_{H_4}	m_{H_5}	$g_{VVH_1}^2$	$g_{VVH_2}^2$	$g_{VVH_3}^2$	$g_{VVH_4}^2$	$g_{VVH_5}^2$
0.9	-0.05	75.0	83.5	145.7	436.0	448.2	0.0094	0	0.9897	0.0009	0
0.95	-0.95	139.6	180.9	360.3	425.4	456.4	0.828	0.170	0	0	0.0028

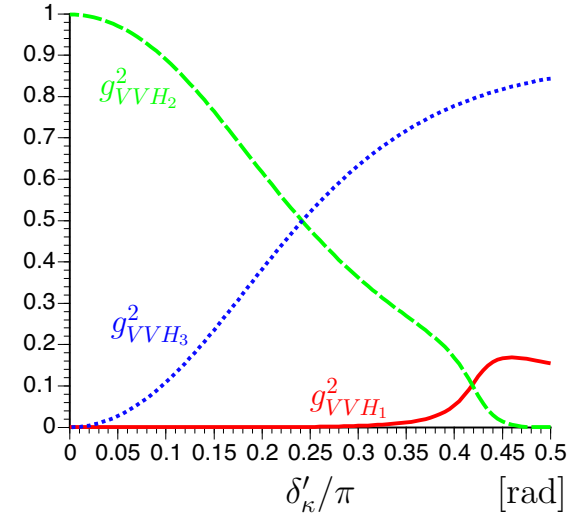
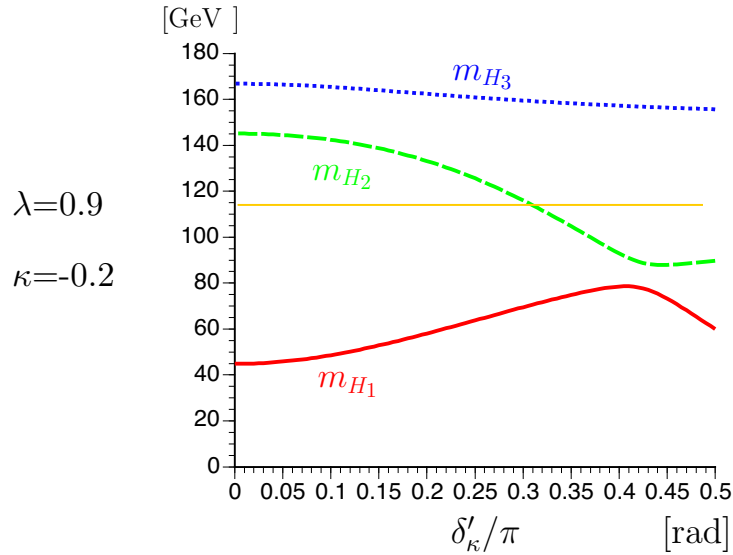
along the line of $\lambda = 0.9$



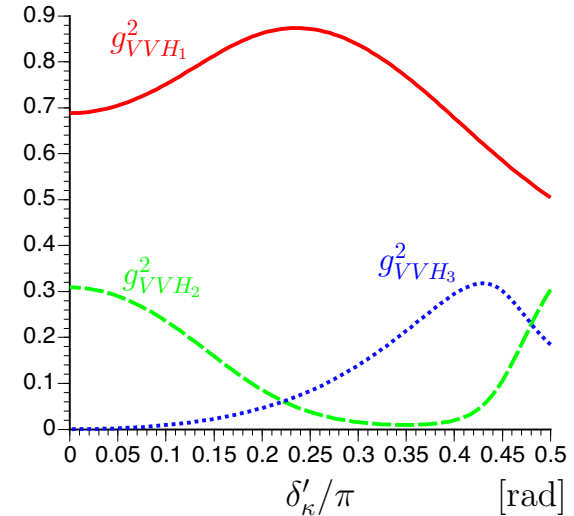
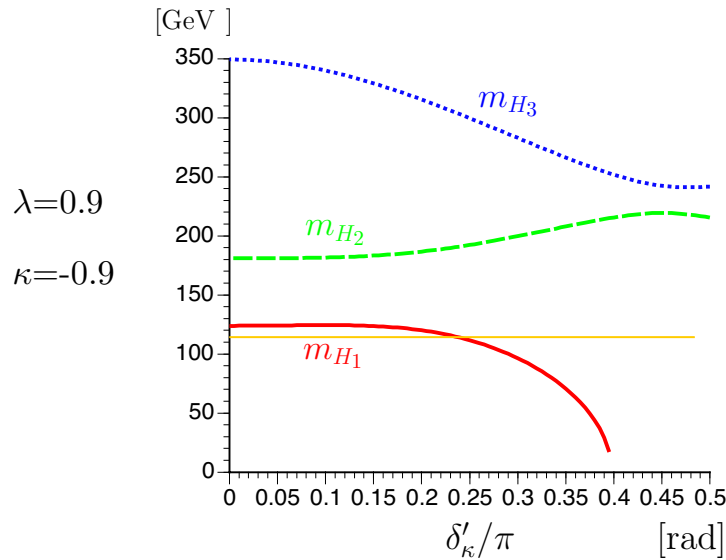
Effects of CP violation

$$\delta'_\kappa \equiv \text{Arg}\kappa + 3\varphi_0 \quad \text{Arg}\lambda + \theta_0 + \varphi_0 = 0 \Leftrightarrow \text{small EDM}$$

Light Higgs



Heavy Higgs



★ Phase transitions in the NMSSM

There has been a belief that the EWPT in the NMSSM is strongly first-order because of the cubic terms in the Higgs potential.

naive (?) argument

[Pietroni, NPB402('93)27]

$$\text{order parameters : } \begin{cases} v_d = v \cos \beta(T) = y \cos \alpha(T) \cos \beta(T), \\ v_u = v \sin \beta(T) = y \cos \alpha(T) \sin \beta(T), \\ v_n = y \sin \alpha(T) \end{cases}$$

$$V_0 = \frac{1}{2} \left((m_1^2 \cos^2 \beta + m_2^2 \sin^2 \beta) \cos^2 \alpha + m_N^2 \sin^2 \alpha \right) y^2 \\ - \left(R_\lambda \cos^2 \alpha \sin \alpha \cos \beta \sin \beta + \frac{1}{3} R_\kappa \sin^3 \alpha \right) y^3 + \dots$$

strongly 1st order PT by the tree-level cubic term ?

Is such a parametrization valid ?

No!

∴ no symmetry between the doublets and the singlet



Indeed, we have found various phases and transitions among them.

possible phases and transitions

phase	order parameters	symmetries
EW	$v \neq 0, v_n \neq 0$	fully broken
I, I'	$v = 0, v_n \neq 0$	local $SU(2)_L \times U(1)_Y$
II	$v \neq 0, v_n = 0$	global $U(1)$
SYM	$v = v_n = 0$	$SU(2)_L \times U(1)_Y$, global $U(1)$

global $U(1)$: $v_u e^{i\theta} = v_2 + iv_3 \mapsto e^{i\alpha}(v_2 + iv_3)$ in the subspace of $v_n = 0$

phase-I : heavy Higgs phase-I': light Higgs

4 types of phase transitions

A: SYM \rightarrow I \Rightarrow EW

B: SYM \rightarrow I' \Rightarrow EW

C: SYM \Rightarrow II \rightarrow EW

D: SYM \Rightarrow EW

“ \Rightarrow ” : EWPT

examples of the phase transitions in the CP-conserving case

common parameters: $\tan \beta_0 = 5$, $v_{0n} = 200\text{GeV}$, $A_\kappa = -100\text{GeV}$

A	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.9, -0.9)$	light-squark-1
B	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.85, -0.1)$	heavy-squark
C	$m_{H^\pm} = 600\text{GeV}$	$(\lambda, \kappa) = (0.82, -0.05)$	light-squark-1
D	$m_{H^\pm} = 700\text{GeV}$	$(\lambda, \kappa) = (0.96, -0.02)$	light-squark-2

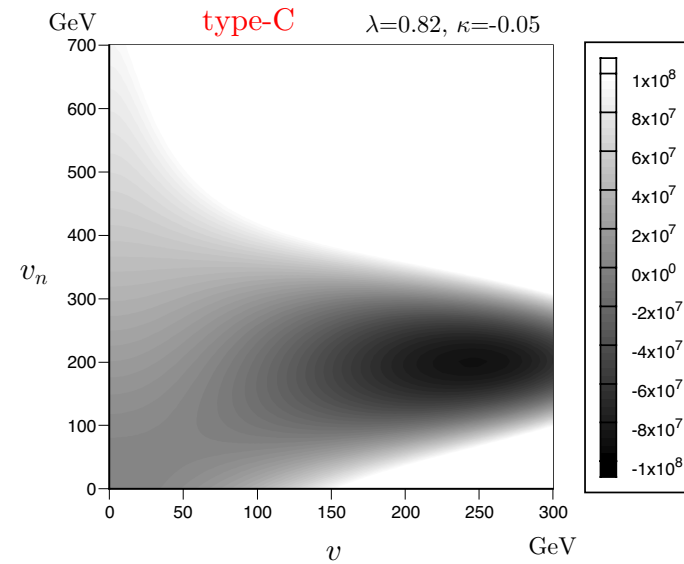
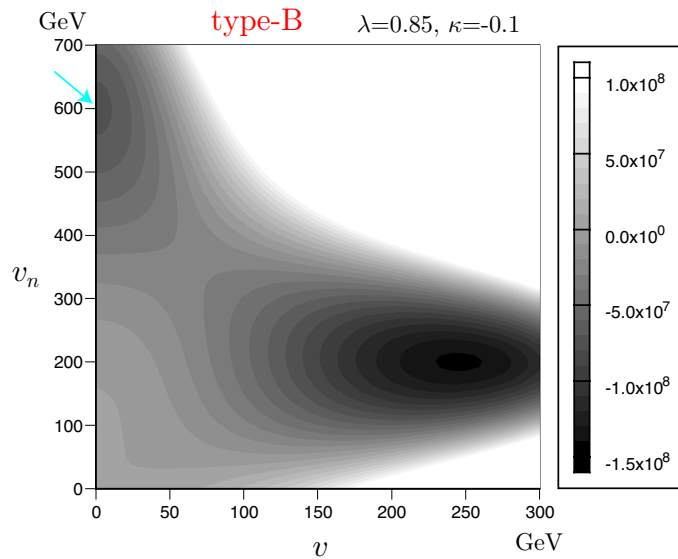
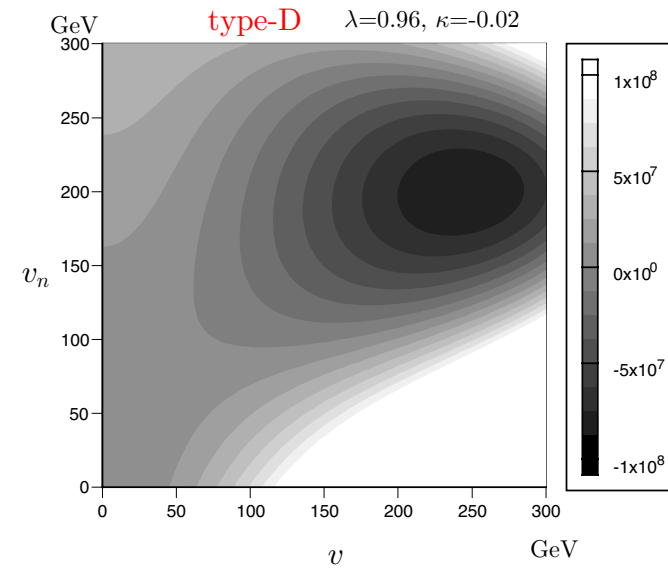
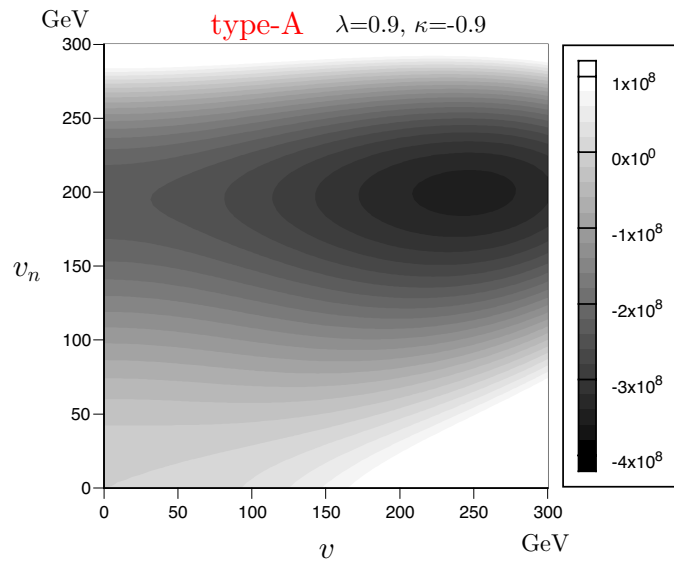
Higgs spectrum and VVH -couplings

		H_1	H_2	H_3	H_4	H_5
A	$m_{H_i}(\text{GeV})$	119.53	203.59	265.74	617.24	637.47
	$g_{VVH_i}^2$	0.9992	5.926×10^{-4}	0	0	1.884×10^{-4}
B	$m_{H_i}(\text{GeV})$	38.89	75.31	131.11	625.61	627.95
	$g_{VVH_i}^2$	6.213×10^{-8}	0	0.9999	6.816×10^{-5}	0
C	$m_{H_i}(\text{GeV})$	42.24	63.49	117.25	625.09	627.44
	$g_{VVH_i}^2$	0.00188	0	0.9980	9.541×10^{-5}	0
D	$m_{H_i}(\text{GeV})$	41.88	58.62.08	115.15	730.51	734.58
	$g_{VVH_i}^2$	0	1.015×10^{-4}	0.9997	1.632×10^{-4}	0

A: heavy Higgs (MSSM-like), B, C, D: light Higgs

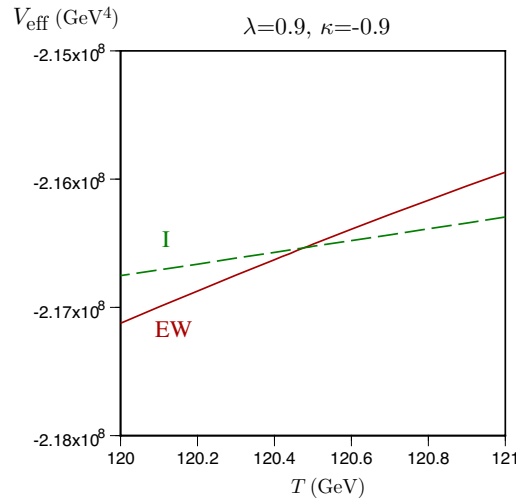
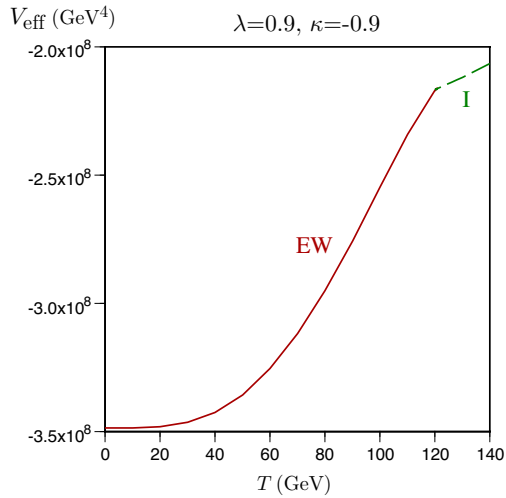
reduced effective potential:

$$\tilde{V}_{\text{eff}}(v, v_n; T) = V_{\text{eff}}(v \cos \beta(T), v \sin \beta(T), 0, v_n, 0; T) - V_{\text{eff}}(0, 0, 0, 0, 0; T)$$



★ How the phase transitions proceed

type-A

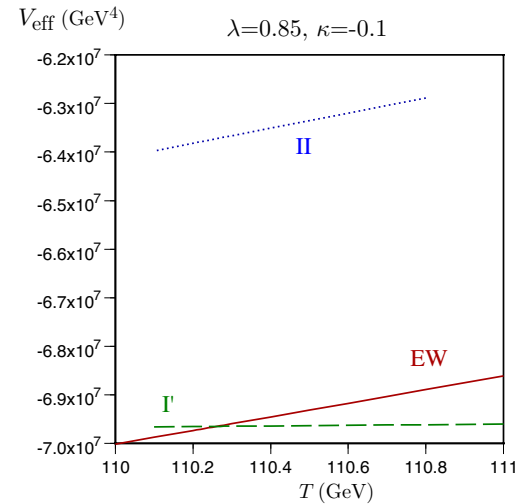
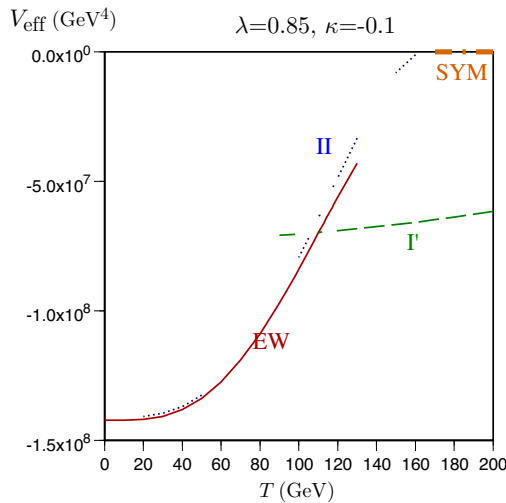


$$(v, v_n) = (106.92, 194.23)(\text{GeV})$$

$$\downarrow T_C = 120.47\text{GeV}$$

$$(0, 192.75)(\text{GeV})$$

type-B

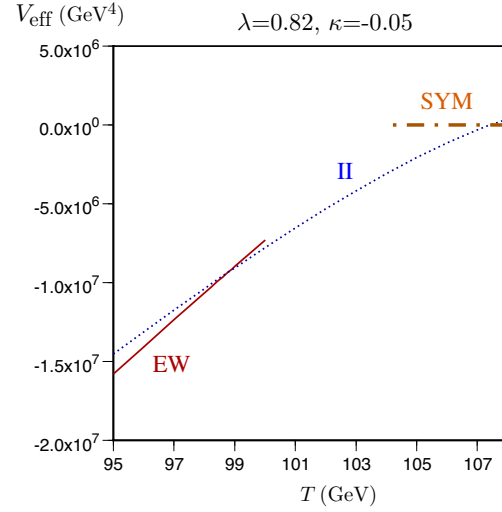
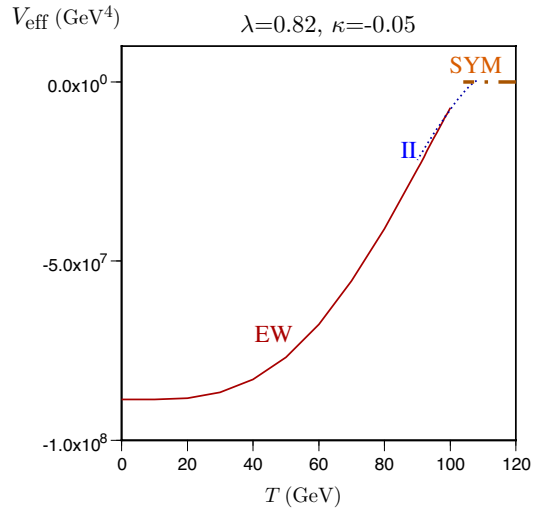


$$(v, v_n) = (208.13, 248.85)(\text{GeV})$$

$$\downarrow T_C = 110.26\text{GeV}$$

$$(0, 599.93)(\text{GeV})$$

type-C



$$(v, v_n) = (194.27, 173.75)(\text{GeV})$$

$$\downarrow T_N = 98.76\text{GeV}$$

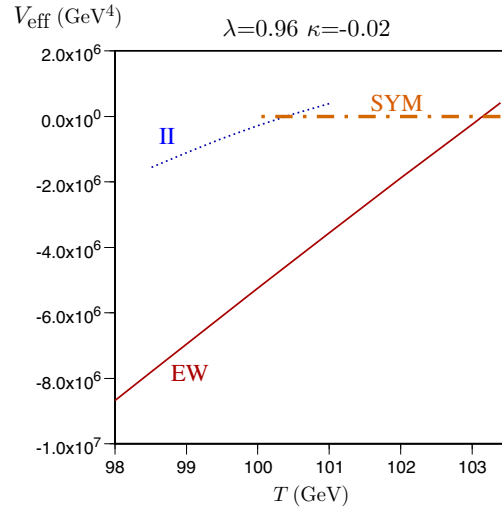
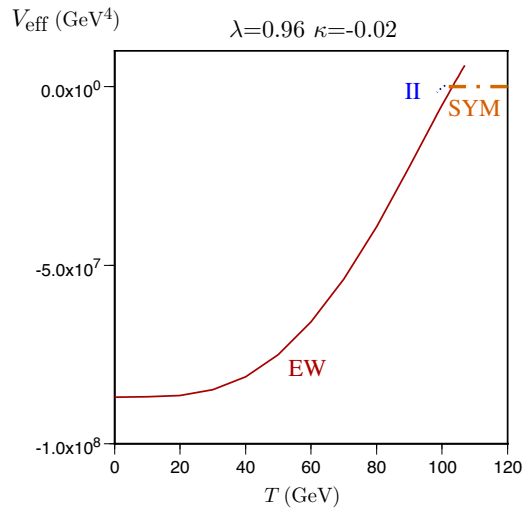
$$(165.97, 0)(\text{GeV})$$

$$(109.54, 0)(\text{GeV})$$

$$\downarrow T_C = 107.44\text{GeV}$$

$$(0, 0)$$

type-D



$$(v, v_n) = (182.49, 192.26)(\text{GeV})$$

$$\downarrow T_C = 103.14\text{GeV}$$

$$(0, 0)$$

type-A MSSM-like EWPT — proceeds along almost constant $v_n \neq 0$

a light stop is needed for it to be strongly first order

type-B new type of 2-stage PT

leap from $(v(T_{C-}), v_n(T_{C-}))$ to $(0, v_n(T_{C+}))$

strongly first order EWPT (no light stop is needed)

type-C new type of 2-stage PT

EWPT proceeds along $v_n = 0$

a light stop is needed for it to be strongly first order

type-D 1-stage PT (so far mainly considered in the NMSSM)

a light stop is needed for the EWPT to be strongly first order

type-B,C,D — light-Higgs scenario — peculiar to NMSSM

★ Phase transitions in the presence of CPV

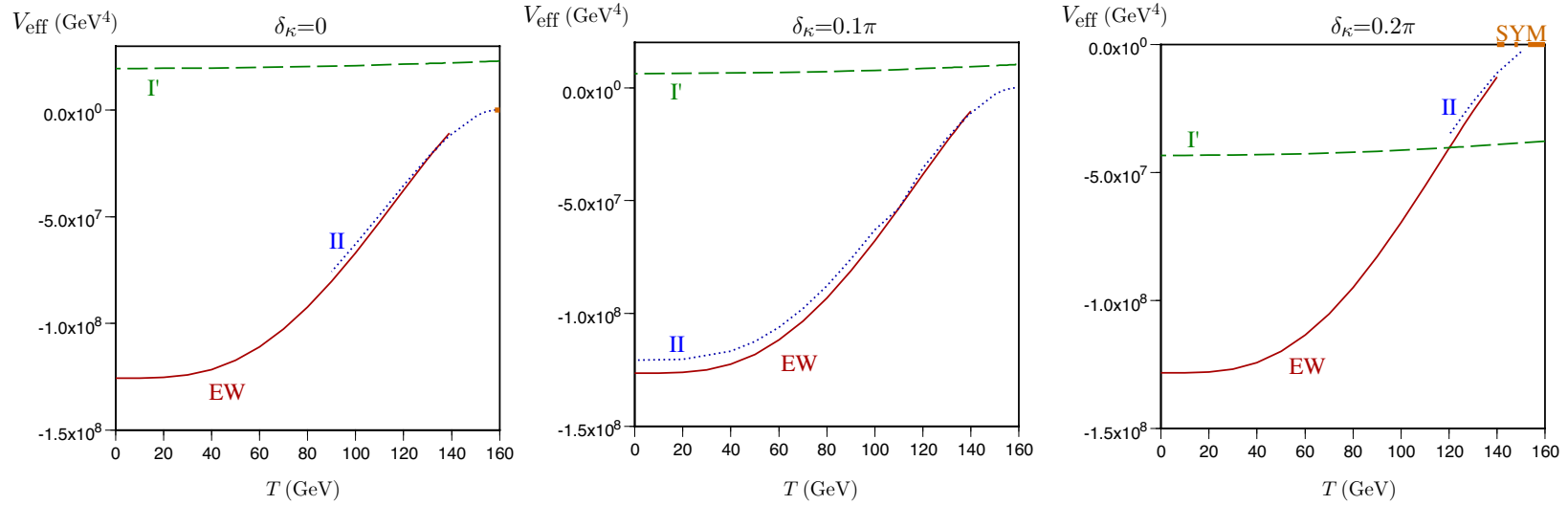
explicit CP violation: δ_κ with $\delta_{\text{EDM}} = 0$ and $\theta_0 = \varphi_0 = 0$

$\tan \beta_0 = 5$, $v_{0n} = 200\text{GeV}$, $A_\kappa = -100\text{GeV}$

$m_{H^\pm} = 600\text{GeV}$, $(\lambda, \kappa) = (0.83, -0.07)$, heavy-squark

Higgs mass and VVH -couplings

δ_κ		H_1	H_2	H_3	H_4	H_5
0	$m_{H_i}(\text{GeV})$	38.89	75.31	131.11	625.61	627.945
	$g_{VVH_i}^2$	6.213×10^{-8}	0	0.9999	6.816×10^{-5}	0
0.1π	$m_{H_i}(\text{GeV})$	40.04	73.24	131.20	625.54	627.56
	$g_{VVH_i}^2$	2.749×10^{-6}	0.00169	0.9982	6.570×10^{-5}	2.363×10^{-6}
0.2π	$m_{H_i}(\text{GeV})$	43.21	66.95	131.38	625.40	627.85
	$g_{VVH_i}^2$	3.133×10^{-5}	0.00531	0.9946	6.132×10^{-5}	6.407×10^{-6}



$\delta_\kappa = 0$; at $T_N = 133.22\text{GeV}$, $(v, v_n) = (180.74\text{GeV}, 195.49\text{GeV}) \rightarrow (163.16\text{GeV}, 0)$

type-C at $T_C = 158.27\text{GeV}$, $(v, v_n) = (26.33\text{GeV}, 0) \rightarrow (0, 0)$

$\delta_\kappa = 0.1\pi$; at $T_N = 136.58\text{GeV}$, $(v, v_n) = (173.99\text{GeV}, 195.88\text{GeV}) \rightarrow (154.78\text{GeV}, 0)$

type-C at $T_C = 158.27\text{GeV}$, $(v, v_n) = (26.33\text{GeV}, 0) \rightarrow (0, 0)$

$\delta_\kappa = 0.2\pi$; at $T_C = 120.16\text{GeV}$, $(v, v_n) = (200.62\text{GeV}, 208.93\text{GeV}) \rightarrow (0, 750.93\text{GeV})$

type-B

In the phase-II of type-C PT, $\theta(T) = \text{Arg}(v_2 + iv_3)$ is **undertermined** (global $U(1)$)

For $\delta_\kappa = 0.2\pi$, $\varphi(T) = 0.05(\text{EW}) \rightarrow 0.215(\text{I}')$ (chiral charge enhanced)

4. Summary

EW Baryogenesis

- ★ based on a testable model
- ★ free from the proton decay problem

needs extensions of the MSM for

- ★ CP violation

new sources of CP violation EDM, precise measurements of CP-viol. BR

μ, A_q , gaugino masses, θ, \dots in SUSY models

- ★ strongly 1st-order EWPT

extra scalars: 2HDM, MSSM, NMSSM, \dots

\implies Higgs spectrum and couplings LHC, ILC, \dots

- $m_H > 120\text{GeV} \implies$ 1st-order EWPT in the MSSM **X**

- $m_H > 135\text{GeV} \implies$ MSSM **X**
 NMSSM (light Higgs for 1st-order EWPT)
 2HDM, etc.

We studied the phase transitions in the NMSSM to find

- there are 4 phases EW, SYM, I, I', II
- 4 types of PT, 3 of which have 2-stage nature
 heavy Higgs \implies MSSM-like EWPT
 light Higgs \implies strongly 1st order EWTP

NMSSM in the light Higgs scenario with heavy charged Higgs
 \simeq Minimal SM with 1st order EWPT (type-B), extra CP violation

How can we distinguish it from the MSM?

— may be by precision measurements of the couplings