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PHASES OF QCD: LATTICE THERMODYNAMICS AND A FIELD THEORETICAL MODEL

C. RATTI[1,2], M. A. THALER[1], W. WEISE[2]

[1] *Physik-Department, Technische Universität München, D-85747 Garching, Germany*
[2] *ECT*, Villazzano (Trento), Italy*

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Abstract. We investigate QCD thermodynamics at zero and finite quark chemical potential by means of a generalized Nambu Jona-Lasinio model, in which quarks couple simultaneously to the chiral condensate and to a background temporal gauge field representing Polyakov loop dynamics. This so-called PNJL model thus includes features of both deconfinement and chiral symmetry restoration.

Key words: QCD thermodynamics, field model, Polyakov loop.

1. INTRODUCTION

QCD thermodynamics has been the subject of deep investigation in recent years: heavy-ion collisions are looking for signals of Quark-Gluon Plasma formation, and meanwhile large scale computer simulations on the lattice have become a major tool to understand the patter of phases in QCD. Accurate computations of lattice QCD thermodynamics in the pure gauge sector have been performed. First simulations at finite quark chemical potential are now available. The equation of state of strongly interacting matter is now at hand as a function of temperature T and in a limited range of quark chemical potential μ. Improved multi-parameter re-weighting techniques [1], Taylor series expansion methods [2, 3] and analytic continuation from imaginary chemical potential [4, 5] provide lattice data for the pressure, entropy density, quark density and selected susceptibilities.

The availability of such a large number of lattice data represents a unique tool to test the effectiveness of phenomenological models to reproduce the main features of QCD thermodynamics: a lot of models have in fact been developed, for example to give an interpretation of the data in terms of quasiparticles.

In our paper [6], we study the thermodynamics of two-flavour QCD at finite quark chemical potential. Our investigation is based on a synthesis of a Nambu Jona-Lasinio (NJL) model and the non-linear dynamics involving the Polyakov loop [7, 8]. In this Polyakov-loop-extended (PNJL) model, quarks develop quasiparticle masses by propagating in the chiral condensate, while they couple at the same time to a homogeneous background (temporal) gauge field representing Polyakov loop dynamics. This model incorporates both chiral symmetry restoration and deconfinement, and can in fact be seen as a minimal synthesis of these two basic features.

2. THE MODEL

Following [8] we introduce a generalized $N_f = 2$ Nambu Jona-Lasinio Lagrangian with quarks coupled to a (spatially constant) temporal background $SU(3)$ gauge field, A_0, representing Polyakov loop dynamics (the PNJL model):

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] - U\left(\Phi[A], \Phi^*[A], T\right), \quad (1)$$

where

$$D_\mu = \partial_\mu + iA_\mu \qquad \text{and} \qquad A_\mu = \delta_{\mu 0}A_0. \quad (2)$$

The quantity $\mathcal{U}(\Phi, \Phi^*, T)$ is the effective potential expressed in terms of the traced Polyakov loop[1], $\Phi = (\mathrm{Tr}_c\, L)/N_c$. The Polyakov loop L is an $SU(N_c)$ matrix in colour space explicitly given by

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta \mathrm{d}\tau\, A_0(\tau, \vec{x})\right], \quad (3)$$

with $\beta = 1/T$ the inverse temperature. The coupling between Polyakov loop and quarks is uniquely determined by the covariant derivative D_μ in the PNJL Lagrangian (1). The effective potential $\mathcal{U}(\Phi, \Phi^*, T)$ depends on temperature. At small temperatures, $\mathcal{U}$ has a single minimum at $\langle\Phi\rangle = 0$, while at high temperatures it develops a second one which turns into the absolute minimum above a critical temperature T_0. In the limit $T \to \infty$ we have $\langle\Phi\rangle \to 1$. The function $\mathcal{U}(\Phi, \Phi^*, T)$ will be fixed by comparison with pure-gauge Lattice QCD. For this purpose we choose the following general form:

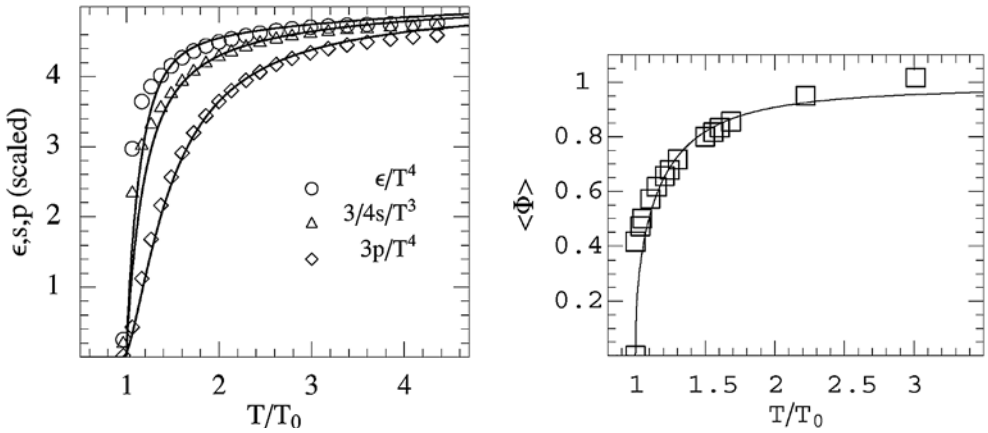
$$\frac{\mathcal{U}\left(\Phi, \Phi^*, T\right)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - \frac{b_3}{6}\left(\Phi^3 + \Phi^{*3}\right) + \frac{b_4}{4}\left(\Phi^*\Phi\right)^2 \quad (4)$$

with

[1] more precisely: the Polyakov line with periodic boundary conditions.

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3, \tag{5}$$

and perform a precision fit of the coefficients a_i, b_i to reproduce the lattice data in the pure gauge sector of QCD thermodynamics: the results of this procedure can be seen in Fig. 1. The parameters of the NJL part of the Lagrangian are fixed by reproducing known physical quantities in the adronic sector, like the pion mass and the pion decay constant (for all details see [6]).



(a) (b)

Fig. 1 – (a) Scaled pressure, entropy density and energy density as functions of the temperature in the pure gauge sector, compared to the corresponding lattice data taken from Ref. [9]. (b)Expectation value of the Polyakov loop as a function of temperature in the pure gauge sector, compared to corresponding lattice data taken from Ref. [10].

We then evaluate the thermodynamic potential of the system through the standard finite-temperature formalism:

$$\Omega(T,\mu) = \mathcal{U}\left(\Phi, \Phi^*, T\right) - \frac{T}{2}\sum_n \int \frac{d^3p}{(2\pi)^3} \mathrm{Tr}\ \ln\left(\frac{1}{T}\tilde{S}^{-1}\left(i\omega_n, \vec{p}\,\right)\right) + \frac{\sigma^2}{2G}. \tag{6}$$

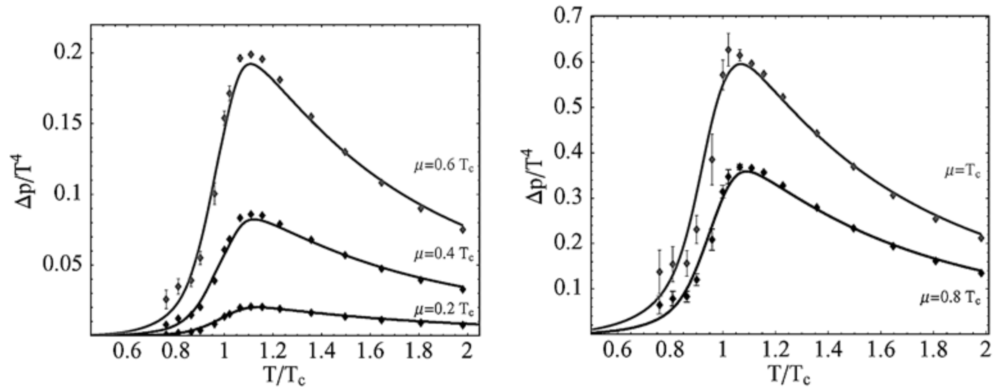
Here $\omega_n = (2n+1)\pi T$ are the Matsubara frequencies for fermions. The inverse quark propagator (in Nambu-Gorkov representation) becomes

$$\tilde{S}^{-1}\left(p^0, \vec{p}\,\right) = \begin{pmatrix} p' - \hat{M} - (\mu - A^0)\gamma_0 & 0 \\ 0 & p' - \hat{M} + (\mu - A^0)\gamma_0 \end{pmatrix}. \tag{7}$$

By minimizing the above thermodynamic potential with respect to the fields σ, Φ and Φ^* we can obtain the behaviour of the chiral condensate and the

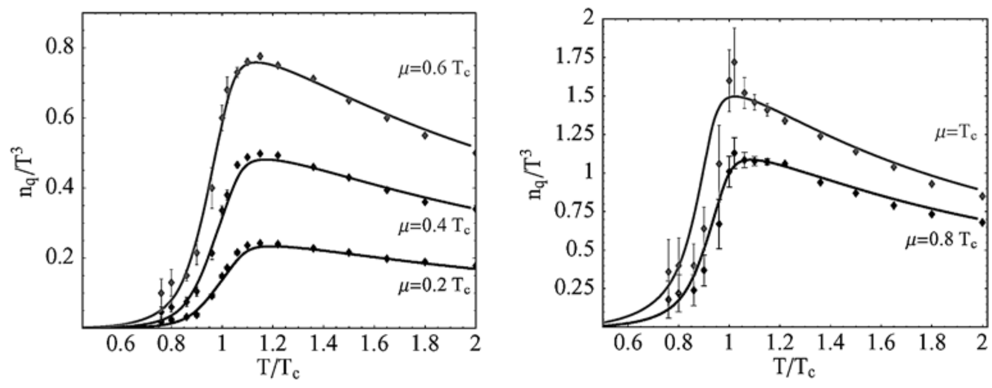
Polyakov loop as functions of temperature and quark chemical potential. We find that by introducing quarks the transition is no longer first-order, but it is turned into a smooth crossover. Besides, the two critical temperatures for chiral symmetry restoration and deconfinement coincide, at a value of $T_c \simeq 200$ MeV.

Now we are able to investigate the thermodynamics of the system: we evaluate the scaled pressure difference and quark number density as functions of the temperature for different values of the chemical potential, and show our results in Figs. 2 and 3. As we can see, the agreement between our results and the corresponding lattice data is really remarkable.



(a) (b)

Fig. 2 – Scaled pressure difference as a function of temperature at different values of the quark chemical potential, compared to lattice data taken from Ref. [3].



(a) (b)

Fig. 3 – Scaled quark number densities as a function of temperature at different values of the chemical potential, compared to lattice data taken from Ref. [3].

3. CONCLUSIONS

We have studied a Polyakov-loop-extended Nambu and Jona-Lasinio (PNJL) model with the aim of exploring whether such an approach can catch essential features of QCD thermodynamics when confronted with results of lattice computations at finite temperature and quark chemical potential.

Once a limited set of input parameters is fitted to Lattice QCD in the pure gauge sector and to pion properties in the hadron sector, the quark-gluon thermodynamics above T_c up to about twice the critical temperature is well reproduced, including quarkdensities up to chemical potentials of about 0.2 GeV. In particular, the PNJL model correctly describes the step from the first-order deconfinement transition observed in pure-gauge Lattice QCD (with $T_c \simeq 270$ MeV) to the crossover transition (with T_c less than 200 MeV) when $N_f = 2$ light quark flavours are added. The non-trivial result is that the crossovers for chiral symmetry restoration and deconfinement almost coincide, as found in lattice simulations. The model also reproduces the quarknumber densities at various chemical potentials remarkably well when confronted with corresponding lattice data.

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