Phenomenological Analysis of Charmless Decays $B_s \to PP, PV$ with QCD Factorization*

Junfeng Sun,[1,†] Guohuai Zhu,[2,‡] and Dongsheng Du[1,§]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, P.O.Box 918(4), Beijing 100039, China*

[2]*Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan*

(Dated: March 25, 2022)

Abstract

We calculated the CP averaged branching ratios and CP-violating asymmetries of two-body charmless hadronic $B_s \to PP$, PV decays with the QCDF approach, including the contributions from the chirally enhanced power corrections and weak annihilations. Only several decay modes, such as $B_s \to K^{(*)}K$, $K^{(*)\pm}\pi^{\mp}$, $K^{\pm}\rho^{\mp}$, $\eta^{(\prime)}\eta^{(\prime)}$, have large branching ratios, which may be observed in the near future. The penguin-to-tree ratio $|P_{\pi\pi}/T_{\pi\pi}|$ and a bound on angle γ are given.

PACS numbers: 13.25.Hw 12.38.Bx

*Supported in part by National Natural Science Foundation of China

†E-mail address: sunjf@mail.ihep.ac.cn

‡E-mail address: zhugh@post.kek.jp.

§E-mail address: duds@mail.ihep.ac.cn

I. INTRODUCTION

Recently there has been remarkable progress in the study of exclusive charmless $B_{u,d}$ decays. Experimentally, many two-body non-leptonic charmless $B_{u,d}$ decays have been observed by CLEO and B-factories at KEK and SLAC (see Refs. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]), and more B decay channels will be measured with great precision soon. With the accumulation of data, the Standard Model can be tested in more detail. Theoretically, several attractive methods have been proposed to study the nonfactorizable effects in hadronic matrix elements from first principles, such as QCD factorization (QCDF) [11], perturbative QCD method (PQCD) [12, 13, 14], and so on. Intensive investigations on hadronic charmless two-body $B_{u,d}$ decays have been studied in detail, for example, in Refs. [15, 16, 17, 18, 19, 20, 21, 22].

The potential B_s decay modes permit us to overconstrain the unitarity CKM matrix. This makes the search for CP violation in the B_s decays highly interesting. The problem is that B_s mesons oscillate at a high frequency, and nonleptonic B_s decays have still remained elusive from observation. Today only some weak upper limits on branching ratios of several charmless hadronic decays are available, mostly from LEP and SLD experiments [23], such as $B_s^0 \to \pi^+\pi^-$, $\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$, K^+K^-, π^+K^-, $\cdots$. Unlike $B_{u,d}$ mesons, the heavier B_s mesons cannot be studied at the B-factories operating at the $\Upsilon(4S)$ resonance. However it is believed that in the future at hadron colliders, such as CDF, D0, HERA-B, BTeV, and LHCb, the signs of CP violation in B_s system can be observed with high accuracy in addition to studies of certain $B_{u,d}$ modes [24].

The early theoretical studies of two-body charmless nonleptonic decays of B_s meson can be found in Refs. [25, 26, 27, 28, 29]. The investigation on the exclusive charmless B_s decays into final states containing $\eta^{(\prime)}$ meson was given within the generalized factorization framework [30]. Chen, Cheng, and Tseng calculated carefully the branching ratios for charmless decays $B_s \to PP$, PV, VV (here P and V denote pseudoscalar and vector mesons respectively) [31]. And new physics effects in B_s decays was considered in [32]. It is found that the electroweak penguin contributions can be large for some decays modes [29, 31], and that branching ratios for $B_s \to \eta\eta'$ and several other decay modes can be as large as 10^{-5} [30, 31, 32] which is measurable at future experiments.

Few years ago, Beneke, Buchalla, Neubert, and Sachrajda gave a QCDF formula to compute the hadronic matrix elements $\langle M_1 M_2|O_i|B\rangle$ in the heavy quark limit, so that the hadronic uncertainties enter only at the level of power corrections of $1/m_b$. This basic formula is presumed to be valid for B decays into two light final states [11, 33]. We made a comprehensive analysis on exclusive hadronic $B_{u,d}$ decay using the QCDF approach, and calculated the branching ratios and CP asymmetries for decays $B_{u,d} \to PP$ [21] and PV [22]. We find that with appropriate parameters, most of our predictions are in agreement with the present experimental data. In this paper, we would like to apply the QCDF approach to the case of B_s mesons.

This paper is organized as follow: In section II, we discuss the theoretical framework and define the relevant matrix elements for $B_s \to PP$, PV decays. In section III, we list the theoretical input parameters used in our analysis. Section IV and section V are devoted to the numerical results and some remarks of CP averaged branching ratios and CP-violating asymmetries, respectively. In the mean time, the theoretical uncertainties due to the variation of inputs are investigated. In section VI, we give the values of the penguin-to-tree ratio $P_{\pi\pi}/T_{\pi\pi}$ and a constraint on weak angle γ. Finally, we conclude with a summary in section VII.

II. THEORETICAL FRAMEWORK FOR B DECAYS

A. The effective Hamiltonian

Using the operator product expansion and renormalization group equation, the low energy effective Hamiltonian relevant to nonleptonic B decays can be written as [34]:

$$\begin{aligned}\mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}} \sum_{q=u,c} v_q \Big\{ C_1(\mu) Q_1^q(\mu) + C_2(\mu) Q_2^q(\mu) + \sum_{k=3}^{10} C_k(\mu) Q_k(\mu) \\ &+ C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \Big\} + \text{H.c.}, \end{aligned} \tag{1}$$

where $v_q = V_{qb}\, V_{qd}^*$ (for $b{\to}d$ transition) or $v_q = V_{qb}\, V_{qs}^*$ (for $b{\to}s$ transition) are CKM factors. $C_i(\mu)$ are Wilson coefficients which have been reliably evaluated to the next-to-leading logarithmic order. Their numerical values in the naive dimensional regularization scheme at three different scales are listed in Table I. The effective operators, Q_i, can be expressed explicitly as follows:

$$Q_1^u = (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A}, \qquad Q_1^c = (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta c_\beta)_{V-A}, \tag{2}$$

$$Q_2^u = (\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \qquad Q_2^c = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{q}_\beta c_\alpha)_{V-A}, \tag{3}$$

$$Q_3 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \qquad Q_4 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A}, \tag{4}$$

$$Q_5 = (\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \qquad Q_6 = (\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A}, \tag{5}$$

$$Q_7 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \qquad Q_8 = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A}, \tag{6}$$

$$Q_9 = \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \qquad Q_{10} = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A}, \tag{7}$$

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu}(1+\gamma_5) b_\alpha F_{\mu\nu}, \qquad Q_{8g} = \frac{g}{8\pi^2} m_b \bar{q}_\alpha \sigma^{\mu\nu}(1+\gamma_5) t^a_{\alpha\beta} b_\beta G^a_{\mu\nu}, \tag{8}$$

where q' denotes all the active quarks at scale $\mu = \mathcal{O}(m_b)$, i.e. $q' = u$, d, s, c, b.

B. Hadronic matrix elements within the QCDF framework

To get the decay amplitudes, the most difficult theoretical work is to compute the hadronic matrix elements of the effective operators, i.e. $\langle M_1 M_2 | O_i | B \rangle$. Phenomenologically, these hadronic matrix elements are usually parameterized into the product of the decay constants and the transition form factors based on the naive factorization scheme (NF) [35]. However, one main defect of the rough NF approach is that hadronic matrix elements cannot make compensation for the renormalization scheme- and scale- dependence of Wilson coefficients, in this sense NF's results are unphysical. This indicates that "nonfactorizable" contributions from high order corrections to the hadronic matrix elements must be taken into account.

The QCDF approach is one of novel methods to evaluate these hadronic matrix elements relevant to B decays systematically. In the heavy quark limit $m_b \gg \Lambda_{QCD}$, up to power corrections of order of Λ_{QCD}/m_b, the basic QCDF formula is [11]

$$\begin{aligned}\langle M_1 M_2 | O_i | B \rangle &= \sum_j F_j^{B\to M_1} \int_0^1 dx\, T_{ij}^I(x) \Phi_{M_2}(x) + (M_1 \leftrightarrow M_2) \\ &\quad + \int_0^1 d\xi \int_0^1 dx \int_0^1 dy\, T_i^{II}(\xi, x, y) \Phi_B(\xi) \Phi_{M_1}(x) \Phi_{M_2}(y) \\ &= \langle M_1 M_2 | J_1 \otimes J_2 | B \rangle_F \Big[1 + \sum r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b)\Big]\end{aligned} \tag{9}$$

where $T_i^{I,II}$ denote hard-scattering kernels. At leading order, $T_i^I = 1$, $T_i^{II} = 0$, the QCDF formula (9) shows that there is no long-distance interaction between M_2 meson and (BM_1) system, and reproduces the NF's results. Neglecting the power corrections of $\mathcal{O}(\Lambda_{QCD}/m_b)$,

$T_i^{I,II}$ are hard gluon exchange dominant, and therefore calculable order by order with perturbative theory. Nonperturbative effects are either suppressed by $1/m_b$ or parameterized in terms of mesons decay constants, form factors $F^{B\to M}$, and meson light-cone distribution amplitudes $\Phi_B(\xi)$, $\Phi_M(x)$. The factorized matrix elements $\langle M_1 M_2 | J_1 \otimes J_2 | B \rangle_F$ is the same as the definition of the BSW approximation [35]. Through the QCDF formula, the hadronic matrix elements can be separated into short-distance part and long-distance part, and the "residual" renormalization scheme- and scale- dependence of hadronic matrix elements could be extracted to cancel those of the corresponding Wilson coefficients, so that physical results at least at the order of α_s level are renormalization scheme and scale independent [33]. Through the QCDF formula, "nonfactorizable" effects can be evaluated, and partial information about the strong phases can be obtained.

It is important to note that some power suppression might fail in some cases because the b quark mass is not asymptotically large. For example, power correction proportional to $2m_M^2/(m_b m_q)$ with $q = u$, d, s, which is formally power suppressed, is now chirally enhanced and numerically important to penguin dominated B rare decays. Therefore it is necessary to include at least the chirally enhanced corrections consistently for phenomenological application of QCDF in B decays. However, the twist-3 corrections to hard scattering kernels T^{II} cannot provide sufficient endpoint suppression, so there appears infrared logarithmic divergence, $\int dx/x \sim \ln(m_b/\Lambda_{QCD})$. In PQCD method, this singularity can be smoothed out by introducing the partonic intrinsic transverse momentum and the mechanism of Sudakov suppression. It should be interesting to investigate the possibility of incorporating the Sudakov form factor into the QCDF approach [36]. But it is really uneasy because the Sudakov suppression is one of the key ideas of the PQCD method while PQCD and QCDF have different power expansions which lead to completely different understanding on B decays. Therefore, to take the chirally enhanced corrections into account, in this paper we adopt phenomenological treatment for the divergent integral $\int dx/x$ [20].

For weak annihilation contributions, they are believed to be very small with the naive factorization assumption (see, for example Ref. [15]). Within the QCDF approach, the weak annihilation amplitudes are also formerly suppressed by $(f_B f_{M_1})/(F^{B\to M_1} m_B^2) \sim \Lambda_{QCD}/m_b$ [11]. But as emphasized in the PQCD method [14, 37, 38], annihilation contributions with QCD corrections could give potentially large strong phases, hence large CP violation could be expected. In addition, the phenomenological investigations on B decays [20, 21, 22] within the QCDF framework also suggest that their effects could be sizable when large model uncertainties are considered. So annihilation contributions cannot be simply neglected. In the previous work [20, 21, 22] it has been shown that annihilation contributions exhibit endpoint singularities even with leading twist distribution amplitudes for the final states, and these infrared divergence must be parameterized, so extra theoretical uncertainties and model dependence are introduced. In spite of these problems, it is still interesting to estimate the weak annihilation effects in this paper.

With the QCDF approach, matrix elements for two-body B_s decays can be written as,

$$\langle M_1 M_2|\mathcal{H}_{eff}|B_s\rangle = \mathcal{A}^f(B_s \to M_1 M_2) + \mathcal{A}^a(B_s \to M_1 M_2), \tag{10}$$

$$\mathcal{A}^f(B_s \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \sum_{i=1}^{10} v_q a_i^q \langle M_1 M_2|J_1 \otimes J_2|B_s\rangle_F, \tag{11}$$

$$\mathcal{A}^a(B_s \to M_1 M_2) \propto \frac{G_F}{\sqrt{2}} \sum_{q=u,c} \sum_{i} f_{B_s} f_{M_1} f_{M_2} v_q b_i, \tag{12}$$

where $\mathcal{A}^a$ arises from weak annihilation contributions. f_{B_s}, f_M are decay constants for B_s, and M mesons, respectively. (M can be either pseudoscalar meson P or vector meson V) The explicit expressions of decay amplitudes $\mathcal{A}^{f,a}$ for $B_s \to PP$, PV are listed in Appendix A, B, C, D. Summary of the dynamical quantities a_i, b_i is given in Refs. [20, 21, 22].

III. INPUT PARAMETERS

Theoretical expressions for decay amplitudes using the QCDF approach are complicated and depend on many input parameters including the SM parameters (such as CKM matrix elements, quark masses), Wilson coefficients and the renormalization scale μ, and some soft and nonperturbative hadronic quantities (such as meson decay constants, form factors, and meson light cone distribution amplitudes), and so on. If quantitative predictions are to be made, the values for various parameters employed in this paper must be specified. It has been shown that the renormalization scale dependence has been greatly reduced compared to the NF's coefficients $a_{i,I}$ obtained at leading order [20, 21, 22], and the residual scale dependence should be further reduced when the higher order radiative corrections are considered. In calculations we use $\mu = m_b$. The rest parameters are discussed below.

A. The CKM matrix elements

One widely used approximate form of the CKM matrix is the Wolfenstein parameterization [39] which emphasizes the hierarchy among its elements and expresses them in terms of powers of $\lambda = |V_{us}|$,

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{13}$$

The values of four Wolfenstein parameters (A, λ, ρ, and η) are given by several analysis methods from the best knowledge of the experimental and theoretical inputs (for example, see Table II). Within one standard deviation, the results from different approaches [23, 40, 41, 42] are virtually consistent with each other. In this paper, we shall take $\lambda = 0.2236 \pm 0.0031$, $A = 0.824 \pm 0.046$, $\bar{\rho}= 0.22 \pm 0.10$, $\bar{\eta}= 0.35 \pm 0.05$, and $\gamma = (59{\pm}13)^\circ$ [23].

However, it is not the right time to draw definite conclusions on the parameters ρ, η and γ. Some interesting hints seem to favour $\gamma{\gtrsim}90^\circ$, which is in conflict with the data in Table II. For example, it is assumed that it is possible to derive constraint on the angle γ from a global analysis B decays. Bargiotti, *et al.* obtain the bound $|\gamma - 90^\circ| > 21^\circ$ at 95% C.L. from $B \to K\pi$ decay rates and CP asymmetries [43]. Combining the QCDF approach with a global CKM matrix analysis — Rfit scenario advocated in [41], Beneke and Neubert make a fit of the Wolfenstein parameters $(\bar{\rho}, \bar{\eta})$ to six $B \to \pi\pi$, $K\pi$ decays with updated measurements, and their fit tend to favor $\gamma > 90^\circ$ [44]. In analogy with works [44], we also make a fit of the CKM matrix parameters to $B \to PP$, PV decays, and the preliminary results are $\lambda = 0.22$, $A = 0.82$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, and $\gamma = 78.8^\circ$ [45]. For comparison, we shall take the results in [45] as the CKM matrix inputs.

B. Quark masses

There are two different classes of quark masses. One type is pole mass for constituent quark, which appears in the penguin loop corrections with the functions $G_M(s_q)$ and $\hat{G}_M(s_q)$, where $s_q = m_q^2/m_b^2$. The definitions of $G_M(s_q)$ and $\hat{G}_M(s_q)$ can be found in [20]. In this paper, we take

$$m_u = m_d = m_s = 0, \qquad m_c = 1.47 \text{ GeV}, \qquad m_b = 4.66 \text{ GeV}. \tag{14}$$

The other is current quark mass which appears in the equations of motions, and is renormalization scale dependent. Their values are [23]

$$\frac{1}{2}\Big[\overline{m}_u(2\,\mathrm{GeV})+\overline{m}_d(2\,\mathrm{GeV})\Big]=(4.2\pm1.0)\,\mathrm{MeV}, \tag{15}$$

$$\overline{m}_s(2\,\mathrm{GeV})=(105\pm25)\,\mathrm{MeV}, \tag{16}$$

$$\overline{m}_b(\overline{m}_b)=(4.26\pm0.15\pm0.15)\,\mathrm{GeV}. \tag{17}$$

Here we would like to use their central values for discussion. And using the renormalization group equation, their corresponding values at the scale of $\mu=\mathcal{O}(m_b)$ can be obtained. Because the current masses of light quarks are determined with large uncertainties, for illustration, we take approximation

$$r_\chi=\frac{2\mu_P}{\overline{m}_b}=r_\chi^\pi=r_\chi^{\eta^{(\prime)}}\Big(1-\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}\Big)=r_\chi^K=\frac{2m_K^2}{\overline{m}_b(\overline{m}_s+\overline{m}_q)}, \tag{18}$$

C. Nonperturbative hadronic quantities

Nonperturbative hadronic quantities, such as meson decay constants, form factors, and meson light cone distribution amplitudes, appear as inputs in the QCDF formula (9). In principle, information about decay constants and form factors can be determined form experiments and/or theoretical estimations. Now we specify these parameters. In this paper, we assume ideal mixing between ω and ϕ, i.e. $\omega=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $\phi=s\bar{s}$. As to η and η', we take the convention in [15, 46], using two-mixing-angle formula for the decay constants, but without the charm quark content in η and η'.

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|\eta^{(\prime)}(p)\rangle=if^q_{\eta^{(\prime)}}p_\mu, \qquad (q=u,d,s) \tag{19}$$

$$\frac{\langle 0|\bar{u}\gamma_5 u|\eta^{(\prime)}\rangle}{\langle 0|\bar{s}\gamma_5 s|\eta^{(\prime)}\rangle}=\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}}, \qquad \langle 0|\bar{s}\gamma_5 s|\eta^{(\prime)}\rangle=-i\frac{m^2_{\eta^{(\prime)}}}{2m_s}(f^s_{\eta^{(\prime)}}-f^u_{\eta^{(\prime)}}), \tag{20}$$

$$f^u_{\eta'}=\frac{f_8}{\sqrt{6}}\sin\theta_8+\frac{f_0}{\sqrt{3}}\cos\theta_0, \qquad f^s_{\eta'}=-2\frac{f_8}{\sqrt{6}}\sin\theta_8+\frac{f_0}{\sqrt{3}}\cos\theta_0, \tag{21}$$

$$f^u_{\eta}=\frac{f_8}{\sqrt{6}}\cos\theta_8-\frac{f_0}{\sqrt{3}}\sin\theta_0, \qquad f^s_{\eta}=-2\frac{f_8}{\sqrt{6}}\cos\theta_8-\frac{f_0}{\sqrt{3}}\sin\theta_0, \tag{22}$$

And for $B_s\to\eta^{(\prime)}$ transition form factors, we take [31]

$$F_0^{B_s\eta}=-\Big(\frac{2}{\sqrt{6}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\Big)F_0^{B_s\eta_{s\bar{s}}}, \qquad F_0^{B_s\eta'}=\Big(-\frac{2}{\sqrt{6}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\Big)F_0^{B_s\eta'_{s\bar{s}}}, \tag{23}$$

The values of these parameters are collected in Table III.

In this paper, we consider the contributions from chirally enhanced twist-3 light cone distribution amplitudes of a light pseudoscalar meson. As to vector mesons, only the longitudinally polarized twist-2 terms are taken into account, and the effects from transversely polarized and higher twist parts are neglected because they are power suppressed. In calculation, we shall take their asymptotic forms, as displayed in [22], i.e. for a light pseudoscalar meson, we have [20, 48]:

$$\begin{aligned} &\langle P(k)|\bar{q}(z_2)q(z_1)|0\rangle \\ &= \frac{if_P}{4}\int_0^1 dx\ e^{i(xk\cdot z_2+\bar{x}k\cdot z_1)}\Big\{\not{k}\gamma_5\Phi_P(x) - \mu_P\gamma_5\Big[\Phi_P^p(x) - \sigma_{\mu\nu}k^\mu z^\nu\frac{\Phi_P^\sigma(x)}{6}\Big]\Big\}, \end{aligned} \tag{24}$$

$$\begin{aligned} &\text{twist-2 asymptotic forms: } \Phi_P(x) = 6x\bar{x}, \\ &\text{twist-3 asymptotic forms: } \Phi_P^p(x) = 1, \qquad \Phi_P^\sigma(x) = 6x\bar{x}. \end{aligned} \tag{25}$$

where f_P is a decay constant; $z = z_2 - z_1$, $\bar{x} = 1 - x$.

For a longitudinally polarized vector meson, we have [48, 49]:

$$\langle 0|\bar{q}(0)\gamma_\mu q(z)|V(k,\lambda)\rangle = k_\mu\frac{\epsilon^\lambda\cdot z}{k\cdot z}f_V m_V\int_0^1 dx\ e^{-ixk\cdot z}\Phi_V^\parallel(x), \tag{26}$$

$$\text{twist-2 asymptotic forms: } \Phi_V^\parallel(x) = 6x\bar{x}. \tag{27}$$

where ϵ is a polarization vector, and $\epsilon_\parallel = k/m_V$.

For the wave function of B_s meson, we take [14, 38, 50]:

$$\Phi_B(\xi) = N_B\xi^2(1-\xi)^2\text{exp}\Big[-\frac{m_B^2\xi^2}{2\omega_B^2} - \frac{\omega_B^2 b^2}{2}\Big], \tag{28}$$

where N_B is the normalization constant. $\Phi_B(\xi)$ is peaked around $\xi \approx 0.1$ with $\omega_B = 0.4\,\text{GeV}$ and $b = 0$.

As to the divergent endpoint integral $\int dx/x$, in analogy with the treatment in works [20], we parameterize it as

$$X = \int_0^1\frac{dx}{x} = (1+\varrho\, e^{i\phi})\ln\frac{m_b}{\Lambda_h}, \qquad \varrho\leq 1, \qquad 0^\circ\leq\phi\leq 360^\circ. \tag{29}$$

In numerical calculation, we take their default values as

decay modes	ϱ_H	ϕ_H	ϱ_A [45]	ϕ_A [45]	Λ_h [20]
$B_s\to PP$	0	0	0.5	10°	0.5 GeV
$B_s\to PV$	0	0	1.0	330°	0.5 GeV

where (ϱ_H, ϕ_H) and (ϱ_A, ϕ_A) are related to the contributions from hard spectator scattering and weak annihilations, respectively.

IV. BRANCHING RATIOS

The branching ratios for charmless $B_s \to PP$, PV decays in B_s meson rest frame can be written as:

$$BR(B_s \to M_1 M_2) = \frac{\tau_{B_s}}{8\pi} \frac{|p|}{m_{B_s}^2} |\mathcal{A}(B_s \to M_1 M_2)|^2. \tag{30}$$

where

$$|p| = \frac{\sqrt{[m_{B_s}^2 - (m_{M_1} + m_{M_2})^2][m_{B_s}^2 - (m_{M_1} - m_{M_2})^2]}}{2m_{B_s}}, \tag{31}$$

The lifetime and mass for B_s meson are $\tau_{B_s} = 1.461$ ps, and $m_{B_s} = 5369.6$ MeV [23]. And since the QCDF approach works in the heavy quark limit, we take the masses of light mesons as zero in the computation of phase space, then $|p| = m_{B_s}/2$.

The numerical results of CP averaged branching ratios for B_s decays are listed in Table IV and Table V, which are calculated at the scale of $\mu = m_b$ with two sets of CKM matrix parameters. The data in BR columns are calculated within the NF framework and $BR \propto |\mathcal{A}^f|^2$; the data in BR^f and BR^{f+a} columns are estimated with the QCDF approach, and $BR^f \propto |\mathcal{A}^f|^2$, $BR^{f+a} \propto |\mathcal{A}^f + \mathcal{A}^a|^2$. In the following there are some remarks

- Only several interesting decay modes, such as $B_s \to K^{(*)}K$, $K^{(*)\pm}\pi^\mp$, $K^\pm\rho^\mp$, $\eta^{(\prime)}\eta^{(\prime)}$, have large branching ratios, which might be observed potentially in the near future. Branching ratios of other decay modes are small, not exceeding 1×10^{-6}. Especially for decays $B_s \to \pi\eta^{(\prime)}$, $\pi\phi$, $\rho\eta^{(\prime)}$, $\omega\eta^{(\prime)}$, whose tree contributions are suppressed by both CKM factor and color, and penguin contributions are electroweak coefficient a_9 dominant, so their CP averaged branching ratios are very small, around $\mathcal{O}(10^{-7})$. As to these pure weak annihilation decays, such as $B_s \to \pi\pi$, $\pi\rho$, $\pi\omega$, their branching ratios are extremely small, around $\mathcal{O}(10^{-8})$.

- For those $b \to s$ transition decay modes, such as $B_s \to \eta^{(\prime)}\eta^{(\prime)}$, $K^{(*)}K$, their tree contributions are CKM suppressed, and "nonfactorizable" effects contribute a large portion to penguin coefficients $a_{4,6}$, so penguin contributions and tree ones are either competitive, or penguin dominant. So we can see large "nonfactorizable" effects in these decays. In addition, the coefficients b_1 and/or b_3 appear in these decay amplitudes, and the data in Tables IV and V show that weak annihilation contributions are sizeable ($\gtrsim$50%). For those $b \to d$ transition decay modes, such as $B_s \to K^{(*)\pm}\pi^\mp$, $K^\pm\rho^\mp$, they are a_1 dominant, and the radiative corrections are α_s suppressed compared with leading order contributions, so the data in Table IV and V show no large difference between the results obtained with the QCDF approach and the NF's ones.

- There are hierarchy among some decay modes, such as

$$BR(\overline{B}_s^0 \to K^+K^-) > BR(\overline{B}_s^0 \to K^+K^{*-}) > BR(\overline{B}_s^0 \to K^-K^{*+}), \tag{32}$$

$$BR(\overline{B}_s^0 \to \overline{K}^0 K^0) > BR(\overline{B}_s^0 \to K^0 \overline{K}^{*0}) > BR(\overline{B}_s^0 \to \overline{K}^0 K^{*0}), \tag{33}$$

$$BR(\overline{B}_s^0 \to K^+\pi^-) > BR(\overline{B}_s^0 \to K^{*+}\pi^-), \tag{34}$$

  There are two lines of reason for the above relations. One main line of reason is that the penguin contributions are important or/and dominant for these decays. Their decay amplitudes involve the QCD penguin parameters a_4 and a_6 in the form of $a_4 + Ra_6$, where $R > 0$, for $B_s \to PP$ decays, and $R = 0$ ($R < 0$) for $B_s \to PV$ decays with $B \to P$ ($B \to V$) transition, respectively, as stated in [31]. The other line for the second inequality of Eq. (32) and Eq. (33) is $f_{K^*} F_1^{B_s^0 \to K} > f_K A_0^{B_s^0 \to K^*}$. The numerical data in Tables IV and V confirm the above relations in general. Here we would like to point out that because the weak annihilation parameters $b_3(P,V) = -b_3(V,P)$, and the combination of $b_1 + b_3$ (or $b_3 + 2b_4$) is destructive for $\overline{B}_s^0 \to K^+K^{*-}$ (or $K^0\overline{K}^{*0}$), and constructive for $\overline{B}_s^0 \to K^-K^{*+}$ (or $\overline{K}^0 K^{*0}$), the weak annihilation have more effects to decays of $\overline{B}_s^0 \to K^-K^{*+}$ ($\overline{K}^0 K^{*0}$) than to decays of $\overline{B}_s^0 \to K^+K^{*-}$ ($K^0\overline{K}^{*0}$), (for example, see the data in Table V) which might be an explanation of why the data $BR^{f+a}(\overline{B}_s^0 \to K^-K^{*+}) > BR^{f+a}(\overline{B}_s^0 \to K^+K^{*-})$ in column 4 of Table V that violate the second inequality of Eq. (32).

- It is interesting to note that $B_s \to \eta^{(\prime)}\eta^{(\prime)}$ decays have large branching ratios. In fact, their $SU(3)$ counterpart, $B_{u,d} \to K\eta'$ have been reported to have the largest branching ratios among the two-body charmless rare B decays,

Decay modes	CLEO [51]	BABAR [2]	Belle [6]
$BR(B_d \to K_s\eta') \times 10^6$	$89^{+18}_{-16} \pm 9$	$42^{+13}_{-11} \pm 4$	$55^{+19}_{-16} \pm 8$
$BR(B_u \to K^\pm\eta') \times 10^6$	$80^{+10}_{-9} \pm 7$	$70 \pm 8 \pm 5$	$79^{+12}_{-11} \pm 9$

  The abnormally large branching functions for $B_{u,d} \to K\eta'$ decays have triggered intense theoretical interests in understanding the special property of meson $\eta^{(\prime)}$. Several mechanisms have been proposed (for example, Refs [52, 53, 54, 55]). There are many works devoted to the study of exclusive B decays into two-body final states containing $\eta^{(\prime)}$, such as Refs. [30, 56]. It is generally believed that this problem is related to the axial anomaly in QCD, but the dynamical details remain unclear. It is now commonly believed that maybe there is large coupling between two gluons and η' which might have important contributions for η' production [54, 55]. It is shown in [17] that the contributions of $g^*g^* \to \eta'$ to the formfactors can give a good explanation for experimental data. Recently, M. Beneke and M. Neubert computed the exclusive $B_{u,d} \to \eta^{(\prime)} + X$ decays using the QCDF approach [56]. Their novel idea is to consider the flavor-singlet amplitudes for producing $\eta^{(\prime)}$ meson from not only a quark-antiquark pair but also a pair of gluons. Their analysis including three effects: $b{\to}sgg$ amplitude, spectator scattering involving two gluons, and weak annihilation, could qualitatively account for the measurements with inputs of specified values, but with large theoretical uncertainties. Their conclusion is that it is the constructive or destructive interference of non-singlet penguin amplitudes that is the key factor in explaining the exclusive $B \to \eta^{(\prime)} + X$ decays.

- From Table IV and V, we can see that CP averaged branching ratios for many decay modes are stable against the choices of the CKM matrix parameters. Only those decays which have large interference between tree contributions and penguin ones, such as $B_s \to K^{(*)}\pi$, $K^{(*)\pm}K^{\mp}$, $\cdots$, are sensitive to the choice of the angle γ.

Of course, theoretical uncertainties from input parameters (such as the CKM matrix elements, quark masses, form factors, X, and so on) should be taken into account when discussing B_s decays, which has been studied in detail in Refs. [20, 21, 22]. In Figures 1 and 2, we consider the effects of the variation of inputs on the CP averaged branching ratios of $B_s \to K^+K^-$, $K^\pm\pi^\mp$, $K^\pm\rho^\mp$, $\overline{K}^0K^0$ and $\pi^+\pi^-$ decays on the weak phase γ. In each plot, the dashed lines, solid lines, and doted lines give the QCDF's predictions with default values of various input parameters for $\overline{m}_s(2\,\mathrm{GeV}) = 90$ MeV, 105 MeV, and 120 MeV, respectively, but keep the ratio of light quark masses fixed, $\overline{m}_q/\overline{m}_s = 4.2/105$. For discussion, we also vary the form factor $F_{0,1}^{B_s\to K}$ by $\pm 10\%$, i.e. $F_{0,1}^{B_s\to K} = 0.250 \sim 0.300$. It also includes the uncertainties from the CKM matrix parameters. From Figures 1 and 2, we can see that

- There exist sizeable theoretical uncertainties which smear some helpful information on the angle γ and demote the predictive power of the QCDF approach.

- The variation of form factors brings very large uncertainties (see the 2nd row of Figure 1) which in principle could be reduced by the ratios of branching ratios, while the uncertainties from X_H which parameterizes the divergent end-point integral in hard spectator scattering corrections are very small (see the 3rd row of Figure 1).

- For tree dominant decay modes (such as $B_s \to K^{\pm}\pi^{\mp}$, $K^{\pm}\rho^{\mp}$), the theoretical uncertainties mainly come from the formfactors and CKM matrix inputs. While for those decays with large interference between tree and penguin amplitudes, such as $B_s \to K^+K^-$, and penguin dominant decays, such as $B_s \to \overline{K}^0 K^0$, the theoretical uncertainties originate mainly from the variation of light quark masses and parameter X_A besides formfactors.

- $B_s \to \pi^+\pi^-$ decay is pure annihilation process. Its amplitude is free from transition form factors and hard spectator scattering corrections. Hence the dominant theoretical uncertainties would come form the weak annihilation effects, more precisely the quantity X_A (see Figure 2). Experimentally, it is worth searching for pure annihilation processes which may be helpful to learn more about the annihilation mechanism and to provide some useful information about final states interactions and nonperturbative parameters, such as X_A.

V. CP ASYMMETRIES

CP-violating asymmetries for B_s decays has been studies in [29]. In this paper, we shall evaluate them with the QCDF approach. In principle, the calculation of CP-violating asymmetries for B_s are similar with those for B_d decays. Due to flavor-changing interactions, $\overline{B}_s^0$ and B_s^0 can oscillate into each other with time evolution. The time dependent CP asymmetries $\mathcal{A}_{CP}$ for B_s decays is defined as

$$\mathcal{A}_{CP}(t) = \frac{\Gamma(\overline{B}_s^0(t) \to \bar{f}) - \Gamma(B_s^0(t) \to f)}{\Gamma(\overline{B}_s^0(t) \to \bar{f}) + \Gamma(B_s^0(t) \to f)}. \tag{35}$$

As discussed previously in Refs. [21, 22], the $B_s \to PP$, PV decays can be classified into three cases according to the properties of the final states,

- case-I: $B_s^0 \to f$, $\overline{B}_s^0 \to \bar{f}$, but $B_s^0 \not\to \bar{f}$, $\overline{B}_s^0 \not\to f$, for example, $\overline{B}_s^0 \to K^+\rho^-$, π^-K^{*+},

$\cdots$, the CP-violating asymmetry for these decays is time independent,

$$\mathcal{A}_{CP} = \frac{\Gamma(\overline{B}_s^0 \to \bar{f}) - \Gamma(B_s^0 \to f)}{\Gamma(\overline{B}_s^0 \to \bar{f}) + \Gamma(B_s^0 \to f)}. \tag{36}$$

- case-II: $B_s^0 \to (f = \bar{f}) \leftarrow \overline{B}_s^0$, for example, $B_s \to K^{\pm}K^{\mp}$, $\eta^{(\prime)}\eta^{(\prime)}$, $\cdots$, the time-integrated CP-violating asymmetry for these decays is

$$\mathcal{A}_{CP} = \frac{1}{1+x_s^2} a_{\epsilon'} + \frac{x_s}{1+x_s^2} a_{\epsilon+\epsilon'}, \tag{37}$$

$$a_{\epsilon'} = \frac{1-|\lambda_{CP}|^2}{1+|\lambda_{CP}|^2}, \quad a_{\epsilon+\epsilon'} = \frac{-2\,\mathrm{Im}(\lambda_{CP})}{1+|\lambda_{CP}|^2}, \quad \lambda_{CP} = \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \frac{\mathcal{A}(\overline{B}_s^0(0) \to \bar{f})}{\mathcal{A}(B_s^0(0) \to f)}, \tag{38}$$

where $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ are direct and mixing-induced CP-violating asymmetries, respectively. The parameter $x_s = \Delta m_{B_s}/\Gamma_{B_s}$ is considerably large for B_s system, $x_s > 19.0$ at 95% C.L. [23]. In our calculation, we shall take the preferred value in the SM $x_s \simeq 20$ [57]. Clearly, CP-violating asymmetry $\mathcal{A}_{CP}$ should be very small because $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ in Eq. (37) are strongly suppressed by $1/x_s^2$ and $1/x_s$, respectively.

- case-III: $B_s^0 \to (f \& \bar{f}) \leftarrow \overline{B}_s^0$, for example, $B_s \to (K_S^0\overline{K}^{*0}$ & $K_S^0 K^{*0})$, $(K^+K^{*-}$ & $K^-K^{*+})$, $(\pi^+\rho^-$ & $\pi^-\rho^+)$. Analogous to the notations for B_d decays in [15], the time dependent decay widths for this case of B_s decays are written as:

$$\Gamma(B_s^0(t) \to f) = \frac{e^{-\Gamma_{B_s}t}}{2}\Big(|g|^2+|h|^2\Big)\Big[1 + a_{\epsilon'}\cos(\Delta m_{B_s}t) + a_{\epsilon+\epsilon'}\sin(\Delta m_{B_s}t)\Big], \tag{39}$$

$$\Gamma(\overline{B}^0(t) \to \bar{f}) = \frac{e^{-\Gamma_{B_s}t}}{2}\Big(|\bar{g}|^2+|\bar{h}|^2\Big)\Big[1 - a_{\bar{\epsilon}'}\cos(\Delta m_{B_s}t) - a_{\epsilon+\bar{\epsilon}'}\sin(\Delta m_{B_s}t)\Big], \tag{40}$$

$$\Gamma(B^0(t) \to \bar{f}) = \frac{e^{-\Gamma_{B_s}t}}{2}\Big(|\bar{g}|^2+|\bar{h}|^2\Big)\Big[1 + a_{\bar{\epsilon}'}\cos(\Delta m_{B_s}t) + a_{\epsilon+\bar{\epsilon}'}\sin(\Delta m_{B_s}t)\Big], \tag{41}$$

$$\Gamma(\overline{B}^0(t) \to f) = \frac{e^{-\Gamma_{B_s}t}}{2}\Big(|g|^2+|h|^2\Big)\Big[1 - a_{\epsilon'}\cos(\Delta m_{B_s}t) - a_{\epsilon+\epsilon'}\sin(\Delta m_{B_s}t)\Big], \tag{42}$$

where

$$g = \mathcal{A}(B_s^0(0) \to f), \qquad \bar{g} = \mathcal{A}(\overline{B}_s^0(0) \to \bar{f}), \tag{43}$$

$$h = \mathcal{A}(\overline{B}_s^0(0) \to f), \qquad \bar{h} = \mathcal{A}(B_s^0(0) \to \bar{f}), \tag{44}$$

and with $q/p = V_{ts}V_{tb}^*/V_{ts}^*V_{tb}$,

$$a_{\epsilon'} = \frac{|g|^2-|h|^2}{|g|^2+|h|^2}, \qquad a_{\epsilon+\epsilon'} = \frac{-2\,\mathrm{Im}[(q/p)\times(h/g)]}{1+|h/g|^2}, \tag{45}$$

$$a_{\bar{\epsilon}'} = \frac{|\bar{h}|^2-|\bar{g}|^2}{|\bar{h}|^2+|\bar{g}|^2}, \qquad a_{\epsilon+\bar{\epsilon}'} = \frac{-2\,\mathrm{Im}[(q/p)\times(\bar{g}/\bar{h})]}{1+|\bar{g}/\bar{h}|^2}. \tag{46}$$

Our numerical results of CP-violating asymmetries for $B_s \to PP$, PV decays are listed in Tables VI–IX, which are calculated with two sets of CKM matrix parameters. In the following there are some remarks.

- From Tables VIII–IX, we can see that as expected, due to the large parameter x_s suppression, CP-violating asymmetry $\mathcal{A}_{CP}$ for these case-II decay modes are indeed very small, not exceeding 5%.

- From the QCDF formula Eq. (9), we know that radiative corrections and "nonfactorizable" contributions should be either at the order of α_s or power suppressed in Λ_{QCD}/m_b, therefore, the CP-violating asymmetries $\mathcal{A}_{CP}$ for these a_1 dominant decays, such as $B_s \to K^{(*)\pm}\pi^{\mp}$, $K^{\pm}\rho^{\mp}$, are not large because of the small strong phases.

- From the previous $B_{u,d} \to PP$, PV analysis [21, 22], we know that "nonfactorizable" effects contribute a large imaginary part to the coefficients $a_{2,4,6}$. So from Tables VI–IX, we see that for these decays which have large interference between tree contributions dominated by a_2 and QCD penguin ones, and large interference between CKM suppressed a_2 dominant tree contributions and electroweak penguin ones, there exist large direct CP-violating asymmetries, for example, $\mathcal{A}_{CP}$ for $\overline{B}_s^0 \to K^{*0}\pi^0$, $K^{*0}\eta^{(\prime)}$ decays, $a_{\epsilon'}$ for $B_s \to K_S^0\pi^0$, $K_S^0\eta^{(\prime)}$, $K_S^0\rho^0$, $K_S^0\omega$ decays, and $a_{\epsilon'}$ for $B_s \to \pi^0\phi$, $\pi^0\eta^{(\prime)}$, $\eta^{(\prime)}\rho^0$, $\eta^{(\prime)}\omega$ decays.

- The CP-violating asymmetries for the most decay modes is not keen on the variation of the CKM matrix parameters. The large and direct CP-violating asymmetries which are sensitive to the angle γ are only display for some of these decays, such as $\mathcal{A}_{CP}$ for $\overline{B}_s^0 \to K^{*0}\pi^0$, $K^{*0}\eta^{(\prime)}$, and $a_{\epsilon'}$ for $B_s \to K_S^0\pi^0$, $K_S^0\omega$,

- If we assume that B_s^0-$\overline{B}_s^0$ mixing phase is negligible, or $q/p = 1$, it should be convenient to determine the weak angle γ or β from measurements of CP-violating asymmetries for B_s decays. Unfortunately, the very rapid B_s^0-$\overline{B}_s^0$ oscillations are expected due to the large mixing parameter x_s in SM, which makes the experimental studies of CP violation in B_s meson system difficult, and only bounds on few decays modes are given for the moment. In addition, there exist large theoretical uncertainties. Within the QCDF approach, the subleading power corrections in $1/m_b$ are might be as important as the radiative corrections numerically because m_b is not infinitely large.

So the QCDF approach can only give the order of magnitude of the CP-violating asymmetries, as stated in [22].

VI. EXTRACTING WEAK PHASES FROM $B_s \to KK$ DECAYS

It is necessary to test the self-consistency of the CKM description of CP violation through a variety of processes. One test involves the $B_d(t) \to \pi^+\pi^-$ decays which are potentially rich sources of information of both strong and weak phases. Experimentally, BABAR and Belle have reported the measurements of CP-violating asymmetries in $B_d(t) \to \pi^+\pi^-$ decays,

	BABAR [4]	Belle [7]	Average
$S_{\pi\pi}$	$-0.02\pm0.34\pm0.05$	$1.21^{+0.27+0.13}_{-0.38-0.16}$	0.48 ± 0.35[5]
$C_{\pi\pi}$	$-0.30\pm0.25\pm0.04$	$-0.94^{+0.31}_{-0.25}\pm0.09$	-0.54 ± 0.20

which has triggered high theoretical interest. Theoretically, if we assume that the penguin amplitudes are zero for $B_d(t) \to \pi^+\pi^-$ decays, then it is expected to determine the weak angle α from the $S_{\pi\pi} = -\sin 2\alpha$. Unfortunately, this relation is strongly polluted by penguin effects. The nature of tree and penguin amplitudes lead to some indeterminacy in determining the weak and strong phases. Hence, it will be an interesting work to investigate the ratio of penguin to tree amplitudes. Using the U-spin symmetry, we can get some additional information on the penguin-to-tree ratio, $P_{\pi\pi}/T_{\pi\pi}$, from its counterpart $B_s \to K^+K^-$ as a cross check. To illustrate, we now describe the expressions for the decay amplitudes of $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$ as follows:

$$\mathcal{A}(\overline{B}^0 \to \pi^+\pi^-) = |T_{\pi\pi}|e^{-i\delta_T}e^{-i\gamma} + |P_{\pi\pi}|e^{-i\delta_P}, \tag{47}$$

$$\mathcal{A}(\overline{B}^0_s \to K^+K^-) = |T_c|e^{-i\delta^c_T}e^{-i\gamma} - |P_c|e^{-i\delta^c_P}, \tag{48}$$

Using the $SU(3)$ flavor symmetry, we have [58]

$$\frac{|T_{\pi\pi}|}{|T_c|} = \frac{|V_{ub}V^*_{ud}|}{|V_{ub}V^*_{us}|} = \frac{1-\lambda^2/2}{\lambda}, \qquad \delta_T = \delta^c_T, \tag{49}$$

$$\frac{|P_{\pi\pi}|}{|P_c|} = \frac{|V_{cb}V^*_{cd}|}{|V_{cb}V^*_{cs}|} = \frac{\lambda}{1-\lambda^2/2}, \qquad \delta_P = \delta^c_P, \tag{50}$$

$$\frac{|P_{\pi\pi}|}{|T_{\pi\pi}|} = \tan^2\theta_c \frac{|P_c|}{|T_c|}, \qquad \delta_T - \delta_P = \delta^c_T - \delta^c_P = \delta', \tag{51}$$

[5] the error includes a scale factor of 1.3

and with $r_A = \frac{f_{B_s} f_K}{F^{B_s \to K} m_{B_s}^2}$,

$$\frac{P_c}{T_c} = \frac{-|V_{cb}V_{cs}^*|\{a_4^c + a_{10}^c + r_\chi(a_6^c + a_8^c) + r_A(b_3 + 2b_4 - \frac{1}{2}b_3^{ew} + \frac{1}{2}b_4^{ew})\}}{|V_{ub}V_{us}^*|\{a_1^u + a_4^u + a_{10}^u + r_\chi(a_6^u + a_8^u) + r_A(b_1 + b_3 + 2b_4 - \frac{1}{2}b_3^{ew} + \frac{1}{2}b_4^{ew})\}}, \tag{52}$$

Compared with $B_d \to \pi^+\pi^-$ decay, the contribution of T_c (P_c) for $B_s \to K^+K^-$ decay is reduced (enhanced) by $\tan\theta_c$. Of course, the relations of Eq. (49) and Eq. (50) are affected by U-spin breaking effects, such as factor $\frac{(m_{B_d}^2 - m_\pi^2)F^{B\to\pi} f_\pi}{(m_{B_s}^2 - m_K^2)F^{B_s\to K} f_K}$, and so on. But $SU(3)$ flavor breaking effects are expected to be very small in Eq. (51) within factorization approach. As stated in [59, 60], assuming that B_s^0-$\overline{B}_s^0$ mixing phase is negligible, and taking the angle β as one known input which can be determined from $B_d \to J/\Psi K_S$ decays and has been tentatively given by BABAR [61] and Belle [62], the strong phase $\delta_T - \delta_P$ and $|P_{\pi\pi}/T_{\pi\pi}|$ (or $\delta_T^c - \delta_P^c$ and $|P_c/T_c|$) as a function of weak angle γ or/and α can be determined from measurements of $S_{\pi\pi}$ and $C_{\pi\pi}$ (or S_{KK} and C_{KK}). And employing the relation Eq. (51) within the U-spin symmetry, the penguin-to-tree ratio can be overconstrained from $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ decays. And using the value of the penguin-to-tree ratios, some information on weak phases γ or/and α can be extracted from the measurements,

$$\lambda_{\pi\pi} = \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \frac{\mathcal{A}(\overline{B}^0(0)\to\pi^+\pi^-)}{\mathcal{A}(B^0(0)\to\pi^+\pi^-)} = e^{i2\alpha}\frac{1 + |P_{\pi\pi}/T_{\pi\pi}|e^{i\delta'}e^{i\gamma}}{1 + |P_{\pi\pi}/T_{\pi\pi}|e^{i\delta'}e^{-i\gamma}}, \tag{53}$$

$$S_{\pi\pi} = \frac{-2\,\mathrm{Im}(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} = \frac{-\sin2\alpha + 2|P_{\pi\pi}/T_{\pi\pi}|\cos\delta'\cos(\alpha - \beta) + |P_{\pi\pi}/T_{\pi\pi}|^2\sin2\beta}{1 - 2|P_{\pi\pi}/T_{\pi\pi}|\cos\delta'\cos(\alpha + \beta) + |P_{\pi\pi}/T_{\pi\pi}|^2}, \tag{54}$$

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} = \frac{2|P_{\pi\pi}/T_{\pi\pi}|\sin\delta'\sin\gamma}{1 + 2|P_{\pi\pi}/T_{\pi\pi}|\cos\delta'\cos\gamma + |P_{\pi\pi}/T_{\pi\pi}|^2}, \tag{55}$$

$$\lambda_{KK} = \frac{V_{ts}V_{tb}^*}{V_{ts}^*V_{tb}} \frac{\mathcal{A}(\overline{B}_s^0(0)\to K^+K^-)}{\mathcal{A}(B_s^0(0)\to K^+K^-)} = e^{-i2\gamma}\frac{1 - |P_c/T_c|e^{i\delta'}e^{i\gamma}}{1 - |P_c/T_c|e^{i\delta'}e^{-i\gamma}}, \tag{56}$$

$$S_{KK} = \frac{-2\,\mathrm{Im}(\lambda_{KK})}{1 + |\lambda_{KK}|^2} = \frac{\sin2\gamma - 2|P_c/T_c|\cos\delta'\sin\gamma}{1 - 2|P_c/T_c|\cos\delta'\cos\gamma + |P_c/T_c|^2}, \tag{57}$$

$$C_{KK} = \frac{1 - |\lambda_{KK}|^2}{1 + |\lambda_{KK}|^2} = \frac{-2|P_c/T_c|\sin\delta'\sin\gamma}{1 - 2|P_c/T_c|\cos\delta'\cos\gamma + |P_c/T_c|^2}. \tag{58}$$

Here the numerical values of penguin-to-tree ratios are given in Table X. Although using different derivation and inputs, within the range of one σ, the results of penguin-to-tree ratio $|P_{\pi\pi}/T_{\pi\pi}|$ that are calculated with Eq. (51) are in agreement with the result of (28.5 $\pm$ 5.1 $\pm$ 5.7) % [20] which is calculated with $X_A = X_H = \ln(m_b/\Lambda_h)$, and the result of (27.6 $\pm$ 6.4) % [63] (including $SU(3)$ breaking effects), but not as large as 0.41 [45]. In addition, the value of the strong phase δ' is also consistent with $(8.2\pm3.8)^\circ$ [20]. The uncertainties of penguin-to-tree ratio $P_{\pi\pi}/T_{\pi\pi}$ are shown in Figure 3, which indicates $\delta' \in (-7^\circ, 22^\circ)$.

In addition, if the penguin-to-tree ratios are determined, then it is expected to extract some information or give bound on weak angle γ from the measurements of $B_s \to K^+K^-$, $\overline{K}^0K^0$ decays in the future. Now let's illustrate this point. The QCD penguin terms for these two decays should be equal according to factorization, so the decay amplitude for $B_s \to \overline{K}^0K^0$ could be written as,

$$\mathcal{A}(\overline{B}_s^0 {\to} \overline{K}^0 K^0) \simeq -\, |P_c| e^{-i\delta_P^c}, \tag{59}$$

There are two approximations in Eq. (59), as stated in [58]: (1) The color suppressed terms which are proportional to the CKM matrix factor $V_{ub}V_{us}^*$ in Eq. (A1) and Eq. (C1) are safely neglected because of $|V_{ub}V_{us}^*/V_{cb}V_{cs}^*| \simeq 2\%$. (2) The tiny isospin breaking effects are disregarded, and the small difference of electroweak penguin contributions between these two decay modes are neglected. The ratio of CP averaged branching ratios is defined as

$$\begin{aligned} R_{KK} &= \frac{BR(\overline{B}_s^0 {\to} K^+K^-) + BR(B_s^0 {\to} K^+K^-)}{BR(\overline{B}_s^0 {\to} \overline{K}^0 K^0) + BR(B_s^0 {\to} \overline{K}^0 K^0)} \\ &= \frac{1 - 2|P_c/T_c|\cos\delta'\cos\gamma + |P_c/T_c|^2}{|P_c/T_c|^2}, \end{aligned} \tag{60}$$

Then we can get a constraint on γ,

$$\cos\gamma \gtrsim \left|\frac{P_c}{T_c}\right| \left(1 - \sqrt{R_{KK}}\right), \tag{61}$$

In addition, Gronau and Ronsner also gave a bound on γ from the decays $B_s \to K^+K^-$, $\overline{K}^0K^0$ without prior knowledge of the penguin-to-tree ratio, $\sin^2\gamma \leq R_{KK}$ [58].

From the above discussion, we can see that sufficient measurements of B_s decays in the future can resolve ambiguities on the determination of the CKM angles. Here, we estimate the bounds on γ with the QCDF approach. The data in Table X indicates that the weak angle γ given in Refs. [42, 45] and the bound from Eq. (61) are consistent with each other, which might be tested in future measurements.

VII. SUMMARY AND CONCLUSION

In this paper, we calculated the CP averaged branching ratios and CP-violating asymmetries of two-body charmless hadronic $B_s \to PP$, PV decays at next-to-leading order in α_s with the QCDF approach, including "nonfactorizable" corrections, as well as those from weak annihilation topologies. We find

- Only several decays, such as $B_s \to K^{(*)}K$, $K^{(*)\pm}\pi^{\mp}$, $K^{\pm}\rho^{\mp}$, $\eta^{(\prime)}\eta^{(\prime)}$, have large branching ratios, which might be accessible at hadron colliders potentially in the near future to allow for detailed phenomenological analysis. The a_1 dominant decays $B_s \to K^{(*)\pm}\pi^{\mp}$, $K^{\pm}\rho^{\mp}$ are generally insensitive to the contributions from both "nonfactorizable" effects and weak annihilation, so the numerical predictions of the QCDF approach on their CP averaged branching ratios are similar to their NF's counterparts; the main uncertainties come mainly from the CKM matrix parameters and form factors. And their direct CP-violating asymmetries A_{CP} are only a few percent because of their small strong phases. But for the branching ratios of $B_s \to K^{(*)}K$, $\eta^{(\prime)}\eta^{(\prime)}$ decays, the contributions of the "nonfactorizable" effects and weak annihilation can be sizeable.

- Large direct CP-violating asymmetries occur in some decays whose decay amplitudes are related to coefficient a_2, because a_2 obtains a large imaginary part from "nonfactorizable" effects, and some can even reach 80% or so, for example, $a^{f}_{\epsilon'}(K^0_S\eta) \approx -85\%$ in Table VI. But $\mathcal{A}_{CP}$ for case-II CP decays is very small because of large x_s. Of course, it is assumed that the contribution of power corrections in $1/m_b$ are as important as radiative ones to strong phase numerically, so the quantitative predictions on CP-violating asymmetries of B_s decays with the QCDF approach should not be taken too seriously.

- Although we can overconstrain the penguin-to-tree ratio, $|P_{\pi\pi}/T_{\pi\pi}|$, and give a bound on γ from B_s decay into charged and neutral kaons, too many input parameters bring large theoretical uncertainties, what's more, experimental studies on B_s decays are very limited so far, so it is not the time to extract some useful information on the angle γ from two-body charmless B_s decays. We look forward to future measurements and theoretical developments to give some insight into these parameters.

Acknowledgments

This work is supported in part by National Natural Science Foundation of China. G. Zhu thanks JSPS of Japan for financial support.

APPENDIX A: THE DECAY AMPLITUDES FOR $\overline{B}_s^0 \to PP$

$$\mathcal{A}^f(\overline{B}_s^0 \to K^0\overline{K}^0) = -i\frac{G_F}{\sqrt{2}} f_K F_0^{B_s^0\to K}\Big(m_{B_s}^2 - m_K^2\Big)\Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big) \times\Big\{a_4 - \frac{1}{2}a_{10} + R_1\Big(a_6 - \frac{1}{2}a_8\Big)\Big\}, \tag{A1}$$

where $R_1 = \frac{2m_{\overline{K}^0}^2}{(m_s+m_d)(m_b-m_d)}$.

$$\mathcal{A}^f(\overline{B}_s^0 \to K^0\pi^0) = -i\frac{G_F}{2} f_\pi F_0^{B_s^0\to K}\Big(m_{B_s}^2 - m_K^2\Big)\Big\{V_{ub}V_{ud}^* a_2 + \Big(V_{ub}V_{ud}^* + V_{cb}V_{cd}^*\Big)\Big[-a_4 + \frac{1}{2}a_{10} - \frac{3}{2}\Big(a_7 - a_9\Big) - R_2\Big(a_6 - \frac{1}{2}a_8\Big)\Big]\Big\}, \tag{A2}$$

where $R_2 = \frac{2m_{\pi^0}^2}{(m_u+m_d)(m_b-m_d)}$.

$$\begin{aligned}\mathcal{A}^f(\overline{B}_s^0 \to K^0\eta^{(\prime)}) = & -i\frac{G_F}{\sqrt{2}} f_K F_0^{B_s^0\to \eta^{(\prime)}}\Big(m_{B_s}^2 - m_{\eta^{(\prime)}}^2\Big)\Big(V_{ub}V_{ud}^* + V_{cb}V_{cd}^*\Big) \\ & \times\Big\{a_4 - \frac{1}{2}a_{10} + R_3\Big(a_6 - \frac{1}{2}a_8\Big)\Big\} \\ & -i\frac{G_F}{\sqrt{2}} f_{\eta^{(\prime)}}^u F_0^{B_s^0\to K}\Big(m_{B_s}^2 - m_K^2\Big)\Big\{V_{ub}V_{ud}^* a_2 \\ & +\Big(V_{ub}V_{ud}^* + V_{cb}V_{cd}^*\Big)\Big[2\Big(a_3 - a_5\Big) + a_4 - \frac{1}{2}a_{10} - \frac{1}{2}\Big(a_7 - a_9\Big) \\ & +R_4^{(\prime)}\Big(a_6 - \frac{1}{2}a_8\Big)\Big(1 - \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s}\Big) + \Big(a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9\Big)\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u}\Big]\Big\},\end{aligned} \tag{A3}$$

where $R_3 = \frac{2m_{K^0}^2}{(m_s+m_d)(m_b-m_s)}$, and $R_4^{(\prime)} = \frac{2m_{\eta^{(\prime)}}^2}{(m_s+m_s)(m_b-m_d)}$.

$$\mathcal{A}^f(\overline{B}_s^0 \to \pi^0\eta^{(\prime)}) = -i\frac{G_F}{2} f_\pi F_0^{B_s^0\to \eta^{(\prime)}}\Big(m_{B_s}^2 - m_{\eta^{(\prime)}}^2\Big)\Big\{V_{ub}V_{us}^* a_2 + \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[-\frac{3}{2}\Big(a_7 - a_9\Big)\Big]\Big\}. \tag{A4}$$

$$\begin{aligned}\mathcal{A}^f(\overline{B}_s^0 \to \eta^{(\prime)}\eta^{(\prime)}) = & -i\sqrt{2}G_F f_{\eta^{(\prime)}}^u F_0^{B_s^0\to \eta^{(\prime)}}\Big(m_{B_s}^2 - m_{\eta^{(\prime)}}^2\Big)\Big\{V_{ub}V_{us}^* a_2 \\ & +\Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[2\Big(a_3 - a_5\Big) - \frac{1}{2}\Big(a_7 - a_9\Big)\Big] \\ & +\Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\frac{f_{\eta^{(\prime)}}^s}{f_{\eta^{(\prime)}}^u}\Big[a_3 - a_5 + a_4 - \frac{1}{2}a_{10} + \frac{1}{2}a_7 \\ & -\frac{1}{2}a_9 + R_5^{(\prime)}\Big(a_6 - \frac{1}{2}a_8\Big)\Big(1 - \frac{f_{\eta^{(\prime)}}^u}{f_{\eta^{(\prime)}}^s}\Big)\Big]\Big\},\end{aligned} \tag{A5}$$

where $R_5^{(\prime)} = \frac{2m^2_{\eta^{(\prime)}}}{(m_s+m_s)(m_b-m_s)}$.

$$\begin{aligned}
\mathcal{A}^f(\overline{B}^0_s \to \eta\eta') \;=\; & -i\frac{G_F}{\sqrt{2}} f^u_\eta F_0^{B^0_s\to\eta'}\Big(m^2_{B_s} - m^2_{\eta'}\Big)\Big\{V_{ub}V^*_{us}a_2 \\
& +\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\Big[2\Big(a_3 - a_5\Big) - \frac{1}{2}\Big(a_7 - a_9\Big)\Big] \\
& +\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\frac{f^s_\eta}{f^u_\eta}\Big[a_3 - a_5 + a_4 - \frac{1}{2}a_{10} + \frac{1}{2}a_7 \\
& -\frac{1}{2}a_9 + R_5\Big(a_6 - \frac{1}{2}a_8\Big)\Big(1 - \frac{f^u_\eta}{f^s_\eta}\Big)\Big]\Big\} \\
& -i\frac{G_F}{\sqrt{2}} f^u_{\eta'} F_0^{B^0_s\to\eta}\Big(m^2_{B_s} - m^2_{\eta}\Big)\Big\{V_{ub}V^*_{us}a_2 \\
& +\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\Big[2\Big(a_3 - a_5\Big) - \frac{1}{2}\Big(a_7 - a_9\Big)\Big] \\
& +\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\frac{f^s_{\eta'}}{f^u_{\eta'}}\Big[a_3 - a_5 + a_4 - \frac{1}{2}a_{10} + \frac{1}{2}a_7 \\
& -\frac{1}{2}a_9 + R'_5\Big(a_6 - \frac{1}{2}a_8\Big)\Big(1 - \frac{f^u_{\eta'}}{f^s_{\eta'}}\Big)\Big]\Big\}.
\end{aligned} \tag{A6}$$

$$\begin{aligned}
\mathcal{A}^f(\overline{B}^0_s \to \pi^- K^+) \;=\; & -i\frac{G_F}{\sqrt{2}} f_\pi F_0^{B^0_s\to K}\Big(m^2_{B_s} - m^2_K\Big)\Big\{V_{ub}V^*_{ud}a_1 \\
& +\Big(V_{ub}V^*_{ud} + V_{cb}V^*_{cd}\Big)\Big[a_4 + a_{10} + R_6\Big(a_6 + a_8\Big)\Big]\Big\},
\end{aligned} \tag{A7}$$

where $R_6 = \frac{2m^2_{\pi^+}}{(m_u+m_d)(m_b-m_u)}$.

$$\begin{aligned}
\mathcal{A}^f(\overline{B}^0_s \to K^- K^+) \;=\; & -i\frac{G_F}{\sqrt{2}} f_K F_0^{B^0_s\to K}\Big(m^2_{B_s} - m^2_K\Big)\Big\{V_{ub}V^*_{us}a_1 \\
& +\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\Big[a_4 + a_{10} + R_7\Big(a_6 + a_8\Big)\Big]\Big\},
\end{aligned} \tag{A8}$$

where $R_7 = \frac{2m^2_{K^-}}{(m_u+m_s)(m_b-m_u)}$.

APPENDIX B: THE DECAY AMPLITUDES FOR $\overline{B}^0_s \to PV$

$$\mathcal{A}^f(\overline{B}^0_s \to K^0 \overline{K}^{*0}) \;=\; \sqrt{2}G_F f_{K^*} F_1^{B^0_s\to K} m_{K^*}(\epsilon\cdot p_{K^0})\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big)\Big\{a_4 - \frac{1}{2}a_{10}\Big\}, \tag{B1}$$

$$\begin{aligned}
\mathcal{A}^f(\overline{B}^0_s \to \overline{K}^0 K^{*0}) \;=\; & \sqrt{2}G_F f_K A_0^{B^0_s\to K^*} m_{K^*}(\epsilon\cdot p_{\overline{K}^0})\Big(V_{ub}V^*_{us} + V_{cb}V^*_{cs}\Big) \\
& \times\Big\{a_4 - \frac{1}{2}a_{10} - Q_1\Big(a_6 - \frac{1}{2}a_8\Big)\Big\},
\end{aligned} \tag{B2}$$

where $Q_1 = \frac{2m^2_{\overline{K}^0}}{(m_s+m_d)(m_b+m_d)}$.

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to K^0\rho^0) \;=\; & G_F f_\rho F_1^{B^0_s\to K} m_\rho(\epsilon\cdot p_{K^0})\Big\{V_{ub}V^*_{ud}a_2 \\ & +\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big[-a_4+\frac{1}{2}a_{10}+\frac{3}{2}a_7+\frac{3}{2}a_9\Big]\Big\}.\end{aligned} \tag{B3}$$

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to K^0\omega) \;=\; & G_F f_\omega F_1^{B^0_s\to K} m_\omega(\epsilon\cdot p_{K^0})\Big\{V_{ub}V^*_{ud}a_2 \\ & +\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big[2\Big(a_3+a_5\Big)+a_4-\frac{1}{2}a_{10}+\frac{1}{2}a_7+\frac{1}{2}a_9\Big]\Big\}.\end{aligned} \tag{B4}$$

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to K^0\phi) \;=\; & \sqrt{2}G_F m_\phi(\epsilon\cdot p_{K^0})\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big) \\ & \times\Big\{f_\phi F_1^{B^0_s\to K}\Big[a_3+a_5-\frac{1}{2}a_7-\frac{1}{2}a_9\Big] \\ & +f_K A_0^{B^0_s\to\phi}\Big[a_4-\frac{1}{2}a_{10}-Q_2\Big(a_6-\frac{1}{2}a_8\Big)\Big]\Big\},\end{aligned} \tag{B5}$$

where $Q_2 = \frac{2m^2_{K^0}}{(m_s+m_d)(m_b+m_s)}$.

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to \pi^0 K^{*0}) \;=\; & G_F f_\pi A_0^{B^0_s\to K^{*0}} m_{K^{*0}}(\epsilon\cdot p_{\pi^0})\Big\{V_{ub}V^*_{ud}a_2 \\ & +\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big[-a_4+\frac{1}{2}a_{10}-\frac{3}{2}a_7 \\ & +\frac{3}{2}a_9+Q_3\Big(a_6-\frac{1}{2}a_8\Big)\Big]\Big\},\end{aligned} \tag{B6}$$

where $Q_3 = \frac{2m^2_{\pi^0}}{(m_u+m_d)(m_b+m_d)}$.

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to \pi^0\phi) \;=\; & G_F f_\pi A_0^{B^0_s\to\phi} m_\phi(\epsilon\cdot p_{\pi^0})\Big\{V_{ub}V^*_{us}a_2 \\ & +\Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[-\frac{3}{2}a_7+\frac{3}{2}a_9\Big]\Big\},\end{aligned} \tag{B7}$$

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s \to \eta^{(\prime)} K^{*0}) \;=\; & \sqrt{2}G_F m_{K^{*0}}(\epsilon\cdot p_{\eta^{(\prime)}})\Big\{f^u_{\eta^{(\prime)}} A_0^{B^0_s\to K^{*0}}\Big[V_{ub}V^*_{ud}a_2 \\ & +\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big(2(a_3-a_5)+a_4-\frac{1}{2}a_{10}-\frac{1}{2}a_7 \\ & +\frac{1}{2}a_9-Q_4^{(\prime)}(a_6-\frac{1}{2}a_8)(1-\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}})\Big) \\ & +\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Big(a_3-a_5+\frac{1}{2}a_7-\frac{1}{2}a_9\Big)\Big] \\ & +f_{K^*}F_1^{B^0_s\to\eta^{(\prime)}}\Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big(a_4-\frac{1}{2}a_{10}\Big)\Big\},\end{aligned} \tag{B8}$$

where $Q_4^{(\prime)} = \frac{2m^2_{\eta^{(\prime)}}}{(m_s+m_s)(m_b+m_s)}$.

$$\mathcal{A}^f(\overline{B}^0_s\to\eta^{(\prime)}\rho^0) \;=\; G_F f_\rho F_1^{B^0_s\to\eta^{(\prime)}} m_\rho(\epsilon{\cdot}p_{\eta^{(\prime)}})\Big\{V_{ub}V^*_{us}a_2 + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[\frac{3}{2}\Big(a_7+a_9\Big)\Big]\Big\}, \tag{B9}$$

$$\mathcal{A}^f(\overline{B}^0_s\to\eta^{(\prime)}\omega) \;=\; G_F f_\omega F_1^{B^0_s\to\eta^{(\prime)}} m_\omega(\epsilon{\cdot}p_{\eta^{(\prime)}})\Big\{V_{ub}V^*_{us}a_2 + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[2\Big(a_3+a_5\Big)+\frac{1}{2}\Big(a_7+a_9\Big)\Big]\Big\}, \tag{B10}$$

$$\begin{aligned}\mathcal{A}^f(\overline{B}^0_s\to\eta^{(\prime)}\phi) \;=\; & \sqrt{2}G_F m_\phi(\epsilon{\cdot}p_{\eta^{(\prime)}})\Big\{f^u_{\eta^{(\prime)}}A_0^{B^0_s\to\phi}\Big[V_{ub}V^*_{us}a_2 \\ & +\Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big(2(a_3-a_5)-\frac{1}{2}(a_7-a_9)\Big) \\ & +\Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\frac{f^s_{\eta^{(\prime)}}}{f^u_{\eta^{(\prime)}}}\Big(a_3-a_5+a_4-\frac{1}{2}a_{10} \\ & +\frac{1}{2}a_7-\frac{1}{2}a_9-Q_4^{(\prime)}(a_6-\frac{1}{2}a_8)(1-\frac{f^u_{\eta^{(\prime)}}}{f^s_{\eta^{(\prime)}}})\Big)\Big] \\ & +f_\phi F_1^{B^0_s\to\eta^{(\prime)}}\Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big(a_3+a_5 \\ & +a_4-\frac{1}{2}a_{10}-\frac{1}{2}a_7-\frac{1}{2}a_9\Big)\Big\},\end{aligned} \tag{B11}$$

$$\mathcal{A}^f(\overline{B}^0_s\to K^+\rho^-) \;=\; \sqrt{2}G_F f_\rho F_1^{B^0_s\to K} m_\rho(\epsilon{\cdot}p_K)\Big\{V_{ub}V^*_{ud}a_1 + \Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big(a_4+a_{10}\Big)\Big\}, \tag{B12}$$

$$\mathcal{A}^f(\overline{B}^0_s\to K^+K^{*-}) \;=\; \sqrt{2}G_F f_{K^*} F_1^{B^0_s\to K} m_{K^*}(\epsilon{\cdot}p_K)\Big\{V_{ub}V^*_{us}a_1 + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big(a_4+a_{10}\Big)\Big\}, \tag{B13}$$

$$\mathcal{A}^f(\overline{B}^0_s\to K^-K^{*+}) \;=\; \sqrt{2}G_F f_K A_0^{B^0_s\to K^*} m_{K^*}(\epsilon{\cdot}p_K)\Big\{V_{ub}V^*_{us}a_1 + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[a_4+a_{10}-Q_5\Big(a_6+a_8\Big)\Big]\Big\}, \tag{B14}$$

where $Q_5 = \frac{2m^2_{K^-}}{(m_u+m_s)(m_b+m_u)}$.

$$\mathcal{A}^f(\overline{B}^0_s\to \pi^-K^{*+}) \;=\; \sqrt{2}G_F f_\pi A_0^{B^0_s\to K^*} m_{K^*}(\epsilon{\cdot}p_\pi)\Big\{V_{ub}V^*_{ud}a_1 + \Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big[a_4+a_{10}-Q_6\Big(a_6+a_8\Big)\Big]\Big\}, \tag{B15}$$

where $Q_6 = \frac{2m^2_{\pi^-}}{(m_u+m_d)(m_b+m_u)}$.

APPENDIX C: THE WEAK ANNIHILATION AMPLITUDES FOR $\overline{B}_s^0 \to PP$

$$\mathcal{A}^a(\overline{B}_s^0 \to K^0\overline{K}^0) = -i\frac{G_F}{\sqrt{2}} f_{B_s} f_K^2 \Big\{ \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[b_3(\overline{K}^0, K^0) + b_4(\overline{K}^0, K^0) + b_4(K^0, \overline{K}^0) - \frac{1}{2}b_3^{ew}(\overline{K}^0, K^0) - \frac{1}{2}b_4^{ew}(\overline{K}^0, K^0) - \frac{1}{2}b_4^{ew}(K^0, \overline{K}^0)\Big]\Big\}, \quad \text{(C1)}$$

$$\mathcal{A}^a(\overline{B}_s^0 \to K^0\pi^0) = i\frac{G_F}{2} f_{B_s} f_K f_\pi \Big(V_{ub}V_{ud}^* + V_{cb}V_{cd}^*\Big)\Big\{b_3(\pi^0, K^0) - \frac{1}{2}b_3^{ew}(\pi^0, K^0)\Big\}, \quad \text{(C2)}$$

$$\mathcal{A}^a(\overline{B}_s^0 \to K^0\eta^{(\prime)}) = -i\frac{G_F}{\sqrt{2}} f_{B_s} f_K \Big(V_{ub}V_{ud}^* + V_{cb}V_{cd}^*\Big)\Big\{ f_{\eta^{(\prime)}}^u \Big[b_3(\eta^{(\prime)}, K^0) - \frac{1}{2}b_3^{ew}(\eta^{(\prime)}, K^0)\Big] + f_{\eta^{(\prime)}}^s \Big[b_3(K^0, \eta^{(\prime)}) - \frac{1}{2}b_3^{ew}(K^0, \eta^{(\prime)})\Big]\Big\}, \quad \text{(C3)}$$

$$\mathcal{A}^a(\overline{B}_s^0 \to \pi^0\pi^0) = -i\frac{G_F}{\sqrt{2}} f_{B_s} f_\pi^2 \Big\{ V_{ub}V_{us}^* b_1(\pi^0, \pi^0) + \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[2b_4(\pi^0, \pi^0) + \frac{1}{2}b_4^{ew}(\pi^0, \pi^0)\Big]\Big\}, \quad \text{(C4)}$$

$$\mathcal{A}^a(\overline{B}_s^0 \to \pi^0\eta^{(\prime)}) = -i\frac{G_F}{2} f_{B_s} f_\pi f_{\eta^{(\prime)}}^u \Big\{ V_{ub}V_{us}^* \Big[b_1(\pi^0, \eta^{(\prime)}) + b_1(\eta^{(\prime)}, \pi^0)\Big] + \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[\frac{3}{2}b_4^{ew}(\pi^0, \eta^{(\prime)}) + \frac{3}{2}b_4^{ew}(\eta^{(\prime)}, \pi^0)\Big]\Big\}, \quad \text{(C5)}$$

$$\begin{aligned}
\mathcal{A}^a(\overline{B}_s^0 \to \eta^{(\prime)}\eta^{(\prime)}) = {} & -i\sqrt{2}G_F f_{B_s} \Big\{ f_{\eta^{(\prime)}}^u f_{\eta^{(\prime)}}^u \Big[V_{ub}V_{us}^* b_1(\eta^{(\prime)}, \eta^{(\prime)}) \\
& + \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big(2b_4(\eta^{(\prime)}, \eta^{(\prime)}) + \frac{1}{2}b_4^{ew}(\eta^{(\prime)}, \eta^{(\prime)})\Big)\Big] \\
& + f_{\eta^{(\prime)}}^s f_{\eta^{(\prime)}}^s \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[b_3(\eta^{(\prime)}, \eta^{(\prime)}) + b_4(\eta^{(\prime)}, \eta^{(\prime)}) \\
& - \frac{1}{2}b_3^{ew}(\eta^{(\prime)}, \eta^{(\prime)}) - \frac{1}{2}b_4^{ew}(\eta^{(\prime)}, \eta^{(\prime)})\Big]\Big\}, \qquad \text{(C6)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}^a(\overline{B}_s^0 \to \eta\eta') = {} & -i\frac{G_F}{\sqrt{2}} f_{B_s} f_\eta^u f_{\eta'}^u \Big\{ V_{ub}V_{us}^* \Big[b_1(\eta', \eta) + b_1(\eta, \eta')\Big] \\
& + \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big[2b_4(\eta', \eta) + 2b_4(\eta, \eta') \\
& + \frac{1}{2}b_4^{ew}(\eta', \eta) + \frac{1}{2}b_4^{ew}(\eta, \eta')\Big]\Big\} \\
& - i\frac{G_F}{\sqrt{2}} f_{B_s} f_\eta^s f_{\eta'}^s \Big(V_{ub}V_{us}^* + V_{cb}V_{cs}^*\Big)\Big\{ b_3(\eta', \eta) + b_3(\eta, \eta') \\
& + b_4(\eta', \eta) + b_4(\eta, \eta') - \frac{1}{2}b_3^{ew}(\eta', \eta) - \frac{1}{2}b_3^{ew}(\eta, \eta') \\
& - \frac{1}{2}b_4^{ew}(\eta', \eta) - \frac{1}{2}b_4^{ew}(\eta, \eta')\Big\}, \qquad \text{(C7)}
\end{aligned}$$

$$\mathcal{A}^a(\overline{B}_s^0\to\pi^-K^+) \;=\; -i\frac{G_F}{\sqrt{2}}f_{B_s}f_\pi f_K\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(\pi^-,K^+)-\frac{1}{2}b_3^{ew}(\pi^-,K^+)\Big\}, \tag{C8}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to K^-K^+) \;=\; & -i\frac{G_F}{\sqrt{2}}f_{B_s}f_K^2\Big\{V_{ub}V_{us}^*b_1(K^+,K^-) \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[b_3(K^-,K^+)+b_4(K^+,K^-)+b_4(K^-,K^+) \\ & -\frac{1}{2}b_3^{ew}(K^-,K^+)+b_4^{ew}(K^+,K^-)-\frac{1}{2}b_4^{ew}(K^-,K^+)\Big]\Big\},\end{aligned} \tag{C9}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\pi^-\pi^+) \;=\; & -i\frac{G_F}{\sqrt{2}}f_{B_s}f_\pi^2\Big\{V_{ub}V_{us}^*b_1(\pi^+,\pi^-)+\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[b_4(\pi^+,\pi^-) \\ & +b_4(\pi^-,\pi^+)+b_4^{ew}(\pi^+,\pi^-)-\frac{1}{2}b_4^{ew}(\pi^-,\pi^+)\Big]\Big\},\end{aligned} \tag{C10}$$

APPENDIX D: THE WEAK ANNIHILATION AMPLITUDES FOR $\overline{B}_s^0\to PV$

$$\mathcal{A}^a(\overline{B}_s^0\to K^0\rho^0) \;=\; -\frac{G_F}{2}f_{B_s}f_K f_\rho\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(\rho^0,K^0)-\frac{1}{2}b_3^{ew}(\rho^0,K^0)\Big\}, \tag{D1}$$

$$\mathcal{A}^a(\overline{B}_s^0\to K^0\omega) \;=\; \frac{G_F}{2}f_{B_s}f_K f_\omega\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(\omega,K^0)-\frac{1}{2}b_3^{ew}(\omega,K^0)\Big\}, \tag{D2}$$

$$\mathcal{A}^a(\overline{B}_s^0\to K^0\phi) \;=\; \frac{G_F}{\sqrt{2}}f_{B_s}f_K f_\phi\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(K^0,\phi)-\frac{1}{2}b_3^{ew}(K^0,\phi)\Big\}, \tag{D3}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to K^0\overline{K}^{*0}) \;=\; & \frac{G_F}{\sqrt{2}}f_{B_s}f_K f_{K^*}\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big\{b_3(\overline{K}^{*0},K^0)+b_4(K^0,\overline{K}^{*0}) \\ & +b_4(\overline{K}^{*0},K^0)-\frac{1}{2}b_3^{ew}(\overline{K}^{*0},K^0)-\frac{1}{2}b_4^{ew}(K^0,\overline{K}^{*0})-\frac{1}{2}b_4^{ew}(\overline{K}^{*0},K^0)\Big\},\end{aligned} \tag{D4}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to \overline{K}^0K^{*0}) \;=\; & \frac{G_F}{\sqrt{2}}f_{B_s}f_K f_{K^*}\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big\{b_3(\overline{K}^0,K^{*0})+b_4(K^{*0},\overline{K}^0) \\ & +b_4(\overline{K}^0,K^{*0})-\frac{1}{2}b_3^{ew}(\overline{K}^0,K^{*0})-\frac{1}{2}b_4^{ew}(K^{*0},\overline{K}^0)-\frac{1}{2}b_4^{ew}(\overline{K}^0,K^{*0})\Big\},\end{aligned} \tag{D5}$$

$$\mathcal{A}^a(\overline{B}_s^0\to\pi^0K^{*0}) \;=\; -\frac{G_F}{2}f_{B_s}f_\pi f_{K^*}\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(\pi^0,K^{*0})-\frac{1}{2}b_3^{ew}(\pi^0,K^{*0})\Big\}, \tag{D6}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\pi^0\rho^0) \;=\; & \frac{G_F}{2\sqrt{2}}f_{B_s}f_\pi f_\rho\Big\{V_{ub}V_{us}^*\Big[b_1(\pi^0,\rho^0)+b_1(\rho^0,\pi^0)\Big] \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[2b_4(\pi^0,\rho^0)+2b_4(\rho^0,\pi^0) \\ & +\frac{1}{2}b_4^{ew}(\pi^0,\rho^0)+\frac{1}{2}b_4^{ew}(\rho^0,\pi^0)\Big]\Big\},\end{aligned} \tag{D7}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\pi^0\omega) \;=\; & \frac{G_F}{2\sqrt{2}}f_{B_s}f_\pi f_\omega\Big\{V_{ub}V_{us}^*\Big[b_1(\pi^0,\omega)+b_1(\omega,\pi^0)\Big] \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[\frac{3}{2}b_4^{ew}(\pi^0,\omega)+\frac{3}{2}b_4^{ew}(\omega,\pi^0)\Big]\Big\},\end{aligned} \tag{D8}$$

$$\mathcal{A}^a(\overline{B}_s^0\to\pi^0\phi)=0, \tag{D9}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\eta^{(\prime)}K^{*0}) \;=\; & \frac{G_F}{\sqrt{2}}f_{B_s}f_{K^*}\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{f_{\eta^{(\prime)}}^u\Big[b_3(\eta^{(\prime)},K^{*0}) \\ & -\frac{1}{2}b_3^{ew}(\eta^{(\prime)},K^{*0})\Big]+f_{\eta^{(\prime)}}^s\Big[b_3(K^{*0},\eta^{(\prime)})-\frac{1}{2}b_3^{ew}(K^{*0},\eta^{(\prime)})\Big]\Big\},\end{aligned} \tag{D10}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\eta^{(\prime)}\phi) \;=\; & \frac{G_F}{\sqrt{2}}f_{B_s}f_\phi f_{\eta^{(\prime)}}^s\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big\{b_3(\eta^{(\prime)},\phi)+b_3(\phi,\eta^{(\prime)}) \\ & +b_4(\eta^{(\prime)},\phi)+b_4(\phi,\eta^{(\prime)})-\frac{1}{2}b_3^{ew}(\eta^{(\prime)},\phi)-\frac{1}{2}b_3^{ew}(\phi,\eta^{(\prime)}) \\ & -\frac{1}{2}b_4^{ew}(\eta^{(\prime)},\phi)-\frac{1}{2}b_4^{ew}(\phi,\eta^{(\prime)})\Big\},\end{aligned} \tag{D11}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\eta^{(\prime)}\rho^0) \;=\; & \frac{G_F}{2}f_{B_s}f_\rho f_{\eta^{(\prime)}}^u\Big\{V_{ub}V_{us}^*\Big[b_1(\eta^{(\prime)},\rho^0)+b_1(\rho^0,\eta^{(\prime)})\Big] \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[\frac{3}{2}b_4^{ew}(\eta^{(\prime)},\rho^0)+\frac{3}{2}b_4^{ew}(\rho^0,\eta^{(\prime)})\Big]\Big\},\end{aligned} \tag{D12}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to\eta^{(\prime)}\omega) \;=\; & \frac{G_F}{2}f_{B_s}f_\omega f_{\eta^{(\prime)}}^u\Big\{V_{ub}V_{us}^*\Big[b_1(\eta^{(\prime)},\omega)+b_1(\omega,\eta^{(\prime)})\Big] \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[2b_4(\eta^{(\prime)},\omega)+2b_4(\omega,\eta^{(\prime)}) \\ & +\frac{1}{2}b_4^{ew}(\eta^{(\prime)},\omega)+\frac{1}{2}b_4^{ew}(\omega,\eta^{(\prime)})\Big]\Big\},\end{aligned} \tag{D13}$$

$$\mathcal{A}^a(\overline{B}_s^0\to K^+\rho^-) \;=\; \frac{G_F}{\sqrt{2}}f_{B_s}f_\rho f_K\Big(V_{ub}V_{ud}^*+V_{cb}V_{cd}^*\Big)\Big\{b_3(\rho^-,K^+)-\frac{1}{2}b_3^{ew}(\rho^-,K^+)\Big\}, \tag{D14}$$

$$\begin{aligned}\mathcal{A}^a(\overline{B}_s^0\to K^+K^{*-}) \;=\; & \frac{G_F}{\sqrt{2}}f_{B_s}f_K f_{K^*}\Big\{V_{ub}V_{us}^*\Big[b_1(K^+,K^{*-}) \\ & +\Big(V_{ub}V_{us}^*+V_{cb}V_{cs}^*\Big)\Big[b_3(K^{*-},K^+)+b_4(K^+,K^{*-})+b_4(K^{*-},K^+) \\ & -\frac{1}{2}b_3^{ew}(K^{*-},K^+)+b_4^{ew}(K^+,K^{*-})-\frac{1}{2}b_4^{ew}(K^{*-},K^+)\Big]\Big\},\end{aligned} \tag{D15}$$

$$\mathcal{A}^a(\overline{B}^0_s\to K^- K^{*+}) = \frac{G_F}{\sqrt{2}} f_{B_s} f_K f_{K^*} \Big\{ V_{ub}V^*_{us}\Big[b_1(K^{*+},K^-) + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[b_3(K^-,K^{*+})+b_4(K^{*+},K^-)+b_4(K^-,K^{*+}) -\frac{1}{2}b^{ew}_3(K^-,K^{*+})+b^{ew}_4(K^{*+},K^-)-\frac{1}{2}b^{ew}_4(K^-,K^{*+})\Big]\Big\}, \tag{D16}$$

$$\mathcal{A}^a(\overline{B}^0_s\to \pi^+\rho^-) = \frac{G_F}{\sqrt{2}} f_{B_s} f_\pi f_\rho \Big\{ V_{ub}V^*_{us}\Big[b_1(\pi^+,\rho^-) + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[b_4(\pi^+,\rho^-)+b_4(\rho^-,\pi^+) +b^{ew}_4(\pi^+,\rho^-)-\frac{1}{2}b^{ew}_4(\rho^-,\pi^+)\Big]\Big\}, \tag{D17}$$

$$\mathcal{A}^a(\overline{B}^0_s\to \pi^-\rho^+) = \frac{G_F}{\sqrt{2}} f_{B_s} f_\pi f_\rho \Big\{ V_{ub}V^*_{us}\Big[b_1(\rho^+,\pi^-) + \Big(V_{ub}V^*_{us}+V_{cb}V^*_{cs}\Big)\Big[b_4(\rho^+,\pi^-)+b_4(\pi^-,\rho^+) +b^{ew}_4(\rho^+,\pi^-)-\frac{1}{2}b^{ew}_4(\pi^-,\rho^+)\Big]\Big\}, \tag{D18}$$

$$\mathcal{A}^a(\overline{B}^0_s\to \pi^- K^{*+}) = \frac{G_F}{\sqrt{2}} f_{B_s} f_\pi f_{K^*} \Big(V_{ub}V^*_{ud}+V_{cb}V^*_{cd}\Big)\Big\{b_3(\pi^-,K^{*+})-\frac{1}{2}b^{ew}_3(\pi^-,K^{*+})\Big\}, \tag{D19}$$

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TABLE I: Wilson coefficients C_i in the NDR scheme. Input parameters in numerical calculations are: $\alpha_s(m_Z) = 0.117$, $\alpha_{em}(m_W) = 1/128$, $m_W = 80.42\,\mathrm{GeV}$, $m_Z = 91.188\,\mathrm{GeV}$, $m_t = 178.1\,\mathrm{GeV}$, $m_b = 4.66\,\mathrm{GeV}$.

	$\mu = m_b/2$		$\mu = m_b$		$\mu = 2m_b$	
	NLO	LO	NLO	LO	NLO	LO
C_1	1.130	1.171	1.078	1.111	1.042	1.071
C_2	-0.274	-0.342	-0.176	-0.239	-0.102	-0.161
C_3	0.021	0.019	0.014	0.012	0.009	0.007
C_4	-0.048	-0.047	-0.034	-0.032	-0.024	-0.022
C_5	0.010	0.010	0.008	0.008	0.007	0.006
C_6	-0.061	-0.058	-0.039	-0.037	-0.026	-0.023
C_7/α_{em}	-0.005	-0.105	-0.011	-0.097	0.035	-0.081
C_8/α_{em}	0.086	0.023	0.055	0.014	0.036	0.009
C_9/α_{em}	-1.419	-0.091	-1.341	-0.087	-1.277	-0.075
C_{10}/α_{em}	0.383	-0.021	0.264	-0.016	0.176	-0.011
$C_{7\gamma}$		-0.342		-0.306		-0.276
C_{8g}		-0.160		-0.146		-0.133

TABLE II: The values of the Wolfenstein parameters, A, λ, ρ, and η

Refs.	[40]	[41]	[42]	[23]
λ	0.2237 ± 0.0033	0.222 ± 0.004	0.2210 ± 0.0020	0.2236 ± 0.0031 [a]
A	0.819 ± 0.040 [b]	0.83 ± 0.07	0.831 ± 0.022 [b]	0.824 ± 0.046 [b]
$\bar{\rho}$ [c]	0.224 ± 0.038	0.21 ± 0.12	0.173 ± 0.046	0.22 ± 0.10
$\bar{\eta}$ [d]	0.317 ± 0.040	0.38 ± 0.11	0.357 ± 0.027	0.35 ± 0.05
γ	$(54.8\pm6.2)^\circ$	$(62\pm15)^\circ$	$(63.5\pm7.0)^\circ$	$(59\pm13)^\circ$

[a]Determined from the measurements of $|V_{ud}| = 0.9734 \pm 0.0008$ and $|V_{us}| = 0.2196 \pm 0.0026$, and the error includes scale factor of 1.5

[b]$A = |V_{cb}|/\lambda$, and $|V_{cb}| = (40.6\pm0.8)\times10^{-3}$ [42], $(41.2\pm2.0)\times10^{-3}$ [23].

[c]$\bar{\rho} = \rho(1-\lambda^2/2)$

[d]$\bar{\eta} = \eta(1-\lambda^2/2)$

TABLE III: Values of meson decay constants, form factors, and η-η' mixing parameters.

Form factor			Decay constants						η-η' mixing angles		
$F_0^{B_sK}$	0.274	[28]	f_π	131 MeV	[23]	f_{K^*}	214 MeV	[15]	θ	- 15.4°	[46]
$F_0^{B_s\eta_{s\bar{s}}}$	0.335	[28]	f_K	160 MeV	[23]	f_ρ	210 MeV	[15]	θ_0	- 9.2°	[46]
$F_0^{B_s\eta'_{s\bar{s}}}$	0.282	[28]	f_0	1.17 f_π	[46]	f_ω	195 MeV	[15]	θ_8	- 21.2°	[46]
$A_0^{B_sK^*}$	0.236	[28]	f_8	1.26 f_π	[46]	f_ϕ	233 MeV	[15]			
$A_0^{B_s\phi}$	0.272	[28]	f_{B_s}	236 MeV	[47]						

TABLE IV: The CP-averaged branching ratios (in the unit of 10^{-6}) of decays $B_s \to PP$ calculated with $\mu = m_b$, and default values of parameters. The data in the column 2∼4 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 5∼7 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

decay	NF	QCDF		NF	QCDF	
modes	BR	BR^f	BR^{f+a}	BR	BR^f	BR^{f+a}
$\overline{B}_s^0 \to K^0\overline{K}^0$	8.724	12.06	18.81	7.995	11.06	17.25
$\overline{B}_s^0 \to K^0\pi^0$	0.189	0.154	0.200	0.247	0.210	0.277
$\overline{B}_s^0 \to K^0\eta$	0.108	0.061	0.071	0.125	0.077	0.091
$\overline{B}_s^0 \to K^0\eta'$	0.434	0.497	0.717	0.396	0.522	0.777
$\overline{B}_s^0 \to \pi^0\pi^0$	—	—	0.011	—	—	0.011
$\overline{B}_s^0 \to \pi^0\eta$	0.052	0.078	0.087	0.058	0.066	0.071
$\overline{B}_s^0 \to \pi^0\eta'$	0.055	0.059	0.056	0.061	0.062	0.062
$\overline{B}_s^0 \to \eta\eta$	4.570	7.084	10.52	4.087	6.535	9.720
$\overline{B}_s^0 \to \eta\eta'$	9.190	10.60	16.44	8.401	9.655	14.96
$\overline{B}_s^0 \to \eta'\eta'$	4.622	6.034	10.65	4.324	5.583	9.846
$\overline{B}_s^0 \to K^+\pi^-$	9.653	10.23	10.44	7.311	7.667	7.700
$\overline{B}_s^0 \to K^+K^-$	6.949	9.762	15.63	7.807	10.65	16.58
$\overline{B}_s^0 \to \pi^+\pi^-$	—	—	0.022	—	—	0.023

TABLE V: The CP-averaged branching ratios (in unit of 10^{-6}) of decays $B_s \to PV$ calculated with $\mu = m_b$, and default values of parameters. The data in the column 2∼4 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 5∼7 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

decay	NF	QCDF		NF	QCDF	
modes	BR	BR^f	BR^{f+a}	BR	BR^f	BR^{f+a}
$\overline{B}_s^0 \to K^0 \overline{K}^{*0}$	2.163	2.812	4.985	1.982	2.581	4.577
$\overline{B}_s^0 \to \overline{K}^0 K^{*0}$	0.422	0.630	2.381	0.387	0.577	2.182
$\overline{B}_s^0 \to K^+ K^{*-}$	1.818	2.254	3.819	2.598	3.119	4.992
$\overline{B}_s^0 \to K^- K^{*+}$	1.262	1.606	3.906	0.863	1.103	2.864
$\overline{B}_s^0 \to \pi^+ \rho^-$	—	—	0.002	—	—	0.001
$\overline{B}_s^0 \to \pi^- \rho^+$	—	—	0.002	—	—	0.001
$\overline{B}_s^0 \to K^0 \rho^0$	0.368	0.367	0.376	0.349	0.350	0.402
$\overline{B}_s^0 \to K^0 \omega$	0.422	0.428	0.480	0.292	0.314	0.341
$\overline{B}_s^0 \to K^0 \phi$	0.029	0.057	0.154	0.036	0.070	0.188
$\overline{B}_s^0 \to \pi^0 K^{*0}$	0.137	0.068	0.102	0.095	0.052	0.082
$\overline{B}_s^0 \to \pi^0 \rho^0$	—	—	0.002	—	—	0.001
$\overline{B}_s^0 \to \pi^0 \omega$	—	—	0.001	—	—	0.001
$\overline{B}_s^0 \to \pi^0 \phi$	0.086	0.100	—	0.095	0.098	—
$\overline{B}_s^0 \to \eta K^{*0}$	0.121	0.120	0.230	0.169	0.154	0.300
$\overline{B}_s^0 \to \eta' K^{*0}$	0.062	0.028	0.038	0.043	0.021	0.028
$\overline{B}_s^0 \to \eta \rho^0$	0.122	0.207	0.223	0.136	0.166	0.174
$\overline{B}_s^0 \to \eta' \rho^0$	0.128	0.128	0.125	0.144	0.146	0.148
$\overline{B}_s^0 \to \eta \omega$	0.042	0.043	0.055	0.052	0.031	0.039
$\overline{B}_s^0 \to \eta' \omega$	0.044	0.012	0.016	0.055	0.014	0.017
$\overline{B}_s^0 \to \eta \phi$	0.259	0.530	0.456	0.216	0.483	0.417
$\overline{B}_s^0 \to \eta' \phi$	0.007	0.266	0.367	0.005	0.238	0.328
$\overline{B}_s^0 \to K^+ \rho^-$	23.69	24.12	24.36	19.02	19.29	19.18
$\overline{B}_s^0 \to \pi^- K^{*+}$	6.643	6.949	6.910	5.709	6.008	6.161

TABLE VI: The CP-violating asymmetry parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ for $B_s \to PP$ decays (in the unit of percent) calculated with the QCDF approach, with $\mu = m_b$, and default values of various parameters. The data in the column 2∼5 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 6∼9 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

modes	$a^{f}_{\epsilon'}$	$a^{f+a}_{\epsilon'}$	$a^{f}_{\epsilon+\epsilon'}$	$a^{f+a}_{\epsilon+\epsilon'}$	$a^{f}_{\epsilon'}$	$a^{f+a}_{\epsilon'}$	$a^{f}_{\epsilon+\epsilon'}$	$a^{f+a}_{\epsilon+\epsilon'}$
$B_s \to K^0\overline{K}^0$	-0.90	-0.72	3.38	3.43	-0.98	-0.79	3.66	3.72
$B_s \to K^0_S\pi^0$	-64.72	-59.92	-61.67	-74.32	-47.54	-43.46	-86.01	-89.93
$B_s \to K^0_S\eta$	-85.03	-82.76	-37.31	-48.06	-67.24	-64.29	-71.02	-75.70
$B_s \to K^0_S\eta'$	54.84	47.19	-32.54	-42.34	52.27	43.60	-39.40	-47.14
$B_s \to \pi^0\pi^0$	—	0	—	-27.71	—	0	—	-28.25
$B_s \to \pi^0\eta$	-16.27	-15.71	24.07	35.66	-19.20	-19.39	26.41	40.18
$B_s \to \pi^0\eta'$	-22.84	-22.75	-31.39	-44.93	-21.67	-20.47	-32.69	-45.21
$B_s \to \eta\eta$	1.08	0.86	1.97	1.57	1.18	0.93	2.12	1.69
$B_s \to \eta\eta'$	-0.69	-0.54	5.14	5.49	-0.76	-0.59	5.59	5.98
$B_s \to \eta'\eta'$	-3.25	-2.47	1.22	1.40	-3.52	-2.67	1.29	1.50
$B_s \to K^{\pm}K^{\mp}$	-6.89	-5.33	-40.89	-33.57	-6.32	-5.04	-40.74	-33.89
$B_s \to \pi^{\pm}\pi^{\mp}$	—	0	—	-27.71	—	0	—	-28.25

TABLE VII: The CP-violating asymmetry parameters $a_{\epsilon'}$ and $a_{\epsilon+\epsilon'}$ for $B_s \to PV$ decays (in the unit of percent) calculated with the QCDF approach, with $\mu = m_b$, and default values of various parameters. The data in the column 2∼5 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 6∼9 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

modes	$a^f_{\epsilon'}$	$a^{f+a}_{\epsilon'}$	$a^f_{\epsilon+\epsilon'}$	$a^{f+a}_{\epsilon+\epsilon'}$	$a^f_{\epsilon'}$	$a^{f+a}_{\epsilon'}$	$a^f_{\epsilon+\epsilon'}$	$a^{f+a}_{\epsilon+\epsilon'}$
$B_s \to K^0_S \rho^0$	-27.38	-21.76	68.58	45.24	-28.72	-20.39	4.69	-24.52
$B_s \to K^0_S \omega$	38.51	32.73	91.14	87.34	52.52	46.18	79.67	88.70
$B_s \to K^0_S \phi$	9.29	6.07	-76.99	-75.32	7.58	4.99	-74.30	-72.78
$B_s \to \pi^0 \rho^0$	—	0	—	99.48	—	0	—	92.23
$B_s \to \pi^0 \omega$	—	0	—	89.01	—	0	—	39.27
$B_s \to \pi^0 \phi$	-20.88	—	-13.09	—	-21.30	—	-14.58	—
$B_s \to \eta \rho^0$	-15.06	-11.51	38.43	45.56	-18.77	-14.75	43.57	52.53
$B_s \to \eta' \rho^0$	-25.73	-30.00	-52.01	-60.37	-22.49	-25.30	-51.79	-59.53
$B_s \to \eta \omega$	-18.83	-12.24	68.45	74.11	-26.04	-17.28	78.68	85.81
$B_s \to \eta' \omega$	-71.72	-59.77	12.78	38.52	-63.65	-57.69	-47.14	-27.12
$B_s \to \eta \phi$	6.56	7.14	4.68	4.13	7.19	7.81	5.00	4.37
$B_s \to \eta' \phi$	10.95	8.88	9.91	9.71	12.25	9.93	10.60	10.48

TABLE VIII: The CP-violating asymmetry parameters $\mathcal{A}_{CP}$ (%) for $B_s \to PP$ decays calculated with the QCDF approach, with $\mu = m_b$, and default values of various parameters. The data in the column 3∼4 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 5∼6 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

modes	case	$\mathcal{A}_{CP}^{f}$	$\mathcal{A}_{CP}^{f+a}$	$\mathcal{A}_{CP}^{f}$	$\mathcal{A}_{CP}^{f+a}$
$B_s \to K^0 \overline{K}^0$	II	0.17	0.17	0.18	0.18
$B_s \to K_S^0 \pi^0$	II	-3.24	-3.86	-4.41	-4.59
$B_s \to K_S^0 \eta$	II	-2.07	-2.60	-3.71	-3.94
$B_s \to K_S^0 \eta'$	II	-1.49	-1.99	-1.83	-2.24
$B_s \to \pi^0 \pi^0$	II	—	-1.38	—	-1.41
$B_s \to \pi^0 \eta$	II	1.16	1.74	1.27	1.96
$B_s \to \pi^0 \eta'$	II	-1.62	-2.30	-1.68	-2.31
$B_s \to \eta \eta$	II	0.10	0.08	0.11	0.09
$B_s \to \eta \eta'$	II	0.25	0.27	0.28	0.30
$B_s \to \eta' \eta'$	II	0.053	0.064	0.056	0.068
$B_s \to K^{\pm} \pi^{\mp}$	I	-4.42	-5.09	-5.91	-6.91
$B_s \to K^{\pm} K^{\mp}$	II	-2.06	-1.69	-2.05	-1.70
$B_s \to \pi^{\pm} \pi^{\mp}$	II	—	-1.38	—	-1.41

TABLE IX: The CP-violating asymmetry parameters $\mathcal{A}_{CP}$ (%) for $B_s \to PV$ decays calculated with the QCDF approach, with $\mu = m_b$, and default values of various parameters. The data in the column 3∼4 are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in the column 5∼6 are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

modes	case	$\mathcal{A}_{CP}^{f}$	$\mathcal{A}_{CP}^{f+a}$	$\mathcal{A}_{CP}^{f}$	$\mathcal{A}_{CP}^{f+a}$
$B_s^0 \to K_S^0 \overline{K}^{*0}$	III	0.68	0.27	0.74	0.29
$B_s^0 \to K_S^0 K^{*0}$	III	-0.96	-0.58	-1.05	-0.63
$B_s^0 \to K^+ K^{*-}$	III	14.28	-7.98	12.02	-8.50
$B_s^0 \to K^- K^{*+}$	III	-13.09	9.51	-13.04	8.71
$B_s^0 \to \pi^+ \rho^-$	III	—	4.96	—	4.60
$B_s^0 \to \pi^- \rho^+$	III	—	4.96	—	4.60
$B_s \to K_S^0 \rho^0$	II	3.35	2.20	0.16	-1.27
$B_s \to K_S^0 \omega$	II	4.64	4.44	4.10	4.54
$B_s \to K_S^0 \phi$	II	-3.82	-3.74	-3.69	-3.62
$\overline{B}_s^0 \to \pi^0 K^{*0}$	I	-63.28	-51.12	-81.61	-63.76
$B_s \to \pi^0 \rho^0$	II	—	4.96	—	4.60
$B_s \to \pi^0 \omega$	II	—	4.44	—	1.96
$B_s \to \pi^0 \phi$	II	-0.70	—	-0.78	—
$\overline{B}_s^0 \to \eta K^{*0}$	I	49.72	30.43	38.61	23.34
$\overline{B}_s \to \eta' K^{*0}$	I	-38.13	-45.57	-50.43	-62.38
$B_s \to \eta \rho^0$	II	1.88	2.24	2.13	2.58
$B_s \to \eta' \rho^0$	II	-2.66	-3.09	-2.64	-3.03
$B_s \to \eta \omega$	II	3.37	3.67	3.86	4.24
$B_s \to \eta' \omega$	II	0.46	1.77	-2.51	-1.50
$B_s \to \eta \phi$	II	0.25	0.22	0.27	0.24
$B_s \to \eta' \phi$	II	0.52	0.51	0.56	0.55
$B_s \to K^{\pm} \rho^{\mp}$	I	-2.09	0.79	-2.61	1.00
$B_s \to \pi^{\pm} K^{*\mp}$	I	0.06	-5.10	0.07	-5.73

TABLE X: Penguin-to-tree ratios and bound on γ, using the default values of various parameters, with the QCDF approach. The bound on γ is from Eq. (61). The data in row a are computed with $A = 0.824$, $\lambda = 0.2236$, $\bar{\rho} = 0.22$, $\bar{\eta} = 0.35$, $\gamma = 59°$, while the data in row b are computed with $A = 0.82$, $\lambda = 0.22$, $\bar{\rho} = 0.086$, $\bar{\eta} = 0.39$, $\gamma = 78.8°$.

	$\|P_c/T_c\|$	$\|P_{\pi\pi}/T_{\pi\pi}\|$	δ'	R_{KK}	γ
a	5.359	0.282	$8.13°$	0.831	$< 61.79°$
b	5.740	0.292	$8.13°$	0.961	$< 83.52°$

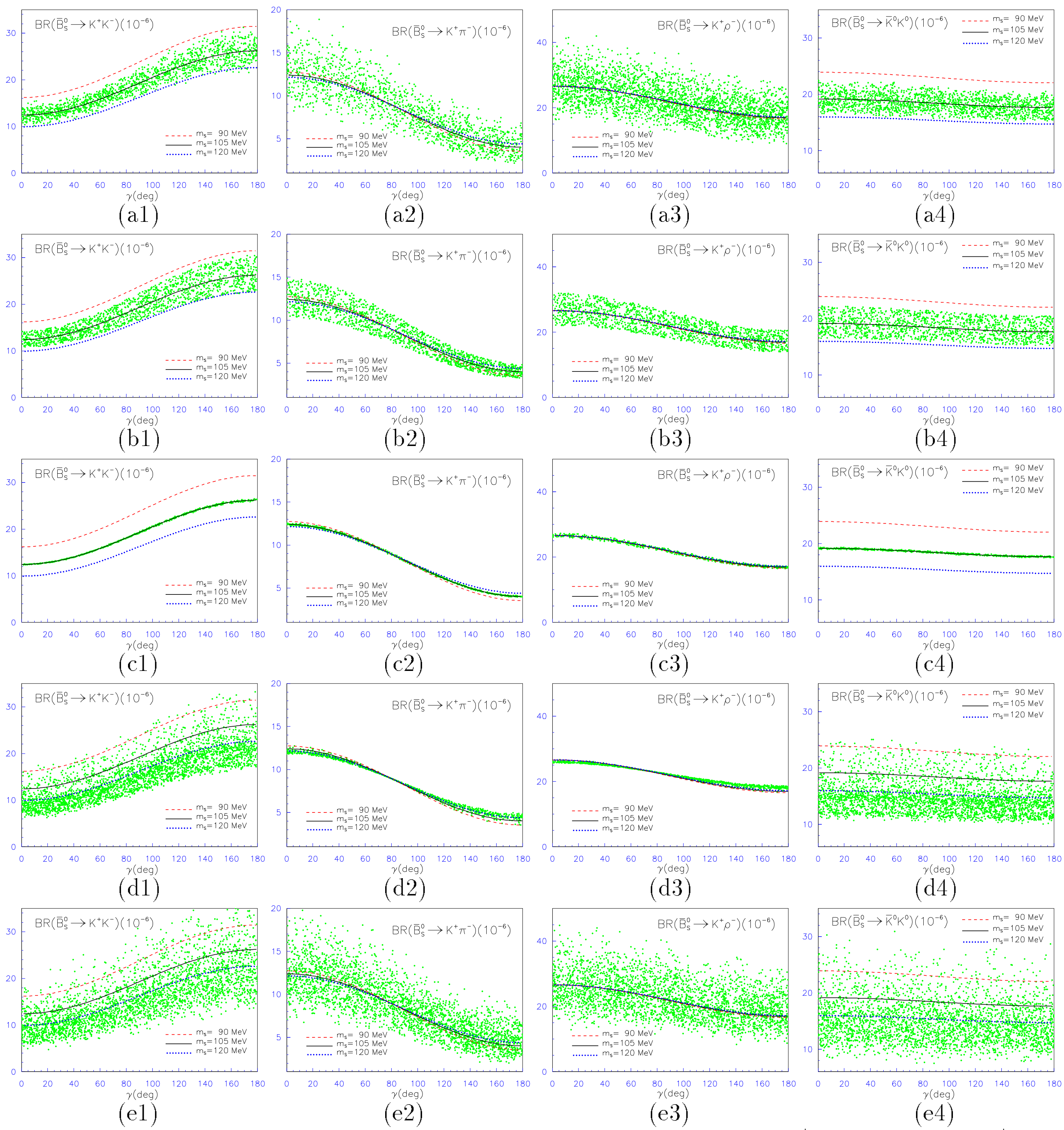

FIG. 1: CP-averaged branching ratios of $B_s \to K^+K^-$ (column 1), $K^{\pm}\pi^{\mp}$ (column 2), $K^{\pm}\rho^{\mp}$ (column 3), and $\overline{K}^0 K^0$ (column 4) decays within the QCDF approach versus the angle γ. The dashed lines, solid lines, and doted lines correspond to the default values of various theory inputs, wherein $\overline{m}_s(2\,\mathrm{GeV}) = 90$ MeV, 105 MeV, and 120 MeV, respectively. The dot-shades demonstrate the uncertainties due to the variations of the CKM matrix parameters A, λ, $\bar{\rho}$, $\bar{\eta}$ (row 1), formfactor $F_{0,1}^{B_s \to K}$ (row 2), $(\varrho_H,\, \phi_H)$ (row 3), $(\varrho_A,\, \phi_A)$ (row 4), and overall inputs (row 5).

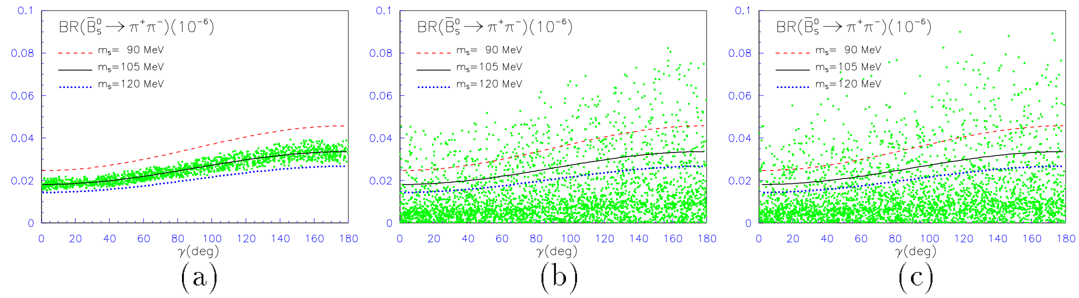

FIG. 2: CP-averaged branching ratios of $B_s \to \pi^+\pi^-$ decays within the QCDF approach versus the angle γ. The legends on the (dashed, solid, and doted) lines are the same as those in Fig. 1. The dot-shades demonstrate the uncertainties due to the variations of the CKM matrix parameters A, λ, $\bar{\rho}$, $\bar{\eta}$ (a), weak annihilation parameters (ϱ_A, ϕ_A) (b), and overall inputs (c).

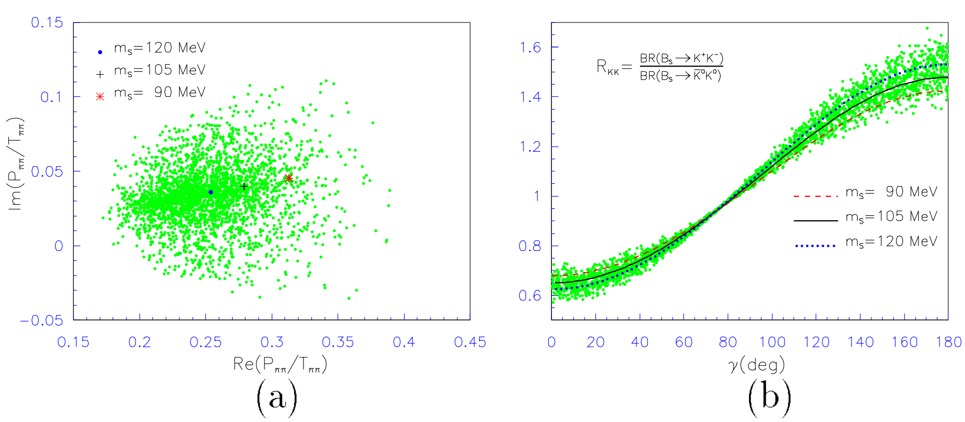

FIG. 3: Penguin-to-tree ratio $|P_{\pi\pi}/T_{\pi\pi}|$ (a) and R_{KK} versus γ. The legends on the (dashed, solid, and doted) lines are the same as those in Fig. 1. The dot-shades demonstrate the uncertainties due to the variations of overall inputs.