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Phenomenological modelling of adiabatic shear band localization fracture of solids in dynamic loading processes

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ABSTRACT

The main objective of the present paper is the application of a recently developed viscoplastic–damage type constitutive theory for high strain rate flow process and ductile fracture to the problem of shear band localization and fracture of dynamically loaded inelastic bodies experiencing strain rates ranging between $10^3 - 10^5$ s^{-1}. In the first part of the paper an adiabatic inelastic flow process is formulated and investigated. The Cauchy problem is examined and the conditions for well–posedness are discussed. The relaxation time is used as a regularization parameter. The viscoplastic regularization procedure assures the unconditionally stable integration algorithm by using the finite element method. The second part of the paper is devoted to the numerical investigation of the three–dimensional dynamic adiabatic deformations of a steel thin tube twisted in a split Hopkinson bar at nominal strain rates ranging $10^3 - 10^5$ s^{-1}.

INTRODUCTION

In technological dynamical processes fracture can occur as a result of an adiabatic shear band localization generally attributed to a plastic instability generated by thermal softening during plastic deformation.

Hartley, Duffy and Hawley [4], Marchand and Duffy [11], Marchand, Cho and Duffy [10] and Cho, Chi and Duffy [2] made microscopic observations of the shear band localization on the thin–welled steel tubes in a split Hopkinson torsion bar Three different steels were tested. Dynamic deformation in shear was imposed to produce shear bands. It was found whenever the shear band led to fracture of the specimen, the fracture occurred by a process of void nucleation, growth and coalescence. No cleavage was observed

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on any fracture surface, including the most brittle of the steel tested.

This is presumably due to the thermal softening of the shear band material that results from the local temperature rise occurring during deformation process.

In recent years Zbib and Jurban [23] have investigated numerically a three–dimensional problem involving the development of shear bands in a steel bar pulled in tension and Batra and Zhang [1] the three–dimensional dynamic thermomechanical deformations of a 4340 steel thin tube twisted in a split Hopkinson bar at nominal strain rate of 1000, 2500 and 25000 s^{-1}.

The main objective of the present paper is the application of a recently developed viscoplastic–damage type constitutive theory for high strain rate flow process and ductile fracture to the problem of shear band localization and fracture of dynamically loaded thin–walled tubes experiencing strain rates ranging between $10^3 - 10^5$ s^{-1}.

In chapter 2 a constitutive model is developed within a thermodynamic framework of the rate type material structure with internal state variables. Such important effects as the micro–damage mechanism and thermomechanical coupling are taken into consideration. It has been assumed that the intrinsic micro–damage mechanism consists of the nucleation, growth and coalescence of microvoids. The rate dependent evolution equation for the porosity parameter has been postulated.

In chapter 3 the formulation of an adiabatic inelastic flow process is given. The Cauchy problem is investigated and the conditions which guarantee its well–posedness are examined. Main feature of rate dependent plastic model have been discussed. Particular attention has been focused on the viscoplastic regularization procedure for the solution of the dynamical initial–boundary value problems with localization of plastic deformation. Some simplifications are introduced and a particular elastic–viscoplastic constitutive model for damaged solids is developed.

Chapter 4 is devoted to the numerical investigation of the three–dimensional dynamic adiabatic deformations of a steel thin tube twisted in a split Hopkinson bar at nominal strain rates ranging $10^3 - 10^5$ s^{-1}. A thin shear band region of finite width along the circumference of the tube which undergoes significant deformations and temperature rise has been determined. Its evolution until occurrence of fracture has been simulated. Numerical results are discussed and compared with available experimental observation data.

CONSTITUTIVE STRUCTURE

Rate type constitutive structure for an elastic–viscoplastic material

The main objective is to develop the rate type constitutive structure for an elastic–viscoplastic material in which the effects of the micro–damage mechanism and thermomechanical coupling are taken into consideration.

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Let us introduce the axioms as follows:

(i) Axiom of the existence of the energy function in the form

$$\psi = \hat{\psi}(\mathbf{e}, \mathbf{F}, \vartheta; \boldsymbol{\mu}), \tag{1}$$

where $\mathbf{e}$ is the Eulerian strain tensor, $\mathbf{F}$ the deformation gradient, ϑ a temperature field and $\boldsymbol{\mu}$ denotes the internal state variable vector.

(ii) Axiom of objectivity (spatial covariance). The constitutive structure should be invariant with respect to any diffeomorphism $\boldsymbol{\xi} : \mathcal{S} \to \mathcal{S}$ [11].

(iii) The axiom of entropy production. For any regular process ϕ_t, ϑ_t, $\boldsymbol{\mu}_t$ of a body $\mathcal{B}$ the constitutive functions are assumed to satisfy the reduced dissipation inequality

$$\frac{1}{\rho_{Ref}}\boldsymbol{\tau} : \mathbf{d} - (\eta\dot{\vartheta} + \dot{\psi}) - \frac{1}{\rho\vartheta}\mathbf{q} \cdot \mathrm{grad}\vartheta \geq 0, \tag{2}$$

where ρ and ρ_{Ref} denote the mass density in the actual and reference configuration, respectively, $\boldsymbol{\tau}$ is the Kirchhoff stress tensor, $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$ the rate of total deformation, η denotes the specific (per unit mass) entropy and $\mathbf{q}$ is the heat vector field.

Let us postulate $\boldsymbol{\mu} = (\boldsymbol{\zeta}, \xi)$, where $\boldsymbol{\zeta}$ denotes the new internal state vector which describes the dissipation effects generated by viscoplastic flow phenomena and ξ is the volume fraction porosity parameter and takes account for micro–damage mechanism.

Let us introduce the plastic potential function for damaged material in the form

$$f = J_2 + n\xi J_1^2, \quad \text{where} \quad J_1 = \tau^{ab}g_{ab}, \quad J_2 = \frac{1}{2}\tau'^{ab}\tau'^{cd}g_{ac}g_{bd}, \tag{3}$$

$n = n(\vartheta)$ is the temperature dependent material function and $\mathbf{g}$ denotes the metric tensor in $\mathcal{S}$.

Let us postulate the evolution equations as follows ($\mathrm{L}_{\boldsymbol{v}}$ defines the Lie derivative with respect to the velocity field and the dot denotes the material derivative)

$$\mathbf{d}^p = \Lambda\mathbf{P}, \quad \mathrm{L}_{\boldsymbol{v}}\boldsymbol{\zeta} = \Lambda\mathbf{Z}, \quad \dot{\xi} = \Xi, \tag{4}$$

where for the elastic–viscoplastic model of a material we assume (cf. Perzyna [14,15,17,18])

$$\Lambda = \frac{1}{T_m}\langle\Phi(f - \kappa)\rangle, \tag{5}$$

T_m denotes the relaxation time for mechanical disturbances and κ is the isotropic work–hardening parameter, Φ is the empirical overstress function and the bracket $\langle\cdot\rangle$ defines the ramp function, $\mathbf{P} = \frac{1}{2\sqrt{J_2}}\frac{\partial f}{\partial \boldsymbol{\tau}}$, the material function $\mathbf{Z}$ is intrinsically determined by the constitutive assumptions postulated and the scalar valued evolution function Ξ has to be determined. Thus, we have

$$P_{ab} = \frac{1}{2\sqrt{J_2}}\tau'^{cd} g_{ca} g_{db} + A g_{ab}, \qquad A = \frac{1}{\sqrt{J_2}} n\xi \tau^{ab} g_{ab}. \tag{6}$$

The isotropic hardening–softening material function κ is assumed in the form as follows

$$\kappa = \kappa_0^2 \left\{q + (1-q)\exp\left[-h(\vartheta)\in^p\right]\right\}^2 \left[1 - \left(\frac{\xi}{\xi^F}\right)^{\frac{1}{2}}\right](1 - b\vartheta), \tag{7}$$

where $q = \frac{\kappa_1}{\kappa_0}$, κ_0 and κ_1 denote the yield and saturation stress of the matrix material, respectively, $h = h(\vartheta)$ is the temperature dependent strain hardening function for the matrix material, $\in^p = \int_0^t (\frac{2}{3}\mathbf{d}^p : \mathbf{d}^p)^{\frac{1}{2}} dt$ is the equivalent plastic deformation, ξ^F denotes the value of porosity at which the incipient fracture occurs and b is a material coefficient; the overstress viscoplastic function Φ is postulated in the form (cf. Perzyna [14,15])

$$\Phi(f - \kappa) = \left(\frac{f}{\kappa} - 1\right)^m, \qquad \text{where } m = 1, 3, 5, \ldots \tag{8}$$

The axioms (i)–(iii) and the evolutions equations (4) lead to the rate equations as follows

$$\begin{aligned} \mathrm{L}_{\boldsymbol{\upsilon}}\boldsymbol{\tau} &= \mathcal{L}^e : \mathbf{d} - \mathcal{L}^{th}\dot{\vartheta} - \left[(\mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}) : \mathbf{P}\right]\frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)^m \right\rangle, \\ \dot{\vartheta} &= -\frac{1}{\rho c_p}\mathrm{div}\mathbf{q} + \frac{\vartheta}{c_p \rho_{Ref}}\frac{\partial \boldsymbol{\tau}}{\partial \vartheta} : \mathbf{d} + \frac{\chi^*}{\rho c_p}\boldsymbol{\tau} : \mathbf{d}^p + \frac{\chi^{**}}{\rho c_p}\dot{\xi}, \end{aligned} \tag{9}$$

where

$$\mathcal{L}^e = \rho_{Ref}\frac{\partial^2 \hat{\psi}}{\partial \mathbf{e}^2}, \qquad \mathcal{L}^{th} = -\rho_{Ref}\frac{\partial^2 \hat{\psi}}{\partial \mathbf{e}\partial \vartheta}, \qquad c_p = -\vartheta\frac{\partial^2 \hat{\psi}}{\partial \vartheta^2}, \tag{10}$$

χ^* and χ^{**} are the irreversibility coefficients.

To make possible numerical investigation of the three–dimensional dynamic adiabatic deformations of a body for different ranges of strain rate we introduce some simplifications of the constitutive model.

(i) By analogy with the infinitesimal theory of elasticity we postulate linear elastic properties of the material, i.e.

$$(\mathcal{L}^e)^{abcd} = \overline{G}\left(g^{ac}g^{db} + g^{cb}g^{da}\right) + \left(\overline{K} - \frac{2}{3}\overline{G}\right)g^{ab}g^{cd} + \tau^{bd}g^{ac}, \tag{11}$$

where $\overline{G}$ and $\overline{K}$ denote the shear and bulk modulus of damaged material, respectively.

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(ii) It is assumed that

$$\mathcal{L}^{e^{-1}} : \mathcal{L}^{th} = \theta \mathbf{g}, \tag{12}$$

where θ is the thermal expansion coefficient in elastic range.

Because of the presence of microvoids, the elastic shear and bulk moduli $\overline{G}$ and $\overline{K}$, respectively, are assumed to be degraded according to the model proposed by Mac Kenzie [8]

$$\overline{G} = G(1-\xi)\left(1 - \frac{6K+12G}{9K+8G}\xi\right), \quad \overline{K} = \frac{4GK(1-\xi)}{4G+3K\xi}, \tag{13}$$

where G and K are the elastic moduli of unvoided material.

Intrinsic micro–damage process

The intrinsic micro–damage process consists of nucleation, growth and coalescence of microvoids (microcracks). Recent experimental observation results (cf. Shockey et al. [21]) have shown that coalescence mechanism can be treated as nucleation and growth process on a smaller scale. This conjecture simplifies very much the description of the intrinsic micro–damage process by taking account only of the nucleation and growth mechanisms. Then the porosity or the void volume fraction parameter ξ can be determined by $\dot{\xi} = \left(\dot{\xi}\right)_{nucl} + \left(\dot{\xi}\right)_{grow}$.

Physical considerations (cf. Curran et al. [3]) and Perzyna [16]) have shown that the nucleation of microvoids in dynamic loading processes which are characterized by very short time duration is governed by the thermally–activated mechanism. Based on this heuristic suggestion we postulate for rate dependent plastic flow

$$\left(\dot{\xi}\right)_{nucl} = \frac{1}{T_m} h^*(\xi, \vartheta)\left[\exp \frac{m^*(\vartheta) \mid \sigma - \sigma_N(\xi, \vartheta, \in^p) \mid}{k\vartheta} - 1\right], \tag{14}$$

where k denotes the Boltzmann constant, $h^*(\xi, \vartheta)$ represents a void nucleation material function which is introduced to take account of the effect of microvoid interaction, $m^*(\vartheta)$ is a temperature dependent coefficient, $\sigma = (1/3)J_1$ is the mean stress and $\sigma_N(\xi, \vartheta, \in^p)$ is the porosity, temperature and equivalent plastic strain dependent threshold stress for microvoid nucleation.

For the growth mechanism we postulate (cf. Johnson [6], Perzyna [16], Perzyna and Drabik [19,20] and Nemes et al. [13])

$$\left(\dot{\xi}\right)_{grow} = \frac{1}{T_m}\frac{g^*(\xi, \vartheta)}{\sqrt{\kappa}}\left[\sigma - \sigma_{eq}(\xi, \vartheta, \in^p)\right], \tag{15}$$

where $T_m\sqrt{\kappa}$ denotes the dynamic viscosity of a material, $g^*(\xi, \vartheta)$ represents a void growth material function and takes account for void interaction and $\sigma_{eq}(\xi, \vartheta, \in^p)$ is the porosity, temperature and equivalent plastic strain dependent void growth threshold mean stress.

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ADIABATIC INELASTIC FLOW PROCESS

Formulation of an adiabatic inelastic flow process

Let us define an adiabatic inelastic flow process as follows (cf. Perzyna [17,18]). Find ϕ, $\boldsymbol{v}$, ρ_M, $\boldsymbol{\tau}$, ξ and ϑ as function of t and $\mathbf{x}$ such that

(i) the field equations

$$
\begin{aligned}
\dot{\phi} &= \boldsymbol{v},\\
\dot{\boldsymbol{v}} &= \frac{1}{\rho_M^0(1-\xi_0)}\left(\frac{\boldsymbol{\tau}}{\rho_M}\mathrm{grad}\rho_M + \mathrm{div}\boldsymbol{\tau} - \frac{\boldsymbol{\tau}}{1-\xi}\mathrm{grad}\xi\right),\\
\dot{\rho}_M &= \frac{\rho_M}{1-\xi}\frac{1}{T_m}\left\{h^*(\xi,\vartheta)\left[\exp\frac{m^*(\vartheta)\mid\sigma-\sigma_N(\xi,\vartheta,\in^p)\mid}{k\vartheta}-1\right]\right.\\
&\quad\left.+\frac{g^*(\xi,\vartheta)}{\sqrt{\kappa}}\left[\sigma-\sigma_{eq}(\xi,\vartheta,\in^p)\right]\right\} - \rho_M\mathrm{div}\boldsymbol{v},\\
\dot{\boldsymbol{\tau}} &= \left[\mathcal{L}^e - \frac{1}{c_p\rho_{Ref}}\vartheta\mathcal{L}^{th}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta}\right] : \mathrm{sym}D\boldsymbol{v} + 2\mathrm{sym}\left(\boldsymbol{\tau} : \frac{\partial\boldsymbol{v}}{\partial\mathbf{x}}\right)\\
&\quad-\left[\left(\frac{\chi^*}{\rho_M(1-\xi)c_p}\mathcal{L}^{th}\boldsymbol{\tau} + \mathcal{L}^e + \mathbf{g}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{g}\right) : \mathbf{P}\right]\frac{1}{T_m}\left\langle\left(\frac{f}{\kappa}-1\right)^m\right\rangle\\
&\quad-\frac{\chi^{**}\mathcal{L}^{th}}{\rho_M(1-\xi)c_p}\frac{1}{T_m}\left\{h^*(\xi,\vartheta)\left[\exp\frac{m^*(\vartheta)\mid\sigma-\sigma_N(\xi,\vartheta,\in^p)\mid}{k\vartheta}-1\right]\right.\\
&\quad\left.+\frac{g^*(\xi,\vartheta)}{\sqrt{\kappa}}\left[\sigma-\sigma_{eq}(\xi,\vartheta,\in^p)\right]\right\},\\
\dot{\xi} &= \frac{1}{T_m}\left\{h^*(\xi,\vartheta)\left[\exp\frac{m^*(\vartheta)\mid\sigma-\sigma_N(\xi,\vartheta,\in^p)\mid}{k\vartheta}-1\right]\right.\\
&\quad\left.+\frac{g^*(\xi,\vartheta)}{\sqrt{\kappa}}\left[\sigma-\sigma_{eq}(\xi,\vartheta,\in^p)\right]\right\},\\
\dot{\vartheta} &= \frac{\vartheta}{c_p\rho_{Ref}}\frac{\partial\boldsymbol{\tau}}{\partial\vartheta} : \mathrm{sym}D\boldsymbol{v} + \frac{\chi^*}{\rho_M(1-\xi)c_p}\boldsymbol{\tau} : \mathbf{P}\frac{1}{T_m}\left\langle\left(\frac{f}{\kappa}-1\right)^m\right\rangle\\
&\quad+\frac{\chi^{**}}{\rho_M(1-\xi)c_p}\frac{1}{T_m}\left\{h^*(\xi,\vartheta)\left[\exp\frac{m^*(\vartheta)\mid\sigma-\sigma_N(\xi,\vartheta,\in^p)\mid}{k\vartheta}-1\right]\right.\\
&\quad\left.+\frac{g^*(\xi,\vartheta)}{\sqrt{\kappa}}\left[\sigma-\sigma_{eq}(\xi,\vartheta,\in^p)\right]\right\},
\end{aligned}
\tag{16}
$$

(ii) the boundary conditions

(a) displacement ϕ is prescribed on a part ∂_ϕ of $\partial\phi(\mathcal{B})$ and tractions $(\boldsymbol{\tau}\cdot\mathbf{n})^a$ are prescribed on part $\partial_{\boldsymbol{\tau}}$ of $\partial\phi(\mathcal{B})$, where $\partial_\phi \cap \partial_{\boldsymbol{\tau}} = 0$ and $\overline{\partial_\phi \cup \partial_{\boldsymbol{\tau}}} = \partial\phi(\mathcal{B})$;

(b) heat flux $\mathbf{q}\cdot\mathbf{n} = 0$ is prescribed on $\partial\phi(\mathcal{B})$;

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(iii) the initial conditions
ϕ, $\boldsymbol{v}$, σ_M, ϑ, ξ and $\boldsymbol{\tau}$ are given at each particle $X \in \mathcal{B}$ at $t = 0$;

are satisfied.

In the field equations (16) a mapping $\mathbf{x} = \phi(\mathbf{X}, t)$ represents a motion of a body, ρ_M and ρ_M^0 denote the actual and reference mass density of the matrix material, respectively, ξ_0 is the initial porosity of a material and $D\boldsymbol{v}$ denotes the spatial velocity gradient.

The Cauchy problem

Let us consider the Cauchy problem

$$\dot{\varphi} = \mathcal{A}(t, \varphi)\varphi + \mathbf{f}(t, \varphi), \qquad t \in [0, t_f], \qquad \varphi(0) = \varphi^0, \tag{17}$$

where $\mathcal{A}$ is a spatial differential operator and $\mathbf{f}$ is a nonlinear function, both defined by the governing equations (16), (cf. Perzyna [17,18]).

In order to examine the existence, uniqueness and well-posedness of the Cauchy problem (17) let us assume that the spatial differential operator $\mathcal{A}$ has domain $\mathcal{D}(\mathcal{A})$ and range $\mathcal{R}(\mathcal{A})$, both contained in a real Banach space E and the nonlinear function $\mathbf{f}$ is as follows $\mathbf{f} : \mathrm{E} \to \mathrm{E}$. To investigate the existence as well as the stability of solutions to (17) it is necessary to characterize their properties without actually constructing the solutions. This can be done by considering the properties of a nonlinear semi-group because if the operator $\mathcal{A} + \mathbf{f}(\cdot)$ generates a nonlinear semi-group $\{I\!F_t; t \geq 0\}$, then a solution to (17) starting at $t = 0$ from any element $\varphi^0 \in \mathcal{D}(\mathcal{A})$ is given by

$$\varphi(t, \mathbf{x}) = I\!F_t \varphi^0(\mathbf{x}) \quad \text{for } t \in [0, t_f]. \tag{18}$$

We say the problem (17) is well posed if $I\!F_t$ is continuous (in the topology on $\mathcal{D}(\mathcal{A})$ and $\mathcal{R}(\mathcal{A})$ assumed) for each $t \in [0, t_f]$.

Let us postulate as follows:

(i) the strong ellipticity condition in the form:

$$I\!E = \mathcal{L}^e - \frac{1}{c_p \rho_{Ref}} \vartheta \mathcal{L}^{th} \frac{\partial \boldsymbol{\tau}}{\partial \vartheta} \tag{19}$$

is strongly elliptic (at a particular deformation ϕ) if there is an $\varepsilon > 0$ such that

$$I\!E^{abcd} \zeta_a \zeta_c \xi_b \xi_d \geq \varepsilon \|\boldsymbol{\zeta}\|^2 \ \|\boldsymbol{\xi}\|^2 \tag{20}$$

for all vectors $\boldsymbol{\zeta}$ and $\boldsymbol{\xi} \in I\!R^3$;

(ii) for positive numbers $\lambda_{\mathbf{f}}^1$ and $\lambda_{\mathbf{f}}^2$ and for $T_m > 0$

$$\mathbf{f}(t, \varphi) \in \mathrm{E}, \quad \|\mathbf{f}(t, \varphi)\|_{\mathrm{E}} \leq \lambda_{\mathbf{f}}^1, \quad \|\mathbf{f}(t, \varphi') - \mathbf{f}(t, \varphi)\|_{\mathrm{E}} \leq \lambda_{\mathbf{f}}^2 \|\varphi' - \varphi\|_{\mathrm{E}}, \tag{21}$$

and

$$t \to \mathbf{f}(t, \varphi) \in \mathrm{E} \quad \text{is continuous.} \tag{22}$$

Using the results presented by Hughes et al. [5] and Marsden and Hughes [11] it is possible to show (cf. Perzyna [17,18]) that the conditions (i) and (ii) guarantee the existence of (locally defined) evolution operators $\mathbb{F}_t : \mathrm{E} \rightarrow \mathrm{E}$ that are continuous in all variables. In other words the solution of the Cauchy problem (17) in the form (18) exists, is unique and well-posed.

Fundamental features of rate dependent plastic model

It has been proved that the localization of plastic deformation phenomenon in an elastic–viscoplastic solid body can arise only as the result of the reflection and interaction of waves. It has different character then that which occurs in a rate independent elasto–plastic solid body (cf. Perzyna [17,18]). Rate dependency (viscosity) allows the spatial difference operator in the governing equations to retain its ellipticity and the initial value problem is well–posed. Viscosity introduces implicitly a length–scale parameter into the dynamical initial–boundary value problem and hence it implies that the localization region is diffused when compared with an inviscid plastic material. In the dynamical initial–boundary value problem the stress and deformation due to wave reflections and interactions are not uniformly distributed, and this kind of heterogeneity can lead to strain localization in the absence of geometrical or material irregularities. This kind of phenomenon has been recently noticed by Nemes and Eftis [12] (cf. also the results by Sluys et al. [22]).

The theory of viscoplasticity gives the possibility to obtain mesh–insensitive results in localization problems with respect to the width of the shear band and the wave reflection and interaction patterns (cf. Sluys et al. [22]).

Since the rate independent plastic response is obtained as the limit case when the relaxation time T_m tends to zero (cf. Perzyna [17,18]) hence the theory of viscoplasticity offers the regularization procedure for the solution of the dynamical initial–boundary value problems with localization of plastic deformation.

SHEAR BAND LOCALIZATION FRACTURE

Formulation of the initial–boundary value problem for a thin steel tube

Cho, Chi and Duffy [2] tested the specimens machined in the shape of thin–walled tubes with integral hexagonal flanges for gripping. Torsional loading at high strain rates was applied in a torsional Kolsky bar (split–Hopkinson bar).

We idealize the initial–boundary value problem (cf. Batra and Zhang [1]) by assuming the specimen in the shape of thin–walled tube.

The initial conditions are taken in the form

$$\begin{aligned} &\phi(\mathbf{x},0) = 0, \quad \boldsymbol{v}(\mathbf{x},0) = 0, \quad \rho(\mathbf{x},0) = \rho_{Ref} = \rho_M^0(1-\xi_0), \\ &\boldsymbol{\tau}(\mathbf{x},0) = 0, \quad \xi(\mathbf{x},0) = \xi_0, \quad \vartheta(\mathbf{x},0) = \vartheta_0 = \text{constant in } \mathcal{B}. \qquad (23) \end{aligned}$$

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That is, the body is initial at rest, is stress free at a uniform temperature ϑ_0 and the initial porosity at every material point is ξ_0.

For the boundary conditions, we assume

$$\begin{aligned} \boldsymbol{\tau}\cdot\mathbf{n} &= 0 \text{ on the inner and outer surfaces of the tube,} \\ \mathbf{q}\cdot\mathbf{n} &= 0 \implies \operatorname{grad}\vartheta\cdot\mathbf{n} = 0 \text{ on all bounding surfaces,} \\ \boldsymbol{v}(x_1,x_2,0,t) &= 0, \quad \boldsymbol{v}(x_1,x_2,\mathrm{L},t) = \omega^*(t)\left(x_1^2+x_2^2\right)^{\frac{1}{2}}\mathbf{n}^*, \end{aligned} \tag{24}$$

where $\mathbf{n}$ is a unit outward normal to the respective surfaces, $\omega^*(t)$ is the angular speed of the end surface $x_3 = \mathrm{L}$ of the tube, and $\mathbf{n}^*$ is a unit vector tangent to the surface $x_3 = \mathrm{L}$. It is assumed that

$$\omega^*(t) = \begin{cases} \omega_0^* t/20, & 0 \le t \le 20\mu\mathrm{s}, \\ \omega_0^*, & t > 20\mu\mathrm{s}. \end{cases} \tag{25}$$

The rise time of 20 μs is typical for torsional tests done in a split Hopkinson bar (cf. Batra and Zhang [1]).

Computation and discussion of the results

The aforestated initial–boundary value problem has been solved by using the wide spectrum of ABAQUS possibilities (cf. Lodygowski et al. [7]).

The finite element mesh consisted by 8–noded brick elements with 400 uniform elements along the gage length of the tube, 5 uniform across the thickness, and 100 uniform elements along the circumference.

It has been assumed following values to various material parameters (HY–100 steel)

$$\begin{array}{llll} \rho_M = 7860\ \mathrm{kg/m^3}, & G = 80\ \mathrm{GPa}, & \vartheta_0 = 20\ {}^0\mathrm{C}, & \xi_0 = 0.001, \\ c_p = 473\ \mathrm{J/kg^0C}, & K = 210\ \mathrm{GPa}, & T_m = 5\cdot 10^{-6}\ \mathrm{s}, & \xi^F = 0.25, \\ b = b^*(\frac{1}{\vartheta_0} - \frac{1}{\vartheta}), & \kappa_0 = 580\ \mathrm{MPa} & \kappa_1 = 1.2\cdot\kappa_0, & \chi^* = 0.85, \\ b^* = 0.01, & h = 5.15, & m = 7, & n = 1.25, \end{array}$$

The tube has been twisted at nominal strain rates ranging $10^3 - 10^5\ \mathrm{s}^{-1}$. Particular forms of the material functions h^*, g^*, m^*, σ_N and σ_{eq} which affect the micro–damage mechanism and have the influence on the final fracture of the tube have been postulated and discussed during the computation process.

A thin shear band region of finite width along the circumference of the tube which undergoes significant deformations and temperature rise has been determined. Its evolution until occurrence of fracture has been simulated.

It has been found that the width of the shear band region and the temperature rise vary with the nominal strain rate as well as with the relaxation time assumed.

The numerical results obtained are in good agreement with experimental observation data of Cho, Chi and Duffy [2]. An exhaustive discussion of the results obtained and the comparison with the experimental observation data will be published elsewhere.

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