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PHOTON-PHOTON SCATTERING CONTRIBUTION TO THE DECAY RATE OF ORTHOPOSITRONIUM

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A B S T R A C T

We present the result of a calculation of the photon-photon scattering correction to the 3γ ray annihilation rate of orthopositronium.

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The annihilation rate of orthopositronium (i.e., the $^3S_1$ state of the atom consisting of an electron and a positron) has been measured to an accuracy of 0.2% [1]. The experimental result is

$$\Gamma_{exp}\,(^3S_1) = (0.7262 \pm 0.0015)\; 10^7 \; sec^{-1} \tag{1}$$

The present theoretical value [2] for the 3γ ray annihilation rate of orthopositronium, which includes only the lowest order contribution described by the Feynman diagrams shown in Fig. 1, is

$$\Gamma_{th}\,(^3S_1 \to 3\gamma) = \frac{\alpha^6}{\pi}\, m \,\frac{2}{9}\,(\pi^2 - 9) \tag{2}$$

Numerically, using the values $\alpha^{-1} = 137.03602(21)$, $m = 0.5110041(16)$ MeV, and $\hbar = 6.582183(22)\; 10^{-22}$ MeV·sec, which are based on the adjustment of the fundamental physical constants made by Taylor, Parker and Langenberg [3], we get

$$\Gamma_{th}\,(^3S_1 \to 3\gamma) = (0.72112 \pm 0.00001)\; 10^7 \; sec^{-1} \tag{3}$$

Since the discrepancy between this result and the experimental value given above is of the order of magnitude expected from the higher order corrections to Eq. (1), it is necessary to calculate these corrections in order to make an adequate comparison with the experimental value.

The correction term to Eq. (2) of relative order α includes contributions which involve photon-photon scattering with one photon off shell. The corresponding Feynman diagrams are shown in Fig. 2. The purpose of this note is to present the result of a calculation of these contributions. If we write

$$\Gamma\,(^3S_1 \to 3\gamma) = \frac{\alpha^6}{\pi}\, m \,\frac{2}{9}\,(\pi^2 - 9)\left[1 + \frac{\alpha}{\pi}\,(A + B) + O(\alpha^2)\right] \tag{4}$$

where A denotes the contribution from the photon-photon scattering terms and B the contribution of all other first order radiative corrections, we find

$$A = -\ 0.741 \pm 0.017 \tag{5}$$

The calculation of A has been done using two different methods. One method consisted in taking the Karplus and Neuman [4] expression for the vacuum polarization tensor [5] and accommodate it to the calculation of the interference between the diagrams shown in Figs. 1 and 2. Here, all the algebraic manipulations were made by hand. Another method was to use a completely different expression for the vacuum polarization tensor with one photon off shell. This was obtained applying Feynman rules to each diagram with subtraction of the corresponding regularization counter terms. Terms in the loop integration which lead to formal logarithmic ultra- violet divergences were handled using Pauli-Villars regularization. Integrating by parts over one of the Feynman parameters it is still possible to rewrite the final integrand as a rational function of three Feynman parameters and two other parameters which stem from the three-photon phase space integration. Here, all the algebraic manipulations were made using the computer program SCHOONSCHIP developed by Veltman [6].

We thus have two different integrands, each being a rational function of five parameters. The final integrations over these five parameters were performed numerically using the LSD integration sub-routine [7]. The results are as follows

$$A = -\ 0.712 \pm 0.039 \tag{6}$$

using the first method mentioned above, and

$$A = -\ 0.748 \pm 0.019 \tag{7}$$

using the second method. The value quoted in Eq. (5) is a weighted average of these two.

As a partial check of our results we have also made an analytic calculation of the photon-photon scattering correction in the case where the fermion line in the loop has a mass $M \gg m$ (e.g., the case of a muon loop). In this case we obtain

$$A = -\frac{11}{135}\,\frac{29-3n^2}{9-n^2}\left(\frac{m}{M}\right)^4 \tag{8}$$

This result was reproduced by numerical integration.

Due to the fact that A appears to be negative the disagreement with the experimental value is even worse when the photon-photon scattering corrections alone are taken into account. However, as noted in Eq. (4), the photon-photon scattering correction is only a part of the complete correction of relative order α . The calculation of the other terms, which is now clearly necessary, is in progress.

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5) This expression has been checked by one of us (P.P.); except for a few misprints it agrees with the one obtained by Karplus and Neuman.

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7) This subroutine was originally written by G.C. Sheppey at CERN and it has been subsequently developed by A.J. Dufner at SLAC and B.E. Lautrup at CERN. A detailed description by these authors is under preparation.

FIGURE CAPTIONS

Figure 1 : Feynman diagram contributing to the lowest order 3 ray annihilation rate of orthopositronium. There are five other diagrams obtained by permutation of the photon lines.

Figure 2 : Feynman diagram contributing to the photon-photon scattering correction to the decay rate of orthopositronium. There are five other diagrams obtained by permutation of the photon lines.

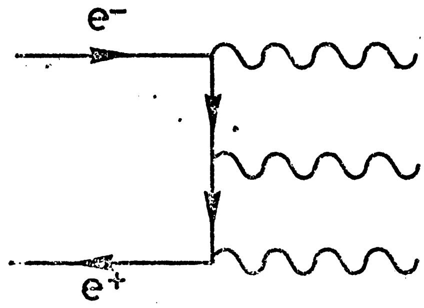

FIG.1

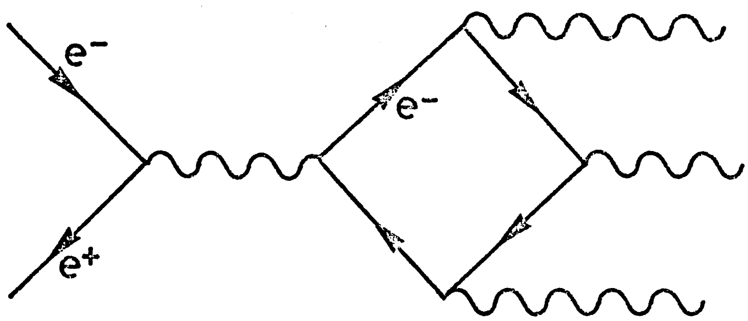

FIG.2