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## PAPER

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## OPEN ACCESS

## RECEIVED

28 July 2015

## REVISED

9 October 2015

## ACCEPTED FOR PUBLICATION

26 October 2015

## PUBLISHED

23 December 2015

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Keywords: topological order, photonic crystals, metamaterials

## Abstract

We demonstrate that topologically nontrivial states of light can be engineered in periodic photonic structures containing media with a Tellegen-type bianisotropic response. Whilst in such bianisotropic materials the time-reversal symmetry is broken, they are characterized by an intrinsic magnetic order which does not require macroscopic magnetization. Our design can therefore be considered as a direct analog of the solid state Chern insulator which exhibits a topological order in the absence of an external bias. Numerical simulations of such Tellegen photonic crystals reveal the existence of one-way edge transport at domain walls and perfectly conducting boundaries not sensitive to structural imperfections such as local defects and disorder. We demonstrate a scheme for achieving robust steering of the edge modes by controlling the phase and amplitude of the source.

## 1. Introduction

The past three decades have witnessed the discovery of condensed matter systems characterized by topological order, in particular topological insulators [1–4]—a development which has significantly enriched our understanding of wave phenomena in numerous branches of physics [5–10]. Topological insulators are characterized by the simultaneous presence of suppressed wave propagation in the bulk and gap-traversing edge states which exhibit immunity to backscattering in the presence of defects and disorder. In the last few years a number of studies have demonstrated that analogs of the topological insulators can be realized in electromagnetic systems, which has provided a new means for achieving robust transport in photonics [7–24].

Systems with topological order can be classified into two main categories, according to whether or not time-reversal (TR) symmetry is broken. Photonic structures with broken TR symmetry, such as magneto-optical photonic crystals (PCs) [25] with gyrotropic permittivity and permeability were the first to be investigated both theoretically [11–13], and experimentally [14, 26] with a view to creating topological insulators for photons. However, gyroelectric and gyromagnetic activity do not exhaust the class of electromagnetic responses which violate TR symmetry. Indeed, the introduction of gain and loss, as well as some forms of bianisotropic response also allow the removal of TR symmetry and it is therefore natural to investigate these in the context of realizing topological order for light.

Here we demonstrate that non-trivial band topology can be realized in PCs with bianisotropic inclusions of the Tellegen type, referred to hereafter as Tellegen photonic crystals (TPC). The Tellegen response is defined by constitutive relations of the form  $\mathbf{D} = \hat{\epsilon}\mathbf{E} + \hat{\chi}\mathbf{H}$  and  $\mathbf{B} = \hat{\mu}\mathbf{H} + \hat{\zeta}\mathbf{E}$  [27, 28] where  $\hat{\zeta}$  and  $\hat{\chi}$  are real valued tensors, and therefore breaks TR symmetry, setting it apart from the conventional bianisotropic responses in electromagnetics (exhibited by for example by split-ring-resonators and chiral meta-molecules) which do not [19, 24]. Although exotic, this kind of magnetoelectric response can be found in certain naturally occurring compounds [29] such as multiferroics [30–32], anisotropic antiferromagnetic materials such as  $\text{Cr}_2\text{O}_3$  [27, 33, 34], electronic topological insulators [35], as well as in composite electromagnetic structures [36] where particularly strong Tellegen responses can be achieved, albeit at low frequencies.

One of the main advantages of considering TR broken systems stems from the fact that the resulting topological protection is extremely robust [7, 14, 26]. This is in sharp contrast to the alternative approach based on realizing symmetry protected topological phases (photonic analogs of the quantum spin Hall effect), including that found in bianisotropic meta-crystals, where topological properties exist only to the extent that  $\hat{\epsilon} = \hat{\mu}$  or similar constraints can be engineered [19, 24], a constraint which ensures pseudo-spin conservation in the structure. This crucial difference between electronic and photonic insulators follows from that fact that bosonic TR symmetry is not sufficient to protect any nontrivial topological phases in three or less dimensions [7], unlike fermionic systems which can support robust topological phases protected by TR symmetry alone in two and three dimensions. In practice both approaches to creating a topologically non-trivial PC suffer from frequency limitations, due to the challenge of matching  $\hat{\epsilon} = \hat{\mu}$  over a significant range in the pseudo-spin conserving systems, and to the difficulty of attaining a wide topological band gap in TR-broken systems.

## 2. Theoretical approach

We follow the standard procedure of studying a PC which possesses point degeneracies within its photonic band structure in the absence of any symmetry-breaking perturbation. For this purpose we employ a 2D triangular lattice structure that exhibits linear Dirac-like dispersion with 2-fold degeneracies at the K and K' points of its Brillouin zone in the absence of the bianisotropy. Such degeneracies are guaranteed at these locations by the simultaneous presence of rotational and PT symmetry. We then introduce a Tellegen-type response into the system which couples in-plane magnetic fields to out-of-plane electric fields (and vice-versa), and is described by the constitutive relations  $D_z = \epsilon E_z + \chi_{xz} H_x + \chi_{yz} H_y$ ,  $B_x = \mu H_x + \chi_{xz} E_z$  and  $B_y = \mu H_y + \chi_{yz} E_z$ , in which  $\hat{\zeta}$  was eliminated by invoking the conservation of energy  $\hat{\zeta} = \hat{\chi}^T$  [37]. Such a response mixes modes of like polarization and can therefore be expected to immediately open a gap at the degenerate Dirac points, whereas the alternative choice of a diagonal  $\hat{\chi}$  would mix modes of opposite polarization and therefore give zero splitting to first order in the coupling strength. Whilst our choice to retain just two non-zero elements in the magneto-electric tensor implies that its effect will only be felt in the TM polarization, one could also consider the dual choice of having  $\zeta_{xz/yz} \neq 0$ ,  $\chi_{zx/zy} \neq 0$  and for the TE polarization, or any combination of the two. In what follows we also assume  $\hat{\mu} = 1$ .

When substituted into the Maxwell's equations, the above constitutive relations lead to a wave equation of the form:

$$\begin{aligned} \nabla \times \nabla \times \mathbf{E}(\mathbf{r}) + k_0^2 \left[ \hat{\epsilon}_r(\mathbf{r}) - \hat{\chi}(\mathbf{r})^T \hat{\chi}(\mathbf{r}) \right] \mathbf{E}(\mathbf{r}) \\ + ik_0 \left\{ \nabla \times \left[ \hat{\chi}(\mathbf{r}) \mathbf{E}(\mathbf{r}) \right] - \hat{\chi}(\mathbf{r}) \nabla \times \mathbf{E}(\mathbf{r}) \right\} = 0, \end{aligned} \quad (1)$$

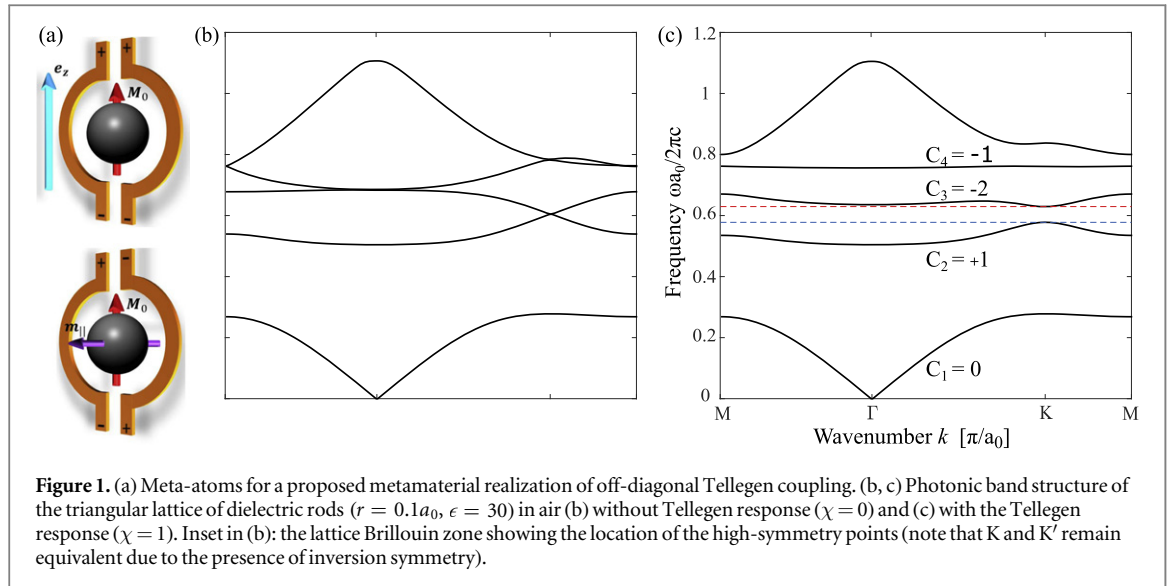
where  $k_0 = \omega/c$  is the magnitude of the free-space wave-vector.

One can simplify equation (1) by introducing an in-plane vector  $\boldsymbol{\chi} = \{-\chi_{yz}(\mathbf{r}), \chi_{xz}(\mathbf{r}), 0\}$  constructed from the components of the bianisotropy tensor  $\chi_{xz}$  and  $\chi_{yz}$ . With the use of this notation equation (1) reduces to scalar equation for  $E_z$ :

$$\left( \nabla + ik_0 \boldsymbol{\chi} \right)^2 E_z + \left( ik_0 \nabla \cdot \boldsymbol{\chi} \right) E_z = k_0^2 \hat{\epsilon}_r E_z, \quad (2)$$

which is reminiscent of the Schrodinger equation for an electron in presence of a magnetic potential. In what follows we assume an effective Coulomb gauge  $\nabla \cdot \boldsymbol{\chi} = 0$  by employing a uniformly azimuthal distribution of the vector viz.  $\boldsymbol{\chi}(\mathbf{r}) = \chi \hat{\phi}$  within the rod with constant  $\chi$ . This particular choice, although by no means unique, ensures that PT symmetry (the product of time-reversal with in-plane inversion) is broken as well as TR, a necessary condition for the nucleation of non-zero Chern numbers [7]. Indeed, the arguably simpler choice  $\boldsymbol{\chi}(\mathbf{r}) = \chi \hat{c}$  with  $\hat{c}$  standing for any constant vector also breaks TR but does so in a way which retains PT-symmetry, forcing the Berry curvature to vanish.

To our knowledge there is no single-phase material known to exhibit significant off-diagonal Tellegen coupling, despite tensors with this form being allowed for by at least three magnetic symmetry (point) groups ( $\mathbf{2}$ ,  $\mathbf{m}$ , and  $\mathbf{(2/m)}$ ) [38, 39]. Although it is therefore conceivable that such materials might be discovered in the near future, it seems unlikely that the coupling strength will greatly exceed that in known materials with a diagonal magnetoelectric response, where components of  $\hat{\chi}$  are at most of the order  $10^{-3}$  [38]. It bears mentioning that off-diagonal magnetoelectric coupling can be straightforwardly engineered in composites of piezoelectric and magnetic materials [40] where elastic forces mediate the coupling between electric and magnetic orders, however this feature limits their application to low frequencies. Putting aside the requirement of zero external bias allows one to consider metamaterial possibilities such as the one sketched in figure 1(a), in which omega-ga-particle elements are used to couple the ac magnetic dipole moment of the double-wire



metamolecule  $m_{||}$  (which corresponds to asymmetric mode of the antennas) and its ac electric dipole moment  $e_{zz}$  (which corresponds to the symmetric mode of the antennas) through natural magneto-optical response of a ferrite sphere placed in the center [41]. This proposal bears some relation to the concept in [36, 41] and will not be investigated in detail, but supports the notion that a practical TPC design may be within reach. In the following we focus on demonstrating the general concept leaving issues of implementation to the future.

### 3. Numerical results

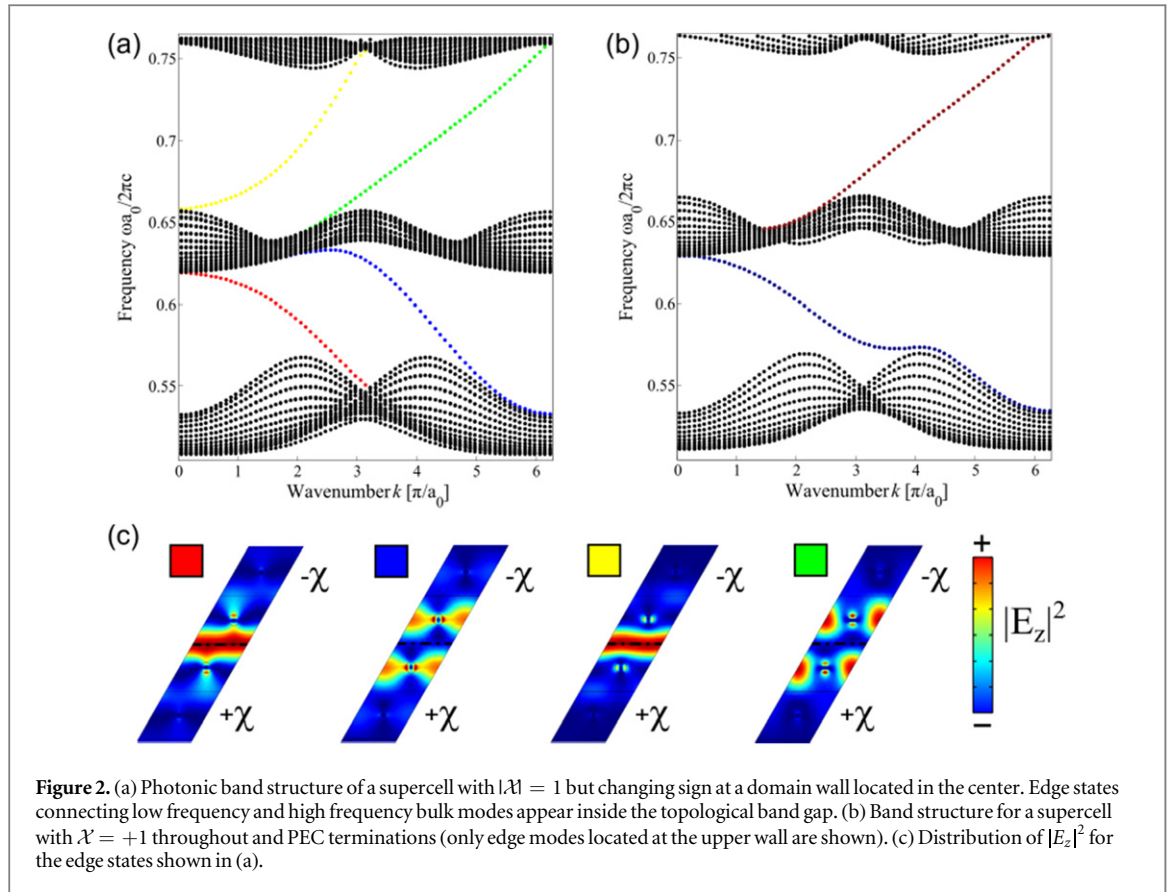
Two complimentary numerical approaches were used to calculate band structure and eigenmodes of our TPC: (i) the plane wave expansion (PWE) method and (ii) commercial finite element method software COMSOL Multiphysics. The PWE was best suited to calculating topological invariants of the bulk bands, whilst the study of topological edge states in different configurations (including disorder) was most conveniently carried out using COMSOL. Both approaches were used to calculate and confirm the bulk band structure.

To begin we calculated the band structure of the crystal without bianisotropy as shown in figure 1(b), which reveals the anticipated presence of Dirac degeneracies at K and K'. These features occur near the dimensionless frequency  $f = \omega a_0 / 2\pi c = 0.6$  (second and third bands) and correspond to dipolar modes of the circular rods.

Next a Tellegen response was introduced within the rods, lifting both degeneracies at K and K' due to the consequent reduction in symmetry (we note that our choice of  $\chi$  preserves inversion symmetry, so that K and K' remain equivalent). Figure 1(c) demonstrates that our particular choice of bianisotropy opens a full photonic band gap within the bulk spectrum. To ensure that this opening corresponds to the photonic states acquiring a topologically nontrivial character we calculated the Berry connection  $\mathcal{A}_n(k_x, k_y) = i\langle E_z | \nabla_k | E_z \rangle_n$  and the Berry curvature  $\Omega_n(k_x, k_y) = \partial_y \mathcal{A}_{nx} - \partial_x \mathcal{A}_{ny}$  for bands 1–4. It was found that adopting a gauge in which  $\mathcal{A}_{nx} = 0$  avoided most of the numerical error associated with the eigenvector's phase ambiguity. The Berry curvature of the lowest dipolar band exhibited two peaks located at the K and K' points of the same sign, giving the Chern number  $C_n = \frac{1}{2\pi} \int_{BZ} \Omega_n(k_x, k_y) d\mathbf{k}$  calculated by the integration of the Berry curvature over the Brillouin zone a non-zero value of  $C_2 = 1$ . The calculation of the Chern number for the upper dipolar band gives the value of  $C_3 = -2$  due to an opposite contribution from K and K' and an additional contribution of  $-1$  from  $\Gamma$ .

The physical hallmark of topological order is the presence of topologically protected edge states robust to structural imperfections and disorder, a property which stems from the one-way character of such modes. According to the bulk-boundary correspondence (BBC), the number of gap-traversing states  $N$  found along a particular edge is determined by the difference in the Chern numbers across the two domains summed over all bands of lower frequency, i.e.  $N = |C_{\text{gap}}^I - C_{\text{gap}}^{II}|$ , where  $C_{\text{gap}}^{I,II} = \sum_n C_n^{I,II}$  is the 'gap Chern number' [23] in which the sum is over all the bands below  $i$ th gap, and roman numerals indicate the domain. In addition, the direction of propagation for these modes is predicted by the sign in  $C_{\text{gap}}^I - C_{\text{gap}}^{II}$ .

We first consider a domain wall formed between two TPCs with reversed signs of the gauge field  $\chi$ , in which case the BBC predicts  $N = 2$  modes for the interface. To confirm the presence of topological edge states we performed numerical simulations of a  $30 \times 1$  supercell with a reversal of  $\chi$  occurring in the center. As can be seen from figure 2(a), a set of two modes occurs inside the complete photonic band gap opened by the Tellegen response near the former Dirac degeneracy. These modes (highlighted in blue and red) share a clear



unidirectional ‘one-way’ character, but differ in their field profiles. In particular, one can see from figure 2(c) (two left panels) that one of these modes is concentrated at the domain wall while the other one is concentrated at the center of the unit cells adjacent to the domain wall. Another set of one-way edge states emerges at higher frequencies where the Tellegen bianisotropy also opens a complete band gap (highlighted in green and yellow), however, for these higher bands the direction of the one-way propagation is reversed as predicted by the BBC outlined above. Simple inspection of their dispersion reveals that none of the edge states shown in figure 2 can be removed by deformations of the band structure unless the band gaps in which they lie are closed; such states are therefore said to be topologically protected.

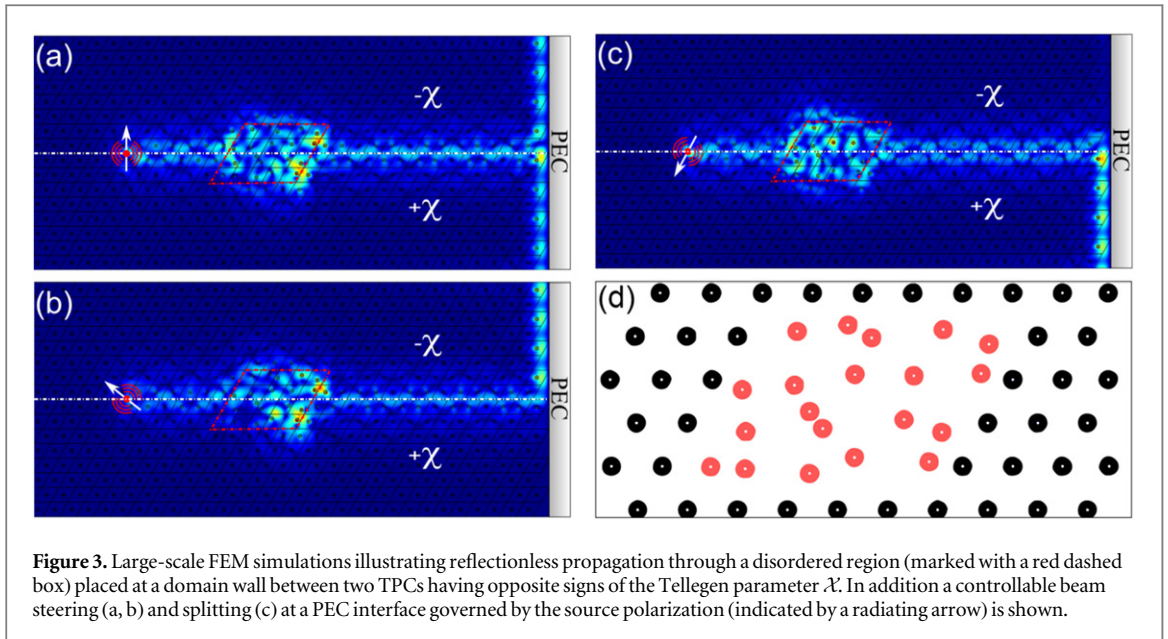
In addition to domain walls where the Tellegen parameter  $\chi$  switches sign we have found that one-way edge modes also appear at terminations of the TPC by a perfect electric conductor (PEC) (figure 2(b), field profiles not shown). By contrast with the domain wall we find only a single gap-traversing state at such boundaries, behavior which is consistent with interpreting the PEC as a trivial ( $C = 0$ ) photonic material.

A property of special interest is the robustness of the edge states with respect to backscattering. To illustrate such robustness we performed a numerical study on the scattering of one-way edge modes by disorder [19, 42]. The edge state was excited by an in-plane magnetic dipole source placed at the domain wall, as indicated by radiating arrow in figure 3. This figure shows that despite the presence of a strongly disordered region along the domain wall (bounded by red dashed box in figures 3(a)–(c), and detailed in figure 3(d)), the edge state continues to propagate in the same direction without back-reflection, although it is clear from the field profiles that some inter-modal scattering occurs as a result.

Topological edge states in systems with higher values of Chern number have recently been proposed for selectively routing light along different topological interfaces [23]. In our system, a transition between two distinct topological interfaces can be engineered by terminating the domain wall configuration considered above with a topologically trivial insulating region, such as a PEC. This configuration is shown in figure 3 where the domain wall supporting two states ( $N = 2$ ), meets two PEC interfaces with only a single state ( $N = 1$ ) propagating in opposite directions. Therefore, edge modes of the domain wall will split into two edge modes along the PEC boundary which continue to flow in opposite upward and downward directions.

Note that similar steering control should also be realizable in systems with higher Chern numbers that can be achieved by starting with a system that has a large number of degeneracies with non-zero Berry flux, and this applies equally to all TR-breaking topological insulators. Square lattices [23] seem to be better for this than hexagonal, and a wide variety of other lattice types exhibiting high point symmetry and with multiple degrees of





freedom per unit cell (including hexagonal/honeycomb [43], Kagome [44] and Lieb lattices [45, 46]) could also be considered, along with a more common approach to utilize higher frequency bands of the PC [23].

The splitting between modes at a ‘topological junction’ can be controlled by adjusting the relative phase and amplitude of edge modes as they enter it. In figure 3 we demonstrate how, by controlling source polarization, a reflectionless steering can be achieved regardless of the presence the disorder in the intermediate region. This scheme takes advantage of the fact that there are two edge states at the domain wall which both couple to the edge states at the PEC boundaries. These two contributions can be tuned to be out of phase in order to give rise to destructive interference along either arm of the splitter. In figure 3 we depict particular choices of the source polarization which lead to complete steering of the wave in the upward and downward directions, as well as to equal division of energy between the two arms.

#### 4. Conclusions

Our results show that topologically nontrivial PCs can in principle be engineered with a variety of constitutive relations. Here we have explored a new avenue towards this aim based on the use of magneto-electric coupling as a source of non-reciprocity. Together with the gyrotropic permittivity and permeability, which have formed the basis of similar investigations into photonic analogs of the quantum Hall effect, the Tellegen material response exhausts the number possibilities for achieving non-reciprocity at the level of linear constitutive relations in systems without gain or loss. To achieve this end we have developed appropriate numerical techniques for computing photonic eigenmodes, which were then used to design and characterize topological order in TPC. Finally, we have shown that by controlling the phase and amplitude of the edge states in the systems with multiple topological channels allows robust steering of electromagnetic radiation, a feature which could prove useful for applications. Since a strong Tellegen response can be engineered in metamaterials comprised of magnetic ferrites with bianisotropic elements, such as chiral antennas and split-rings, artificial media of this kind could be used to design the proposed system. While it would be more straightforward to implement this proposal experimentally in the microwave domain where the nonreciprocal magneto-optical response is naturally large, it can also be of interest in the IR and visible spectral domains, where strongly resonant optical elements can lead to an enhanced time-reversal symmetry violating bianisotropic response.

#### Acknowledgments

We acknowledge a partial support from the Australian Research Council.

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