Photosynthetic Solar constant

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Abstract

An important concept of photosynthetic solar constant is proposed. This signifies the value of power produced in the form of biomass through the photosynthetic engine by making use of the visible band present in the standard solar constant under ideal conditions. Its value is estimated as 70 W/m^2 which corresponds to an equivalent biomass of $4.5x10^{-6}Kg/m^2/s$.

Keywords: Solar constant, visible band, photosynthetic engine, harvested value, upper limit.

Resumen

El objetivo del presente trabajo es proponer un concepto importante en la constante de fotosíntesis solar. Esto significa que el valor de la potencia producida en forma de biomasa a través de la fotosíntesis es medido por el uso de una banda visible que se encuentra presente en la constante solar estándar, bajo condiciones ideales. Se tiene el valor estimado de 70 W/m^2 que corresponde al equivalente en biomasa de $4.5x10^{-6}Kg/m^2/s$.

Palabras clave: Constante solar, banda visible, máquina fotosintética, mayor valor, límite superior.

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I. INTRODUCTION

It is well known [1] that the yearly mean solar irradiance on the surface of the earth oriented towards the Sun above the atmosphere is 1340 W/m^2. This is the well known solar constant comprising of electromagnetic radiation which extends from below the radio frequencies at the long-wavelength end through gamma radiation at the short-wavelength end covering wavelengths from thousands of kilometers down to a fraction of the size of an atom. The above value is distributed [2] over all the bands of the electromagnetic spectrum as follows:

TABLE I. The distribution of power in the various bands of electromagnetic radiation present in the solar constant.

Gamma-rays	$0 \rightarrow 10^{-14}$ m	~ 0 W/m^2
X-rays	$10^{-14} \rightarrow 10^{-10}$ m	~ 0 W/m^2
Ultra-violet	$10^{-10} \rightarrow 400x10^{-9}$ m	160 W/m^2
Visible	$400\ x10^{-9} \rightarrow 750\ x10^{-9}$ m	560 W/m^2
Infrared	$750\ x10^{-9} \rightarrow 10^{-3}$ m	620 W/m^2
Microwave	$10^{-3} \rightarrow 0.1$ m	$8.1x10^{-6}$ W/m^2
Amateur	$0.1 \rightarrow 10^{2}$ m	$5.6x10^{-5}$ W/m^2
Radio waves	$10^{2} \rightarrow 10^{4}$ m	$3.0x10^{-5}$ W/m^2
Long waves	$10^{4} \rightarrow \infty$ m	$3.7x10^{-7}$ W/m^2
	TOTAL	1340 W/m^2

Here the contribution of the visible part of the spectrum in the value of solar constant corresponds to 560 W/m^2. This visible band of the solar radiation plays two important roles. Firstly, it illuminates the earth directly during the day with a solar luminous constant [3] of magnitude 122.7 k lux and indirectly via the moon during night with a lunar luminous constant [3] of 0.2 lux – a rather too dim light. Secondly, this band is also utilized by photosynthetic engine [4] to produce the biomass for the existence of life on the earth. The corresponding equivalent energy in Watt per square meter may be termed as photosynthetic constant or photosynthetic solar constant to convey the message the availability of photosynthetic energy in the standard solar constant. The aim of the present paper is to report this value for the benefit of undergraduate students.

II. FORMULATION

The solar energy being electromagnetic in nature is characterized by wavelength λ, frequency ν and velocity c satisfying the relation

$$c = \lambda \nu , 0 \leq \lambda \leq \infty , \infty \leq \nu \leq 0 . \quad (1)$$

Under the assumption that the Sun has as a uniform temperature T over its surface the Planck's radiation law [1, 5] says that

$$I(\lambda,T)d\lambda = \frac{\varepsilon(\lambda,T)A(2\pi hc^2)d\lambda}{\lambda^5\{\exp(hc/\lambda kT)-1\}} \text{ W.} \quad (2)$$

$I(\lambda, T)d\lambda$, is the power radiated between the wavelengths λ and $\lambda + d\lambda$, A is the surface area, ε is the the emissivity and the constants h and k, respectively are Planck's constant and Boltzmann's constant. For simplicity, considering the Sun to be an ideal blackbody (ε =1) the solar flux Q emitted over all the wavelengths from the unit area (A=1 m^2) of its surface will be given by

$$Q = \int_0^\infty I(\lambda,T)d\lambda = \sigma T^4 \text{ W/m}^2, \quad (3)$$

where σ is the Stefan-Boltzmann constant. When this flux reaches the earth this is diluted by a factor [4]

$$f = \frac{R_S^2}{d^2}, \quad (4)$$

giving the value of standard solar constant as

$$S = \sigma T^4 f. \quad (5)$$

Here R_s is the radius of the Sun and d is the yearly mean distance between the earth and the Sun. Now the calculation of the value of solar flux in between wavelengths λ_i and λ_f present in the solar radiation will be taken up.

A. Expression of solar flux in the region λ_i and λ_f

The photosynthetically active region (PAR) λ_i =400nm to λ_f =750 nm is responsible for the process of photosynthesis [6] in plants and algae as shown in figure 1 where the light absorption (in percent) for the pigments chlorophyll, carotenoids and phycocyanin are depicted. The collective curve corresponding to these three pigments over the said wavelengths region can be displayed as shown [6] in figure 2 enclosing the ABCDE shaded area in green. It is clear that the light absorption is not uniform over the region and this absorption factor will be denoted by $\alpha(\lambda)$ % in the sequel. The expression for the solar flux $Q(\lambda_i \rightarrow \lambda_f)$ emitted from an unit area of the Sun in between wavelengths λ_i and λ_f according to (3) would be

$$Q(\lambda_i \rightarrow \lambda_f) = \int_{\lambda_i}^{\lambda_f} I(\lambda,T)d\lambda \text{ W/m}^2. \quad (6)$$

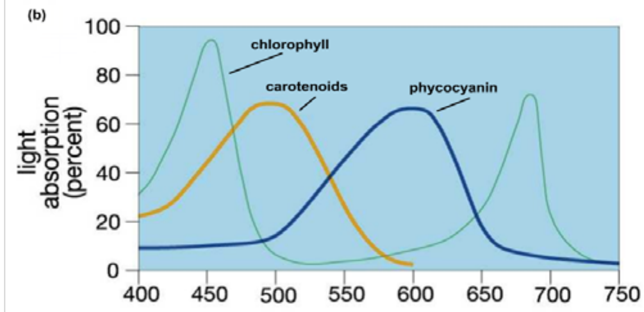

FIGURE 1. Plot of light absorption in percentage for chlorophyll, carotenoids and phycocynanin against the wavelength.

B. Solar Flux into Photosynthetic engine

The flux $Q(\lambda_i \rightarrow \lambda_f)$ will be diluted by the factor f [cf. Eq. (4)] when it reaches the earth's atmosphere. If this is further multiplied by the absorption factor $\alpha(\lambda)$ the amount of *input solar energy* into the photosynthetic engine comes out as

$$Q_P = \int_{\lambda_i}^{\lambda_f} \frac{2\pi hc^2 d\lambda}{\lambda^5\{\exp(hc/\lambda kT)-1\}}\alpha(\lambda)f \text{ W/m}^2. \quad (7)$$

The photosynthetic engine also works like any other engine in accordance with the well known relation

$$OutputPower\ (P_P) = Efficiency(\eta) x InputPower(Q_P). \quad (8)$$

In the present case the absorbed solar energy in the photosynthetically active region corresponds to

$$Input\ Power = Q_P \text{ W/m}^2. \quad (9)$$

If the efficiency of the photosynthetic engine is η then the actual energy being utilized in the production of biomass and the value of biomass so produced would be

$$Output\ Power = P_P = \eta Q_P \text{ W/m}^2, \quad (10)$$

$$M_P = 6.45 x 10^{-8} P_P \text{ Kg/m}^2\text{/s}. \quad (11)$$

The factor [4] 6.45x10^{-8} Kg/J corresponds to production of biomass for every Joule of photosynthetically active radiation being utilized by the photosynthetic engine.

C. Parameterization of absorption factor $\alpha(\lambda')$

The parameterization of the absorption factor $\alpha(\lambda')$ present in the integrand of Eq. (7) is approximated by dividing the rate of photosynthesis curve of figure 2 in four blocks with the straight lines AB, BC, CD, and DE [cf. Fig. 3] whose equations are, respectively

1. Line AB: $\alpha(\lambda') = 1.2698\lambda' - 487.9\ \%$; $400 \le \lambda' < 463$,
2. Line BC: $\alpha(\lambda') = -0.6437\lambda' + 398.0\ \%$; $463 \le \lambda' < 550$,
3. Line CD: $\alpha(\lambda') = 0.3231\lambda' - 133.7\ \%$; $550 \le \lambda' < 680$,
4. Line DE: $\alpha(\lambda') = -1.2285\lambda' + 921.4\ \%$; $680 \le \lambda' < 750$. (12a)

Here λ' satisfies the relation

$$\lambda = 10^{-9}\lambda' \text{ m}, \tag{12b}$$

and the coordinates of the points A, B, C, D, and E, respectively being (400, 20), (463,100), (550, 44), (680, 86), and (750, 0). Now we turn to the analytical solution of Eq. (7).

D. Analytical solution of Eq. (7)

Substituting the value of $\alpha(\lambda')$ from (12a) and that of λ' from (12b) one gets

$$Q_P = 2\pi hc^2 f\left[\int_{400x10^{-9}}^{463x10^{-9}} \frac{(1.2698x10^9\lambda - 487.9)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT)-1\}} + \int_{463x10^{-9}}^{550x10^{-9}} \frac{(-0.6437x10^9\lambda + 398.0)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT)-1\}} + \int_{550x10^{-9}}^{680x10^{-9}} \frac{(0.3231x10^9 - 133.7)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT)-1\}} + \int_{680x10^{-9}}^{750x10^{-9}} \frac{(-1.2285x10^9\lambda + 921.4)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT)-1\}}\right]. \tag{13}$$

Here all the four integrals have the following typical form

$$\text{Typical Integral} \equiv 2\pi hc^2 f \int_{\lambda_1}^{\lambda_2} \frac{(a\lambda + b)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT) - 1\}}, \tag{14}$$

whose analytical solution is worked out in the Appendix *Solution of the typical integral (14)*.

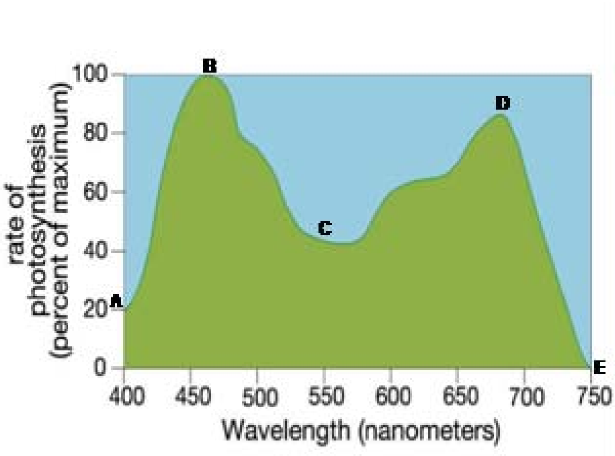

FIGURE 2. The collective rate of photosynthesis curve for the three pigments against the wavelength shown in figure 1.

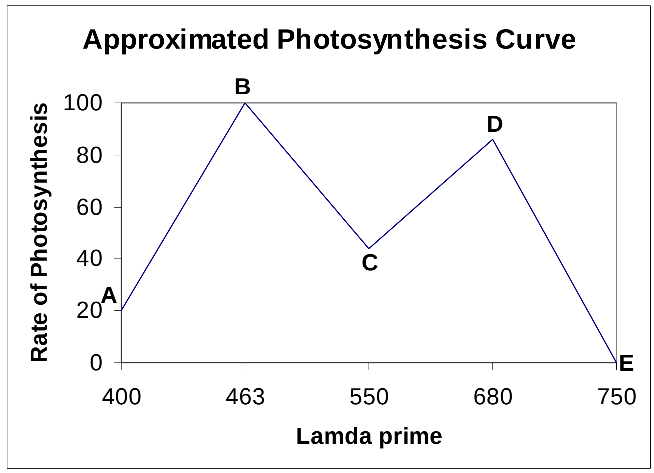

FIGURE 3. The straight line approximation of rate of photosynthesis curve shown in figure 2.

III. NUMERICAL WORK

Employing [1, 7]

$$h = 6.63x10^{-34} J.s : c = 3.0x10^8 m/s; k = 1.38x10^{-23} J/K,$$

$$R_S = 6.96x10^8 m; d = 1.50x10^{11} m : T = 5776K, \tag{15}$$

along with the relation (4) all the four terms in Eq. (13) were evaluated as per analytical solution (A4) given in the Appendix which gives the calculated value of solar energy going into the photosynthetic channel out of the standard solar constant as

$$Input\ Power\ Q_P = 63.53 + 110.1 + 134.6 + 41.86 = 350\ \text{W/m}^2. \tag{16}$$

The upper theoretical limit of the efficiency of the photosynthetic engine reported in the literature [8] is

$$\eta = 20\% . \quad (17)$$

The above efficiency provides an estimate for maximum possible utilization of solar energy under ideal conditions through photosynthesis [cf. Eq. (10)] as

$$Output\ Power\ P_P = 0.2x350 = 70 \text{ W/m}^2, \quad (18)$$

and the corresponding equivalent biomass [cf. Eq. (11)] produced would be

$$M_P = 6.45x10^{-8}x70 = 4.5x10^{-6} \text{ Kg/m}^2\text{/s} . \quad (19)$$

IV. CONCLUSIONS & DISCUSSION

The salient conclusions of the present work are discussed below.

- An important concept of photosynthetic solar constant is proposed. This signifies the value of power produced in the form of biomass through the photosynthetic engine by making use of the visible band present in the standard solar constant.
- The photosynthetically active region 400 nm to 750 nm of the standard solar constant 1340 W/m^2 provides 350 W/m^2 of input power to the photosynthetic engine [cf. Eq. (16)].
- The theoretical upper limit [8] on the efficiency of this engine being 20% the maximum output power that can be achieved under ideal conditions would be 70 W/m^2 [cf. Eq. (18)]. This is the value of photosynthetic solar constant.
- The theoretical upper limit on the equivalent biomass harvested would be 4.5x10^{-6} Kg/m^2/s [cf. Eq. (19)]. This value may be called solar biomass constant as it can be achieved in principle out of standard solar constant under ideal conditions.
- The students can utilize these concepts, derivation and values for a better understanding of the natural photosynthetic engine.
- They can also estimate yearly yield of the biomass over the globe for comparison with actual biomass produced.
- The present work is a very useful application of the standard solar constant.

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APPENDIX

Solution of the typical integral (14)

Typical Integral

$$\equiv 2\pi hc^2 f \int_{\lambda_1}^{\lambda_2} \frac{(a\lambda + b)d\lambda}{100\lambda^5\{\exp(hc/\lambda kT) - 1\}}$$

$$= 2\pi hc^2 f \left[\int_{\lambda_1}^{\lambda_2} \frac{a.d\lambda}{100\lambda^4\{\exp(hc/\lambda kT) - 1\}} + \int_{\lambda_1}^{\lambda_2} \frac{b.d\lambda}{100\lambda^5\{\exp(hc/\lambda kT) - 1\}} \right] \quad \text{(A1)}$$

Making the variable change, $y = hc/\lambda kT$, Eq. (A1) becomes

Typical Integral

$$= \frac{2\pi(kT)^3}{100h^2c} fa \int_{y_1}^{y_2} \left[-\frac{y^2.dy}{\exp(y) - 1} \right] + \frac{2\pi(kT)^4}{h^3c^2} fb \int_{y_1}^{y_2} \left[-\frac{y^3.dy}{\exp(y) - 1} \right]. \quad \text{(A2)}$$

Here $y_1 = hc/\lambda_1 kT$ and $y_2 = hc/\lambda_2 kT$. Now the denominator of the both the integrands can be written and expanded as the infinite series given below

$$\frac{1}{\exp(y) - 1} = \exp(-y)\{1 - \exp(-y)\}^{-1}$$
$$= \exp(-y) + \exp(-2y) + \exp(-3y) + ... \exp(-ny)....... \quad \text{(A3)}$$

Substituting this series in integrands of (A1) and integrating term by term [5] leads to the result

Typical Integral

$$= \frac{2\pi (kT)^3}{100h^2c} fa\left[\sum_{n=1}^{\infty} \exp(-ny_2)\left\{\frac{y_2^2}{n}+\frac{2y_2}{n^2}+\frac{2}{n^3}\right\}\right.$$
$$\left. -\sum_{n=1}^{\infty} \exp(-ny_1)\left\{\frac{y_1^2}{n}+\frac{2y_1}{n^2}+\frac{2}{n^3}\right\}\right]$$
$$+\frac{2\pi (kT)^4}{h^3c^2} fb\left[\sum_{n=1}^{\infty} \exp(-ny_2)\left\{\frac{y_2^3}{n}+\frac{3y_2^2}{n^2}+\frac{6y_2}{n^3}+\frac{6}{n^4}\right\}\right. \quad \text{(A4)}$$
$$\left. -\sum_{n=1}^{\infty} \exp(-ny_1)\left\{\frac{y_1^3}{n}+\frac{3y_1^2}{n^2}+\frac{6y_1}{n^3}+\frac{6}{n^4}\right\}\right].$$

This series converges rapidly and can be evaluated with the help of a computer by taking a finite number of terms; truncation at $n = 5$ gives excellent results.