Physical-Layer Multicasting by Stochastic Beamforming and Alamouti Space-Time Coding

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Outline

Physical-Layer Multicasting

Nowadays, there is an explosive growth of multimedia services:

- live mobile TV
- multiparty video conferencing
- multimedia streaming for a group of paid users

From http://money.cnn.com

From http://www.telepresenceoptions.com

From http://www.slashgear.com

Physical-layer multicasting: an important class of techniques for resource-efficient massive content delivery

System Model for Physical-Layer Multicasting

- Scenario: common information broadcast to M users, MISO downlink

$$y_i(t) = \mathbf{h}_i^H \mathbf{x}(t) + n_i(t); \quad t = 1, 2, \ldots; \quad i = 1, 2, \ldots, M, \qquad (1)$$

where

- $y_i(t)$ is the received signal of user i at time t,
- $\mathbf{h}_i \in \mathbb{C}^N$ is the downlink channel to user i,
- $\mathbf{x}(t) \in \mathbb{C}^N$ is the multi-antenna transmit signal, and
- $n_i(t) \sim \mathcal{CN}(0, 1)$ is a standard complex Gaussian noise.

- A simple and efficiently realizable physical-layer scheme: Transmit Beamforming

(Sidiropoulos, Davidson, and Luo, "Transmit Beamforming for Physical Layer Multicasting", IEEE TSP 54(6), 2006.)

Transmit Beamforming (BF)

Transmitted signal:

$$\mathbf{x}(t) = \mathbf{w}s(t); \quad t = 1, 2, \ldots$$

- $s(t)$: symbol stream, with $E[|s(t)|^2] = 1$
- $\mathbf{w} \in \mathbb{C}^N$: beamformer

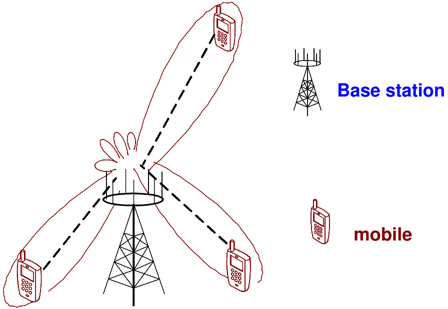


Figure: Transmit Beamforming

Beamformer Design: Max-Min-Fair (MMF) Formulation

- Given the beamformer $\mathbf{w} \in \mathbb{C}^N$, the receiver SNR for user i is $|\mathbf{h}_i^H \mathbf{w}|^2$.
- BF design using the Max-Min-Fair (MMF) formulation:

$$\begin{aligned} \gamma_{\mathsf{BF}} = \max_{\mathbf{w}\in\mathbb{C}^N} \ & \min_{i=1,\dots,M} |\mathbf{h}_i^H \mathbf{w}|^2 \\ \text{s.t.} \ & \|\mathbf{w}\|^2 \le P, \end{aligned} \tag{MMF}$$

  where
  - $P =$ maximum allowable transmission power,
  - $\gamma_{\mathsf{BF}} =$ best achievable worst-user receiver SNR by (MMF).
- Unfortunately, Problem (MMF) is NP-hard.

Semidefinite Relaxation (SDR) of Problem (MMF)

- To tackle Problem (MMF), apply semidefinite relaxation (SDR).
- Key observation:

$$\mathbf{W} = \mathbf{w}\mathbf{w}^H \quad \Longleftrightarrow \quad \mathbf{W} \succeq \mathbf{0} \text{ and } \operatorname{rank}(\mathbf{W}) \leq 1.$$

- Thus, Problem (MMF) is equivalent to

$$\begin{aligned} \gamma_{\mathsf{BF}} = \max_{\mathbf{W} \in \mathbb{H}^N} \; & \min_{i=1,\ldots,M} \operatorname{Tr}(\mathbf{W}\mathbf{h}_i\mathbf{h}_i^H) \\ \text{s.t. } & \operatorname{Tr}(\mathbf{W}) \leq P, \;\; \mathbf{W} \succeq \mathbf{0}, \;\; \operatorname{rank}(\mathbf{W}) \leq 1. \end{aligned}$$

- Now, drop the non-convex rank constraint to obtain the convex problem

$$\begin{aligned} \max_{\mathbf{W} \in \mathbb{H}^N} \; & \min_{i=1,\ldots,M} \operatorname{Tr}(\mathbf{W}\mathbf{h}_i\mathbf{h}_i^H) \\ \text{s.t. } & \operatorname{Tr}(\mathbf{W}) \leq P, \;\; \mathbf{W} \succeq \mathbf{0}. \end{aligned} \qquad \text{(SDR)}$$

Properties of (SDR)

- Let $\mathbf{W}^\star$ be an optimal solution to (SDR). If $\mathbf{W}^\star = \hat{\mathbf{w}}\hat{\mathbf{w}}^H$ (i.e., $\text{rank}(\mathbf{W}^\star) = 1$), then $\hat{\mathbf{w}}$ is optimal for (MMF).
- In general, $\text{rank}(\mathbf{W}^\star) > 1$ because (SDR) is a relaxation. However, if $M \leq 3$, then one can find a rank-one optimal solution $\mathbf{W}^\star$ to (SDR) in polynomial time.

  (Sidiropoulos et al. 2006, Huang and Zhang 2007)

Properties of (SDR) (Cont'd)

- For $M \geq 3$, if $\text{rank}(\mathbf{W}^\star) > 1$, then a Gaussian randomization procedure can be used to find a feasible solution $\hat{\mathbf{w}}$ to (MMF):

$$\hat{\mathbf{w}} = \sqrt{P}\boldsymbol{\xi}/\|\boldsymbol{\xi}\|, \quad \boldsymbol{\xi} \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}^\star).$$

- It is known that the worst-user receiver SNR achieved by $\hat{\mathbf{w}}$ is no worse than $1/8M$ times γ_{BF} (Luo et al. 2007).
  - That is, compared to the optimum, the worst-user receiver SNR achieved by the classical transmit beamforming scheme degrades at the rate of M in the worst case.

Motivations of Our Work

- From the signal processing perspective:

  Transmit beamforming is just one way of specifying the transmit structure of $\mathbf{x}(t)$.

  Are there other simple and efficiently realizable physical-layer strategies?

- From the optimization perspective:

  We saw that transmit beamforming corresponds to finding a rank-one solution to (SDR).

  Is there any physical-layer strategy that corresponds to finding a higher-rank solution?

Our Contributions

- We develop an SDR-based transmit beamformed Alamouti scheme, which can be seen as a rank-two generalization of the previous (rank-one) beamforming scheme.

- We provide a theoretical analysis of the proposed scheme, which shows that in the worst case,

$$\text{worst-user receiver SNR of beamformed Alamouti}$$
$$\geq \frac{1}{12.22\sqrt{M}} \times \text{optimal worst-user receiver SNR.}$$

- Simulation results show a marked improvement over transmit beamforming, both in terms of the achieved worst-user receiver SNR and worst-user BER.

Outline

System Model for Rank-2 BF Alamouti Scheme

- Idea: Apply a space-time code on the message.
- Specifically, the transmitted signal is

$$\boldsymbol{X}(n) = [\ \mathbf{x}(2n)\ \mathbf{x}(2n+1)\ ] = \mathbf{BC}(\boldsymbol{s}(n)),$$

where

  - $\boldsymbol{s}(n) = [\ s(2n)\ s(2n+1)\ ]^T$ is a block of data symbols,
  - $\mathbf{B} \in \mathbb{C}^{N \times 2}$ is the transmit beamforming matrix,
  - $\mathbf{C} : \mathbb{C}^2 \to \mathbb{C}^{2 \times 2}$ is the Alamouti space-time code given by

$$\mathbf{C}(\boldsymbol{s}) = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}.$$

- At the receiver side, we have

$$\boldsymbol{y}_i(n) = [\ y_i(2n)\ y_i(2n+1)\ ] = \mathbf{h}_i^H \mathbf{BC}(\boldsymbol{s}(n)) + \boldsymbol{n}_i(n).$$

Why the Alamouti Code?

- Given the beamforming matrix $\mathbf{B}$, the receiver SNR of user i can be characterized as $\mathbf{h}_i^H \mathbf{B}\mathbf{B}^H \mathbf{h}_i$.
- It is easy to implement and does not require sophisticated detection mechanism.
- In particular, the problem of finding an MMF beamforming matrix can be formulated as

$$\gamma_{\mathsf{BF-ALAM}} = \max_{\mathbf{B} \in \mathbb{C}^{N \times 2}} \min_{i=1,\ldots,M} \mathbf{h}_i^H \mathbf{B}\mathbf{B}^H \mathbf{h}_i \quad \text{s.t.} \ \mathrm{Tr}(\mathbf{B}\mathbf{B}^H) \le P. \qquad \text{(MMF-ALAM)}$$

  - Again, this problem is NP-hard.

SDR of Problem (MMF-ALAM)

- To apply SDR to Problem (MMF-ALAM), we observe

$$\mathbf{W} = \mathbf{B}\mathbf{B}^H \quad \Longleftrightarrow \quad \mathbf{W} \succeq \mathbf{0} \text{ and } \mathrm{rank}(\mathbf{W}) \leq 2,$$

which implies that (MMF-ALAM) is equivalent to

$$\gamma_{\mathrm{BF-ALAM}} = \max_{\mathbf{W} \in \mathbb{H}^N} \min_{i=1,\ldots,M} \mathrm{Tr}(\mathbf{W}\mathbf{h}_i\mathbf{h}_i^H)$$
$$\text{s.t. } \mathrm{Tr}(\mathbf{W}) \leq P, \;\; \mathbf{W} \succeq \mathbf{0}, \;\; \mathrm{rank}(\mathbf{W}) \leq 2.$$

- In particular, we have $\gamma_{\mathrm{BF-ALAM}} \geq \gamma_{\mathrm{BF}}$, i.e., the performance of the beamformed Alamouti scheme cannot be worse than that of transmit beamforming.

- Dropping the non-convex rank constraint yields the same SDR as that for transmit beamforming!

Rank-2 BF Alamouti: Theoretical Guarantees

- Let $\mathbf{W}^\star$ be an optimal solution to (SDR). If $\text{rank}(\mathbf{W}^\star) = r \leq 2$, then $\mathbf{W}^\star = \hat{\mathbf{B}}\hat{\mathbf{B}}^H$ for some $\hat{\mathbf{B}} \in \mathbb{C}^{N \times r}$, and $\hat{\mathbf{B}}$ is optimal for (MMF-ALAM).

Proposition

When $M \leq 8$, one can find an optimal solution $\mathbf{W}^\star$ to (SDR) of rank at most 2 in polynomial time.

- Thus, the beamformed Alamouti scheme achieves the optimal worst-user receiver SNR when there are no more than 8 users.
- Recall that transmit beamforming is guaranteed to achieve the optimal worst-user receiver SNR only when there are at most 3 users.

Rank-2 BF Alamouti: Theoretical Guarantees (Cont'd)

- For $M > 8$, if $\text{rank}(\mathbf{W}^\star) > 2$, then we can generate a feasible solution $\hat{\mathbf{B}}$ to (MMF-ALAM) as follows:

$$\hat{\mathbf{B}} = \sqrt{P \big/ \text{Tr}(\tilde{\mathbf{B}}\tilde{\mathbf{B}}^H)} \cdot \tilde{\mathbf{B}}; \;\; \tilde{\mathbf{B}} = \frac{1}{\sqrt{2}}[\; \boldsymbol{\xi}_1 \;\; \boldsymbol{\xi}_2 \;]; \;\; \boldsymbol{\xi}_1, \boldsymbol{\xi}_2 \sim \mathcal{CN}(\mathbf{0}, \mathbf{W}^\star).$$

Theorem

With constant probability,

$$\min_{i=1,\ldots,M} \text{Tr}(\mathbf{h}_i^H \hat{\mathbf{B}}^H \hat{\mathbf{B}} \mathbf{h}_i^H) \geq \frac{1}{12.22\sqrt{M}} \gamma_{\text{BF-ALAM}}.$$

- In particular, compared to the optimum, the worst-user receiver SNR achieved by the beamformed Alamouti scheme degrades only at the rate of $\sqrt{M}$ in the worst case.

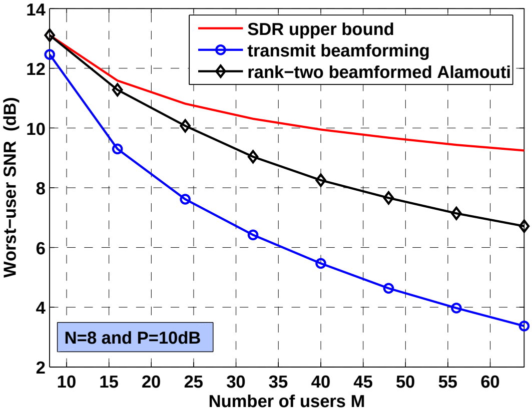
- The proof utilizes the SDP rank reduction theory developed in So et al. 2008.

A “Rank-n” Beamforming Scheme?

- It is tempting to extend our arguments to construct a “rank-n” beamforming scheme. Essentially, we need
  - an n-dimensional orthogonal space-time code (OSTBC), and
  - a rank-n SDR approximation procedure.
- The latter can be developed using the SDP rank reduction theory (So et al. 2008).
- Unfortunately, full rate OSTBCs do not exist when $n > 2$ (Liang and Xia 2003).
  - For instance, when $n = 3$, the maximal OSTBC code rate is only 3/4.
- Such a rate loss can erase the gain obtained by adopting a higher-rank SDR solution.

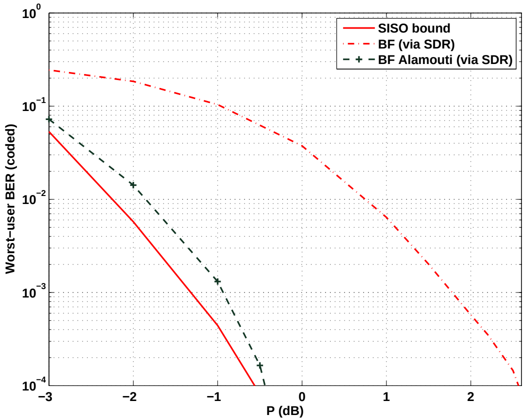
Outline

Worst-User SNR vs Number of Users



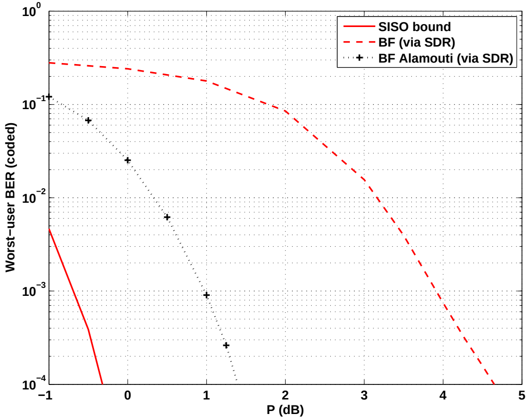
The rank-two BF Alamouti scheme is more capable of handling large number of users than the BF scheme.

Worst-User BER vs Transmission Power, $M = 16$ Users



- Note: “SISO Bound” is a performance lower bound.
- In this 16-user setting, the SDR beamformed Alamouti scheme is quite close to the performance lower bound (SISO bound).

Worst-User BER vs Transmission Power, $M = 32$ Users



- At BER = 1e-4, the beamformed Alamouti scheme is 2dB away from the performance lower bound, but 3dB better than beamforming.

Outline

Conclusions

- Physical-layer multicasting has become increasingly important in modern communication systems.
- The worst-user receiver SNR of the traditional transmit beamforming scheme degrades at the rate of M, where M is the number of users.
- We proposed a rank-2 transmit beamformed Alamouti scheme for physical-layer multicasting.
- The proposed scheme achieves the optimal worst-user receiver SNR when $M \leq 8$, which compares favorably with transmit beamforming's $M \leq 3$.
- Moreover, the worst-user receiver SNR only degrades at the rate of $\sqrt{M}$.

Further Remarks

- A natural question is, can we do even better by changing the transmit structure?
  - Recall that the transmit structures considered so far are

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{w}s(t) && \text{for transmit beamforming,} \\ \boldsymbol{X}(n) &= \mathbf{BC}(\boldsymbol{s}(n)) && \text{for rank-2 BF Alamouti.} \end{aligned}$$

- Key insight: Use a time-varying beamforming strategy!
  - For transmit beamforming, we can consider

$$\mathbf{x}(t) = \mathbf{w}(t)s(t),$$

where $\mathbf{w}(t)$ is generated randomly according to a common distribution.

Further Remarks

- What are the candidate distributions? We generate $\mathbf{w}(t)$ so that its covariance matrix matches the optimal solution $\mathbf{W}^\star$ to (SDR).
- With carefully chosen distributions, it can be shown that the above scheme achieves a rate that is within a constant of the optimal multicast capacity.
- This should be contrasted with the fixed beamforming schemes, where the gap between achievable rate and optimal multicast capacity deteriorates logarithmically as the number of users increases.
- The same idea applies to the rank-2 BF Alamouti scheme, with even better results.

Thank You!