PHYSICAL PROPERTIES OF CEPHEIDS AND RR LYRAE STARS

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1. INTRODUCTION

At the Leiden Southern Station a large program on galactic pulsating variables has been carried out with the Walraven five-channel photometer. For 170 Cepheids and 100 RR Lyrae stars south of $\delta = +15^\circ$ there are now available 30000 VBLUW measurements (Walraven et al., 1964; Pel, 1976; Lub, 1977b). These data are used to derive physical properties of the variables. A general discussion of the VBLUW system and its calibration for A-G stars by means of the theoretical fluxes of Kurucz (1975) has been given by Lub and Pel (1977). Following this calibration, reddening and blanketing corrections, as well as effective temperatures, gravities, and their cyclical variations were determined, and luminosity and radius curves were derived (Cepheids: Pel, 1977; RR Lyrae stars: Lub, 1977a). We will present here some interesting results that have been obtained from these data. A more detailed discussion is in preparation.

2. CEPHEIDS

The absence of gravity effects in (B-L) for F-G stars is the basis of the locus method used to derive reddenings for the Cepheids (Pel, 1977). The Cepheid locus in the (V-B)-(B-L) diagram can only be applied to Population I Cepheids, however, as (B-L) is

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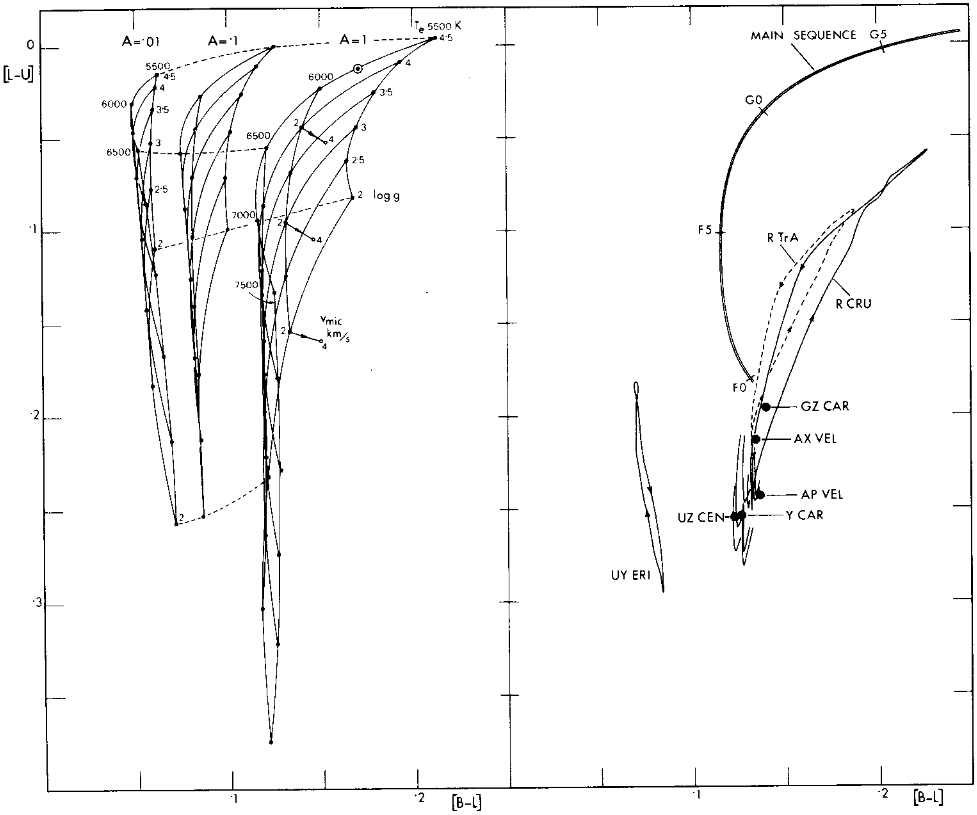


Fig. 1. The [B-L] - [L-U] diagram for F-G stars. Left: theoretical colors for 1x, 0.1x, and 0.01x the solar heavy elements abundance (A=1, 0.1, 0.01), computed from Kurucz (1975). In the A=1 grid the effect of raising the microturbulence from 2 to 4 km/s has been indicated. Right: observed colors for main-sequence stars, the Pop II Cepheid UY Eri, and for short-period Pop I Cepheids with T_{max} 6600 K (two complete loops are shown, for the other 8 Cepheids only the maxima are drawn). Dots indicate the mean maxima of five double-mode Cepheids. The observed main-sequence was used to match the zeropoints of observed and theoretical colors. As is customary in the VBLUW system, all colors are on a $\log_{10}$ (intensity) scale.

sensitive to differences in line blanketing. Extra information about the chemical homogeneity of the Pop I Cepheids is therefore very important. For this purpose the [B-L] - [L-U] diagram (Fig. 1) is useful. [B-L] = (B-L)-0.43 (V-B) and [L-U] = (L-U)-0.21 (V-B) are reddening-independent indices.

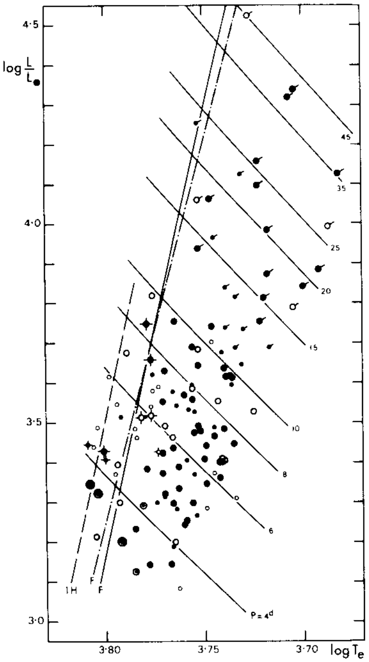
The theoretical colors in Fig. 1 indicate that around T_{eff} = 6500-7000 K [B-L] is a reddening-free composition parameter that is virtually gravity and temperature independent. Provided that a Cepheid reaches the range 6500-7000 K, we can estimate its composition from the bluest [B-L] value along the cycle. Indeed we get a clear separation between UY Eri and the Pop I Cepheids. For UY Eri we estimate A=0.03; a similar value ($\log Z/Z_{\odot}=-1.66$) was used by Böhm-Vitense et al. (1974). The Cepheid loops agree even in detail with the theoretical colors: the Pop I loops are described counter-clockwise and fit well to the cool branch of the A=1 grid, while UY Eri runs in the opposite sense corresponding to the high-temperature branch of the A=0.03 grid. Considering the fact that composition is not the only source of scatter in [B-L], Fig. 1 indicates a remarkably small range in composition for the short-period Pop I Cepheids. This is strong support for the use of one intrinsic locus to derive reddenings for the "classical" Cepheids. Furthermore, Fig. 1 shows that the double-mode Cepheids have Pop I compositions, a result which is important in view of the theoretical problems for these stars (e.g. Petersen, 1973; Cox et al., 1977).

The observed periods and temperatures of the Cepheids can be used to derive luminosities from the pulsation relation, which gives the period as a function of mass M, luminosity L, and temperature. In order to eliminate M, one has to assume a M-L relation. Once T_{eff} and L are known, we can construct the theoretical HR diagram for the Cepheids. This is shown in Fig. 2. Here we used the pulsation relation and the theoretical M-L relation for Cepheids from Iben and Tuggle (1975). For the Cepheids with P=11 days the equilibrium temperatures from Pel (1977) were applied. The theoretical VBLUW colors do not cover the long-period Cepheids over their whole cycle, however. We therefore designed a recipe which gives the colors of the equilibrium configuration directly from the color curves. These equilibrium colors lie within the range of the theoretical colors for all periods. As this algorithm works very accurately for all short-period Cepheids, we applied it also to the long-period Cepheids to obtain the temperatures for those stars.

Both in the HR diagram of Fig. 2 and in the P-T_{eff} diagram (not shown) we get an excellent fit of the data to the fundamental blue edge for Y=0.28 and Z=0.02. A number of stars lie to the left of this blue edge, but most of these are suspected to have blue companions (open symbols) while the others are double-mode or likely first-harmonic pulsators (see below). The Cepheid strip seems to become wider at higher L, an effect also observed by Butler (1976a, 1977) in the Magellanic Clouds (most clearly in the LMC).

The present Cepheid temperatures are significantly cooler than most of the Cepheid temperature scales currently in use. It is therefore interesting to see what the consequences are of our T_{eff} scale for the well-known Cepheid "mass-discrepancy". As an example we take the cluster Cepheid S Nor. Using the cluster data

Fig.2 The Cepheids in the HR diagram. The M-L and pulsation relations of Iben and Tuggle (1975) were used to compute luminosities and lines of constant period. Composite fundamental (F, full curve) and first-harmonic (1H, dashed) blue edges from Iben and Tuggle are given, as well as the fund.blue edge (F, dot-dash) from King et al. (1975); all for Y=0.28, Z=0.02, and the M-L relation of Iben and Tuggle (1975). Open symbols: stars with known or suspected companions or with peculiarities; ✓ : long-period Cepheids for which the equilibrium-algorithm (see text) was used to derive T_e ; ◉ double-mode Cepheids; ✦ : likely overtone pulsators (see text), plotted at the position corresponding to the fundamental period.



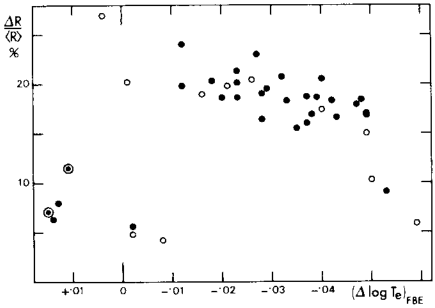


Fig.3 Radius amplitude ΔR/<R> versus $(\Delta \log T_e)_{FBE}$, the distance in T_{eff} measured at constant L from the fundamental blue edge (for Y=0.28, Z=0.02) of Iben and Tuggle (1975). Symbols as in Fig. 2.

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from Breger (1970), and (m-M) = $3^m\!.20$ as distance modulus for the Hyades, we find an "evolution mass" M=6.48 $M_\odot$ and a "pulsation mass" M=5.73 $M_\odot$ for this Cepheid (again with the M-L and pulsation relations of Iben and Tuggle). This is a mass-discrepancy of only 12%, three times smaller than the discrepancies usually found (e.g. Fricke et al., 1972), and not very disturbing in view of the many uncertainties involved in these mass estimates.

The behavior of the pulsation amplitude in the instability strip is important for several reasons. In particular, many authors have tried to use the pulsation amplitude to locate Cepheids in the strip (e.g. Kraft, 1960; Sandage and Tammann, 1971). Conflicting evidence on this point has been put forward by Madore (1976) and Butler (1976b). In Fig. 3 we have therefore plotted the radius amplitudes from Pel (1977) versus distance in T_{eff} measured at constant L from Iben and Tuggle's fundamental blue edge. Only Cepheids with accurate radius curves (Pel, 1977; Table 5^a) were used, and only in the interval $3.24 < \log L/L_\odot < 3.60$ in order to avoid effects from the bumps around P=9-10 days or from the decrease in amplitude at the shortest periods.

The pronounced amplitude discontinuity in Fig. 3 around the position of the fundamental blue edge reminds us very much of the behaviour of the Type-c and Type-ab RR Lyrae stars, and suggests strongly that the blue small-amplitude Cepheids are first-harmonic pulsators. Contrary to the theoretical computions of Stellingwerf (1975b), Fig. 3 indicates that the double-mode Cepheids are probably related to the fundamental-overtone transition. Finally, we see that the pulsation amplitude is not well suited to measure the position of a Cepheid inside the strip: for the large-amplitude pulsators $\Delta R/<R>$ varies too little with temperature, and at the small amplitudes we can not distinguish the red-edge Cepheids from the overtone pulsators by amplitude alone.

3. RR LYRAE STARS

In contrast to the classical Cepheids, the RR Lyrae stars show a wide range in metal abundances. A discussion of their properties is thus preferably done by deblanketed or blanketing-independent colors. Again we use the absence of gravity effects in the (V-B) - (B-L) diagram, but this time to determine the amount of metal line blanketing from [B-L] measured at minimum light (for the RRab stars). A study of the empirical and theoretical blanketing vectors shows that $(V-B)^* = (V-B)-0.370(B-L)$ and (L-U) are the appropriate blanketing-free temperature and gravity indicators in this case (Lub, 1977a). Blanketing is measured with respect to the most metal-poor stars by $\Delta[B-L] = (B-L)-0.51(V-B)^*-0.083$. This $\Delta[B-L]$ correlates very well with conventional blanketing indicators such as ΔS (Preston, 1959) and $\delta(U-B)_s$ (Sturch, 1966); it is however measured to a much higher accuracy (± 0.002 on a total range of 0.1 from A=0.01 to 1).

Using blanketing-independent colors, the reddening is determined from:

1) the correlation between $(V-B)^*$ at minimum and period for RRab stars (cf. Sturch, 1966)

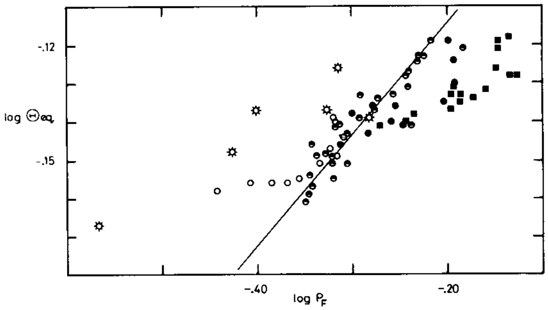

Fig.4 The period-temperature plane for the RRab stars.

■ : $\Delta[B-L] < 0.020$, $\Delta S \geq 9$
● : $\Delta[B-L] = 0.020-0.025$, $\Delta S \simeq 8$
◒ : $\Delta[B-L] = 0.025-0.045$, $\Delta S \simeq 6$
o : $\Delta[B-L] = 0.045-0.075$, $\Delta S = 2, 3$
☼ : $\Delta[B-L] \geq 0.075$, $\Delta S \leq 1$

The line is the theoretically expected relation for $\log M^{0.81}/L = -1.89$.

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2) the blue envelope in the $\langle V-B\rangle^* - \langle\beta_J\rangle$ plane (<>: time average) for all variables; $\langle\beta_J\rangle$ has been recalculated from the H_β data of Jones (1973).

Both methods give identical results.

The complete coverage of all light- and color curves (Lub, 1977b) allows a detailed study of the luminosity, temperature, gravity and radius variations. Typical radius amplitudes of 14 and 5% are found for the RRab and RRc stars, respectively, in good agreement with the calculations by Stellingwerf (1975b). The equilibrium position of the variables in the T_{eff}-log g plane is derived by taking the appropriate average over the cycle (van Albada and de Boer, 1975).

The most useful comparison with pulsation theory can be made in the period-temperature plane, which we show in Fig. 4 (here $\theta_{eff}=5040/T_{eff}$. We have grouped the stars in intervals of $\Delta[B-L]$ and a regular trend with metallicity is immediately evident in this diagram. Moreover, from pulsation theory (e.g. van Albada and Baker, 1971) it is predicted that stars of a given mass-luminosity ratio will lie along a linear log θ - log P relation in Fig. 4; the stars with $\Delta S \simeq 6$ are seen to fall almost exactly along such a line with $\log(M^{0.81}/L) = -1.89 \pm 0.01$. It is remarkable that in the inhomogeneous sample of field variables it is possible to isolate groups with such a small internal spread in physical properties. Even though there are transition cases, we are strongly reminded of the Oosterhoff phenomenon (van Albada and Baker, 1973) by the average parameters of the two main groups present: $\Delta S=6$ and 9, $P_{ab}=0\overset{d}{.}55$ and $0\overset{d}{.}65$, $P_{trans}=0\overset{d}{.}44$ and $0\overset{d}{.}53$, log $(M^{0.81}/L)=-1.89$ and -1.97. A discussion of the best estimates of mass and luminosity, taking into account constraints set by stellar evolution theory, leads to values at $\Delta S=6$ of $M=0.6\pm0.03$, log $L=1.70\pm0.03$, or $M_V=0\overset{m}{.}45\pm0\overset{m}{.}08$ (Lub, 1977a). The agreement with recent results from statistical parallax determinations (Hemenway, 1975; Heck, 1977) seems good.

A second important feature of the period-temperature diagram is the fact that the fundamental and first-harmonic blue edges do not depend upon mass and luminosity (Tuggle and Iben, 1972). This makes this diagram ideal for the determination of the helium abundance in the envelopes of pulsating stars. At $\Delta[B-L] = 0.030$ both from the RRc stars (not shown in Fig. 4) and the RRab stars we find $Y=0.28\pm0.02$. This value agrees with the one derived from a comparison of the mass-luminosity ratio $(M^{0.81}/L)$ with the horizontal-branch evolutionary tracks by Sweigart and Gross (1976). The behavior of the transition period appears to be in agreement with the hypothesis by van Albada and Baker (1973) and with the detailed stability calculations of Stellingwerf (1975a).

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DISCUSSION

BUSCOMBE: How are masses for classical Cepheids estimated, and what are typical values?

PEL: The so-called evolution masses are taken from the observed luminosities and the theoretical mass-luminosity relation for the Cepheid region. "Pulsation masses" are derived from the observed period, luminosity and temperature and the theoretical pulsation relation.

Both estimates can only be made when direct information about the luminosity is available, which is only for the Cepheids in clusters.

Typical evolution masses are 3 solar masses at the shortest periods to 12 solar masses for the long-period Cepheids.

KRAFT: What do you get for $<M_V>$ when you go from $\Delta S = 0$ or $\Delta S = 10$?

LUB: We estimate the absolute magnitude at $\Delta S \sim 10$ to be about $0\overset{m}{.}20$ lower than at $\Delta S = 6$; conversely at $\Delta S \sim 0$, say, to be $0\overset{m}{.}30$ higher than at $\Delta S = 6$.

BUTLER: Do you assume constant mass in deriving these luminosity differences?

LUB: Not quite, stellar evolution theory puts constraints on the possible masses of stars which after reaching the horizontal branch will evolve through the instability strip. This has been taken into account in the values quoted in my answer to Kraft.

DICKENS: Are any of the first harmonic pulsators amongst the Cepheids in clusters and if so are their pulsation masses more in accord with evolutionary masses?

PEL: There is one likely first harmonic Cepheid which is member of an open cluster, and that is EV Sct. I have not determined evolution and pulsation masses for that star yet, but it is clear that the puslation mass-estimate comes out wrong when the period is taken for the wrong mode.

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