

## PICARD SEQUENCES IN $b$ -METRIC SPACES

SUZANA ALEKSIĆ\*, ZORAN D. MITROVIĆ\*\* AND STOJAN RADENOVIĆ\*\*\*

\*University of Kragujevac, Faculty of Science, Department of Mathematics and Informatics,  
Radoja Domanovića 12, 34000 Kragujevac, Serbia  
E-mail: suzanasimic@kg.ac.rs

\*\*University of Banja Luka, Faculty of Electrical Engineering,  
Patre 5, 78000 Banja Luka, Bosnia and Herzegovina  
E-mail: zoran.mitrovic@etf.unibl.org

\*\*\*King Saud University, College of Science, Department of Mathematics,  
Riyadh 11451, Saudi Arabia  
E-mail: stojan.radenovic@tdtu.edu.vn

**Abstract.** The purpose of this article is to provide much simpler and shorter proofs of some important results in the framework of  $b$ -metric spaces. Namely, we show that the given contractive condition implies  $b$ -Cauchyness of the corresponding Picard sequence. The obtained results improve well-known comparable results in the literature.

**Key Words and Phrases:**  $b$ -metric space,  $b$ -complete,  $b$ -Cauchy,  $b$ -continuous, Picard sequence.

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