PID Passivity-Based Control o Port-Hamiltonian Systems

Zhang, Meng; Borja Rosales, Luis Pablo; Ortega, Romeo; Liu, Zhitao; Su, Hongye

Published in:
IEEE Transactions on Automatic Control

DOI:
10.1109/TAC.2017.2732283

Publication date:
2018

PID Passivity-Based Control of Port-Hamiltonian Systems

Meng Zhang, Pablo Borja, Romeo Ortega, *Fellow, IEEE*, Zhitao Liu, and Hongye Su

***Abstract*—In this note, we address the problem of stabilization of *port-Hamiltonian* systems via the ubiquitous proportional-integral-derivative (PID) controller. The design is based on passivity theory, hence the first step is to identify *all passive outputs* of the system, which is the first contribution of the paper. Adding a PID around this signal ensures that the closed-loop system is $\mathcal{L}_2$-stable for *all* positive PID gains. Global stability (and/or global attractivity) of a desired *constant equilibrium* is also guaranteed for a new class of systems for which a Lyapunov function can be constructed. A second contribution is to prove that this class—that is identified via some easily verifiable integrability conditions—is strictly larger than the ones previously reported in the literature. Comparisons of the proposed PID controller with control-by-interconnection passivity-based control are also discussed.**

***Index Terms*—Hamiltonian systems, nonlinear systems, passivity, passivity-based control (PBC), stabilization.**

I. Introduction

PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) controllers overwhelmingly dominate engineering applications where the control objective is to regulate some signal around a desired value. Commissioning of PIDs reduces to the suitable selection of the controller gains, which is a difficult task for wide ranging operating systems, where the validity of a linearized approximation is limited. Although gain scheduling, auto tuning and adaptation provide some help to overcome this problem, they suffer from well-documented drawbacks that include being time consuming and fragility of the design [1]. In contrast with this scenario in PID passivity-based control (PBC), where the PID is wrapped around a passive output, the gain tuning step is trivialized, as convergence of the output to zero and $\mathcal{L}_2$-stability of the closed-loop system is guaranteed *for all* positive gains—among which the designer selects those that ensure best *transient performance*.

However, it is often the case that the signal to be regulated is not a passive output or its reference output is nonzero. Another scenario of practical interest is when the control objective cannot be captured by the behavior of an output signal, for instance, when it is desired to drive the full system *state* to a desired constant value. A classical example is underactuated mechanical systems, whose passive outputs are the actuated velocities, but in most applications the objective is to drive *all positions* to some desired constant values. To address these problems, two approaches have been adopted in the literature; first, to identify passive systems for which the PID controller on the original passive outputs assigns the equilibrium and preserves the passivity but with a *new storage function* that has a minimum at the desired equilibrium, which then qualifies as a Lyapunov function for the latter. The identification of these systems boils down to imposing some integrability conditions that allows us to express the integral term of the PID as a function of the systems state. Second, to give conditions under which the *incremental model* of the system is also passive [10], [12], [14], property called "shifted passivity" in [30]. In this case, adding the PID around the incremental variables ensures not only that the incremental output goes to zero, but also that the desired equilibrium is assigned to the closed loop. The first approach has been pursued in [7] and [26] for mechanical systems and in [2] for general *port-Hamiltonian* (pH) systems. PID-PBCs have been designed following the second line of research in [5], [12], and [27] for power converters, in [17] for photovoltaic systems, and in [4] for general *RLC* circuits. The addition of integral actions has also been proposed to robustify PBCs, *vis-à-vis* external disturbances, in [6], [10], [22], and [25].

Motivated by the application of PID in physical systems, we restrict our attention in this paper to pH systems. As shown in [11], [30], and [31], pH models describe the behavior of many physical processes and have the central feature of underscoring the importance of the energy function, the interconnection pattern, and the dissipation of the system, which are the essential

Manuscript received November 25, 2016; revised April 28, 2017; accepted July 14, 2017. Date of publication July 26, 2017; date of current version March 27, 2018. This work was supported in part by the National Basic Research Program of China (973 Program 2013CB035406) and in part by the Science Fund for Creative Research Groups of the National Natural Science Foundation of China under Grant 61621002. The work of L. P. Borja was supported in part by the Mexican National Council of Science and Technology (CONACyT) and in part by the Mexican Secretary of Public Education (SEP). The work of R. Ortega was supported in part by the Government of Russian Federation [Grant 074-U01, GOSZADANIE 2014/190 (Project 2118)], and in part by the Ministry of Education and Science of Russian Federation (Project 14.Z50.31.0031). Recommended by Associate Editor C. M. Kellett. *(Corresponding author: Zhitao Liu.)*

M. Zhang, Z. Liu, and H. Su are with the State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China (e-mail: mengzhang2009@163.com; ztliu@zju.edu.cn; hysu@iipc.zju.edu.cn).

P. Borja and R. Ortega are with the Laboratoire des Signaux et Systèmes, CNRS UMR 8506, Supélec, Gif-sur-Yvette 91192, France (e-mail: borja@lss.supelec.fr; ortega@lss.supelec.fr).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2017.2732283

ingredients of PBC. The purpose of this paper is to report some new results on PID-PBC of pH systems. Our main contributions are the following.

1) The identification of *all* the passive outputs of a pH system that allows us to define a general framework for the design of PID-PBCs. The outputs are parameterized in terms of two free mappings and all passive outputs reported in the literature are obtained for some particular choices of them.
2) The extension of the, so-called, *power-shaping* passive output first reported in [19] to the case when the systems matrix is not full rank.
3) The inclusion of an *input and output* change of coordinate for energy shaping that relaxes the integrability conditions imposed in the literature [2].
4) The proposition of a new class of PID-PBCs that does not rely on the generation of Casimir functions for the pH system [20], [30], but identifies instead *first integrals of motion* for it. Since the Casimir functions are a particular case of the latter, the resulting design is applicable to a broader class of systems.
5) Give more flexibility to the PID-PBC design replacing the linear integral term by an arbitrary function that can be used to shape the Lyapunov function and/or improve the transient performance.

The remainder of this paper is structured as follows. In Section II, the problem addressed in this paper is formulated, whereas in Section III, all passive outputs for a given pH system are identified. In Section IV, we carry out the $\mathcal{L}_2$-stability analysis, and in Section V we show that a PID control can shape the energy function. Section VI discusses how to assign the desired equilibrium and to make the shaped energy function a Lyapunov function. In Section VII, we illustrate the results with some examples. We wrap up the paper with concluding remarks in Section VIII.

Notation: I_n is the $n \times n$ identity matrix and $0_{n\times s}$ is the $n \times s$ matrix of zeros. For $x \in \mathbb{R}^n$, $S \in \mathbb{R}^{n\times n}$, $S = S^\top > 0$, we denote the Euclidean norm $|x|^2 := x^\top x$, and the weighted-norm $\|x\|_S^2 := x^\top S x$. All mappings are supposed smooth enough. Given a function $f : \mathbb{R}^n \to \mathbb{R}$, we define the differential operator $\nabla_x f := (\frac{\partial f}{\partial x})^\top$ and $\nabla_x^2 f := \frac{\partial^2 f}{\partial x^2}$. For a mapping $F : \mathbb{R}^n \to \mathbb{R}^m$, $m > 1$, we define the ijth element of its $m \times n$ Jacobian matrix as $(\nabla_x F)_{ji} := \frac{\partial F_i}{\partial x_j}$. When clear from the context, the subindex in ∇ is omitted. For any mapping $T : \mathbb{R}^n \to \mathbb{R}^{p\times q}$ and the distinguished element $x^\star \in \mathbb{R}^n$, we define the constant matrix $T^\star := T(x^\star)$.

II. Formulation of the PID-PBC Problem

Throughout this paper, we consider pH systems whose dynamics is described via the standard input-state-output representation [11], [30]

$$\dot{x} = [\mathcal{J}(x) - \mathcal{R}(x)]\nabla H(x) + g(x)u \tag{1}$$

$$y = h(x) + j(x)u \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$, $m \le n$, is the control vector, $y \in \mathbb{R}^m$ is an output of the system defined via the mappings $h : \mathbb{R}^n \to \mathbb{R}^m$ and $j : \mathbb{R}^n \to \mathbb{R}^{m\times m}$, $H : \mathbb{R}^n \to \mathbb{R}$ is the system's Hamiltonian, which we assume is bounded from below,[1] $\mathcal{J}, \mathcal{R} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, with $\mathcal{J}(x) = -\mathcal{J}^\top(x)$ and $\mathcal{R}(x) = \mathcal{R}^\top(x) \ge 0$, are the interconnection and damping matrices, respectively, and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is the input matrix, which is full rank. To simplify the notation in the sequel, we define the matrix $F : \mathbb{R}^n \to \mathbb{R}^{n\times n}$

$$F(x) := \mathcal{J}(x) - \mathcal{R}(x).$$

The *control objective* is to stabilize an equilibrium $x^\star \in \mathbb{R}^n$, which is an element of the set of assignable equilibria defined as

$$\mathcal{E} := \{x \in \mathbb{R}^n \mid g^\perp(x)F(x)\nabla H(x) = 0\} \tag{3}$$

where $g^\perp : \mathbb{R}^n \to \mathbb{R}^{(n-m)\times n}$ is a full rank, left annihilator of $g(x)$, that is, $g^\perp(x)g(x) = 0$ and $\text{rank}\{g^\perp(x)\} = n - m$. See [35] for a parameterization of all full rank left annihilators of a matrix.

We restrict our attention to PID controllers of the form

$$\begin{aligned} u &= -K_P y - K_I x_c - K_D \dot{y} \\ \dot{x}_c &= y,\ x_c(0) = x_c^0 \in \mathbb{R}^m \end{aligned} \tag{4}$$

where $K_P, K_I, K_D \in \mathbb{R}^{m\times m}$ with $K_P, K_I > 0$ and $K_D \ge 0$ are the PID tuning gains. Notice that we have also specified an initial condition for the controller state x_c^0 that will be required to assign the desired equilibrium point to the closed loop for Lyapunov stabilization. As explained in Section VI, an alternative—*static* state feedback—realization of the PID-PBC that removes the aforementioned initial condition constraint is also proposed.

PID-PBC Stabilization Problem: Given the pH system (1) with full state measurement and a desired equilibrium $x^\star \in \mathcal{E}$, with $\mathcal{E}$ defined in (3). Identify a class of pH systems for which there exist mappings $h(x), j(x)$ such that the following statement holds.

1) The mapping $\Sigma : u \mapsto y$ is passive.
2) The PID-PBC (2), (4) ensures $x^\star$ is an (asymptotically) stable equilibrium of the closed-loop system.

As is well known [21], *integral* control can be represented as a pH system. Indeed, defining a pH controller system as

$$\begin{aligned} \dot{x}_c &= u_c \\ y_c &= \nabla H_c(x_c) \end{aligned} \tag{5}$$

with state $x_c \in \mathbb{R}^m$, input and output $u_c, y_c \in \mathbb{R}^m$, respectively, and Hamiltonian function

$$H_c(x_c) := \frac{1}{2}\|x_c\|_{K_I}^2$$

[1]This assumption is made to simplify the presentation, since in this case we deal with passivity of the pH system instead of cyclopassivity, see [30, Proposition 6.1.8].

we see that the action of the integral term of the PID-PBC (4) is obtained with the interconnection

$$\begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} 0_{m\times m} & -I_m \\ I_m & 0_{m\times m} \end{bmatrix} \begin{bmatrix} y \\ y_c \end{bmatrix}. \tag{6}$$

Since the interconnection is power preserving [11], the closed-loop system is still pH with total energy function $H(x) + H_c(x_c)$. This is the point of view adopted in control by interconnection (CbI), which is an approach for the design of PBCs widely studied in the literature, cf., [10], [11], [20]. In CbI, the stabilization problem mentioned above is solved generating Casimir functions that relate the states of the plant and the controller to generate the required Lyapunov function. See Section V-B for a comparison between CbI and the PID-PBC proposed here.

III. Passivity of pH Systems

The first contribution of this paper is the solution of the task (i) of the problem formulation above, that is, the characterization of *all* passive outputs for the pH system (1), which is given in the next section. The relationship between the proposed parameterization and the currently reported passive outputs is given in Section III-B, which also contains a generalization of the power-shaping output reported in [19] to the case of not full-rank matrix $F(x)$.

A. Parameterization of All Passive Outputs

The first step for the characterization of all passive outputs is to compute a (nonunique) factorization of the dissipation matrix of the form

$$\mathcal{R}(x) = \phi^\top(x)\phi(x) \tag{7}$$

where $\phi : \mathbb{R}^n \to \mathbb{R}^{q\times n}$, with $q \in \mathbb{N}$ satisfying $q \geq \text{rank}\{\mathcal{R}(x)\}$. We recall the basic linear algebra fact that $\mathcal{R}(x) \geq 0$ *iff* such a factor exists [13].

Proposition 1: Consider the pH system Σ, given by (1) and (2). The following statements are equivalent.

S1) The mapping $\Sigma : u \mapsto y$ is passive with storage function $H(x)$.

S2) For any factorization of the dissipation matrix $\mathcal{R}(x)$ of the form (7), the mappings $h(x)$ and $j(x)$ can be expressed as

$$\begin{aligned} h(x) =&\ [g(x) + 2\phi^\top(x)w(x)]^\top \nabla H(x) \\ j(x) =&\ w^\top(x)w(x) + D(x) \end{aligned} \tag{8}$$

for some mappings $w : \mathbb{R}^n \to \mathbb{R}^{q\times m}$ and $D : \mathbb{R}^n \to \mathbb{R}^{m\times m}$, with $D(x)$ skew symmetric.

Proof: It is well known, e.g., [30, Proposition 4.1.2], that the system (1), (2) is passive if and only if

$$\begin{bmatrix} -2(\nabla H(x))^\top \mathcal{R}(x)\nabla H(x) & (\nabla H(x))^\top g(x) - h^\top(x) \\ g^\top(x)\nabla H(x) - h(x) & -[j(x) + j^\top(x)] \end{bmatrix} \leq 0.$$

To prove that (S2) implies (S1) replace (7) and the definition of $h(x)$ and $j(x)$ of (2) above to get

$$2\begin{bmatrix} -|\phi(x)\nabla H(x)|^2 & -(\nabla H(x))^\top \phi^\top(x)w(x) \\ -w^\top(x)\phi(x)\nabla H(x) & -w^\top(x)w(x) \end{bmatrix} \leq 0$$

which is always satisfied.

The proof that (S1) implies (S2) proceeds as follows [28]. Assume $u \mapsto y$ is passive with storage function $H(x)$ and define the mapping $d : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}_{\geq 0}$

$$d(x, u) := -\dot{H} + u^\top[h(x) + j(x)u] \geq 0. \tag{9}$$

Evaluating $\dot{H}$ along the trajectories of (1) and using (7), we get

$$\begin{aligned} d(x, u) =&\ |\phi(x)\nabla H(x)|^2 + u^\top[h(x) - g^\top(x)\nabla H(x)] \\ &+ \frac{1}{2}u^\top[j(x) + j^\top(x)]u. \end{aligned}$$

Because $d(x, u)$ is quadratic in u and nonnegative for all u and x, there exists a (nonunique) matrix valued function $w(x)$ such that

$$d(x, u) = [\phi(x)\nabla H(x) + w(x)u]^\top[\phi(x)\nabla H(x) + w(x)u].$$

The proof that $h(x)$ and $j(x)$ take the form (8) is established equating the terms of like power in u and invoking the skew symmetry of $D(x)$. ■

B. Particular Cases of Passive Outputs

In this section, we prove that all passive outputs of the pH system (1) reported in the literature can be generated via the output (8).

Proposition 2: Consider the output (2) and its parameterization (8). The following implications hold true.

1) Natural output [18], [30]

$$\left.\begin{aligned} w(x) = 0 \\ D(x) = 0 \end{aligned}\right\} \Rightarrow\ y = g^\top(x)\nabla H(x). \tag{10}$$

2) Power-shaping output of [19] with $F(x)$ full rank

$$\left.\begin{aligned} &w(x) = \phi(x)F^{-1}(x)g(x) \\ &D(x) = -g^\top(x)F^{-\top}(x)\mathcal{J}(x)F^{-1}(x)g(x) \end{aligned}\right\} \\ \Rightarrow\ y = -g^\top(x)F^{-\top}(x)\dot{x}. \tag{11}$$

3) The alternate output of [11], [20], and [33] with generalized damping matrix verifying

$$\mathcal{Z}(x) := \begin{bmatrix} \mathcal{R}(x) & T(x) \\ T^\top(x) & S(x) \end{bmatrix} \geq 0 : \tag{12}$$

$$\left.\begin{aligned} S(x) = w^\top(x)w(x) \\ T(x) = \phi^\top(x)w(x) \end{aligned}\right\} \\ \Rightarrow\ y = [g(x) + 2T(x)]^\top \nabla H(x) + [S(x) + D(x)]u. \tag{13}$$

4) Power-shaping output of [19] with $F(x)$ *not full rank* but verifying

$$F^\top(x)[F^\dagger(x)]^\top(x)F(x) = F(x) \tag{14}$$

$$\text{span}\{g(x)\} \subseteq \text{span}\{F(x)\} \tag{15}$$

where $F^\dagger(x)$ is a pseudoinverse of $F(x)$

$$\left.\begin{array}{l} w(x) = \phi(x)F^\dagger(x)g(x) \\ D(x) = -g^\top(x)[F^\dagger(x)]^\top(x)\mathcal{J}(x)F^\dagger(x)g(x) \end{array}\right\}$$
$$\Rightarrow\ y = -g^\top(x)[F^\dagger(x)]^\top \dot{x}. \tag{16}$$

Proof: The proofs of (10) and (13) follow via direct replacement of the definitions of $w(x)$ and $D(x)$ in (8). For the latter, notice that the generalized damping matrix takes the form

$$\mathcal{Z}(x) = \begin{bmatrix} \phi^\top(x)\phi(x) & \phi^\top(x)w(x) \\ w^\top(x)\phi(x) & w^\top(x)w(x) \end{bmatrix}$$

which clearly satisfies the condition (12). Furthermore, since $q \geq \text{rank}\{\mathcal{R}(x)\}$ and $w(x)$ is free, taking the integer q large enough it is possible to construct any matrix $T(x)$ such that

$$\text{rank}\{T(x)\} \leq \max\{\text{rank}\{\mathcal{R}(x)\}, \text{rank}\{S(x)\}\}.$$

To prove (11), replace the definitions of $w(x)$ and $D(x)$ in (8) to get[2]

$$\begin{aligned} y =&\ (g + 2\phi^\top\phi F^{-1}g)^\top\nabla H + g^\top F^{-\top}(\phi^\top\phi - \mathcal{J})F^{-1}gu \\ =&\ (g + 2\mathcal{R}F^{-1}g)^\top\nabla H + g^\top F^{-\top}(\mathcal{R} - \mathcal{J})F^{-1}gu \\ =&\ ((I + 2\mathcal{R}F^{-1})g)^\top\nabla H - g^\top F^{-\top}FF^{-1}gu \\ =&\ ((F + 2\mathcal{R})F^{-1}g)^\top\nabla H - g^\top F^{-\top}gu \\ =&\ ((\mathcal{J} + \mathcal{R})F^{-1}g)^\top\nabla H - g^\top F^{-\top}gu \\ =&\ -(F^\top F^{-1}g)^\top\nabla H - g^\top F^{-\top}gu \\ =&\ -g^\top F^{-\top}F\nabla H - g^\top F^{-\top}gu \\ =&\ -g^\top F^{-\top}(F\nabla H + gu). \end{aligned}$$

Finally, we proceed to establish (16). Toward this end, we recall [3, Lemmata 15 and 16]—see also [24]—that state that (14) is equivalent to the existence of a mapping $Z : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ solution of the equation

$$F^\top ZF = -F \tag{17}$$

and that (15) and (17) imply that

$$F^\top Zg = -g \tag{18}$$

respectively. Now, defining Z as

$$Z := -(F^\dagger)^\top FF^\dagger \tag{19}$$

[2]To simplify notation, in some proofs the argument x is omitted from all mappings.

and with the choice of w given in (16), it is easy to verify that

$$\begin{aligned} (g + 2\phi^\top w)^\top =&\ [g + 2\phi^\top\phi F^\dagger g]^\top \\ =&\ [-F^\top Zg + 2\phi^\top\phi F^\dagger g]^\top \\ =&\ [F^\top(F^\dagger)^\top FF^\dagger g + 2\mathcal{R}F^\dagger g]^\top \\ =&\ [FF^\dagger g + 2\mathcal{R}F^\dagger g]^\top \\ =&\ [(F + 2\mathcal{R})F^\dagger g]^\top \\ =&\ [(\mathcal{J} + \mathcal{R})F^\dagger g]^\top \\ =&\ -(F^\top F^\dagger g)^\top \\ =&\ -g^\top(F^\dagger)^\top F \end{aligned} \tag{20}$$

where we have used (18) in the second equation, (19) in the third equation, and (14) in the fourth and eighth equation. Then, using w and D given in (16), we get

$$\begin{aligned} w^\top w + D =&\ g^\top(F^\dagger)^\top\phi^\top\phi F^\dagger g - g^\top(F^\dagger)^\top\mathcal{J}F^\dagger g \\ =&\ g^\top(F^\dagger)^\top\mathcal{R}F^\dagger g - g^\top(F^\dagger)^\top\mathcal{J}F^\dagger g \\ =&\ -g^\top(F^\dagger)^\top(\mathcal{J} - \mathcal{R})F^\dagger g \\ =&\ -g^\top(F^\dagger)^\top FF^\dagger g \\ =&\ -g^\top(F^\dagger)^\top F^\top(F^\dagger)^\top FF^\dagger g \\ =&\ -g^\top(F^\dagger)^\top g \end{aligned} \tag{21}$$

where we have used (14) in the fifth equation and (18) and (19) in the last equation. Replacing (20) and (21) in (2) and (8), we obtain

$$y = -g^\top(x)[F^\dagger(x)]^\top\dot{x}.$$

■

IV. Well Posedness and $\mathcal{L}_2$-Stability

As is well known, PID controllers define input strictly passive mappings [30]. Thus, the passivity theorem [9], [30] allows to immediately conclude output strict passivity—hence, $\mathcal{L}_2$-stability—of the closed-loop system. In this section, we establish this result for the pH system (1) in the closed loop with the PID-PBC (2), (4), (8) where, we recall, $w(x)$ and $D(x)$ are free parameters.

As explained in Section II, the initial condition in the controller state was fixed to a value $x_c(0) = x_c^0$ to assign the desired equilibrium point to the closed loop for Lyapunov stabilization. It should be underscored that for the input–output analysis carried out in this section, the PID-PBC applies to arbitrary initial conditions for x_c.

A. Well-Posedness Condition

Before proceeding to analyze the stability of the closed loop, it is necessary to ensure that the control law (4) can be computed without differentiation nor singularities that may arise due to the presence of the derivative term $\dot{y}$. Clearly, this term can be added only when the output y has relative degree equal to one, that is, when $w(x) = 0$ and $D(x) = 0$, hence y is the natural output (10). But even in that case, a well-posedness assumption

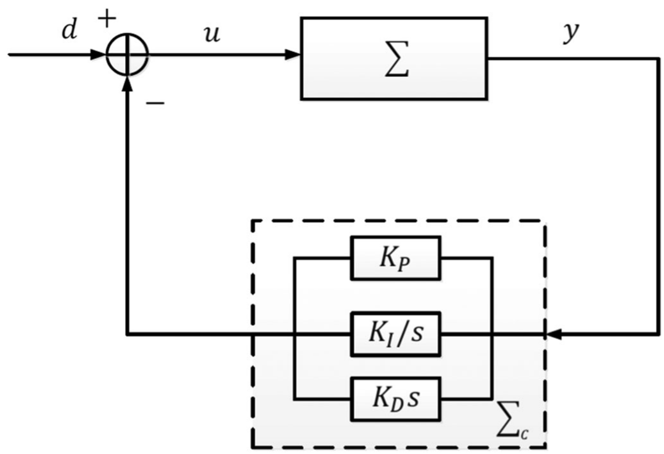


Fig. 1. Block diagram representation of the system (1), represented by Σ, in closed loop with the PID-PBC (2), (4), (8), represented by Σ_c, with an external signal d.

is required. Indeed, we can prove that (2), (4), and (8), with $w(x) = 0$ and $D(x) = 0$, are equivalent to

$$K(x)u = -K_P y - K_I x_c - K_D \nabla h(x) F(x) \nabla H(x) \quad (22)$$

where the mapping $K : \mathbb{R}^n \to \mathbb{R}^{m \times m}$ is given by

$$K(x) := I_m + K_D \nabla h(x) g(x).$$

To ensure that the control law (22)—and consequently (2), (4), (8)—are well defined, we impose the full-rank assumption

$$\det[K(x)] \neq 0.$$

Before closing this section, we note that in [30] PID control is viewed from a different perspective. Namely, assuming that $\dot{y}$ is *computable*, it is shown that the closed-loop system can be represented as a pH system with *algebraic constraints*. However, leaving aside the complexity of computing $\dot{y}$, the stability analysis of this kind of system remains an essentially open question.

B. $\mathcal{L}_2$-Stability Analysis

As indicated in the introduction, PIDs define input strictly passive maps therefore it is straightforward to prove $\mathcal{L}_2$-stability if it is wrapped around a passive output, since we are dealing with the *negative* feedback interconnection of two passive maps. This analysis is summarized in the proposition below that establishes the $\mathcal{L}_2$-stability of the closed-loop system represented in Fig. 1 where, as it is customary [9] (and with a slight abuse of notation), we have added an external signal d to define the closed-loop map.

Proposition 3: Consider the pH system (1) in closed loop with the PID-PBC (4), (2), (8) with an external signal d. The operator $d \mapsto y$ is $\mathcal{L}_2$-stable. More precisely, there exists $\beta \in \mathbb{R}$ such that

$$\int_0^t |y(s)|^2 ds \leq \frac{1}{\lambda_{\min}(K_P)} \int_0^t |d(s)|^2 ds + \beta \qquad \forall t \geq 0.$$

Proof: The proof follows directly from the Passivity Theorem [9], Proposition 1 that ensures passivity of the mapping $\Sigma : u \mapsto y$ defined by the pH system and output strict passivity of the mapping $\Sigma_c : y \mapsto (-u)$ defined by the PID-PBC. To prove the latter, we compute

$$\begin{aligned} y^\top(-u) &= y^\top K_P y + y^\top K_I x_c + y^\top K_D \dot{y} \\ &\geq \lambda_{\min}(K_P)|y|^2 + \dot{x}_c^\top K_I x_c + y^\top K_D \dot{y}. \end{aligned}$$

Integrating the expression above, we get

$$\int_0^t y(s)(-u(s))ds \geq \lambda_{\min}(K_P) \int_0^t |y(s)|^2 ds - \|x_c(0)\|^2_{K_I} - \|y(0)\|^2_{K_D} \qquad \forall t \geq 0.$$

■

V. Energy Shaping

$\mathcal{L}_2$-stability is a rather weak property. For instance, boundedness of trajectories is not guaranteed and the system can be destabilized by a constant external disturbance. Hence, we are interested in establishing a stronger property, e.g., Lyapunov stability of a desired equilibrium, which is the topic addressed in this section. A first step for Lyapunov stabilization of a desired constant state is, obviously, to ensure that it is an *equilibrium* of the closed loop. A second step is the creation of a *Lyapunov function* for this equilibrium that will ensure its stability. Following the terminology used in PBC, we will refer to this second step as *energy shaping* and address it in this section, leaving the equilibrium assignment and Lyapunov analysis problems for the next section.

A. Energy Shaping via Generation of First Integrals

Define the function $U : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$

$$U(x, x_c) := H(x) + \frac{1}{2}\|h(x)\|^2_{K_D} + \frac{1}{2}\|x_c\|^2_{K_I} \quad (23)$$

where we make the observation that $K_D \neq 0$ only if $y = h(x) = g^\top(x)\nabla H(x)$—as discussed in Section IV-A. Differentiating this function, we get

$$\begin{aligned} \dot{U} &\leq y^\top u + h^\top(x) K_D \dot{h}(x) + x_c^\top K_I \dot{x}_c \\ &= -y^\top(K_P y + K_I x_c + K_D \dot{y}) + y^\top K_D \dot{y} + x_c^\top K_I y \\ &= -\|y\|^2_{K_P} \leq 0 \end{aligned} \quad (24)$$

where we used the passivity property (9) in the first inequality and replaced (4) in the second row. A La Salle-based analysis [15] allows us to establish from (24) some properties of the system trajectories, for instance to conclude that $y(t) \to 0$. However, to prove Lyapunov stability, it is necessary to construct a Lyapunov function for the closed-loop system, which is done finding a function $H_d : \mathbb{R}^n \to \mathbb{R}$ such that

$$U(x, x_c) \equiv H_d(x). \quad (25)$$

In view of (24) and (25), we have that $H_d(x(t))$ is a nonincreasing function; therefore it will be a *bona fide* Lyapunov function if we can ensure it is *positive definite*.

It is clear from (23) that to satisfy the identity (25), it is necessary to be able to express x_c as a function of x. Not surprisingly this is tantamount to finding a *first integral* for the system dynamics that, in its turn, involves the solution of a partial differential equation (PDE). This fact is summarized in the following proposition.

Proposition 4: Consider the pH system (1), (2), (7), (8) with $\dot{x}_c = y$. Assume that there exist mappings $w(x)$ and $D(x)$ such that the PDE

$$\begin{bmatrix} (\nabla H(x))^\top F^\top(x) \\ g^\top(x) \end{bmatrix} \nabla\gamma(x) = \begin{bmatrix} (\nabla H(x))^\top [g(x) + 2\phi^\top(x)w(x)] \\ w^\top(x)w(x) - D(x) \end{bmatrix} \tag{26}$$

admits a solution $\gamma : \mathbb{R}^n \to \mathbb{R}^m$. Then

$$x_c = \gamma(x) + \kappa \tag{27}$$

for some $\kappa \in \mathbb{R}$. Consequently, the identity (25) holds with the *shaped* energy function

$$H_d(x) = H(x) + \frac{1}{2}\|h(x)\|^2_{K_D} + \frac{1}{2}\|\gamma(x) + \kappa\|^2_{K_I}. \tag{28}$$

Proof: The proof is established showing that (26) implies

$$y = \dot{\gamma}. \tag{29}$$

Consequently, using $\dot{x}_c = y$ and integrating, we get (27) with

$$\kappa := x_c(0) - \gamma(x(0)).$$

Replacing (1) and (8) in (29) yields

$$[g(x) + 2\phi^\top(x)w(x)]^\top \nabla H(x) + [w^\top(x)w(x) + D(x)]u$$
$$= [\nabla\gamma(x)]^\top [F(x)\nabla H(x) + g(x)u].$$

The proof is completed equating the terms dependent and independent of u, respectively. ■

B. First Integrals Versus Casimir Functions

To the best of the authors' knowledge, Proposition 4 is the first time that it is suggested to shape the energy of the system generating *first integrals*. In the literature of pH systems, the energy shaping is usually achieved generating Casimir functions, which are quantities that are conserved by the open-loop pH system *for all* energy functions $H(x)$ [20], [30]. These functions have the very interesting property that they are solely determined by the interconnection and damping matrices. This makes the Casimir functions physically appealing and endows them with a very nice geometric interpretation, see [30, Sec. 6.4].

In the context of this paper, Casimir-like functions are the solutions of the PDE

$$\begin{bmatrix} F^\top(x) \\ g^\top(x) \end{bmatrix} \nabla\gamma(x) = \begin{bmatrix} g(x) + 2\phi^\top(x)w(x) \\ w^\top(x)w(x) - D(x) \end{bmatrix}. \tag{30}$$

Comparing with the PDE (26), we notice the absence of the term $\nabla H(x)$ in the first set of equations. Clearly, any solution $\gamma(x)$ of (30) is also a solution of (26), but the converse is not true. Whence, the set of solutions of (30) is *strictly contained* in the set of solutions of (26). An example that illustrates this point is given in Section VII.

An important advantage of considering the PDE (30)—instead of (26)—is that it is possible to give *integrability* conditions on the pH system parameters such that the PDE reduces to a simple integration. As reported in [2], a particularly simple condition is obtained when the matrix $F(x)$ is full rank. The following proposition extends the result of [2] to the case when $F(x)$ *is not full rank*.

Proposition 5: Consider the pH system (1) verifying conditions (14) and (15). Assume the Jacobian of the mappings $F^\dagger(x)g_i(x)$, with $g_i : \mathbb{R}^n \to \mathbb{R}^n,\ i = 1, \ldots, m$, the columns of $g(x)$, satisfy the symmetry condition

$$\nabla[F^\dagger(x)g_i(x)] = \left(\nabla[F^\dagger(x)g_i(x)]\right)^\top. \tag{31}$$

Then

$$\gamma(x) = -\int_0^1 [F^\dagger(sx)g(sx)]^\top x ds + \gamma(0)$$

is a solution of (30) with the parameters $w(x)$ and $D(x)$ given in (16).

Proof: For brevity, the argument x has been left out throughout the proof. Recalling Poincare's Lemma—cf., [15, Exercise 4.5]—it is known that (31) is equivalent to the existence of a mapping $\gamma : \mathbb{R}^n \to \mathbb{R}^m$ such that

$$\nabla\gamma = -F^\dagger g. \tag{32}$$

Premultipying by $F^\top$, we get

$$F^\top \nabla\gamma = -F^\top F^\dagger g = g + 2\phi^\top w \tag{33}$$

where the second identity is obtained invoking the seventh equation of (20). This proves that the first equation of (30) is verified.

It only remains to show that w and D given in (16) verify

$$w^\top w + D = (\nabla\gamma)^\top g. \tag{34}$$

Toward this end, we note that using (17) and (18), we have

$$g = F^\top (F^\dagger)^\top F F^\dagger g = F F^\dagger g. \tag{35}$$

Therefore, we can rewrite (34) as

$$w^\top w + D = (\nabla\gamma)^\top F F^\dagger g. \tag{36}$$

Then, replacing (the transpose of) (33) into (36), we obtain

$$w^\top w + D = (g + 2\phi^\top w)^\top F^\dagger g. \tag{37}$$

Computing the symmetric part of (37) and recalling (14), we get

$$\begin{aligned}
&w^\top w \\
&= \frac{1}{2}[g^\top F^\dagger g + g^\top (F^\dagger)^\top g] + w^\top \phi F^\dagger g + g^\top (F^\dagger)^\top \phi^\top w \\
&= \frac{1}{2}[g^\top (F^\dagger)^\top F^\top F^\dagger g + g^\top (F^\dagger)^\top F F^\dagger g] + w^\top \phi F^\dagger g \\
&\quad + g^\top (F^\dagger)^\top \phi^\top w \\
&= \frac{1}{2}[g^\top (F^\dagger)^\top (F^\top + F) F^\dagger g] + w^\top \phi F^\dagger g + g^\top (F^\dagger)^\top \phi^\top w \\
&= -g^\top (F^\dagger)^\top \mathcal{R} F^\dagger g + w^\top \phi F^\dagger g + g^\top (F^\dagger)^\top \phi^\top w \\
&= -g^\top (F^\dagger)^\top \phi^\top \phi F^\dagger g + w^\top \phi F^\dagger g + g^\top (F^\dagger)^\top \phi^\top w
\end{aligned}$$

where (35) is used in the second equation. A simple calculation shows that the equation above is equivalent to

$$(w - \phi F^\dagger g)^\top (w - \phi F^\dagger g) = 0$$

whose only solution is

$$w = \phi F^\dagger g.$$

Replacing the latter into (37) and grouping terms, we obtain

$$D = -g^\top (F^\dagger)^\top \mathcal{J} F^\dagger g.$$

■

Derivations similar to the ones carried out in Proposition 5 are reported in [30, Sec. 7.1] where, following the construction of [16], new passive outputs—called "alternate" in [30]—are used for CbI. There is a relation also with input–output Hamiltonian systems with dissipation studied in [32], for which the integrability condition (31) is implicitly assumed. See these references for further details.

C. Enlarging the Class via Input Change of Coordinates

An additional contribution of this paper is the proof that the class of pH systems for which (30) is solvable can be enlarged via an *input and output* change of coordinates. As far as we know, this is the first time that this kind of modification is used for the design of PBCs.

Toward this end, we introduce a *full-rank* matrix $M : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ and define the new input–output pair

$$\bar{u} := M^{-1}(x)u, \ \bar{y} := M^\top (x) y. \tag{38}$$

It is clear that the power balance inequality is preserved for these new port variables, that is

$$\dot{H} \le u^\top y = \bar{u}^\top \bar{y}.$$

For ease of presentation, we restrict ourselves to the case of full rank $F(x)$, the extension to the nonfull rank case being straightforward. In this case, the passive output of interest is the power-shaping output given in (11). The new output is then

$$\bar{y} = -M^\top (x) g^\top (x) F^{-\top}(x)\dot{x}. \tag{39}$$

The key question of existence of a mapping $\gamma(x)$ such that (29) holds now takes the form $\bar{y} = \dot{\gamma}$.

Similarly to Proposition 5 (with $F^\dagger(x) = F^{-1}(x)$), it is clear that imposing the symmetry condition on the Jacobians of the new mappings $F^{-1}(x)[g(x)M(x)]_i$, where $[g(x)M(x)]_i$ is the ith column of $g(x)M(x)$, guarantees the existence of $\gamma(x)$. The key question is then: Under which conditions does there exists a full rank $M(x)$ such that the required conditions are satisfied? The answer to this question is provided in the following proposition.

Proposition 6: Define a mapping $\Lambda : \mathbb{R}^n \to \mathbb{R}^{n\times(n-m)}$ verifying

C1) $\text{rank}\{\Lambda(x)\} = n - m$.

C2)

$$g^\top (x) F^{-\top}(x) \Lambda(x) = 0. \tag{40}$$

There exists a full-rank matrix $M : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ such that

$$-F^{-1}(x) g(x) M(x) = \nabla \gamma(x)$$

where $\gamma : \mathbb{R}^n \to \mathbb{R}^m$, if and only if for all $1 \le i, j \le n - m$

$$\text{rank}\left\{\left[\Lambda(x) \ \vdots\ [\Lambda_i(x), \Lambda_j(x)]\right]\right\} = n - m \tag{41}$$

where $\Lambda_i(x)$ is the ith column of $\Lambda(x)$ and $[\cdot,\cdot]$ is the standard Lie bracket [29].

Proof: The proof proceeds as follows. First, recall the following version of Frobenius Theorem reported in [34, Th. 7.2.24]. Given the $n - m$ linearly independent vectors $\Lambda_i(x)$, there exist functions $\gamma_i : \mathbb{R}^n \to \mathbb{R}$ such that

1) the vectors $\nabla\gamma_i(x)$ are linearly independent, and
2)

$$[\nabla\gamma_i(x)]^\top \Lambda_j(x) = 0, \ i = 1, \ldots, n - m; j = 1, \ldots, m$$

if and only if (41) is satisfied.

The proof is completed noting that, since $M(x)$ is full rank, we have

$$\ker\left\{M^\top (x) g^\top (x) F^{-\top}(x)\right\} = \ker\left\{g^\top (x) F^{-\top}(x)\right\}$$

and recalling, from (40), that the columns of $\Lambda(x)$ are a basis for this space. ■

VI. Lyapunov Stability Analysis

A first step for Lyapunov stabilization of a desired constant state is, obviously, to ensure that it is an *equilibrium* of the closed loop. It turns out that to establish this fact it is necessary to invoke the condition of solvability of the PDE (26) given in Proposition 4 and to fix, accordingly, the initial condition of the integrator in the PID-PBC. Also, since solvability of the PDE ensures the existence of a function of the pH systems state (28) that is nonincreasing along the solutions of the closed-loop system, it will be a *bona fide* Lyapunov function if we can ensure it is *positive definite*. In this section, these two issues are addressed.

A. Equilibrium Assignment

In the following proposition, we prove that, adequately selecting the initial conditions of the controller, we can assign the desired equilibrium to the closed-loop system.

Proposition 7: Consider the pH system (1), (2), (8) in closed loop with the PID-PBC (4), with the mappings $w(x)$ and $D(x)$ such that the PDE (26) admits a solution $\gamma(x)$. Fix an equilibrium $x^\star \in \mathcal{E}$ and

$$x_c^0 = \gamma(x(0)) - \gamma^\star + K_I^{-1}[(g^\star)^\top g^\star]^{-1}(g^\star)^\top F^\star \nabla H^\star. \quad (42)$$

Then, there exists $x_c^\star \in \mathbb{R}^m$ such that $(x, x_c) = (x^\star, x_c^\star)$ is an equilibrium point of the closed-loop system.

Proof: First, notice that (26) ensures (29), which may be rewritten as

$$y = [\nabla\gamma(x)]^\top \dot{x}.$$

Therefore, at the equilibrium, y equals zero. Consequently, the control (4) at the equilibrium becomes

$$\begin{aligned} u^\star &:= u|_{(x^\star, x_c^\star)} \\ &= -K_I x_c^\star \\ &= -K_I[\gamma^\star + x_c^0 - \gamma(x(0))] \end{aligned} \quad (43)$$

where we have used (27) to get the third identity. On the other hand, since $x^\star \in \mathcal{E}$, we have that

$$\dot{x} = 0 \;\Leftrightarrow\; u^\star = -[(g^\star)^\top g^\star]^{-1}(g^\star)^\top F^\star \nabla H^\star. \quad (44)$$

The proof is completed replacing the expression above in (43). ■

B. Construction of the Lyapunov Function

It may be argued that Proposition 7 imposes a particular initial condition to the controller state x_c making the result "trajectory dependent" and somehow fragile—see [23, Remark 10 and the corresponding sidebar]. Indeed, implementing the PID-PBC with the dynamic extension x_c, it is necessary to ensure that $x_c(t) \equiv \gamma(t) + \kappa$, $\forall t \geq 0$, which requires the particular selection of the initial conditions of x_c. Interestingly, this restriction is avoided if we exploit the generation of the first integral to implement the controller *without dynamic extension*, that is, as a static state feedback. This result is summarized in the following proposition, which is the main stabilization result of the paper.[3]

Proposition 8: Consider the pH system (1), (2), (8) with the mappings $w(x)$ and $D(x)$ such that the PDE (26) admits a solution $\gamma(x)$. Fix an equilibrium $x^\star \in \mathcal{E}$ and define the PID-PBC as

$$u = -K_P y - K_I[\gamma(x) - \gamma^\star] - K_D \dot{y} + u^\star \quad (45)$$

where $u^\star$ is given in (44). Assume that $H_d(x)$, given in (28), with

$$\kappa = -(\gamma^\star + K_I^{-1} u^\star) \quad (46)$$

verifies

$$x^\star = \arg\min H_d(x). \quad (47)$$

1) The closed-loop system has a *stable* equilibrium at $x^\star$ with Lyapunov function (28).
2) The equilibrium is *asymptotically* stable if the signal y is a *detectable* output for the closed-loop system.
3) The stability properties are *global* if $H_d(x)$ is radially unbounded.

Proof: The proof that $x^\star$ is an equilibrium of the closed loop follows noting, as in the proof of Proposition 7, that y equals zero and u in (45) equals $u^\star$ when they are evaluated at the equilibrium.

Now

$$\begin{aligned} \dot{H}_d &\leq y^\top u + h^\top(x) K_D \dot{h}(x) + [\gamma(x) + \kappa]^\top K_I \dot{\gamma} \\ &= -y^\top[K_P y + K_I(\gamma(x) + \kappa) + K_D \dot{y}] \\ &\quad + y^\top K_D \dot{y} + [\gamma(x) + \kappa]^\top K_I y \\ &= -\|y\|_{K_P}^2 \leq 0 \end{aligned} \quad (48)$$

where we used the passivity property (9) in the first inequality and replaced (45) with (46) and used (29) in the second row. From (48), it follows that $H_d(x(t))$ is a nonincreasing function that, moreover, is positive definite because of assumption (47). The proof is completed invoking standard Lyapunov stability theory [15]. ■

Following the derivations of [2], it is possible to prove that $\nabla H_d^\star = 0$ if y is the generalized power-shaping output defined in (16). Therefore, in that case it only remains to verify

$$\nabla^2 H_d^\star > 0 \quad (49)$$

to ensure (47).

VII. Examples

In this section, we present some examples that illustrate the main results of this paper.

A. On the Use of Input Change of Coordinates for Energy Shaping

Consider a pH system with the standard power-shaping output, that is

$$\begin{aligned} \dot{x} &= F\nabla H(x) + g(x)u \\ y &= -g^\top(x) F^{-\top}\dot{x} \end{aligned}$$

with $H(x) = \frac{1}{2}|x|^2$ and

$$F = \begin{bmatrix} -1 & 1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix}, \; g(x) = \begin{bmatrix} -x_3 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Hence

$$-F^{-1} g(x) = \begin{bmatrix} 1 & 0 \\ x_3 & 1 \\ -1 & 1 \end{bmatrix}.$$

It is easy to see that the vectors $F^{-1} g_i(x)$ do not satisfy the symmetry condition (31). Therefore, the PID-PBC design of [2] is not applicable. We investigate now the possibility of extending it with the input–output change of coordinates (38), as proposed in Section V-C.

Since the kernel of $g^\top(x) F^{-\top}(x)$ is of dimension one, the condition (41) is automatically satisfied and thus Proposition

[3] It is easy to see that a similar result holds for the dynamic realization of the PID-PBC (4) but is omitted for brevity.

6 ensures the *existence* of the required full-rank mapping $M : \mathbb{R}^3 \to \mathbb{R}^{2\times 2}$ that defines input–output change of coordinates. To compute $M(x)$, we invoke again Poincare's Lemma and solve the PDEs

$$\nabla(F^{-1}g(x)m_i(x)) = \left[\nabla(F^{-1}g(x)m_i(x))\right]^\top, \; i = 1, 2$$

where $m_i : \mathbb{R}^3 \to \mathbb{R}^2$ are the columns of $M(x)$. A simple solution to these PDEs is given by

$$M(x) = \begin{bmatrix} 1 & 0 \\ x_2 & 1 \end{bmatrix}$$

that yields

$$-F^{-1}g(x)M(x) = \begin{bmatrix} 1 & 0 \\ x_2 + x_3 & 1 \\ -1 + x_2 & 1 \end{bmatrix}. \tag{50}$$

Integrating the columns of (50), we get the desired mapping

$$\gamma(x) = \begin{bmatrix} x_1 + x_2x_3 - x_3 + \frac{1}{2}x_2^2 \\ x_2 + x_3 \end{bmatrix} \tag{51}$$

that satisfies $\bar{y} = \dot{\gamma}$.

We proceed now with the design of the PID-PBC with the new input $\bar{u}$ (38) and the new output $\bar{y}$ (39), that is

$$\begin{aligned} \bar{u} &= -K_P\bar{y} - K_I x_c \\ \dot{x}_c &= \bar{y}, \; x_c(0) = x_c^0 \in \mathbb{R}^3. \end{aligned} \tag{52}$$

As discussed in Proposition 7, x_c^0 must be selected as (42) to assign the desired equilibrium to the closed loop. To select the latter, we need to define first the assignable equilibrium set (3). Hence, we compute $g^\perp(x) = [1 \;\; x_3 \;\; 0]$ and define

$$\mathcal{E} = \left\{x \in \mathbb{R}^3 \mid x_2 - x_3 - x_1(x_3 + 1) = 0\right\}.$$

Notice that after the input change of coordinates, x_c^0 becomes

$$x_c^0 = \gamma(x(0)) - \gamma^\star + K_I^{-1}(M^\star)^{-1}[(g^\star)^\top g^\star]^{-1}(g^\star)^\top F^\star \nabla H^\star. \tag{53}$$

In the sequel, we fix $x^\star = [1 \;\; 3 \;\; 1]^\top$ and get the formula

$$x_c^0 = \begin{bmatrix} x_1(0) + x_2(0)x_3(0) - x_3(0) + \frac{1}{2}x_2^2(0) \\ x_2(0) + x_3(0) \end{bmatrix} + \begin{bmatrix} -\frac{1}{K_I} - 7.5 \\ \frac{1}{K_I} - 4 \end{bmatrix} \tag{54}$$

to assign the equilibrium.

The function $H_d(x)$ given in (28) takes the form

$$\begin{aligned} H_d(x) = \frac{1}{2}|x|^2 + \frac{k_I}{2}\Bigg[\left(x_2 + x_3 + \frac{1}{k_I} - 4\right)^2 \\ + \left(x_1 - x_3 + x_2x_3 - \frac{1}{k_I} + \frac{x_2^2}{2} - 7.5\right)^2\Bigg] \end{aligned} \tag{55}$$

where we have used (46) and, for simplicity, we have taken $K_I = k_I I_2$ and $K_P = k_P I_2$, where k_I and k_P are positive constants. We proceed now to verify the condition (47) of Proposition 8 that ensures $H_d(x)$ is positive definite. Since we are dealing with the power-shaping output, the condition $\nabla H_d^\star = 0$ is automatically satisfied [2]. To complete the design, it only remains to check the Hessian condition (49). Some straightforward calculations

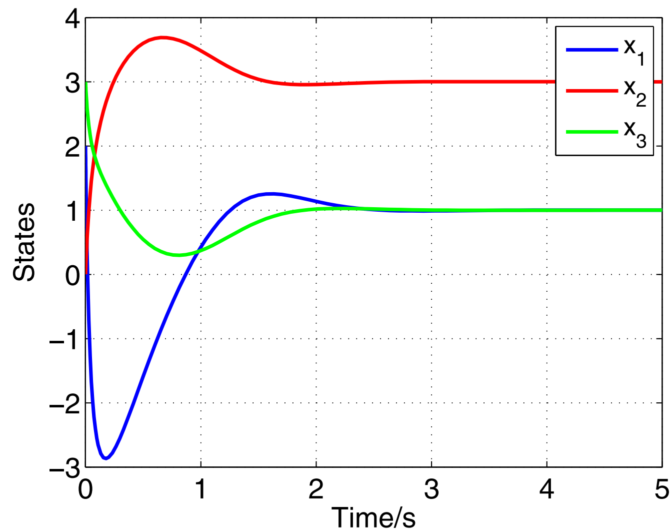


Fig. 2. Simulation results with $K_I = 20I_2$ and $K_P = 5I_2$.

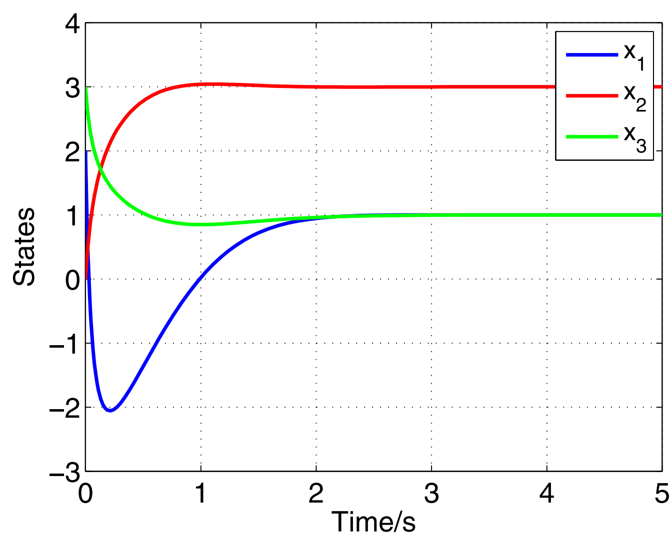


Fig. 3. Simulation results with $K_I = 20I_2$ and $K_P = 10I_2$.

yield

$$\nabla^2 H_d^\star = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + k_I \begin{bmatrix} 1 & 4 & 2 \\ 4 & 17 - \frac{1}{k_I} & 9 - \frac{1}{k_I} \\ 2 & 9 - \frac{1}{k_I} & 5 \end{bmatrix}.$$

The positive definiteness of the second right-hand matrix can be checked via the Schur complement, whence the aforementioned matrix is positive definite if and only if $k_I > 1$. Hence, choosing any $k_I > 1$ and $k_P > 0$, guarantees $(x^\star, x_c^\star)$ is a stable equilibrium of the closed-loop system. The proof of asymptotic stability, being a little lengthy, is given in the appendix.

In Figs. 2 and 3, some simulation results of the PID-PBC (52), (54) for $x(0) = \text{col}(2, 0, 3)$ and two different PI gains are shown. We notice that, as predicted by the theory, the transient performance is improved increasing K_P.

As a final observation, we note that, invoking Proposition 8, we can write the PID-PBC as a static state feedback

$$\bar{u} = [I - K_P M^\top(x) g^\top(x) F^{-\top} g(x) M(x)]^{-1} \times [-K_I(\gamma(x)+\kappa) + K_P M^\top(x) g^\top(x) F^{-\top} F \nabla H(x)]$$

with κ given by (46).

B. First Integrals Versus Casimir Functions and the Use of General y Versus the Power-Shaping Output (11)

In this section, we give an example of a pH system that does not admit Casimir functions, therefore, cannot be stabilized with CbI, but is stabilizable via a PID-PBC using first integrals. Moreover, it is shown that the symmetry condition (31) is not satisfied; therefore, the power-shaping output (11) cannot be used in the PID-PBC. On the other hand, we prove that there exists a choice of $w(x)$ and $D(x)$ to generate a *new output* such that the key identity $y = \dot{\gamma}$ holds for a suitable mapping $\gamma(x)$.

Proposition 9: Consider the system (1) with

$$H(x) = \frac{1}{2}(x_1+x_2)^2 + \frac{1}{2}x_3^2$$

$$\mathcal{J} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathcal{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, g = \begin{bmatrix} x_1 \\ 0 \\ 1 \end{bmatrix}.$$

The control objective is to stabilize the point $x^\star = (0, 0, x_3^\star)$, with $x_3^\star < 0$.

1) The system is not stabilizable via CbI.
2) The system is not stabilizable via PID-PBC with the power-shaping output (11).
3) The system is stabilizable with the PID-PBC using the output

$$y = (g + 2\phi^\top w)^\top \nabla H + w^\top w u \tag{56}$$

with

$$w = \begin{bmatrix} x_1 \\ n0 \\ -1 \end{bmatrix}, \ \phi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{57}$$

Proof: To establish the claim (i), we first notice that the $\mathcal{R}\nabla H^\star \neq 0$; therefore, the system is constrained by the dissipation obstacle [20], [36] and is, therefore, not stabilizable with CbI using the natural output $y = g^\top(x)\nabla H(x)$. Moreover, CbI with the power-shaping output (11) is also not applicable because the interconnected system does not admit Casimir functions. Indeed, we have

$$-F^{-1}g(x) = \begin{bmatrix} 0 \\ -x_1 \\ 1 \end{bmatrix}$$

which is clearly not integrable.

The fact above also proves the claim (ii) because, as stated in Proposition 5, this is also the condition for the existence of the function $\gamma : \mathbb{R}^3 \to \mathbb{R}$ such that $\dot{\gamma} = y$ for the power-shaping output.

To prove claim (iii), we replace (57) in (56) to get

$$y = x_1(x_1+x_2) - x_3 + (x_1^2+1)u.$$

Some simple calculations prove that the function

$$\gamma(x) := \frac{1}{2}x_1^2 + x_3$$

satisfies $\dot{\gamma} = y$.

It only remains to prove that the function

$$H_d(x) = H(x) + \frac{1}{2}K_I[\gamma(x)+\kappa]^2$$

with κ computed from (46) as

$$\kappa := -x_3^\star - \frac{x_3^\star}{K_I}$$

satisfies the condition (47). This is easily verified computing

$$\nabla H_d = \begin{bmatrix} x_1+x_2 \\ x_1+x_2 \\ x_3 \end{bmatrix} + K_I(\gamma+\kappa)\begin{bmatrix} x_1 \\ 0 \\ 1 \end{bmatrix}$$

$$\nabla^2 H_d = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + K_I \begin{bmatrix} x_1^2 & 0 & x_1 \\ 0 & 0 & 0 \\ x_1 & 0 & 1 \end{bmatrix} + K_I(\gamma+\kappa)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and evaluating them at $x^\star$ to get

$$\nabla H_d^\star = \begin{bmatrix} 0 \\ 0 \\ x_3^\star \end{bmatrix} - x_3^\star \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\nabla^2 H_d^\star = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1+K_I \end{bmatrix} - x_3^\star \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} > 0$$

where we have used the fact that $x_3^\star < 0$ to obtain the last inequality.

It is possible to show that convergence to the equilibrium cannot be established with the usual argument of detectability of y. First, we write the PID-PBC (45) as

$$u = -K_I(\gamma+\kappa) - K_P y. \tag{58}$$

Now, we compute the exact expression of $\dot{H}_d$ as

$$\begin{aligned} \dot{H}_d &= -|\phi\nabla H + wu|^2 + yu + yK_I(\gamma+\kappa) \\ &= -|\phi\nabla H + wu|^2 + y(u + K_I(\gamma+\kappa)) \\ &= -|\phi\nabla H + wu|^2 - K_P y^2 \end{aligned}$$

where we have used (58) in the last identity.

The proof is completed recalling the Barbashin–Krasovskii's Theorem [15] and proving that

$$\dot{H}_d = 0 \Longrightarrow x = x^\star.$$

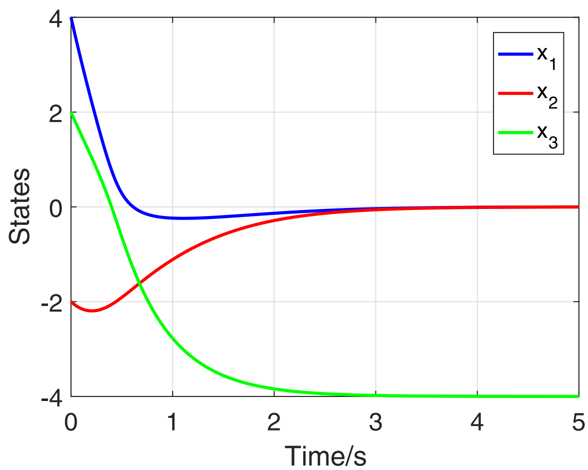


Fig. 4. Simulation results of the example of Proposition 9.

Indeed, evaluating the control (58) at $y = 0$ and replacing it in $\phi\nabla H(x) + w(x)u = 0$, we get the equations

$$\begin{aligned} -x_1 K_I\left(\frac{1}{2}x_1^2 + x_3 + \kappa\right) &= 0 \\ x_3 + K_I\left(\frac{1}{2}x_1^2 + x_3 + \kappa\right) &= 0. \end{aligned} \tag{59}$$

Combining these equations, we obtain

$$\frac{1}{K_I + 1}x_1\left(\frac{1}{2}x_1^2 - x_3^\star - \frac{1}{K_I}x_3^\star\right) = 0$$

whose only solution is $x_1 = 0$. Replacing the latter in the second equation of (59), we get

$$x_3 = \frac{K_I}{1 + K_I}\left(x_3^\star + \frac{1}{K_I}x_3^\star\right) \Longrightarrow x_3 = x_3^\star.$$

Finally, notice that the dynamics of x_1 is given by

$$\dot{x}_1 = x_1 + x_2 + x_1 u$$

therefore

$$x_1 = 0 \Rightarrow \dot{x}_1 = 0 \Rightarrow x_2 = 0$$

completing the proof of asymptotic stability. ∎

Fig. 4 shows the simulation results for $x_3^\star = -4$ with initial conditions $x_0 = (4, -2, 2)$ and choosing the gains as $k_I = 3$ and $k_P = 1$.

C. On the Limitations of PID-PBC

As expected, the proposed PID-PBC is unable to stabilize all linear time-invariant (LTI) systems. The example below shows that this is true even for controllable systems. This should be contrasted with other PBCs—like IDA-PBC—which are applicable to all stabilizable LTI systems [30]. A similar example may be found in [2].

Proposition 10: Consider the single input, LTI, pH system (1) with energy function $H(x) = \frac{1}{2}x^\top Qx$ and

$$F := \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}, Q := \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \; g := \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

in closed loop with the PID-PBC (4), with $K_P = K_D = 0$ and y defined by (2) and (8) with $D = 0$. For all values of the free parameter w and all controller gains K_I, the closed loop is unstable.

Proof: First, notice that the system matrix is given by

$$FQ = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$$

thus the system is controllable. Now, the dissipation matrix is

$$\mathcal{R} = -\frac{1}{2}(F + F^\top) = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}.$$

Hence, $\phi := \mathrm{col}(0, \sqrt{3})$. The passive output (8) takes the form

$$y = -(1 + 2\sqrt{3}w)x_2 + w^2 u.$$

The closed-loop system is given by

$$\begin{bmatrix} \dot{x} \\ \dot{x}_C \end{bmatrix} = \begin{bmatrix} FQ & -gK_I \\ (g + 2\phi^\top w)^\top Q & -w^2 K_I \end{bmatrix}\begin{bmatrix} x \\ x_C \end{bmatrix}.$$

The characteristic polynomial is

$$s^3 + (K_I w^2 - 3)s^2 + [2 - 3K_I w^2 - (1 + 2\sqrt{3}w)K_I]s + 2K_I w^2.$$

A simple Routh–Hurwitz analysis shows that this polynomial is unstable for all values of K_I and w. ∎

VIII. Conclusion

We have identified in this paper a class of pH systems for which any assignable equilibrium can be rendered stable via a PID controller. Given the popularity and simplicity of this controller, the result is of practical interest. It should be underscored that there is a total freedom in the choice of the mappings $w(x)$ and $D(x)$ that define the input to the PID—the key requirement being, as usual, the solvability of the PDE (26).

The relationship between the PID-PBC proposed here and total energy-shaping controllers for the case of mechanical systems has been investigated in [8]. In particular, it has been shown that the PID-PBCs proposed in [7] and [26] are total energy-shaping controllers with generalized forces, that is, allowing for a target dynamics that includes forces, which are more general than the gyroscopic ones. It is interesting to note that in [26], it is shown that the *transient performance* of the PID-PBC may be better than the one of total energy-shaping controllers, rendering it even more attractive.

Current research is under way to extend the realm of applicability of the PID-PBC by checking integrability of the vectors $F^\dagger(x)g_i(x) + z_i(x)$, where the vectors $z_i(x)$ are *free* but should satisfy

$$\begin{aligned} [\nabla H(x)]^\top F^\top(x) z_i(x) &= 0 \\ g^\top(x) z_i(x) &= 0. \end{aligned}$$

Clearly, the inclusion of the vectors $z_i(x)$ relaxes the condition (31) of Proposition 5.

APPENDIX
PROOF OF ASYMPTOTIC STABILITY OF SECTION VII-A

To prove asymptotic stability of the equilibrium $x^\star$, we invoke La Salle's invariance principle [15]. Toward this end, first note that the function $H_d(x)$ given in (55) is radially unbounded, therefore, all trajectories are bounded. The proof is completed if we can show that

$$\dot{H}_d \equiv 0 \implies x = x^\star.$$

After introducing the input and output change of coordinates (38), the pH system can be written as

$$\bar{\Sigma}: \begin{cases} \dot{x} = & F\nabla H(x) + g(x)M(x)\bar{u} \\ \bar{y} = & -M^\top(x)g^\top(x)F^{-\top}\dot{x}. \end{cases} \tag{60}$$

The power balance equation is

$$\dot{H} = \dot{x}^\top F^{-1}\dot{x} + \bar{y}^\top \bar{u}$$

which, using

$$F^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

yields

$$\dot{H} = -\dot{x}_3^2 + \bar{y}^\top \bar{u}.$$

Hence

$$\dot{H}_d = -\dot{x}_3^2 - \|\bar{y}\|^2_{K_P}.$$

Clearly

$$\dot{H}_d = 0 \Longleftrightarrow \dot{x}_3 = 0,\ \bar{y} = 0. \tag{61}$$

Now, after some straightforward calculations, (60) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 - x_3 - x_3\bar{u}_1 \\ -x_1 + \bar{u}_1 \\ -x_1 - x_3 + x_2\bar{u}_1 + \bar{u}_2 \end{bmatrix}$$

$$\bar{y} = \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \end{bmatrix} = \begin{bmatrix} 1 & x_2 + x_3 & x_2 - 1 \\ 0 & 1 & 1 \end{bmatrix} \dot{x}. \tag{62}$$

From (61) and (62), we have that $\dot{x}_2 = 0$ and $\dot{x}_1 = 0$, therefore, x is constant. We will prove now that this constant is $x^\star$.

From the equation above (62) and $\dot{x}_2 = \dot{x}_3 = 0$, we get

$$\bar{u} = \begin{bmatrix} x_1 \\ x_1 + x_3 - x_1 x_2 \end{bmatrix}. \tag{63}$$

On the other hand, (52) together with $\bar{y} = 0$ and $x_c = \gamma(x) + \kappa$ yields

$$\bar{u} = -k_I \begin{bmatrix} x_1 - x_3 + x_2 x_3 - \frac{1}{k_I} + \frac{x_2^2}{2} - 7.5 \\ x_2 + x_3 + \frac{1}{k_I} - 4 \end{bmatrix} \tag{64}$$

where $\gamma(x)$ is given in (51) and κ is calculated from (46) and (53). Setting (63) equal to (64) gives

$$\begin{cases} -k_I(x_1 - x_3 + x_2 x_3 - \frac{1}{k_I} + \frac{x_2^2}{2} - 7.5) = x_1 \\ -k_I(x_2 + x_3 + \frac{1}{k_I} - 4) = x_1 + x_3 - x_1 x_2 \end{cases}$$

which is equivalent to

$$\begin{cases} -k_I(x_1 - x_3 + x_2 x_3 + \frac{x_2^2}{2} - 7.5) = x_1 - 1 \\ -k_I(x_2 + x_3 - 4) = x_1 + x_3 - x_1 x_2 + 1. \end{cases} \tag{65}$$

Since (65) must hold for all $k_I > 1$, we obtain

$$x_1 = 1 \tag{66}$$

from the first equation, and

$$\begin{cases} x_2 + x_3 - 4 = 0 \\ x_1 + x_3 - x_1 x_2 + 1 = 0 \end{cases} \tag{67}$$

from the second one. It is easy to see that the only solution of (66) and (67) is $x = x^\star = (1, 3, 1)$.

ACKNOWLEDGMENT

The authors are grateful to A. van der Schaft for the help in the proof of Proposition 6 and to R. Cisneros and J. G. Romero for many useful discussions on the topic.

REFERENCES

[1] K. J. Astrom and T. Hagglund, "The future of PID control," *Control Eng. Pract.*, vol. 9, no. 11, pp. 1163–1175, 2001.

[2] L. Borja, R. Cisneros, and R. Ortega, "A constructive procedure for energy shaping of port-Hamiltonian systems," *Automatica*, vol. 72, pp. 230–234, 2016.

[3] F. Castaños and R. Ortega, "Energy-balancing passivity-based control is equivalent to dissipation and output invariance," *Syst. Control Lett.*, vol. 58, no. 8, pp. 553–560, 2009.

[4] F. Castaños, B. Jayawardhana, R. Ortega, and E. García-Canseco, "Proportional plus integral control for set point regulation of a class of nonlinear RLC circuits," *Circuits, Syst. Signal Process.*, vol. 28, no. 4, pp. 609–615, 2009.

[5] R. Cisneros, M. Pirro, G. Bergna, R. Ortega, G. Ippoliti, and M. Molinas, "Global tracking passivity-based PI control of bilinear systems and its application to the boost and modular multilevel converters," *Control Eng. Pract.*, vol. 43, pp. 109–119, 2015.

[6] A. Donaire and S. Junco, "On the addition of integral action to port-controlled Hamiltonian systems," *Automatica*, vol. 45, pp. 1910–1916, 2009.

[7] A. Donaire *et al.*, "Shaping the energy of mechanical systems without solving partial differential equations," *IEEE Trans. Automat. Control*, vol. 61, no. 4, pp. 1051–1056, Apr. 2016.

[8] A. Donaire, R. Ortega, and J. G. Romero, "Simultaneous interconnection and damping assignment passivity-based control of mechanical systems using dissipative forces," *Syst. Control Lett.*, vol. 94, pp. 118–126, 2016.

[9] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input–Output Properties*. New York, NY, USA: Academic, 1975.

[10] J. Ferguson, R. H. Middleton, and A. Donaire, "Disturbance rejection via control by interconnection of port-Hamiltonian systems," in *Proc. 54th IEEE Conf. Decis. Control*, Osaka, Japan, 2015, pp. 507–512.

[11] V. Duindam, A. Machelli, S. Stramigioli, and H. Bruyninckx, *Geometric Network Modeling and Control of Complex Physical Systems*. New York, NY, USA: Springer-Verlag, 2009.

[12] M. Hernandez-Gomez, R. Ortega, F. Lamnabhi-Lagarrigue, and G. Escobar, "Adaptive PI stabilization of switched power converters," *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 3, pp. 688–698, May 2010.

[13] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.

[14] B. Jayawardhana, R. Ortega, E. Garcia-Canseco, and F. Castaños, "Passivity of nonlinear incremental systems: Application to PI stabilization of nonlinear RLC circuits," *Syst. Control Lett.*, vol. 56, no. 9, pp. 618–622, 2007.

[15] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 2002.

[16] B. Maschke, R. Ortega, and A. Van Der Schaft, "Energy-based Lyapunov functions for forced Hamiltonian systems with dissipation," *IEEE Trans. Automat. Control*, vol. 45, no. 8, pp. 1498–1502, Aug. 2000.

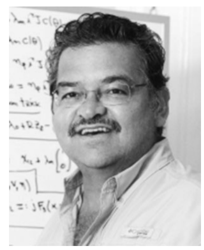
[17] C. Meza, D. Biel, D. Jeltsema, and J. Scherpen, "Lyapunov-based control scheme for single-phase grid-connected PV central inverters," *IEEE Trans. Control Syst. Technol.*, vol. 20, no. 2, pp. 520–529, Mar. 2012.
[18] R. Ortega, A. Van Der Schaft, I. Mareels, and B. Maschke, "Putting energy back in control," *IEEE Control Syst. Mag.*, vol. 21, no. 2, pp. 18–33, Apr. 2001.
[19] R. Ortega, D. Jeltsema, and J. M. A. Scherpen, "Power shaping: A new paradigm for stabilization of nonlinear RLC circuits," *IEEE Trans. Automat. Control*, vol. 48, no. 10, pp. 1762–1767, Oct. 2003.
[20] R. Ortega, A. van der Schaft, F. Castaños, and A. Astolfi, "Control by interconnection and standard passivity-based control of port-Hamiltonian systems," *IEEE Trans. Automat. Control*, vol. 53, no. 11, pp. 2527–2542, Dec. 2008.
[21] R. Ortega and E. Garcia-Canseco, "Interconnection and damping assignment passivity-based control: A survey," *Eur. J. Control*, vol. 10, pp. 432–450, 2004.
[22] R. Ortega and J. Romero, "Robust integral control of port-Hamiltonian systems: The case of non-passive outputs with unmatched disturbances," *Syst. Control Lett.*, vol. 61, no. 1, pp. 11–17, 2012.
[23] R. Ortega and E. Panteley, "Adaptation is unnecessary in $\mathcal{L}_1$-adaptive control," *IEEE Control Syst. Mag.*, vol. 36, no. 1, pp. 47–52, Jan. 2016.
[24] C. Rao and S. Mitra, *Generalized Inverse of Matrices and Its Applications*. New York, NY, USA: Wiley, 1971.
[25] J. G. Romero, A. Donaire, and R. Ortega, "Robust energy shaping control of mechanical systems," *Syst. Control Lett.*, vol. 62, no. 9, pp. 770–780, 2013.
[26] J. G. Romero, R. Ortega, and A. Donaire, "Robust energy shaping control of mechanical systems without solving PDEs and extension to constant speed tracking," *IEEE Trans. Automat. Control*, vol. 61, no. 11, pp. 3551–3556, Jan. 2016.
[27] S. Sanders and G. Verghese, "Lyapunov-based control for switched power converters," *IEEE Trans. Power Electron.*, vol. 7, no. 1, pp. 17–24, Jan. 1992.
[28] R. Sepulchre, M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*. New York, NY, USA: Springer, 1997.
[29] M. Spivak, *A Comprehensive Introduction to Differential Geometry*, 3rd ed. Lombard, IL, USA: Perish, Inc., 1999.
[30] A. van der Schaft, *L_2–Gain and Passivity Techniques in Nonlinear Control*, 3rd ed. Berlin, Germany: Springer, 2016.
[31] A. van der Schaft and D. Jeltsema, *Port-Hamiltonian Systems Theory: An Introductory Overview* (Foundations and Trends in Systems and Control), vol. 1. Hanover, MA, USA: Now Publ. Inc., 2014, pp. 173–378.
[32] A. van der Schaft, "Interconnections of input-output Hamiltonian systems with dissipation," in*Proc. 55th IEEE Conf. Decis. Control*, Las Vegas, NV, USA, Dec. 12–14, 2016, pp. 4686–4691.
[33] A. Venkatraman and A. van der Schaft, "Energy shaping of port-Hamiltonian systems by using alternate passive outputs," in *Proc. Eur. Control Conf.*, Budapest, Hungary, 2009.
[34] M. Vidyasagar, *Nonlinear Systems Analysis*, 2nd ed. Englewood Cliffs, NJ, USA: Prentice-Hall, 1993, pp. 2175–2180.
[35] F. Zhang, H. L. Trentelman, and J. M. A. Scherpen, "Robust cooperative output regulation of heterogeneous Lur'e networks," *Int. J. Robust Nonlinear Control*, 2016, doi: 10.1002/rnc.3725.
[36] M. Zhang, R. Ortega, D. Jeltsema, and H. Su, "Further deleterious effects of the dissipation obstacle in control-by-interconnection of port-Hamiltonian systems," *Automatica*, vol. 61, pp. 227–231, Nov. 2015.

Meng Zhang was born in 1990. He received the Bachelor's degree in engineering from Xi'an Jiaotong University, Xi'an, China, in 2013. He is currently working toward the Ph.D. degree in control science and engineering at Zhejiang University, Hangzhou, China.

His research interests include nonlinear control and passivity-based control.

Pablo Borja was born in Mexico in 1987. He received the B.S. degree in electrical and electronic engineering and the Master's degree in electrical engineering from the National Autonomous University of Mexico, Mexico City, Mexico, in 2011 and 2014, respectively. He is currently working toward the Ph.D. degree in electrical and electronic engineering from the Laboratoire de Signaux et Systèmes (Centrale-Supelec), Gif-sur-Yvette, France.

His current research focuses in the field of nonlinear control and systems, including port-Hamiltonian systems and passivity-based control.



Romeo Ortega (F'99) was born in Mexico. He received the B.Sc. degree in electrical and mechanical engineering from the National University of Mexico, Mexico City, Mexico, Master of Engineering degree in automatic control from the Polytechnical Institute of Leningrad, Saint Petersburg, Russia, and the Docteur D'Etat degree in robust and adaptive control from the Politechnical Institute of Grenoble, Grenoble, France, in 1974, 1978, and 1984, respectively.

He then joined the National University of Mexico, where he worked until 1989. He was a Visiting Professor at the University of Illinois in 1987–1988 and at McGill University in 1991–1992, and a Fellow of the Japan Society for Promotion of Science in 1990–1991. Since June 1992, he has been a member of the French National Research Council (CNRS). He is currently a Directeur de Recherche in the Laboratoire de Signaux et Systèmes (CentraleSupelec), Gif-sur-Yvette, France. He has published three books and more than 300 scientific papers in international journals, with an h-index of 74. He has supervised more than 30 Ph.D. thesis. His current research focuses on the fields of nonlinear and adaptive control, with special emphasis on applications.

Dr. Ortega is an IFAC Fellow since 2016. He has served as a Chairman in several IFAC and IEEE committees and participated in various editorial boards of international journals.

Zhitao Liu received the B.S. degree in information and computing science from Shandong University, Jinan, China, in 2005, and the Ph.D. degree in control science and engineering from Zhejiang University, Hangzhou, China, in 2010.

From 2011 to 2014, he was a Research Fellow in TUM CREATE, Singapore. Since 2015, he has been an Assistant Professor with the Institute of Cyber-Systems and Control, Zhejiang University. His research interests include robust adaptive control, nonlinear control, and battery management systems.

Hongye Su was born in 1969. He received the B.S. degree in industrial automation from Nanjing University of Chemical Technology, Nanjing, China, in 1990, and the M.S. and Ph.D. degrees in control science and engineering from Zhejiang University, Hangzhou, China, in 1993 and 1995, respectively.

From 1995 to 1997, he was a Lecturer with the Department of Chemical Engineering, Zhejiang University, where he was an Associate Professor with the Institute of Advanced Process Control from 1998 to 2000 and where he is currently a Professor with the Institute of Cyber-Systems and Control. His research interests include robust controls, time-delay systems, and advanced process control theory and applications.