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Piezo-damping of light structures: modelling and experimental results

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Abstract

By including piezo-materials into a light structure, it is possible to convert a part of its vibrational energy into electrical energy. If this electrical energy is addressed to a dissipative impedance, a damping effect is obtained.
This paper presents, from modelling results, the performance improvement obtained when the dissipative impedance includes either a negative capacity in order to cancel the capacity of the used piezo-ceramics, or a resonant electrical circuit tuned to the mechanical frequency to be damped.
It is shown how the obtained performances are related to a non-dimensional coupling factor between the used piezo-materials and the structure to be damped.
Piezo-damping is applied to a blade for isostatic mount of satellite equipment and experimental results are presented. The retained technique is of the tuned circuit type. The obtained results emphasise the performance of the piezo-damping, the good correlation with modelling prediction, the low sensitivity to mistuning of the dissipative electrical circuit and its very low power consumption (related to the use of active components for achieving the electrical tuning).

1 Introduction

The simplest form of piezo-damping is achieved by using piezo-materials included in the vibrating structure to be damped and by addressing the corresponding delivered current to a passive electrical resistance. In this case the achievable piezo-damping is strongly limited due to the shunt effect of the electrical capacity of the used piezo-patches versus this resistance.
Then, the efficiency can be strongly improved if the used dissipative impedance also includes either a negative capacity [1] cancelling this shunt effect, or an inductance allowing to achieve an electrical resonant circuit tuned to the mechanical frequency to be damped. The achievable damping effect by using these different kinds of dissipative circuits are presented in the following section.

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2 Modelling of a piezo-damped structure with different dissipative circuits

2.1 Piezo-damping implementation on a vibrating structure and corresponding modelling

Let us consider a vibrating structure S with N embedded or surface mounted piezo-material patches P. These patches are supposed of identical shape and properties. They are wired in parallel and linked to a dissipative electrical impedance Z (see figure 1)

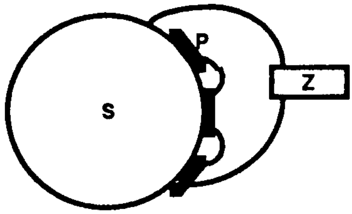

Figure 1 : piezo-damping of a vibrating structure

By using unidimensional relations describing the piezo-electric behaviour of the piezo-patches [2] it coult be established that the modal response of a structural mode to be damped is described in the frequency domain by :

$$-\omega^2 m_s\alpha + k_s'\alpha + j\omega\mu_s\alpha + {}^t\{\phi\}\frac{K}{1-\dfrac{n^2 j\omega Z}{K\left(1+j\omega C_N Z\right)}}\{\phi\}\,\alpha = {}^t\{\phi\}\{F_e\} \qquad (1)$$

where:

m_S, μ_S, k_S' are the modal mass, stiffness and damping of the structure without the used short circuit stiffness K of the piezo-patches ;

$\{\phi\}$ is the modal amplitude vector ;

$\{F_e\}$ is the excitation forces vector;

α is the generalised modal amplitude of the mode to be damped; (response displacement vector is $\{X\} = {}^t\{\phi\}\,\alpha$);

$j^2 = -1$;

t denotes transpose;

ω is the pulsation;

ϕ_{P1}, ϕ_{P2} are the modal amplitudes at extremities 1 and 2 of piezo-patch P ;

d_P is the used piezo-coefficient ;

S_P, l_P are the corresponding surface and length ;

E_P is the corresponding Young's modulus ;

$n = \frac{d_p S_p E_p}{l_p}$ is a transformation coefficient ;

$K = \frac{S_p E_p}{l_p}$ is the used short circuit stiffness ;

C_N is the total electrical capacity of the piezo-patches wired in parallel.

Looking at the order of magnitude of the different coefficients it appears that :

$$\left|\frac{n^2 j\omega Z}{K(1+j\omega C_N Z)}\right| << 1$$

and eqn (1) can be approximated by :

$$-\omega^2 m_s \alpha + k_s \alpha + j\omega\mu_s \alpha + n^2 \left[\sum_N (\phi_{P2} - \phi_{P1})\right]^2 \frac{Z j \omega}{1 + j \omega C_N Z} \alpha = {}^t\{\phi\}\{F_e\} \quad (2)$$

where :

k_s is the modal stiffness of the structure with the piezo-patches short-circuited.

The later equation (2) may by also readily established by considering that the current i_p and the voltage e_p delivered by each patch P are approximately linked to its elongation velocity ΔV_p and delivered force Fp by the transformation coefficient n (see figure 2) :

$$e_p = \frac{F_p}{n} \text{ and } i_p = n \, \Delta V_p \; ;$$

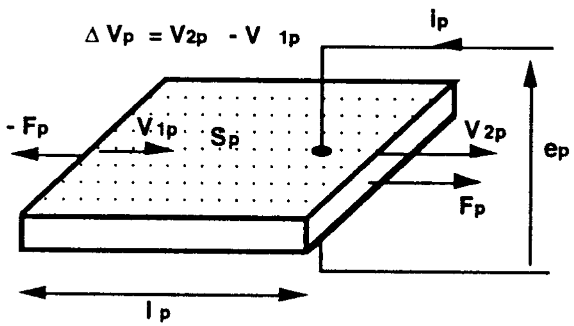

Figure 2 : definition of useful quantities of the piezo-patches

Then as we use N piezo-ceramics wired in parallel, the delivered current i_p is summed up and the voltage e_p is common. By considering that these piezo-patches are of identical shape and properties, and by introducing C_p their common electrical capacity the equivalent electrical circuit is defined as shown figure 3 :

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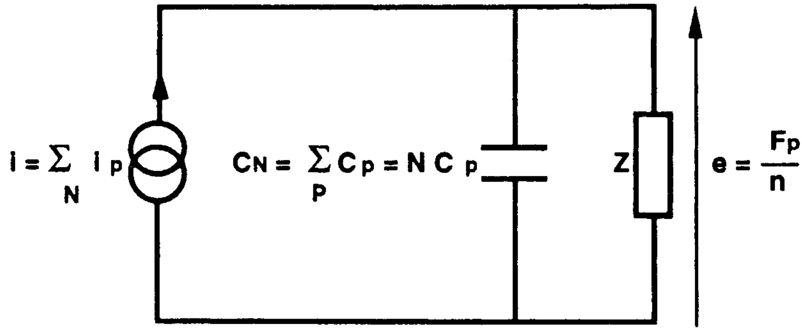

Figure 3 : electrical circuit for N patches wired in parallel

which leads to :

$$e = \frac{Z}{1+j\omega C_N Z}\, i$$

$$F_p = n^2 \frac{Z}{1+j\omega C_N Z} \sum_N \Delta V_P$$

and by introducing the modal response of each piezo-patch :

$$\Delta V_p = j\omega\left(\phi_{p2} - \phi_{p1}\right)\alpha$$

The modal response of the structural mode to be damped is obtain as described by eqn (2).

From (2) the shunting effect of the piezo-patches capacity C_N is obvious :

$$\frac{Zj\omega}{1+j\omega C_N Z} \rightarrow 0 \text{ when } \omega C_N Z >> 1$$

and the behaviour of the structure is equal to its behaviour with the piezo-patches short-circuited : that means without any possibility of piezo-damping.

Here after two approaches for avoiding this shunting effect are presented.

2.2 Use of a negative capacity

Z is equal to a negative capacity - C (achievable with an active circuit) wired in parallel with a resistance R_D (see figure 4).

Then by introducing the following coefficients for the structural mode to be damped:

$\Omega_s = \sqrt{\frac{k_s}{m_s}}$ resonance pulsation, $\xi_s = \frac{\mu_s}{2\sqrt{k_s m_s}}$ initial damping ratio;

and the following coefficients for the electrical part :

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$\beta = \dfrac{C_N - C}{C_N}$ reduction ratio of the electrical capacity; $\xi_D = R_D\, C_N\, \Omega_s$ electrical damping ratio;

a non-dimensional expression of eqn (2) is obtained :

$$\left\{\left[1 - \left(\frac{\omega}{\Omega_s}\right)^2 + 2\, j\, \xi_s \left(\frac{\omega}{\Omega_s}\right)\right] + \frac{n^2 D}{\left(m_s\, C_N\, \Omega_s^{\,2}\right)}\, x\, j \left(\frac{\omega}{\Omega_s}\right) \frac{\xi_D}{1 + j\left(\frac{\omega}{\Omega_s}\right)\beta\, \xi_D}\right\} \alpha$$

$$= \frac{1}{m_s\, \Omega_s^{\,2}}\, {}^t\{\phi\}\, \{F_e\} \qquad (3)$$

$$\text{where : } V_C = \frac{n^2 D}{\left(m_s\, C_N\, \Omega_s^{\,2}\right)} = \frac{n^2\left[\sum_N \left(\phi_{P2} - \phi_{P1}\right)\right]^2}{\left(m_s\, C_N\, \Omega_s^{\,2}\right)} \qquad (4)$$

is a coupling factor between the ceramics and the structure. The greater this coupling factor is, the greater the potential piezo-damping efficiency.

The mechanical equivalency of eqn (3) is shown figure 4.

2.3 Use of tuned resonant circuits

Z is equal to an inductance L wired in parallel with a resistance R_D or wired in series with a resistance R_L (see figure 5). Then by introducing the additional following coefficients :

$\Omega_D = \left(\dfrac{1}{L\, C_N}\right)^{1/2}$ electrical resonance pulsation of the LC circuit, $Q_D = \left(\dfrac{L\, \Omega_D}{R_L}\right)$ quality factor of the self,

$\xi_D = \dfrac{R_D}{\left(2\sqrt{\dfrac{L}{C_N}}\right)}$ electrical damping ratio,

a non-dimensional expression of eqn (2) is obtained :

$$\left\{\left[1 - \left(\frac{\omega}{\Omega_s}\right)^2 + 2\, j\, \xi_s \left(\frac{\omega}{\Omega_s}\right)\right] + \left(\frac{n^2 D L}{m_s}\right)\left(\frac{\Omega_D}{\Omega_s}\right)^2 \frac{j + \frac{1}{Q_D}\frac{\Omega_D}{\omega}}{j\left(\frac{\Omega_D}{\Omega_s}\right)^2\left[-1 + \left(\frac{\omega}{\Omega_D}\right)^2 - \frac{1}{(2 Q_D \xi_D)}\right] + \left(\frac{\Omega_D}{\omega}\right)\left[\frac{1}{(2\xi_D)} + \frac{1}{Q_D}\right]}\right\} x\, \alpha$$

$$= \frac{1}{m_s \Omega_s^{\,2}}\, {}^t\{\phi\}\, \{F_e\} \qquad (5)$$

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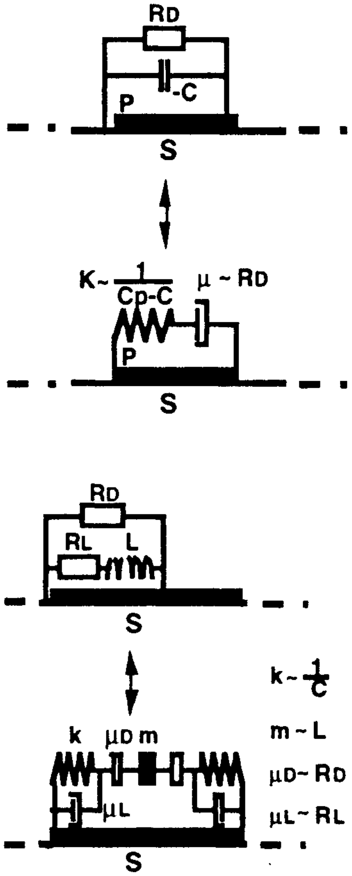
$$\text{where : } V_L = \left(\frac{n^2 D L}{m_s}\right) = \left(n^2 \left[\sum_N \left(\phi_{P2} - \phi_{P1}\right)\right]^2 \frac{L}{m_s}\right) \qquad (6)$$

is a coupling factor between the ceramics and the structure, the greater $\left(\frac{n^2 D L}{m_s}\right)$ is, the greater the potential piezo-damping efficiency. If the tuning of the electrical resonance is correct ($\Omega_S = \Omega_D$), then this coupling factor (6) is equal to the coupling factor of the negative capacitance option (4).

The two options for the tuned circuit are obtained with Q_D reaching infinity for the "L//R" option and ξ_D reaching infinity for the "L+R" option.

The mechanical equivalencies of these two options are shown figure 4.



Dissipative circuit including a negative capacity

Dissipative circuit including a tuned inductance

L//R : $R_L = 0$, $\mu_L = 0$

L + R : $R_D \rightarrow \infty$, $\mu_D \rightarrow \infty$

Figure 4 : mechanical equivalencies of the different dissipative circuits.

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2.4 Performances prediction and discussion

Figure 5 and 6 provides values of modal amplitudes α versus non dimensional frequencies for two coupling factor values (0.5 % and 5 %). From these figures the improvements in term of maximal structural response, above the use of a dissipative circuit limited to a simple resistance is obvious.

The use of a negative capacity provide a broad band efficiency allowing to damp different modes. The efficiency is nevertheless limited by the use of a negative capacity that can generate electrical unstabilities. That means that for practical applications the capacity reduction ratio β must remain above values around 1/3.

The use of a tuned inductance provides an intrinsically stable circuit. The inductance has generally high values that implies the use of a very simple active electronic circuit in order to simulate it rather than to use a real inductance. The mean limitation arises from the tuned approach that means that only one mode can be damped with such an approach.

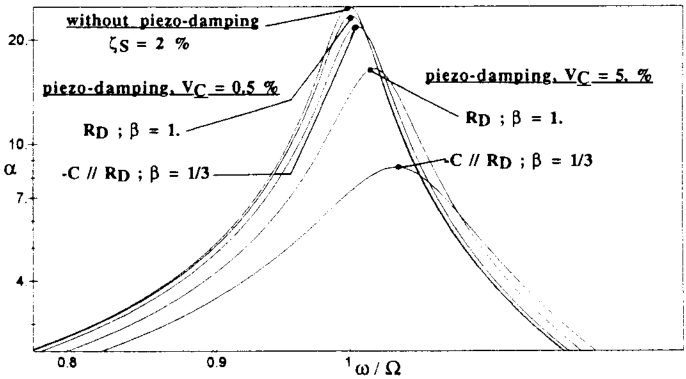

Figure 5 : performances prediction with a dissipative circuit including a negative capacity

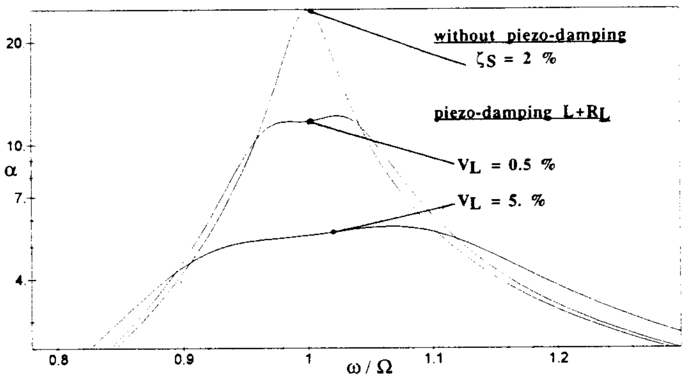

Figure 6 : performances prediction with a dissipative circuit including a tuned inductance

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3 Experimental and numerical results from a case study

3.1 Case study description

Some satellite equipments are isostatically mounted on their receiving structures by using three so-called V blades (see figure 7). Then the equipment suspension modes are mainly related to the tensile stiffness of each side of the V-blades. The purpose is to damp these modes by a piezo-damping technique. A proof of concept has been achieved with the arrangement described in figure 8.

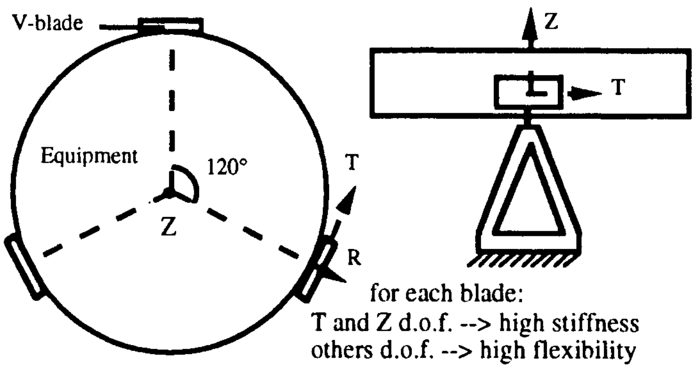

Figure 7 : Isostatic mounting of satellite equipment.

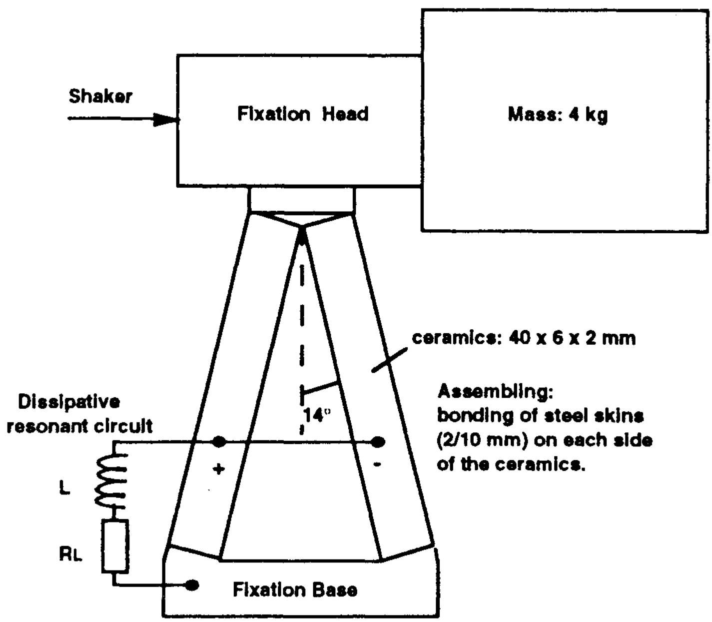

Figure 8 : experimental set-up

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3.2 Numerical results

Equation (5) has been used to predict the piezo-damping performances of the tested V-blades (see figure 9). The needed data have been derived from a finite element model (modal amplitudes) and from the used piezo-ceramics properties. These data lead to a coupling factor VL equal to 2.4 %.

Additionnaly, as the inductance has been achieved with an active circuit, the required imput current and voltage are of interest.

It can be easily demonstrated that the maximum current i and voltage e are obtained at the resonance pulsation Ω_s when using a well tuned circuit and are expressed by :

$$i\,max = n \sum_{N} \left(\phi p_2 - \phi p_1\right) \alpha(\Omega) \tag{7}$$

$$e\,max = \frac{\sqrt{1 + QD^2}}{C_N\,\Omega_s}\; i\,max \tag{8}$$

wich lead for our application to :

$i\,max(mAmp) = 9{,}2\ \ 10^{-3}\ V\ ;\ U\ max(Volts) = 21\ V$

Where V is the velocity of the mass (see figure 8) in mms^{-1}.

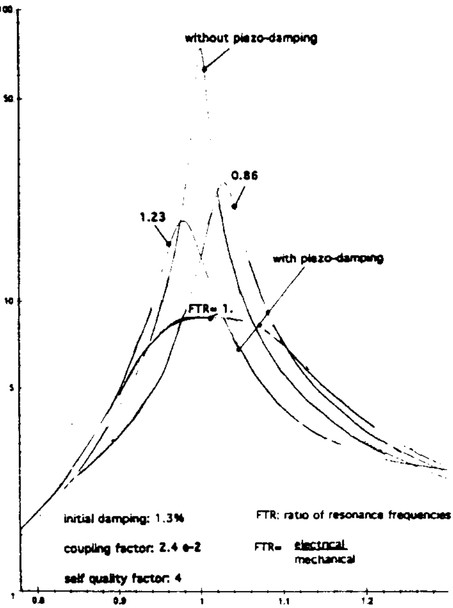

Figure 9 : non-dimensional acceleration versus non-dimensional frequency of the piezo-damped V-blade

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3.3 Experimental results

The effect of the piezo-damping upon the mass acceleration are provided in figure 10 for various tuning frequencies of the inductance L. For all the curves the dissipative resistance R is unchanged.

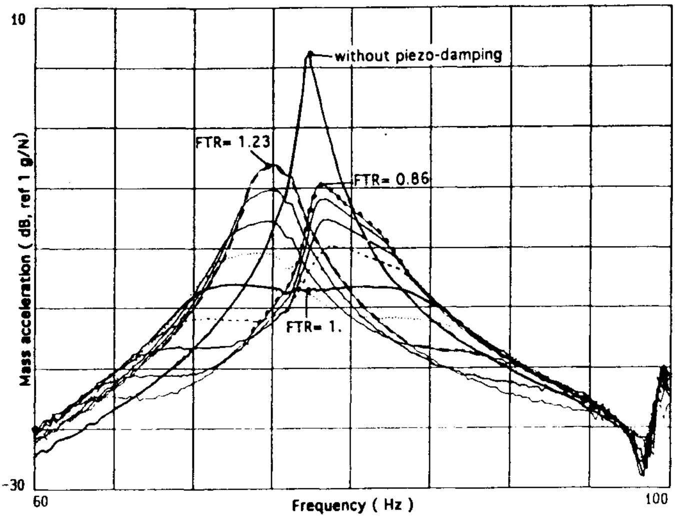

Figure 10 : piezo-damping experimental results

3.4 Comparison of numerical and experimental results

The comparison between the predicted amplification factor and the experimental ones is very good (see table 1).

	MAXIMUM AMPLIFICATION FACTOR	
	PREDICTION	TESTS
well tuned circuit	8	7,5
electrical frequency detuned from mechanical frequency by + 23 % (FTR = 1.23)	16	23
electrical frequency detuned from mechanical frequency by - 16 % (FTR = 0.86)	22	16

Table 1 : test and prediction results

Additionally, the relative mechanical resonance frequency variation induced by these two mistunings of the electric circuit are quite equal :

- experimentation = 5 %
- prediction = 4 %

In the case of the well tuned circuit the predicted and experimental tension e at the resonance are also very closed :

e (prediction) = 21 V/(mms^{-1})
e (tests) = 23 V/(mms^{-1})

3.4 Discussions

This case study illustrates the validity of the used approximated model (2), in particular it perfectly demonstrates the like tuned vibration absorber behaviour.

The computation of the associated current and voltage from the piezoceramics provides very low values of 9,2 mA and 21 V for a vibration velocity at the resonance frequency of 1 mms-1. These values illustrate the very low electrical power needed by the electronic dissipative impedance for applications aiming to control low level vibrations as for satellite applications.

4 Conclusions

The proposed piezo-damping technique is proven efficient. Its main advantages above passive viscoelastic damping techniques are its best efficiency and its very low thermal sensitivity.

The main limitation arise from the need of a high coupling coefficient for achieving a high damping. That means that the location of the piezo-material into the structure to be damped must be defined carefully and that a large amount of such material must be used.

A second limitation may arise from the used piezo-ceramic brightness limit with respect to launch loads.

Acknoledgements

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