# Contents

About the Authors xvii  
Preface xix  

1 Introduction to Piezoelectric Energy Harvesting  
1.1 Vibration-Based Energy Harvesting Using Piezoelectric Transduction 1  
1.2 An Example of a Piezoelectric Energy Harvesting System 4  
1.3 Mathematical Modeling of Piezoelectric Energy Harvesters 6  
1.4 Summary of the Theory of Linear Piezoelectricity 9  
1.5 Outline of the Book 12  
References 14  

2 Base Excitation Problem for Cantilevered Structures and Correction of the Lumped-Parameter Electromechanical Model 19  
2.1 Base Excitation Problem for the Transverse Vibrations of a Cantilevered Thin Beam 20  
2.1.1 Response to General Base Excitation 20  
2.1.2 Steady-State Response to Harmonic Base Excitation 25  
2.1.3 Lumped-Parameter Model of the Harmonic Base Excitation Problem 26  
2.1.4 Comparison of the Distributed-Parameter and the Lumped-Parameter Model Predictions 29  
2.2 Correction of the Lumped-Parameter Base Excitation Model for Transverse Vibrations 31  
2.2.1 Correction Factor for the Lumped-Parameter Model 31  
2.2.2 Effect of a Tip Mass on the Correction Factor 32  
2.3 Experimental Case Studies for Validation of the Correction Factor 35  
2.3.1 Cantilevered Beam without a Tip Mass under Base Excitation 35  
2.3.2 Cantilevered Beam with a Tip Mass under Base Excitation 39  
2.4 Base Excitation Problem for Longitudinal Vibrations and Correction of its Lumped-Parameter Model 39  
2.4.1 Analytical Modal Analysis and Steady-State Response to Harmonic Base Excitation 40  
2.4.2 Correction Factor for Longitudinal Vibrations 42
2.5 Correction Factor in the Electromechanically Coupled Lumped-Parameter Equations and a Theoretical Case Study

2.5.1 An Electromechanically Coupled Lumped-Parameter Model for Piezoelectric Energy Harvesting

2.5.2 Correction Factor in the Electromechanically Coupled Lumped-Parameter Model and a Theoretical Case Study

2.6 Summary

2.7 Chapter Notes

References

3 Analytical Distributed-Parameter Electromechanical Modeling of Cantilevered Piezoelectric Energy Harvesters

3.1 Fundamentals of the Electromechanically Coupled Distributed-Parameter Model

3.1.1 Modeling Assumptions and Bimorph Configurations

3.1.2 Coupled Mechanical Equation and Modal Analysis of Bimorph Cantilevers

3.1.3 Coupled Electrical Circuit Equation of a Thin Piezoceramic Layer under Dynamic Bending

3.2 Series Connection of the Piezoceramic Layers

3.2.1 Coupled Beam Equation in Modal Coordinates

3.2.2 Coupled Electrical Circuit Equation

3.2.3 Closed-Form Voltage Response and Vibration Response at Steady State

3.3 Parallel Connection of the Piezoceramic Layers

3.3.1 Coupled Beam Equation in Modal Coordinates

3.3.2 Coupled Electrical Circuit Equation

3.3.3 Closed-Form Voltage Response and Vibration Response at Steady State

3.4 Equivalent Representation of the Series and the Parallel Connection Cases

3.4.1 Modal Electromechanical Coupling Terms

3.4.2 Equivalent Capacitance for Series and Parallel Connections

3.4.3 Equivalent Representation of the Electromechanical Equations

3.5 Single-Mode Electromechanical Equations for Modal Excitations

3.6 Multi-mode and Single-Mode Electromechanical FRFs

3.6.1 Multi-mode Electromechanical FRFs

3.6.2 Single-Mode Electromechanical FRFs

3.7 Theoretical Case Study

3.7.1 Properties of the Bimorph Cantilever

3.7.2 Frequency Response of the Voltage Output

3.7.3 Frequency Response of the Current Output

3.7.4 Frequency Response of the Power Output

3.7.5 Frequency Response of the Relative Tip Displacement

3.7.6 Parallel Connection of the Piezoceramic Layers

3.7.7 Single-Mode FRFs
3.8 Summary 90
3.9 Chapter Notes 90
References 94

4 Experimental Validation of the Analytical Solution for Bimorph Configurations 97
4.1 PZT-5H Bimorph Cantilever without a Tip Mass 97
  4.1.1 Experimental Setup and Guidelines for Testing an Energy Harvester 97
  4.1.2 Validation of the Electromechanical FRFs for a Set of Resistors 103
  4.1.3 Electrical Performance Diagrams at the Fundamental Short-Circuit and Open-Circuit Resonance Frequencies 107
  4.1.4 Vibration Response Diagrams at the Fundamental Short-Circuit and Open-Circuit Resonance Frequencies 110
4.2 PZT-5H Bimorph Cantilever with a Tip Mass 111
  4.2.1 Experimental Setup 111
  4.2.2 Validation of the Electromechanical FRFs for a Set of Resistors 113
  4.2.3 Electrical Performance Diagrams at the Fundamental Short-Circuit and Open-Circuit Resonance Frequencies 114
  4.2.4 Vibration Response Diagrams at the Fundamental Short-Circuit and Open-Circuit Resonance Frequencies 119
  4.2.5 Model Predictions with the Point Mass Assumption 119
  4.2.6 Performance Comparison of the PZT-5H Bimorph with and without the Tip Mass 121
4.3 PZT-5A Bimorph Cantilever 122
  4.3.1 Experimental Setup 122
  4.3.2 Validation of the Electromechanical FRFs for a Set of Resistors 124
  4.3.3 Comparison of the Single-Mode and Multi-mode Electromechanical FRFs 125
4.4 Summary 128
4.5 Chapter Notes 128
References 130

5 Dimensionless Equations, Asymptotic Analyses, and Closed-Form Relations for Parameter Identification and Optimization 131
5.1 Dimensionless Representation of the Single-Mode Electromechanical FRFs 132
  5.1.1 Complex Forms 132
  5.1.2 Magnitude-Phase Forms 132
  5.1.3 Dimensionless Forms 133
5.2 Asymptotic Analyses and Resonance Frequencies 134
  5.2.1 Short-Circuit and Open-Circuit Asymptotes of the Voltage FRF 134
  5.2.2 Short-Circuit and Open-Circuit Asymptotes of the Tip Displacement FRF 135
  5.2.3 Short-Circuit and Open-Circuit Resonance Frequencies of the Voltage FRF 136
5.2.4 Short-Circuit and Open-Circuit Resonance Frequencies of the Tip Displacement FRF 136
5.2.5 Comparison of the Short-Circuit and Open-Circuit Resonance Frequencies 137
5.3 Identification of Mechanical Damping 138
5.3.1 Identification of the Modal Mechanical Damping Ratio from the Voltage FRF 138
5.3.2 Identification of the Modal Mechanical Damping Ratio from the Tip Displacement FRF 139
5.4 Identification of the Optimum Electrical Load for Resonance Excitation 139
5.4.1 Electrical Power FRF 139
5.4.2 Optimum Values of Load Resistance at the Short-Circuit and Open-Circuit Resonance Frequencies of the Voltage FRF 140
5.5 Intersection of the Voltage Asymptotes and a Simple Technique for the Experimental Identification of the Optimum Load Resistance 141
5.5.1 On the Intersection of the Voltage Asymptotes for Resonance Excitation 141
5.5.2 A Simple Technique for the Experimental Identification of the Optimum Load Resistance 142
5.6 Vibration Attenuation/Amplification from the Short-Circuit to Open-Circuit Conditions 143
5.7 Experimental Validation for a PZT-5H Bimorph Cantilever 144
5.7.1 Identification of Mechanical Damping 144
5.7.2 Fundamental Short-Circuit and Open-Circuit Resonance Frequencies 145
5.7.3 Magnitude and Phase of the Voltage FRF 145
5.7.4 Voltage Asymptotes for Resonance Excitation 146
5.7.5 Power vs. Load Resistance Diagrams and the Optimum Loads 147
5.7.6 Comment on the Optimum Load Resistance Obtained from the Simplified Circuit Representations of a Piezoceramic Layer 147
5.8 Summary 148
5.9 Chapter Notes 149
References 150

6 Approximate Analytical Distributed-Parameter Electromechanical Modeling of Cantilevered Piezoelectric Energy Harvesters 151
6.1 Unimorph Piezoelectric Energy Harvester Configuration 152
6.2 Electromechanical Euler–Bernoulli Model with Axial Deformations 153
6.2.1 Distributed-Parameter Electromechanical Energy Formulation 153
6.2.2 Spatial Discretization of the Energy Equations 157
6.2.3 Electromechanical Lagrange Equations 159
6.2.4 Solution of the Electromechanical Lagrange Equations 163
6.3 Electromechanical Rayleigh Model with Axial Deformations 166
6.3.1 Distributed-Parameter Electromechanical Energy Formulation 166
6.3.2 Spatial Discretization of the Energy Equations 167
6.3.3 Electromechanical Lagrange Equations 167
6.3.4 Solution of the Electromechanical Lagrange Equations 168

6.4 Electromechanical Timoshenko Model with Axial Deformations 168
6.4.1 Distributed-Parameter Electromechanical Energy Formulation 168
6.4.2 Spatial Discretization of the Energy Equations 171
6.4.3 Electromechanical Lagrange Equations 174
6.4.4 Solution of the Electromechanical Lagrange Equations 178

6.5 Modeling of Symmetric Configurations 181
6.5.1 Euler–Bernoulli and Rayleigh Models 181
6.5.2 Timoshenko Model 182

6.6 Presence of a Tip Mass in the Euler–Bernoulli, Rayleigh, and Timoshenko Models 183
6.7 Comments on the Kinematically Admissible Trial Functions 185
6.7.1 Euler–Bernoulli and Rayleigh Models 185
6.7.2 Timoshenko Model 186

6.8 Experimental Validation of the Assumed-Modes Solution for a Bimorph Cantilever 187
6.8.1 PZT-5H Bimorph Cantilever without a Tip Mass 187
6.8.2 PZT-5H Bimorph Cantilever with a Tip Mass 189

6.9 Experimental Validation for a Two-Segment Cantilever 191
6.10 Summary 194
6.11 Chapter Notes 195
References 196

7 Modeling of Piezoelectric Energy Harvesting for Various Forms of Dynamic Loading 199
7.1 Governing Electromechanical Equations 199
7.2 Periodic Excitation 202
7.2.1 Fourier Series Representation of Periodic Base Acceleration 202
7.2.2 Periodic Electromechanical Response 203

7.3 White Noise Excitation 204
7.3.1 Representation of the Base Acceleration 205
7.3.2 Spectral Density and Autocorrelation Function of the Voltage Response 206
7.3.3 Expected Value of the Power Output 206

7.4 Excitation Due to Moving Loads 208
7.4.1 Cantilevered Piezoelectric Energy Harvester Located on a Bridge 208
7.4.2 Thin Piezoelectric Layer Covering a Region on the Bridge 212

7.5 Local Strain Fluctuations on Large Structures 214
7.5.1 Power Output to General Strain Fluctuations 215
7.5.2 Steady-State Power Output to Harmonic Strain Fluctuations 216
7.5.3 Strain Gage Measurements and Strain Transformations 217

7.6 Numerical Solution for General Transient Excitation 218
7.6.1 Initial Conditions in Modal Coordinates 219
7.6.2 State-Space Representation of the Electromechanical Equations 219
7.7 Case Studies
7.7.1 Periodic Excitation of a Bimorph Energy Harvester on a Mechanism Link 222
7.7.2 Analysis of a Piezoceramic Patch for Surface Strain Fluctuations of a Bridge 226

7.8 Summary 230
7.9 Chapter Notes 231
References 232

8 Modeling and Exploiting Mechanical Nonlinearities in Piezoelectric Energy Harvesting 233
8.1 Perturbation Solution of the Piezoelectric Energy Harvesting Problem: the Method of Multiple Scales 234
8.1.1 Linear Single-Mode Equations of a Piezoelectric Energy Harvester 234
8.1.2 Exact Solution 234
8.1.3 Resonance Approximation of the Exact Solution 235
8.1.4 Perturbation Solution 236
8.2 Monostable Duffing Oscillator with Piezoelectric Coupling 239
8.2.1 Analytical Expressions Based on the Perturbation Solution 239
8.2.2 State-Space Representation of the Governing Equations for Numerical Solution 241
8.2.3 Theoretical Case Study 242
8.3 Bistable Duffing Oscillator with Piezoelectric Coupling: the Piezomagnetoelastic Energy Harvester 247
8.3.1 Lumped-Parameter Electromechanical Equations 247
8.3.2 Time-Domain Simulations of the Electromechanical Response 249
8.3.3 Performance Comparison of the Piezomagnetoelastic and the Piezoelectric Configurations in the Phase Space 250
8.3.4 Comparison of the Chaotic Response and the Large-Amplitude Periodic Response 252
8.4 Experimental Performance Results of the Bistable Piezomagnetoelastic Energy Harvester 253
8.4.1 Experimental Setup 253
8.4.2 Performance Results of the Piezomagnetoelastic Configuration 254
8.4.3 Comparison of the Piezomagnetoelastic and the Piezoelectric Configurations for Voltage Generation 256
8.4.4 On the Chaotic and the Large-Amplitude Periodic Regions of the Response 256
8.4.5 Broadband Performance Comparison 258
8.4.6 Vertical Excitation of the Piezomagnetoelastic Energy Harvester 260
8.5 A Bistable Plate for Piezoelectric Energy Harvesting 262
8.5.1 Nonlinear Phenomena in the Bistable Plate 262
8.5.2 Broadband Power Generation Performance 265
8.6 Summary 267
8.7 Chapter Notes 268
References 270
10.4.3 Comparison of Soft and Hard Single Crystals: PMN-PZT vs. PMN-PZT-Mn
10.4.4 Overall Comparison of Ceramics (PZT-5H, PZT-8) and Single Crystals (PMN-PZT, PMN-PZT-Mn)
10.5 Experimental Demonstration for PZT-5A and PZT-5H Cantilevers
   10.5.1 Experimental Setup
   10.5.2 Identification of Mechanical Damping and Model Predictions
   10.5.3 Performance Comparison of the PZT-5A and PZT-5H Cantilevers
10.6 Summary
10.7 Chapter Notes
References

11 A Brief Review of the Literature of Piezoelectric Energy Harvesting Circuits
   11.1 AC–DC Rectification and Analysis of the Rectified Output
   11.2 Two-Stage Energy Harvesting Circuits: DC–DC Conversion for Impedance Matching
   11.3 Synchronized Switching on Inductor for Piezoelectric Energy Harvesting
   11.4 Summary
   11.5 Chapter Notes
References

Appendix A Piezoelectric Constitutive Equations
   A.1 Three-Dimensional Form of the Linear Piezoelectric Constitutive Equations
   A.2 Reduced Equations for a Thin Beam
   A.3 Reduced Equations for a Moderately Thick Beam
   A.4 Reduced Equations for a Thin Plate
References

Appendix B Modeling of the Excitation Force in Support Motion Problems of Beams and Bars
   B.1 Transverse Vibrations
   B.2 Longitudinal Vibrations
Reference

Appendix C Modal Analysis of a Uniform Cantilever with a Tip Mass
   C.1 Transverse Vibrations
      C.1.1 Boundary-Value Problem
      C.1.2 Solution Using the Method of Separation of Variables
      C.1.3 Differential Eigenvalue Problem
      C.1.4 Response to Initial Conditions
      C.1.5 Orthogonality of the Eigenfunctions
      C.1.6 Normalization of the Eigenfunctions
      C.1.7 Response to External Forcing