AIAA Guidance, Navigation, and Control Conference and Exhibit
16 - 19 August 2004, Providence, Rhode Island

AIAA 2004-4998

PILOT-INDUCED OSCILLATION ANALYSIS WITH ACTUATOR RATE LIMITING AND FEEDBACK CONTROL LOOP

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Abstract

Fully-developed pilot-induced oscillation (PIO) is an important issue to be solved in the development of modern fly-by-wire flight control systems. In this paper, the fully-developed PIO is analyzed as a worst case for the safety of piloted airplanes, including actuator rate limiting, feedback control loop, and pilot delay by using describing function method. It is shown that the predictions obtained with this method closely match results of the simulation in the frequency and the amplitude of the PIO limit cycle. And it demonstrates that the feedback control loop has a positive effect on PIO and decreases amplitude of the oscillation.

Introduction

For an understanding of PIOs, McRuer[1] introduced the three PIO categories as follows: Category Ⅰ, essentially linear pilot-vehicle system oscillations; Category Ⅱ, quasi-linear pilot-vehicle system oscillations with rate or position limiting; and Category Ⅲ, essentially nonlinear pilot-vehicle system oscillations with transitions.

The focus of this study is on Category Ⅱ PIO because some aircrafts recorded severe PIOs, such as the YF-22[2], the JAS39[3], and the T-2CCV[4], have shown actuator rate limiting. Dramatically incremental phase lag because of the actuator rate limiting adversely affects flying qualities and does not allow sufficient pilot control of the aircraft. Therefore, a fuller understanding is essential to the prevention of these kinds of PIOs. Smith[5] studied fully-developed PIO with bang-bang pilot control by running simulations. Klyde, McRuer and Myers[6], and Duda[7] studied the fully-developed PIO by using an analytical method by the describing function technique. Hess and Snell[8], and A'Harrah[9] studied methods to design flight control systems with a rate-limited actuator using software-based compensation. But the effects of feedback loop to PIO are not analytically included in these studies.

It is important to analyze the fully-developed PIO as a worst case for the safety of piloted airplanes. In this paper, a PIO analytical model is developed, including actuator rate limiting, feedback control loop, and pilot delay. It demonstrates that the feedback control loop has a positive effect on PIO and decreases amplitude of the oscillation.

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PIO analysis

In the previous paper[10], PIO analysis was made with due consideration to aileron actuator rate limiting, aileron feedback control loop, and pilot delay. Under consideration of these parameters, the PIO limit cycle frequency and amplitude of the oscillation were analyzed.

In this paper, using the analysis method, further investigation is made what kind of feedback control loop decreases the amplitude of the Pilot-Induced Oscillation.

Figure 1 shows the PIO analysis model, which is the worst case scenario for fully-developed PIO, in which a pilot controls the aircraft with continuous and full authority of the control surface. The pilot control is modeled as a relay. Aileron actuator rate limiting, which is important as a cause of PIO, is included in the model. The actuator is modeled as a rate limiting element, but its dynamics is not considered here for the sake of simplicity. The rudder loop is not limited by the actuator rate limiting because its control surface is small and its surface rate is sufficiently high. This hypothesis holds true for conventional flight control systems. When the aircraft starts to roll, the pilot controls the aircraft to maintain a zero roll rate by using a full authority bang-bang type control with time delay. Figure 2 shows the relation between the lagged roll rate p_L, the output of the relay element of the pilot model U_{plt}, the actuator input U_c, and aileron deflection δa.

From Reference 10, the prediction method of the peak amplitude and frequency of the fully developed PIO is given as follows. The δa during PIO is considered to be a periodic function. It can be expressed by the following;

$$\delta a = a_1 \cos \omega t + b_1 \sin \omega t \tag{1}$$

Where

$$\begin{cases} a_1 = \dfrac{\omega}{\pi}\displaystyle\int_{-\pi/\omega}^{\pi/\omega} \delta a \cos \omega t \, dt \\ b_1 = \dfrac{\omega}{\pi}\displaystyle\int_{-\pi/\omega}^{\pi/\omega} \delta a \sin \omega t \, dt \end{cases} \tag{2}$$

The response of the linear system to this harmonic function of δa is expressed as

$$-L(j\omega) = -L_0(\omega)e^{j\lambda(\omega)} = -L_x(\omega) - jL_y(\omega) \tag{3}$$

Where $L(j\omega)$ is the loop transfer function in the aileron control loop and

$$\begin{cases} L_x(\omega) = L_0(\omega)\cos\lambda(\omega) \\ L_y(\omega) = L_0(\omega)\sin\lambda(\omega) \end{cases} \tag{4}$$

The output U_f of the linear system to δa, which is aileron feedback of the aircraft response, can be expressed as

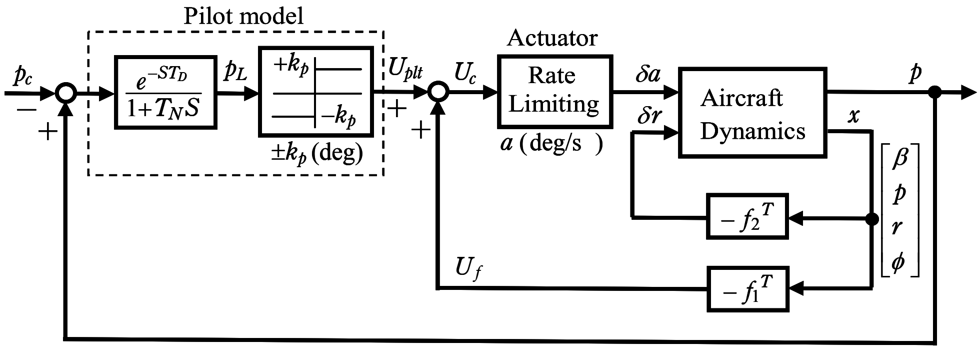


Fig.1 PIO analysis model

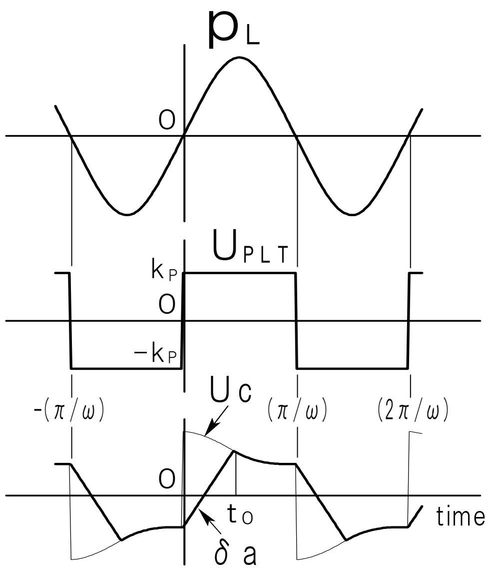


Fig.2 Relation between $P_L, U_{PLT}, U_c, U_{PLT}$, and δa

$$U_f = -L_0(\omega)\left[a_1 \cos\{\omega t + \lambda(\omega)\} + b_1 \sin\{\omega t + \lambda(\omega)\}\right] \quad (5)$$

Aileron deflection δa needed to calculate Eq.(2) is now expressed as follows.

$0 \leqq t \leqq t_0$: Aileron deflection δa is rate limited, thus it is expressed as

$$\delta a = at - k_p - L_0\left[a_1 \cos\lambda + b_1 \sin\lambda\right] \quad (6)$$

where a (deg/s) is the limitation of the control deflection rate of aileron actuator and k_p is the limitation of the pilot control output.

$t_0 \leqq t \leqq \pi/\omega$: δa is expressed as the sum of U_{plt} and U_f ; then

$$\delta a = k_p - L_0\left[a_1 \cos(\omega t + \lambda) + b_1 \sin(\omega t + \lambda)\right] \quad (7)$$

In case of $t<0$, δa is expressed as follows in the same way.

$t_0 - \pi/\omega \leqq t \leqq 0$:

$$\delta a = -k_p - L_0\left[a_1 \cos(\omega t + \lambda) + b_1 \sin(\omega t + \lambda)\right] \quad (8)$$

$-\pi/\omega \leqq t \leqq t_0 - \pi/\omega$:

$$\delta a = -a(t + \pi/\omega) + k_p - L_0\left[a_1 \cos(\pi + \lambda) + b_1 \sin(\pi + \lambda)\right] \quad (9)$$

When these equations are used, the coefficients of Eq.(2) are obtained.

On the other hand, if Eq.(6) and Eq.(7) at $t = t_0$ are equivalent, the following equation can be derived.

$$\begin{aligned} k_p - L_0&\left[a_1 \cos(\omega t_0 + \lambda) + b_1 \sin(\omega t_0 + \lambda)\right] \\ &= a t_0 - k_p - L_0\left[a_1 \cos\lambda + b_1 \sin\lambda\right] \end{aligned} \quad (10)$$

Substituting Eq.(10) for the coefficients of Eq.(2), the following equations can be obtained by eliminating k_p .

$$\begin{cases} P_{11} a_1 + P_{12} b_1 = -\dfrac{2a}{\omega\pi} R_1 \\ P_{21} a_1 + P_{22} b_1 = \dfrac{2a}{\omega\pi} R_2 \end{cases} \quad (11)$$

where

$$R_1 = 1 - \cos\omega t_0 , \quad R_2 = \sin\omega t_0 \quad (12)$$

and

$$\begin{cases} P_{11} = 1 + \dfrac{2\pi - 2\omega t_0 + \sin 2\omega t_0}{2\pi} L_x - \dfrac{1 - \cos 2\omega t_0}{2\pi} L_y \\ P_{12} = \dfrac{2\pi - 2\omega t_0 + \sin 2\omega t_0}{2\pi} L_y + \dfrac{1 - \cos 2\omega t_0}{2\pi} L_x \\ P_{21} = -\dfrac{2\pi - 2\omega t_0 - \sin 2\omega t_0}{2\pi} L_y + \dfrac{1 - \cos 2\omega t_0}{2\pi} L_x \\ P_{22} = 1 + \dfrac{2\pi - 2\omega t_0 - \sin 2\omega t_0}{2\pi} L_x + \dfrac{1 - \cos 2\omega t_0}{2\pi} L_y \end{cases} \quad (13)$$

When Eq.(11) is solved, the coefficients of Eq.(2) can be obtained as follows:

$$\begin{cases} a_1 = -\dfrac{2a}{\omega\pi} d_1 \\ b_1 = \dfrac{2a}{\omega\pi} c_1 \end{cases} \quad (14)$$

where

$$\begin{cases} c_1 = \dfrac{R_1 P_{21} + R_2 P_{11}}{P_{11} P_{22} - P_{12} P_{21}} \\ d_1 = \dfrac{R_1 P_{22} + R_2 P_{12}}{P_{11} P_{22} - P_{12} P_{21}} \end{cases} \quad (15)$$

Because the aileron deflection δa during PIO limit cycle oscillation has been expressed as a describing function, the use of this δa allows the response of roll rate during PIO to be obtained as follows. In case of $p_c = 0$, the response of the lagged roll rate $(p_L / \delta a)_{op}$ in the aileron control loop open would be

$$W(j\omega) = W_0(\omega)e^{j\theta(\omega)} = U(\omega) + jV(\omega) \tag{16}$$

$$\begin{cases} U(\omega) = W_0(\omega)\cos\theta(\omega) \\ V(\omega) = W_0(\omega)\sin\theta(\omega) \end{cases} \tag{17}$$

The response of p_L during PIO and its differentiation with respect to time can be obtained as

$$p_L(t) = -W_0(\omega)\frac{2a}{\omega\pi}\left[-d_1\cos\{\omega t + \theta(\omega)\} + c_1\sin\{\omega t + \theta(\omega)\}\right] \tag{18}$$

$$\dot{p}_L(t) = -W_0(\omega)\frac{2a}{\pi}\left[d_1\sin\{\omega t + \theta(\omega)\} + c_1\cos\{\omega t + \theta(\omega)\}\right] \tag{19}$$

Now from Fig.2,the following equations can be obtained during PIO at $t = \pi/\omega$.

$$p_L(\pi/\omega) = 0, \quad \dot{p}_L(\pi/\omega) < 0 \tag{20}$$

Typkin's parameter[11] J defined by the following equation is introduced to analyze the limit cycle.

$$\begin{aligned} J(\omega) &= \frac{1}{\omega}\dot{p}_L(\pi/\omega) + jp_L(\pi/\omega) \\ &= \frac{2a}{\omega\pi}(c_1 - jd_1)[U(\omega) + jV(\omega)] \end{aligned} \tag{21}$$

When Eq.(20) and Eq.(21) are used, the PIO limit cycle conditions are obtained by the following equations.

$$\mathrm{Re}[J(\omega)] < 0, \quad \mathrm{Im}[J(\omega)] = 0 \tag{22}$$

Therefore, the frequency of the limit cycle ω_0 can be obtained by the following equation.

$$\angle[U(\omega_0) + jV(\omega_0)] = -\pi + \angle\left[\frac{1}{(c_1 - jd_1)}\right] \tag{23}$$

As for t_0, Eq.(10) becomes

$$\begin{aligned} \frac{k_p}{a} = \frac{t_0}{2} &- \frac{c_1}{\omega\pi}[(1-\cos\omega t_0)L_y - (\sin\omega t_0)L_x] \\ &+ \frac{d_1}{\omega\pi}[(1-\cos\omega t_0)L_x + (\sin\omega t_0)L_y] \end{aligned} \tag{24}$$

On the other hand, at $t = \pi/(2\omega)$ the peak amplitude of the PIO limit cycle is expressed as

$$p_{L\,peak} = \frac{2a}{\omega_0\pi}|c_1 - jd_1| \cdot |U(\omega_0) + jV(\omega_0)| \tag{25}$$

Next, $(\omega\pi/2)/(c_1 - jd_1)$, which is necessary to obtain the frequency of the limit cycle ω_0 as well as the $p_{L\,peak}$, is further considered in the following. From Eq.(12)～Eq.(15), c_1 and d_1 are expressed as

$$\begin{cases} c_1 = \dfrac{R_2 + E_1(R_2L_x - R_1L_y)}{1 + 2E_0L_x + E_1E_2(L_x^2 + L_y^2)} \\ d_1 = \dfrac{R_1 + E_1(R_1L_x + R_2L_y)}{1 + 2E_0L_x + E_1E_2(L_x^2 + L_y^2)} \end{cases} \tag{26}$$

where

$$\begin{cases} E_0 = \dfrac{\pi - \omega t_0}{\pi}, \quad E_1 = \dfrac{\pi - \omega t_0 + \sin\omega t_0}{\pi}, \\ E_2 = \dfrac{\pi - \omega t_0 - \sin\omega t_0}{\pi} \end{cases} \tag{27}$$

The magnitude and phase of $(\omega\pi/2)/(c_1 - jd_1)$ are then obtained as follows:

$$\begin{cases} \left|\dfrac{\omega\pi}{2(c_1 - jd_1)}\right| = \dfrac{\omega\pi}{4\sin\dfrac{\omega t_0}{2}} \cdot \dfrac{|1 + 2E_0L_x + E_1E_2(L_x^2 + L_y^2)|}{\sqrt{1 + 2E_1L_x + E_1^2(L_x^2 + L_y^2)}} \\ \angle\left[\dfrac{\omega\pi}{2(c_1 - jd_1)}\right] = \dfrac{\omega t_0}{2} + \angle\left[(L_x + jL_y) + 1/E_1\right] \end{cases} \tag{28}$$

From Eq.(24), the t_0 corresponding to the PIO limit cycle frequency ω_0 can be obtained as

$$\frac{2}{t_0} = \left[1 + (4/\pi)F_2H_2\right]\frac{a}{k_p} \tag{29}$$

where

$$\begin{cases} F_2 = \dfrac{1 - \cos\omega_0 t_0}{\omega_0 t_0} \\ H_2 = \dfrac{L_x + E_1(L_x^2 + L_y^2)}{1 + 2E_0L_x + E_1E_2(L_x^2 + L_y^2)} \end{cases} \tag{30}$$

During PIO, the value of $\omega_0 t_0$ is usually considered as

$$\omega_0 t_0 \fallingdotseq 0 \sim \pi/2 \tag{31}$$

then, E_1 is approximated as follows:

$$E_1 \fallingdotseq 1 \tag{32}$$

Then Eq.(23), which is the phase equation for obtaining the frequency of the limit cycle ω_0, becomes

$$\angle[U(\omega_0) + jV(\omega_0)] \fallingdotseq -\pi + \frac{\omega_0 t_0}{2} + \angle[1 + (L_x + jL_y)] \tag{33}$$

where the third term of the right hand side of this equation is the phase of the sum of 1.0 and the loop transfer function in the aileron loop. As it is the phase of the closed loop, the following relation can be derived.

$$\angle\left[\frac{U + jV}{1 + (L_x + jL_y)}\right] = \angle\left[(p_L/\delta a)_{cl}\right] \tag{34}$$

where $(p_L/\delta a)_{cl}$ is the response of the lagged roll rate with both aileron and rudder control loop closed and without rate limiting. If $p_c = 0$, then p_L is expressed as

$$p_L = \frac{e^{-sT_D}}{1 + T_N s}p \tag{35}$$

where T_D and T_N are time delay and time lag constants in the pilot model. If we write

$$\lambda_{AP} = \angle\left[(p_L/\delta a)_{cl}\right] \tag{36}$$

then λ_{AP} is expressed as

$$\lambda_{AP} = \angle\left[(p/\delta a)_{cl}\right] - \omega_0 T_D - \tan^{-1}(\omega_0 T_N) \tag{37}$$

The frequency of the PIO limit cycle ω_0 can be obtained from Eq.(33), Eq.(36), and Eq.(37), as follows:

$$\omega_0 \fallingdotseq (\pi + \lambda_{AP})\frac{2}{t_0} \tag{38}$$

On the other hand, the $p_{L\,peak}$ is derived from Eq.(25), Eq.(28), and Eq.(29), as follows:

$$p_{L\,peak} = \frac{4k_p}{\pi} \cdot \frac{\sin(\omega_0 t_0/2)}{\omega_0 t_0/2} \cdot \frac{H_1}{1+(4/\pi)F_2H_2} \cdot |U+jV| \tag{39}$$

where

$$H_1 = \frac{\sqrt{1+2E_1L_x+E_1{}^2(L_x{}^2+L_y{}^2)}}{1+2E_0L_x+E_1E_2(L_x{}^2+L_y{}^2)} \tag{40}$$

From Eq.(35), the following relations are obtained.

$$\begin{cases} p_{L\,peak} = \dfrac{p_{peak}}{|1+j\omega_0 T_N|} \\ |U+jV| = \dfrac{|(p/\delta a)_{op}|}{|1+j\omega_0 T_N|} \end{cases} \tag{41}$$

Therefore, the amplitude of the oscillation can be obtained from Eq.(39) and Eq.(41), as follows:

$$\begin{cases} p_{peak} = N_0(\omega_0)\cdot|(p/\delta a)_{op}| \\ \phi_{p-p} = 2N_0(\omega_0)\cdot|(\phi/\delta a)_{op}| \end{cases} \tag{42}$$

where

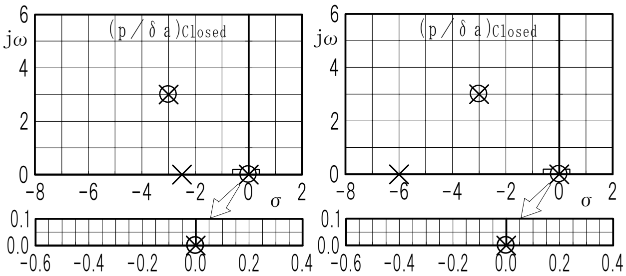
$$N_0(\omega) = \frac{4k_p}{\pi} \cdot \frac{\sin(\omega t_0/2)}{\omega t_0/2} \cdot \frac{H_1}{1+(4/\pi)F_2H_2} \tag{43}$$

Example

To demonstrate the PIO analysis method in this paper, the lateral-directional flight control system is considered. The aircraft dynamics[12] is shown in Eq.(44) and Eq.(45).

$$A = \begin{bmatrix} -0.277 & 0 & -1.0 & 0.0345 \\ -27.6 & -1.890 & 2.59 & 0 \\ 7.50 & -0.0442 & -0.627 & 0 \\ 0 & 1.0 & 0 & 0 \end{bmatrix} \tag{44}$$

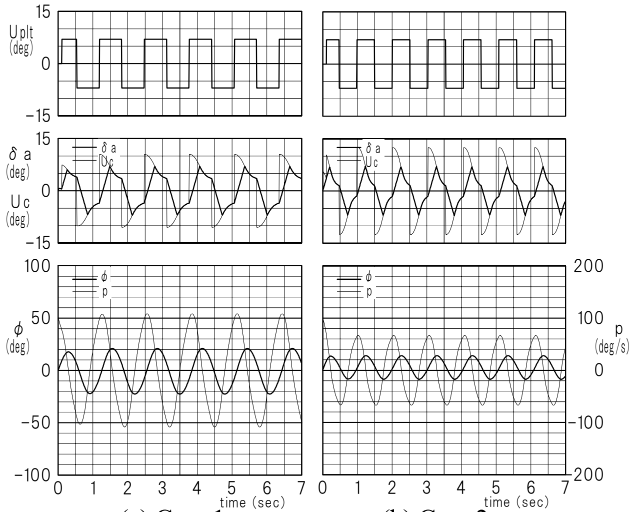
$$B = \begin{bmatrix} 0 & 0.0392 \\ -71.2 & 11.10 \\ -9.34 & -3.28 \\ 0 & 0 \end{bmatrix} \tag{45}$$



(a) Case 1 (b) Case 2

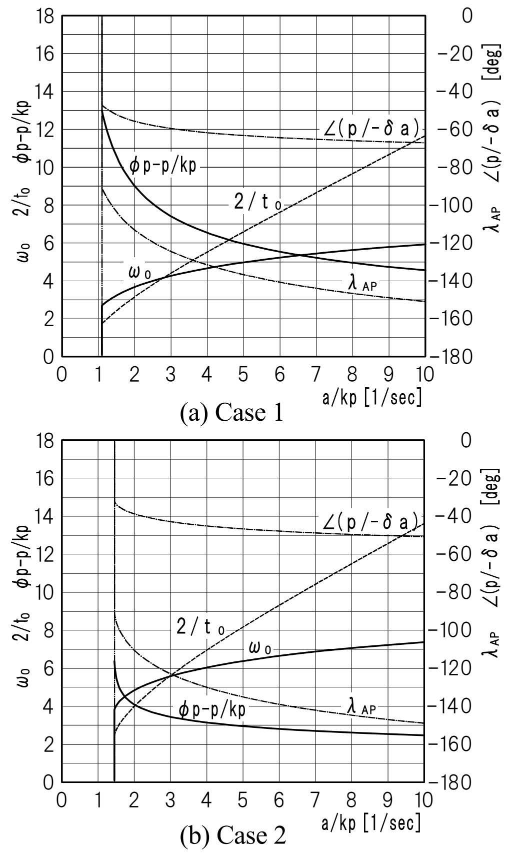
Fig.3 Locations of the poles and zeros of the $(p/\delta a)_{cl}$

Locations of the poles and zeros for the feedback control system (case 1 and case 2) are shown in Fig.3. The limit cycle obtained by the simulation corresponding to these cases are shown in Fig.4 in the case of $T_D = 0.1$ sec, $T_N = 0.2$ sec, limitation of output of the pilot control $k_p = 7.0°$ and the limitation of control deflection rate of actuator $a = 35.0$ (deg/s), that is, $a/k_p = 5.0$ (1/s).



(a) Case 1 (b) Case 2

Fig.4 PIO simulation corresponding to Fig.3
($a = 35.0$ [deg/ s], $k_P = 7.0°$, $a/k_p = 5.0$[1/s])



(a) Case 1

(b) Case 2

Fig.5 PIO analysis diagram
($T_D = 0.1$ sec, $T_N = 0.2$ sec)

The PIO analysis diagram for the two cases, using Eq.(29), Eq.(37), Eq.(38), and Eq.(42), are shown in Fig.5.

The results obtained by the simulation and by the analysis method are as follows:

$$\text{simulation:}\begin{cases} case1:\ \phi = 42.0°^{p-p},\ \omega = 4.8(\text{rad/s}) \\ case2:\ \phi = 22.0°^{p-p},\ \omega = 6.1(\text{rad/s}) \end{cases} \qquad (46)$$

$$\text{analysis:}\begin{cases} case1:\ \phi = 41.3°^{p-p},\ \omega = 5.0(\text{rad/s}) \\ case2:\ \phi = 21.0°^{p-p},\ \omega = 6.3(\text{rad/s}) \end{cases} \qquad (47)$$

It can be found that the results of the analysis method closely match that of the simulation. Comparing case 1 with case 2, the amplitude of the PIO of the former is about twice that of the latter. In the next section, using this analysis method, feedback control law to decrease the amplitude of the oscillation is considered in detail.

feedback control law

Now we consider the relationship between $2/t_0$ and a/k_p in Eq.(29) which is an important parameter in the PIO phenomenon. Control surface rate limit value a and the maximum value of the pilot input k_p have already been decided as fixed values. Therefore, it is needed that the $\left[1+(4/\pi)F_2H_2\right]$ in Eq.(29) has a large value to use the effect of the feedback loop effectively. When the value of $2/t_0$ becomes large, we can get a small value of the time t_0 restricted by the rate limit. For that purpose, it is needed that F_2 and H_2 in Eq.(29) have large values. However, the F_2 is the function of only $\omega_0 t_0$, and it is unchanged by the feedback to mention it later. Therefore, a feedback control law is devised to increase the value of H_2.

Figure 6 shows that the function H_2 varies with the change in (L_x, L_y). From this figure, it can be seen that H_2 increases as L_x and $|L_y|$ increase. This can be interpreted as follows. Assuming that E_1E_2 is approximated as follows:

$$E_1E_2 = \left(\frac{\pi - \omega_0 t_0}{\pi}\right)^2 - \left(\frac{\sin \omega_0 t_0}{\pi}\right)^2 \fallingdotseq E_0{}^2 \qquad (48)$$

then H_2 in Eq.(30) is expressed as

$$H_2 \fallingdotseq \frac{1}{E_0{}^2}\cdot\frac{(L_x+1, L_y)\cdot(L_x, L_y)}{(L_x+1/E_0)^2 + L_y{}^2} = \frac{1}{E_0{}^2}\cdot\frac{|C|}{|D|}\cdot\frac{|L|}{|D|}\cos\theta \qquad (49)$$

where $|C|$, $|D|$, $|L|$ and θ are shown in Fig.7. The E_0 is unchanged by the feedback, because it is the function of only $\omega_0 t_0$. Therefore, it is to make the angle θ small and to make the $|C|/|D|$ and $|L|/|D|$ large to increase the value of H_2 from Eq.(49). In other words, it is understood that it is good that the vector locus (L_x, L_y) of the open-loop transfer function is moved to right hand direction and bottom side direction. When the value of H_2 is increased, the direct effect which makes the peak value ϕ_{p-p} of Eq.(42) small can be also expected.

On the other hand, ϕ_{p-p} of Eq.(42) is in proportion to the function H_1. Therefore, it is needed that the feedback

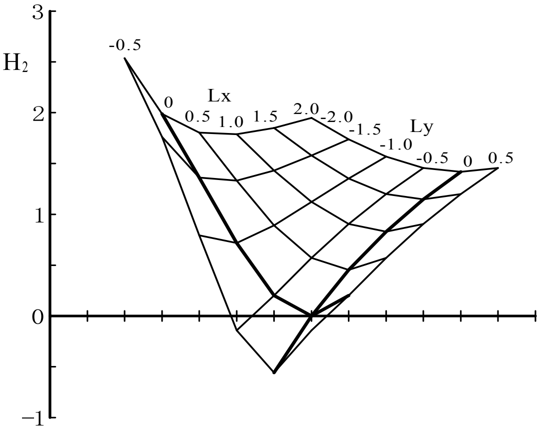


Fig.6 The function H_2 for variation of (L_x, L_y) (ω = 5rad/s = constant) (Case 1)

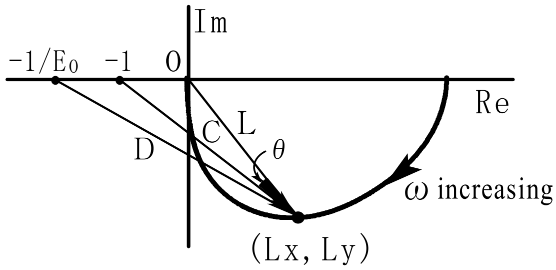


Fig.7 Vector locus of the open-loop transfer function in the aileron control loop

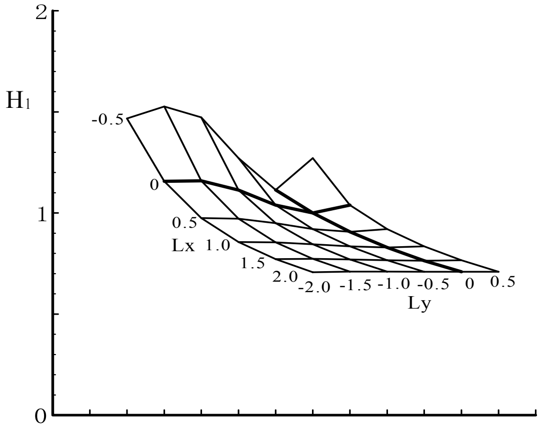


Fig.8 The function H_1 for variation of (L_x, L_y) (ω = 5rad/s = constant) (Case 1)

control law is also devised to decrease the value of H_1. Figure 8 shows that the function H_1 varies with the change in (L_x, L_y). From this figure, it can be seen that H_1 decreases as L_x and $|L_y|$ increase, mainly as L_x increase. Assuming Eq.(32) and Eq.(48), H_1 in Eq.(40) is expressed as

$$H_1 \fallingdotseq \frac{1}{E_0{}^2}\cdot\frac{\sqrt{(L_x+1)^2 + L_y{}^2}}{(L_x+1/E_0)^2 + L_y{}^2} = \frac{1}{E_0{}^2}\cdot\frac{|C|}{|D|^2} \qquad (50)$$

where $|C|$ and $|D|$ are shown in Fig.7. Therefore, it is to make the $|D|$ large to decrease the value of H_1 from the

Eq.(50). In other words, it is understood that it is good that the vector locus (L_x, L_y) of the open-loop transfer function is moved to right hand direction and bottom side direction. Figure 9 shows that the amplitude ϕ_{p-p}/k_p of the PIO varies with the change in (L_x, L_y). From this figure, it can be seen that ϕ_{p-p}/k_p decreases as L_x and $|L_y|$ increase.

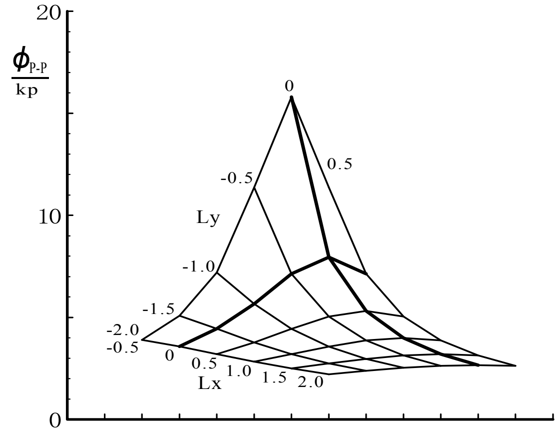


Fig.9 The ϕ_{p-p}/k_p for variation of $(L_x,\ L_y)$ ($\omega = 5$rad/s$=$ constant) (Case 1)

Now we consider the time history data of aileron deflection δa during PIO. Figure 10 shows that the δa is restricted by the rate limit between the point A and the point B. The feedback control is effective between the point B and the point C. The value of the feedback U_f at the point B is given as follows:

$$\ddot{U}_f(t) = U_f(t) = 0 \tag{51}$$

Further, the $\omega_0 t_0$ of the point B becomes the same value even if the feedback control law changes when the value of a/k_p is the same. From Eq.(38), λ_{AP} which is the phase angle of the closed loop of $p_L/\delta a$ is expressed as

$$\lambda_{AP} \doteqdot \frac{\omega_0 t_0}{2} - \pi \tag{52}$$

Therefore, from Fig.5, it can be seen that the λ_{AP} becomes the almost same value even if the feedback control law changes when the value of a/k_p is the same.

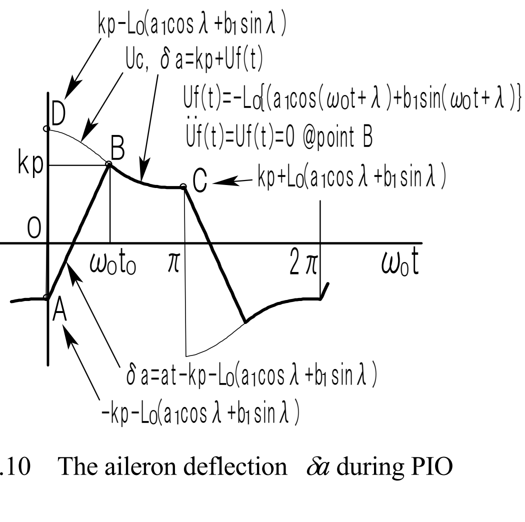
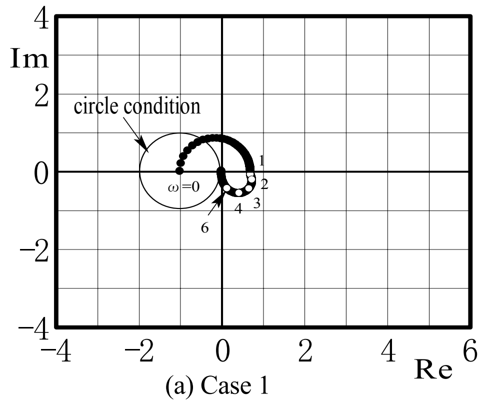
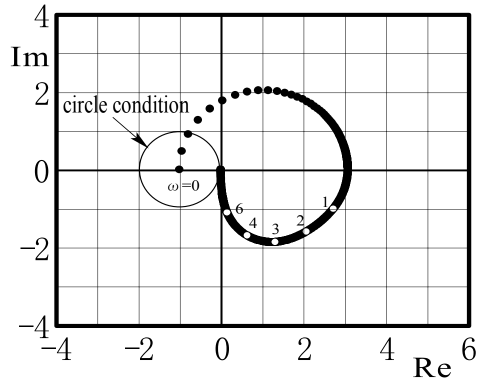


Fig.10 The aileron deflection δa during PIO



(a) Case 1



(b) Case 2

Fig.11 Vector locus of the open-loop transfer function in the aileron control loop

Figure 11 shows the vector locus of the open-loop transfer function $(L_x + jL_y)$ in the aileron control loop for the case 1 and case2. From Fig.11, it can be seen that the L_x and $|L_y|$ for case 2 are larger than for case 1. From Eq.(43), the PIO gain $N_0(\omega)$ which is expressed as a function of L_x and L_y decreases as L_x and $|L_y|$ increase. Therefore it is important to design the feedback control law to suppress the amplitude of the PIO limit cycle.

Conclusions

Based on the findings reported herein, the following conclusions can be drawn.

1) The developed analysis method is a suitable tool to predict the frequency and the amplitude of pilot-induced oscillation(PIO).

2) The conditions, under which the PIO occurs, are delays in actuator rate limiting, delays in aircraft response, delays by the pilot control, and effects of the feedback control.

3) When the open loop transfer function in the control loop is properly designed, it is possible for the feedback control loop to have a positive effect on PIO to decrease the amplitude of the oscillation.

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