Tôhoku Math. J.
40 (1988), 213-220.

PINCHING AND NONEXISTENCE OF STABLE HARMONIC MAPS

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(Received November 25, 1986)

1. Introduction. Let $f: N^m \to M^n$ be a harmonic map from a compact Riemannian manifold N to a Riemannian manifold M. f is said to be stable if its second variation of the energy is non-negative.

Leung [6] proved that if M^n is a unit sphere S^n $(n \geqq 3)$, then constant maps are the only stable harmonic maps for an arbitrary N^m. Considering the above result and some of its generalization (cf. [4], [7] and [8]), we can ask the following:

QUESTION. *Let M^n be a complete simply-connected strictly* $(1/4)$*-pinched Riemannian manifold of dimension* $n(n \geqq 3)$ (i.e., *the sectional curvature* K_M *satisfies* $1/4 < K_M \leqq 1$). *Let N^m be an arbitrary compact Riemannian manifold. Is every stable harmonic map $f: N^m \to M^n$ a constant map?*

This is a "harmonic-version" of the famous conjecture of Lawson and Simons (cf. [5]) on stable minimal submanifolds (or more generally stable currents). To this question, Howard [3] obtained a partial affirmative answer. He showed that for each $n \geqq 3$ there exists a constant $\delta(n)$ satisfying $1/4 < \delta(n) < 1$ such that if M^n is a simply-connected compact strictly $\delta(n)$-pinched Riemannian manifold of dimension n, then there are no nonconstant stable harmonic maps from any compact Riemannian manifold to M. But unfortunately $\lim_{n\to\infty} \delta(n) = 1$.

The purpose of this paper is to give a dimension-independent pinching constant. We prove:

MAIN THEOREM. *Let M^n be a compact simply-connected* 0.83*-pinched Riemannian manifold* $(n \geqq 3)$ (i.e. $0.83 \leqq K_M \leqq 1$). *Then for any compact Riemannian manifold N^m, any stable harmonic map $f: N^m \to M^n$ is a constant map.*

In Section 2 we present some necessary formulas and in Section 3 we prove the main theorem. In Section 4 we use the same technique used in the proof of the main theorem to prove Theorem 3 which is an extension of a theorem of Ohnita [8], and as a corollary we get topological information on minimal submanifolds of sufficiently pinched spheres.

The author would like to express his gratitude to Professors S. Tanno and Y. Hatakeyama for their constant encouragement and advice. The author is also grateful to H. Nakajima who pointed out mistakes in the first draft of this paper and suggested the improvement of Theorems 1 and 3. Lastly the author thanks the referee for his comment and advice.

2. **Preliminaries.** In this section we always assume that M^n is a compact simply-connected δ-pinched Riemannian manifold.

When M^n is a convex hypersurface in the Euclidean space $\boldsymbol{R}^{n+1}$, using the flat connection of $\boldsymbol{R}^{n+1}$ and taking the average of the second variations, Leung [7] proved that for a certain convex hypersurface M^n in $\boldsymbol{R}^{n+1}$ any stable harmonic map $f: N^m \to M^n$ is a constant map. The idea now is to follow the pattern of his calculation. To carry this idea out, we construct a vector bundle E on M and a flat connection ∇' on E instead of $M \times \boldsymbol{R}^{n+1}$ and the flat connection on $M \times \boldsymbol{R}^{n+1}$, respectively. For the construction we follow [1] and [2].

As in [2] we normalize the δ-pinched metric of M by multiplication with $(1+\delta)/2$. Put $E = TM \oplus \varepsilon(M)$, where TM is the tangent bundle of M and $\varepsilon(M)$ is a trivial line bundle on M with a metric. Thus E naturally becomes a Euclidean vector bundle on M. Let e be a cross-section of length one in $\varepsilon(M)$. We define a metric connection ∇'' on E as follows:

$$\nabla''_X Y = \nabla_X Y - \langle X, Y\rangle \cdot e \tag{1}$$

$$\nabla''_X e = X\,, \tag{2}$$

where X and Y are vector fields on M, $\langle\ ,\ \rangle$ and ∇ are the Riemannian metric andt he Riemannian connection of M, respectively. Under the pinching assumption, the curvature R'' of ∇'' is small. We obtain a flat metric connection ∇' close to ∇'' exactly as in [2]. To measure the closeness we define

$$\|\nabla' - \nabla''\| := \mathrm{Max}\,\{\|\nabla'_X Y - \nabla''_X Y\|;\, X \in TM,\, \|X\| = 1,\, Y \in E,\, \|Y\| = 1\}\,.$$

Note that our $\|\nabla' - \nabla''\|$ is half of $\|\nabla' - \nabla''\|$ in [1]. We define $k_1(\delta)$, $k_2(\delta)$ and $k_3(\delta)$ as follows:

$$k_1(\delta) = \frac{4}{3}(1-\delta)\delta^{-1}\left[1 + \left(\delta^{1/2}\cdot \sin\frac{1}{2}\pi\delta^{-1/2}\right)^{-1}\right]. \tag{3}$$

$$k_2(\delta) = \left[\frac{1}{2}(1+\delta)\right]^{-1}\cdot k_1(\delta)\,. \tag{4}$$

$$k_3(\delta) = k_2(\delta)\left\{1 + \left[1 - \frac{1}{24}\pi^2(k_1(\delta))^2\right]^{-2}\right\}^{1/2}. \tag{5}$$

By [1, 4.13], we get

$$\|\nabla' - \nabla''\| \leqq \frac{1}{2}k_3(\delta) . \tag{6}$$

3. Proof of the main theorem. Consider a harmonic map $f: N^m \to M^n$. Let e_a $(a = 1, \cdots, m)$ be a local orthonormal frame on N. The energy of f is defined as

$$E(f) = \frac{1}{2}\int_N \sum_a \|f_* e_a\|^2 .$$

For any vector field V on M, we denote by ϕ_t the flow generated by V. Then we get the following second variational formula (cf. [6]) for the variational vector field V.

$$\begin{aligned} I(V, V) &:= \frac{d^2}{dt^2}\bigg|_{t=0} E(\phi_t \circ f) \\ &= \int_N \sum_a \{\|\nabla_{f_* e_a} V\|^2 - \langle R(V, f_* e_a) f_* e_a, V\rangle\} , \end{aligned} \tag{7}$$

where ∇ and R denote the Riemannian connection and curvature tensor of M, respectively.

THEOREM 1. *Let M^n be a compact simply-connected δ-pinched n-dimensional Riemannian manifold. Suppose that n and δ satisfy*

$$\frac{n+1}{4}(k_3(\delta))^2 + 1 - \frac{2\delta}{1+\delta}(n-1) + (n+1)^{1/2}k_3(\delta) < 0 . \tag{8}$$

Then the only stable harmonic map $f: N^m \to M^n$ for an arbitrary compact Riemannian manifold N^m is a constant map.

PROOF. First we normalize the metric of M by multiplication with $(1+\delta)/2$. Let E be the vector bundle on M constructed in Section 2. For $W \in E$ we denote by W^T and W^ε the TM-component and the $\varepsilon(M)$-component of W, respectively. Let V be a parallel cross-section of E with respect to ∇'. From (7), the second variation corresponding to V^T is given by

$$I(V^T, V^T) = \int_N \sum_a \{\|\nabla_{f_* e_a} V^T\|^2 - \langle R(V^T, f_* e_a) f_* e_a, V^T\rangle\} . \tag{9}$$

Observe that

$$\begin{aligned} \nabla_{f_* e_a} V^T &= \{\nabla''_{f_* e_a}(V - V^\varepsilon)\}^T = (\nabla''_{f_* e_a} V)^T - \{\nabla''_{f_* e_a}(\langle V, e\rangle e)\}^T \\ &= (\nabla''_{f_* e_a} V)^T - \langle V, e\rangle f_* e_a . \end{aligned}$$

Using (6)

$$\|\nabla_{f_*e_a} V^T\|^2 \leqq (1+k)\|\nabla''_{f_*e_a} V\|^2 + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2 \|f_*e_a\|^2 \tag{10}$$
$$\leqq \frac{1+k}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|f_*e_a\|^2 + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2\|f_*e_a\|^2 ,$$

where k is a positive constant fixed later. On the other hand, since we normalized the δ-pinched metric of M by multiplication with $(1+\delta)/2$,

$$\langle R(V^T, f_*e_a)f_*e_a, V^T\rangle \geqq \frac{2\delta}{1+\delta}\{\|V^T\|^2 \cdot \|f_*e_a\|^2 - \langle V^T, f_*e_a\rangle^2\} . \tag{11}$$

Combining (9), (10) and (11), we get

$$I(V^T, V^T) \leqq \int_N \sum_a \left\{\frac{1+k}{4}(k_3(\delta))^2\|V\|^2 \cdot \|f_*e_a\|^2 \right. \tag{12}$$
$$\left. + \left(1+\frac{1}{k}\right)\langle V, e\rangle^2\|f_*e_a\|^2 - \frac{2\delta}{1+\delta}[\|V^T\|^2 \cdot \|f_*e_a\|^2 - \langle V^T, f_*e_a\rangle^2]\right\} .$$

We now define $\mathscr{W} = \{V \in \Gamma(E); \nabla' V = 0\}$, where $\Gamma(E)$ denotes the vector space consisting of all smooth cross-sections of E. Then $\mathscr{W}$ is isomorphic to $\boldsymbol{R}^{n+1}$ and has a natural inner product. We define a quadratic form Q on $\mathscr{W}$ by

$$Q(V) = \text{the right hand side of (12)} := \int_N \sum_a q_a . \tag{13}$$

Take an orthonormal basis $\{W_1, \cdots, W_{n+1}\}$ of $\mathscr{W}$. Then we obtain

$$\sum_{j=1}^{n+1} I(W_j^T, W_j^T) \leqq \sum_{j=1}^{n+1} Q(W_j) = \operatorname{tr} Q = \int_N \sum_a \operatorname{tr} q_a . \tag{14}$$

Since the trace of q_a is independent of the choice of an orthonormal basis for each fiber of E, at each point $x \in M$ we choose an orthonormal basis $\{V_1, \cdots, V_n, e\}$ such that the V_i are tangent to M. Then we get

$$\operatorname{tr} Q = \int_N \sum_a \left\{\frac{(n+1)(1+k)}{4}(k_3(\delta))^2 + 1 + \frac{1}{k} - \frac{2(n-1)\delta}{1+\delta}\right\} \cdot \|f_*e_a\|^2 . \tag{15}$$

Now we take $k = ((n+1)/4)^{-1/2}k_3(\delta)^{-1}$. Then

$$\operatorname{tr} Q = \int_N \sum_a \left\{\frac{n+1}{4}(k_3(\delta))^2 + 1 - \frac{2\delta}{1+\delta}(n-1) \right. \tag{16}$$
$$\left. + (n+1)^{1/2}k_3(\delta)\right\}\|f_*e_a\|^2 .$$

To get the conclusion, we suppose that f is a nonconstant harmonic map and that n and δ satisfy (8). Then we get $\operatorname{tr} Q < 0$. By (14) we obtain $I(W_j^T, W_j^T) < 0$ for some j. Thus f is unstable. q.e.d.

PROOF OF THE MAIN THEOREM. Since we have $k_3(0.83) \risingdotseq 0.964$, (8) is equivalent to $n \geqq 8$ for $\delta = 0.83$. On the other hand, the constant $\delta(n)$ of Howard [3] satisfies $\delta(n) < 0.83$ for $3 \leqq n \leqq 7$. Thus we get the conclusion. q.e.d.

REMARK. The value δ satisfying $k_3(\delta)^2 = 8\delta/(1+\delta)$ is $0.76\cdots$. So our estimate for δ can be improved up to $0.76\cdots$ if n is large.

4. An extension of a theorem of Ohnita and its application to minimal submanifolds. Ohnita [8] proved the following theorem.

THEOREM 2. *Let M^n be an n-dimensional compact minimal submanifold immersed in a unit sphere $S^{N-1}(1)$. If the Ricci curvature Ric_M of M satisfies $\mathrm{Ric}_M > n/2$, then M is harmonically unstable. That is, there exists no nonconstant stable harmonic map from M to any Riemannian manifold nor from any compact Riemannian manifold to M.*

Now we prove the following theorem which is a partial extension of Theorem 2.

THEOREM 3. *Let $(\bar{M}^{N-1}, h)$ be a complete simply-connected δ-pinched $(N-1)$-dimensional Riemannian manifold with $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$. Suppose that $f\colon (M^n, g) \to (\bar{M}^{N-1}, h)$ is an isometric minimal immersion of a complete n-dimensional Riemannian manifold (M^n, g) with $\rho > c(N, n, \delta) := (2n+\delta-1)/4 + \{[(2n+\delta-1)/4]^2 - [(2n+\delta-1)/4 - N(1+\delta)k_3(\delta)^2/16]^2\}^{1/2}$, where ρ is the infimum of the Ricci curvature of M. Then for any compact Riemannian manifold M', any stable harmonic map $\phi\colon M' \to M$ is a constant map.*

PROOF. We normalize the metrics g and h by multiplication with $(1+\delta)/2$. We use the same letters g, h for the normalized metrics. Let $\bar{\nabla}$, $\bar{R}$ be the Riemannian connection and the curvature tensor of $(\bar{M}^{N-1}, h)$. We construct a Euclidean vector bundle E on $(\bar{M}^{N-1}, h)$ and also construct metric connections ∇', ∇'' on E as in Section 2. Let $\langle\ ,\ \rangle$ be the metric on E. Thus we have

$$\nabla''_X Y = \bar{\nabla}_X Y - h(X, Y)\cdot e$$
$$\nabla''_X e = X\,,$$

where X and Y are vector fields on $(\bar{M}^{N-1}, h)$. *Let* σ be the second fundamental form of M^n in $(\bar{M}^{N-1}, h)$ and let ∇ be the Riemannian connection of M. Set $N(M) := \{X \in f^*E;\ X \perp TM\}$. Then we obtain

$$\bar{\nabla}_X Y = \nabla_X Y + \sigma(X, Y)$$
$$\bar{\nabla}_X \xi = -A^{\xi}X + \nabla^{\perp}_X \xi\,,$$

where $X, Y \in TM$, $\xi \in N(M) \cap T\bar{M}^{N-1}$ and A^{ξ}, $\nabla^{\perp}$ are the Weingarten map in the direction of ξ and the normal connection of M in $(\bar{M}^{N-1}, h)$ respectively. Let V be a parallel cross-section of E with respect to ∇'. Let V^T and V^N be the TM-component and the $N(M)$-component of V, respectively. Thus

$$V^N = \langle V, e\rangle e + \sum_j \langle V, \xi_j\rangle \xi_j ,$$

where $\{\xi_1, \cdots, \xi_{N-1-n}\}$ is an orthonormal basis of $N(M) \cap T\bar{M}^{N-1}$. The second variation of $E(\phi)$ corresponding to V^T is given by

$$I(V^T, V^T) = \int_{M'} \sum_a \{\|\nabla_{\phi_* e_a} V^T\|^2 - \langle R(\phi_* e_a, V^T)V^T, \phi_* e_a\rangle\} , \tag{17}$$

where $\{e_a\}$ is a local orthonormal frame of M' and R is the curvature tensor of M. For $W \in E$ we denote by W^{TM} and $W^{T\bar{M}^{N-1}}$ the TM-component and the $T\bar{M}^{N-1}$-component of W, respectively. Observe that

$$\begin{aligned} \nabla_{\phi_* e_a} V^T &= (\bar{\nabla}_{\phi_* e_a} V^T)^{TM} = \{(\nabla''_{\phi_* e_a} V^T)^{T\bar{M}^{N-1}}\}^{TM} \\ &= \{(\nabla''_{\phi_* e_a} V)^{T\bar{M}^{N-1}} - (\nabla''_{\phi_* e_a} V^N)^{T\bar{M}^{N-1}}\}^{TM} \\ &= (\nabla''_{\phi_* e_a} V)^{TM} - (\nabla''_{\phi_* e_a} V^N)^{TM} \end{aligned} \tag{18}$$

and

$$\begin{aligned} (\nabla''_{\phi_* e_a} V^N)^{TM} &= \{\nabla''_{\phi_* e_a}(\langle V, e\rangle e)\}^{TM} + \sum_j \{\nabla''_{\phi_* e_a}(\langle V, \xi_j\rangle \xi_j)\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a + \sum_j \langle V, \xi_j\rangle \{\nabla''_{\phi_* e_a} \xi_j\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a + \sum_j \langle V, \xi_j\rangle \{\bar{\nabla}_{\phi_* e_a} \xi_j\}^{TM} \\ &= \langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a) , \end{aligned} \tag{19}$$

where we abbreviate $A^j = A^{\xi_j}$. Hence we obtain from (18), (19) and (6) that

$$\begin{aligned} \|\nabla_{\phi_* e_a} V^T\|^2 &\leqq (1 + k)\|\nabla''_{\phi_* e_a} V\|^2 + \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 \\ &\leqq \frac{(1 + k)}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|\phi_* e_a\|^2 \\ &\quad + \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 , \end{aligned}$$

where k is a positive constant fixed later. Therefore from (17)

$$\begin{aligned} I(V^T, V^T) \leqq \int_{M'} \sum_a \Big\{ &\frac{(1 + k)}{4}(k_3(\delta))^2 \cdot \|V\|^2 \cdot \|\phi_* e_a\|^2 \\ &+ \left(1 + \frac{1}{k}\right)\left\|\langle V, e\rangle \phi_* e_a - \sum_j \langle V, \xi_j\rangle A^j(\phi_* e_a)\right\|^2 \\ &- \langle R(\phi_* e_a, V^T)V^T, \phi_* e_a\rangle \Big\} . \end{aligned} \tag{20}$$

Denote by $Q(V)$ the right hand side of (20). Then Q is a quadratic form on $\mathscr{W} = \{V \in \Gamma(E); \nabla' V = 0\}$. We take the trace of Q on $\mathscr{W}$. Then we obtain

$$\text{(21)} \qquad \operatorname{tr} Q = \int_{M'} \sum_a \left\{ \frac{(1+k)N}{4} (k_3(\delta))^2 \cdot \|\phi_* e_a\|^2 + \left(1 + \frac{1}{k}\right) \|\phi_* e_a\|^2 + \left(1 + \frac{1}{k}\right) \sum_j \|A^j(\phi_* e_a)\|^2 - \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \right\} .$$

Let $\{V_1, \cdots, V_n\}$ be a local orthonormal basis of TM. Then we get

$$\text{(22)} \qquad \sum_j \|A^j(\phi_* e_a)\|^2 = \sum_{j,i} \langle A^j(\phi_* e_a), V_i \rangle^2 = \sum_{j,i} \langle \xi_j, \sigma(\phi_* e_a, V_i) \rangle^2 = \sum_i \|\sigma(\phi_* e_a, V_i)\|^2 .$$

On the other hand, since

$$\begin{aligned} \langle R(\phi_* e_a, V_i) V_i, \phi_* e_a \rangle &= \langle \bar{R}(\phi_* e_a, V_i) V_i, \phi_* e_a \rangle + \langle \sigma(\phi_* e_a, \phi_* e_a), \sigma(V_i, V_i) \rangle \\ &\quad - \langle \sigma(\phi_* e_a, V_i), \sigma(\phi_* e_a, V_i) \rangle \\ &\leqq \frac{2}{1+\delta} \{ \|V_i\|^2 \cdot \|\phi_* e_a\|^2 - \langle V_i, \phi_* e_a \rangle^2 \} \\ &\quad + \langle \sigma(\phi_* e_a, \phi_* e_a), \sigma(V_i, V_i) \rangle - \|\sigma(\phi_* e_a, V_i)\|^2 , \end{aligned}$$

from the assumption that M is a minimal submanifold of $(\bar{M}^{N-1}, h)$, we obtain

$$\text{(23)} \qquad \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \leqq \frac{2(n-1)}{1+\delta} \|\phi_* e_a\|^2 - \sum_i \|\sigma(\phi_* e_a, V_i)\|^2 .$$

From (22) and (23) we get

$$\text{(24)} \qquad \begin{aligned} \sum_j \|A^j(\phi_* e_a)\|^2 &\leqq \frac{2(n-1)}{1+\delta} \|\phi_* e_a\|^2 - \operatorname{Ric}_M(\phi_* e_a, \phi_* e_a) \\ &\leqq \frac{2(n-1)}{1+\delta} \|\phi_* e_a\|^2 - \frac{2\rho}{1+\delta} \|\phi_* e_a\|^2 . \end{aligned}$$

Thus from (21) and (24)

$$\text{(25)} \qquad \operatorname{tr} Q \leqq \int_{M'} \sum_a \left\{ \frac{(1+k)N}{4} (k_3(\delta))^2 + 1 + \frac{1}{k} + \left(1 + \frac{1}{k}\right) \left[\frac{2(n-1)}{1+\delta} - \frac{2\rho}{1+\delta} \right] - \frac{2\rho}{1+\delta} \right\} \|\phi_* e_a\|^2 .$$

Now we set $k = (N/4)^{-1/2} k_3(\delta)^{-1} \{1 + 2(n-1)/(1+\delta) - 2\rho/(1+\delta)\}^{1/2}$. Then

$$\operatorname{tr} Q \leqq \int_{M'} \sum_a \left\{ \frac{N}{4} (k_3(\delta))^2 + \frac{2n + \delta - 1}{1+\delta} - \frac{4}{1+\delta} \rho + \left[\frac{N}{1+\delta} (2n - 2\rho + \delta - 1) \right]^{1/2} k_3(\delta) \right\} \|\phi_* e_a\|^2 .$$

When $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$, $c(N, n, \delta)$ is a unique solution of the following equation for t.

$$\frac{N}{4}(k_3(\delta))^2 + \frac{2n+\delta-1}{1+\delta} - \frac{4}{1+\delta}t + \left[\frac{N}{1+\delta}(2n-2t+\delta-1)\right]^{1/2} k_3(\delta) = 0\,.$$

Thus $\operatorname{tr} Q < 0$, from which the theorem follows.

We obtain the following corollary as in [8].

COROLLARY. *Let $(\bar{M}^{N-1}, h)$ be a complete simply-connected δ-pinched Riemannian manifold with $(k_3(\delta))^2 \leqq 4(2n+\delta-1)/N(1+\delta)$.*

Suppose that $f\colon M^n \to (\bar{M}^{N-1}, h)$ is a minimal immersion of a complete Riemannian manifold. If the Ricci curvature of M satisfies $\mathrm{Ric}_M > c(N, n, \delta)$, then $\pi_1 M = \{1\}$ and $\pi_2 M = \{1\}$.

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