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Pipe Replacement in a Water Supply Network: Coordinated Versus Uncoordinated Replacement and Budget Effects

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Abstract. Operators of underground water supply networks are challenged with pipe replacement decisions, because pipes are subject to increased failure rates as they age and financial resources are often limited. We study the optimal replacement time and optimal number of pipe replacements such that the expected failure cost and replacement cost are minimized, while satisfying a budget constraint and incorporating uncoordinated and coordinated replacement. Results show that coordinated replacement is economically preferred to uncoordinated replacement. It depends on the size of the budget whether the increase in the number of pipe replacements is sufficient to reduce the total expected failure cost.

Key words: water supply network, pipe replacement, pipe failure, coordinated replacement.

1. Introduction

During his scientific career, Antanas Žilinskas has shown a great interest in the design of specific algorithms for solving practical problems including problems with a dynamic optimization character. The high impact handbook with Aimo Törn on Global Optimization (Törn and Žilinskas, 1989) contained practical problem descriptions like electron trajectories and video beam delays. In further work, Antanas dedicated attention to optimizing biomass growth (Levišauskas *et al.*, 2006) and economic behaviour (Jakaitiene and Žilinskas, 2010). The current dedicated paper describes the decision pattern in water supply networks.

The provision of water to households and industries depends on the quality and functioning of underground water supply networks. These water supply networks are costly to maintain and are subject to increased failure rates as they age (Kleiner, 2001; Kleiner *et al.*, 1998; Rehan *et al.*, 2011). Different parts of the network can be of a different age and may therefore differ in pipe failure behaviour, where pipe failure behaviour is commonly

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expressed in terms of the expected number of failures over a given time interval (Nafi and Kleiner, 2010). Pipe failures may range between small leaks and the complete collapse of pipes. Therefore, when a failure occurs, a cost is initiated that corresponds to the caused damage to the surrounding infrastructure and the loss of water (Hadzilacos *et al.*, 2000; Pelletier *et al*., 2003). Financial resources are often limited (Chang and Hernandez, 2008; Marinoni *et al*., 2012; Papa *et al*., 2013), such that strategies need to be designed for operators of underground water supply networks to manage these pipe failure costs effectively (Kleiner *et al*., 1998) by means of rehabilitation, ranging from repairs (also called relining) to the complete replacement of pipes (Rehan *et al*., 2011). Repairs may be sufficient in the case of relatively small failures, postponing thereby the replacement. Complete replacement can be required when the costs, that are associated with the expected number of pipe failures, i.e. the expected failure costs, exceed the costs of replacement. The operator's replacement decision is in fact an investment decision that is determined by expected pipe failure behaviour, the expected failure cost, the replacement cost and his budget.

In the literature, rehabilitation of underground water networks is often modelled by means of the optimization of pipe replacement and/or replacement time. Optimization studies include (Shamir and Howard, 1979; Kim and Mays, 1994; Kleiner *et al*., 1998; Kleiner, 2001; Nafi and Kleiner, 2010). Shamir and Howard (1979) were one of the first to develop a model to optimize the replacement timing of a homogeneous pipe network such that costs of repairing a break and replacing a pipe are minimized. Kim and Mays (1994) develop a model to decide for each pipe in a network whether to replace, repair or do nothing. The total expected cost for replacement, relining, repair and energy are minimized, while taking into account constraints on mass, energy, water demand and pumping. The objective in Kleiner *et al*. (1998) is to find the rehabilitation strategy for each pipe in the network that minimizes the total costs, while dealing with deterioration of the hydraulic capacity and constraints on the conservation of mass, energy and pressure. The rehabilitation strategy is expressed in terms of relining, replacement of a specific pipe length and the timing thereof, and includes costs associated with relining and replacement. Kleiner (2001) develops an optimal intervention strategy for the timing of rehabilitation and inspection/condition assessment, that minimizes total expected costs of inspection and rehabilitation. More recently, Nafi and Kleiner (2010) recognize that the optimal replacement of a pipe not only depends on failure costs and replacement costs, but also on road work scheduled by other operators of e.g. sewage, telecommunications and street maintenance. Namely, the replacement cost is reduced when pipe replacement is coordinated with such scheduled road work. Nafi and Kleiner (2010) show for different budget constraints how much of the budget is saved each year when scheduled road work by other operators is anticipated over a 5 year planning period. They show that with coordinated replacement significant savings can be made.

Besides economic savings, also from a political point of view a decision maker may desire replacement activities of different underground infrastructure to be coordinated, in order to reduce the frequency of road work and social disruption. However, arguments can be made against coordinated replacement. First, although coordination can reduce the replacement cost, from a point of view of long-term costs and expected pipe failure this may not be beneficial. Namely, it may lead to premature and inefficient replacement of pipes that otherwise would be replaced at a later point in time. Second, with respect to the economic savings due to coordinated replacement, Nafi and Kleiner (2010) show large savings for the case that scheduled road work is anticipated over the entire planning horizon. However, that study ignores that the water network operator may not be informed about planned road work several years in advance. This means that the operator may not be able to incorporate this in the decision making process. It may be more plausible to consider that such planned activities are not anticipated, but rather announced in the same year as when they take place. Keeping in mind the potential advantages and disadvantages of coordinated replacement, based on unannounced road work activities, it remains unknown whether coordinated replacement is economically preferred to uncoordinated replacement. To the best of our knowledge, there are no studies that look at the effects of each of these schemes on pipe replacement, expected failure costs and replacement costs.

Our objective is to find out what the impact is of uncoordinated and coordinated replacement on pipe replacement, the expected failure cost and the replacement cost when a water supply network operator is not informed in time about planned road work by other operators. Considering a policy maker that requires a water supply network operator to make an investment plan for a given planning period, we present a stylized model to find the pipe replacement decision that minimizes total costs, while satisfying a budget constraint. Planned road work by other operators cannot be anticipated by the water supply network operator, but is rather announced on the spot. When this occurs, the replacement cost is reduced, i.e. replacement activities are coordinated. Under uncoordinated replacement, the different operators act without knowledge about each other's activities so that the replacement cost remains unaffected. We address the following research questions. First, what is the optimal replacement time without budget constraint? Second, how many pipes are replaced in each year with budget constraint, under uncoordinated replacement and coordinated replacement? Third, what is the effect of coordinated replacement and a budget constraint on the expected failure cost and replacement cost?

In the decision making process, the operator incorporates pipe failure behaviour, pipe age dynamics, expected failure costs, replacement costs and a budget constraint. Coordinated replacement is studied by means of an unanticipated, instantaneous and temporary reduction in the replacement cost. We analyse the optimal replacement time and optimal number of pipe replacements, as well as expected failure costs and replacement costs, for a high budget and a low budget. The contribution of this paper to the literature is twofold. First, we address planned road work by operators of other underground infrastructure by means of an unanticipated event that instantaneously and temporarily reduces the replacement costs, as opposed to anticipated coordination over a longer period of time in Nafi and Kleiner (2010). Second, we study the effect of a budget constraint on the replacement time, number of pipe replacements and costs under such unanticipated coordinated activities.

Section 2 introduces the model and the settings for the decision maker. The behaviour of the system is analysed in Section 3 and discussed in Section 4. The main findings are summarized in Section 5.

2. Model

We present a dynamic optimization model, where the objective of a water supply operator is to decide for each year how many pipes to replace, so that expected failure costs and replacement costs are minimized. In his decision, the water supply network operator incorporates expected pipe failure behaviour, pipe dynamics and a budget constraint. Under uncoordinated replacement, different operators make decisions without knowledge about each other's activities. Under coordinated replacement, activities are synchronized in return for a reduction in the replacement cost for the water supply network operator. First, pipe failure behaviour is described, followed by a set-up to model optimal pipe replacement. This is followed by a description of how uncoordinated and coordinated replacement is incorporated in the model. Finally, an illustrative data set is introduced to perform a numerical analysis.

2.1. *Pipe Failure Behaviour*

Water network pipes can fail due to continuous aging processes such as corrosion, or due to randomly occurring events such as third party damages. A pipe failure requires immediate action, i.e. the pipe must be either repaired or replaced. As the aging is influenced by many unknown factors, the failure behaviour can only be described stochastically (Kleiner and Rajani, 2001). One way to summarize pipe failure behaviour is to compute the number of failures F that a pipe with age i is expected to experience in a given year. For long-term planning, it is sufficient to model all deterioration processes lumped together as a function of age, because it is the expected number of failures in the entire system that is of interest rather than predictions for a single pipe (Scheidegger *et al.*, 2013). However, if available, further information about the pipes (material, diameter, etc.) can be considered in F_i to improve pipe specific predictions.

2.2. *The Model for Optimal Pipe Replacement*

We consider a water supply network operator who is responsible for a number of pipes A distributed over ages $i = 0, 1, 2, 3, \dots, I$, where I stands for the initially assigned maximum age. All pipes are similar in size and material and each age i has initially a number of $A_{0,i}$ pipes. A replacement strategy comprises the question of how many pipes to replace, knowing that the expected number of failures F_i and corresponding failure costs increase as the pipes age. The replacement strategy determines for each year t the number of pipes A_i of age i to replace, which is denoted by decision $R_{t,i}$. The objective is to minimize the annual expected failure cost and replacement cost over a planning horizon T:

$$\min_R \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{I} \big(c^f F_i A_{t,i} + c^r R_{t,i}\big), \tag{1}$$

where $R_{t,i}$ is the replacement decision expressed in terms of numbers of pipes in period t of segment i. The replacement length is the same for all pipes and is therefore ignored. The expected failure costs of pipes of age i are calculated as the expected number of failures F at age i, multiplied by the number of pipes at that age $A_{t,i}$, times the cost per failure, c^f. The replacement costs are calculated as the cost per pipe replacement, c^r, multiplied by the number of replaced pipes $R_{t,i}$.

The constant unit failure cost c^f includes costs of repair, as well as costs associated with water loss and damage to the adjacent infrastructure and road. Nafi and Kleiner (2010) include indirect damage costs in the failure cost due to accelerated deterioration of the adjacent infrastructure and social costs related to e.g. loss of business, disruption and pollution. As for the replacement cost c^r, in Nafi and Kleiner (2010) this has been defined as the cost of replacing a pipe with a specific length and includes both a fixed term and a term that is variable in the replaced pipe length. They also incorporate economies of scale, based on the assumption that a discount is given on the quantity as the replacement length increases. They further include a reduction on the replacement cost when replacement activities are coordinated with other operators. In Kleiner (2001), besides failure and replacement costs, inspection and condition assessment costs are included as a fixed, time-independent amount, which is related to the decision of the operator to replace immediately or to schedule the next inspection and condition assessment. As we consider all pipe replacements to be of similar size and material, the unit replacement cost c^r is a constant that corresponds to this size and material.

The system is subject to the ageing dynamics $A_{t,i}$ of pipes, where the number of pipes of age i is

$$A_{t,i} = A_{t-1,i-1} - R_{t,i}, \quad \text{for } t = 1, 2, \ldots, T,\ i = 2, 3, \ldots, I \tag{2}$$

and the number of pipes of age 1 in period t is

$$A_{t,1} = \sum_{i=1}^{I} R_{t-1,i} - R_{t,1}, \quad \text{for } t = 1, 2, \ldots, T. \tag{3}$$

In (2), the number of pipes of age i in period t is the number of pipes of age $i-1$ in period $t-1$ minus the number of replaced pipes of age i in period t. We initiate the number of pipes A of age i in period $t = 1$ with $A_{0,i}$. Eq. (3) indicates that all replaced pipes R of age i in period $t-1$ become of age 1 in period t and can directly be replaced again. Age zero, $i = 0$, is therefore not incorporated. Besides expected failure costs and replacement costs, the operator has to deal with a fixed annual budget constraint:

$$\sum_{i=1}^{I} \left(c^f A_{t,i} + c^r R_{t,i}\right) \leqslant b, \quad \text{for } t = 1, 2, \ldots, T. \tag{4}$$

For all pipes together, the operator has a budget b, which is not to be exceeded. In (4), therefore, costs are summed over all ages. If the budget is too low given the age distribution, that is when expected failure costs exceed the budget, the system becomes infeasible.

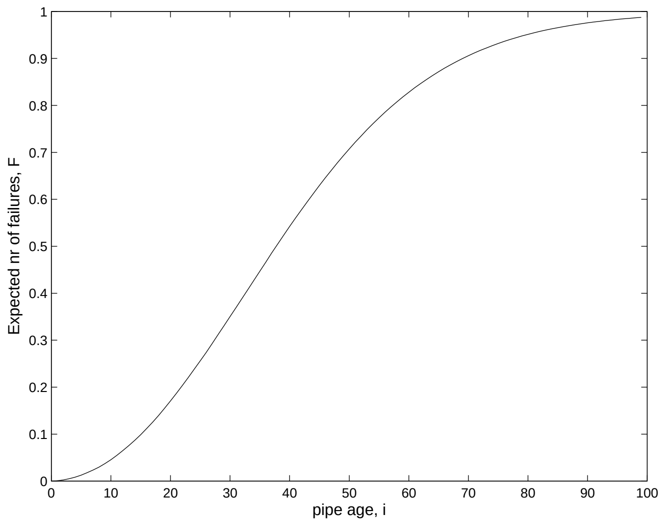

Fig. 1. Fraction of failures F depending on pipe age i, according to the model of Scheidegger *et al*. (2013).

2.3. *Uncoordinated and Coordinated Replacement*

Under uncoordinated replacement, it is considered that the water supply network operator is not informed about scheduled road work by operators of other underground infrastructure. Under coordinated replacement, the water supply network operator is informed about scheduled road work and comes with a reduction in the replacement cost for the operator. This is modelled as an unanticipated, instantaneous and temporary reduction in the replacement cost γc^r for all pipe ages. The behaviour of the operator will follow a rule derived from the model that reacts on the temporary reduction.

2.4. *Illustrative Data*

For this study we derive the fraction of failures F depending on pipe age i from the failure model developed in Scheidegger *et al*. (2013). This function is depicted in Fig. 1 and is based on the observation that the time between the placement of a pipe and its first failure has a different probability distribution than the times between consecutive failures. For the time to the first failure, a Weibull distribution is assumed and for the times between consecutive failures, exponential distributions are considered. Additionally, the mean time between all consecutive failures is assumed to be the same. From here on, F is referred to as the expected number of failures. Note that for the approach presented in this study, F_i may be derived from any other statistical failure model.

With respect to the water supply network operator, we consider an operator who is in charge of a network consisting of $A = 28$ pipes, where the initial maximum age I is 7. The purpose of this small artificial network is to illustrate the change in age composition as the operator makes a replacement decision every year. Each pipe can be replaced at a cost of $c^r = 5 \times 10^3$, but for illustration we also experiment with $c^r = 3 \times 10^3, 4 \times 10^3, 6 \times 10^3, 7 \times 10^3$. The unit cost per failure c^f comprises of a cost of water loss and is assumed to be 100×10^3. When road work is scheduled by operators of neighbouring infrastructure, the replacement cost of the water supply network operator reduces with $\gamma = 50\%$. We consider a finite horizon problem with discrete time steps equal to 1 year. This means that in each year t, the operator makes a replacement decision for the number of pipes of age i, subject to a fixed annual budget of $b = 120 \times 10^3$ or $b = 60 \times 10^3$, referred to as a respective high budget and low budget. All costs are expressed in monetary value.

3. Results

We address the research question what the optimal replacement time is without budget constraint by studying first the optimal replacement time in steady state and by analysing the transition to the steady state replacement time. This optimal replacement time gives insight into the replacement pattern that is consequently addressed with the research question what the optimal number of pipe replacements is with budget constraint. A simulation is performed to study the impact of uncoordinated and coordinated replacement, as well as a high budget and a low budget, on the replacement decision, expected failure costs and replacement costs.

3.1. *Optimal Replacement Time without Budget Constraint*

We address the question what the optimal replacement time is by elaborating on the model behaviour in steady state when there is no budget constraint. The transition to this steady state replacement time is analysed along different initial ages. With the optimal replacement time we are able to better understand the optimal number of pipe replacements.

3.1.1. *Steady State Replacement Time*

Figure 2 depicts the annual replacement cost, c^r/t, and the annual expected failure cost, $(c^f \sum_{i=0}^{t} F_i)/t$, when replacing the pipes every t years. For illustration, the costs are evaluated for different values of the unit replacement cost, $c_r = 3 \times 10^3, 4 \times 10^3, 6 \times 10^3, 7 \times 10^3$ and for replacement time $t = 1, 2, \ldots, 10$.

From the annual replacement cost and annual expected failure cost, the optimal replacement time t^* can be derived as

$$t^* = \arg\min_t \frac{c^f \sum_{i=0}^{t-1} F_i + c^r}{t}, \tag{5}$$

which indicates that the optimal replacement time is at the minimum of the sum of these costs. The total annual cost as a function of the replacement time t is depicted in Fig. 3. It shows for each value of the unit replacement cost c^r at what replacement time t^* the total cost is at its minimum and therefore optimal. For example, at $c^r = 3 \times 10^3$ it is optimal to replace a pipe every 5 years, $t^* = 5$, while at $c^r = 7 \times 10^3$ the optimal replacement time is

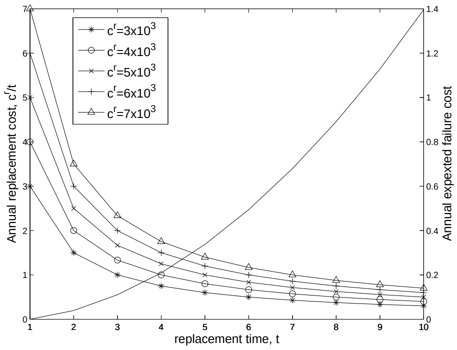

Fig. 2. Annual replacement cost and annual expected failure cost when steady state replacement takes place every t years.

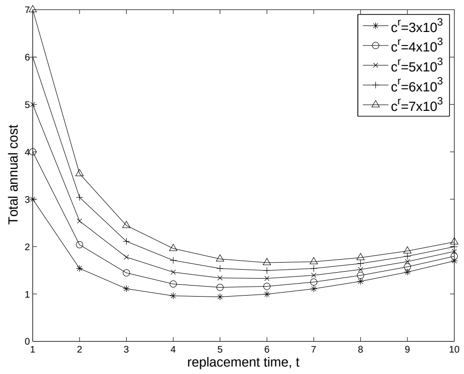

Fig. 3. Total annual cost, including expected failure cost and replacement cost, as function of replacement time t in steady state.

every 6 years, $t^* = 6$. This analytical result of the optimal replacement in discrete time is in line with the continuous time analysis in Kleiner (2001), Kleiner *et al.* (1998) and Nafi and Kleiner (2010). It is confirmed that the total cost function is convex with respect to the timing of replacement and has a point at which costs are minimum and the replacement time is optimal.

When reaching the steady state, the system follows the optimal replacement time t^* as derived in (5) with an annual total cost of

$$V = \min_t \frac{c^f \sum_{i=0}^{t-1} F_i + c^r}{t}, \tag{6}$$

where V can be interpreted as the minimized annual total cost, including the expected failure cost and replacement cost. The Appendix provides an elaboration on the effect of a change in the replacement cost on the optimal replacement time.

3.1.2. *Transition to Steady State Replacement Time*

The transition to the steady state replacement time, of pipes with an initial age j, can be derived from the above derived optimal replacement time t^*. It is known from F_i that older pipes have higher expected failure behaviour and thus higher expected failure costs, i.e. the expected failure is monotonously increasing in the age i. These pipes receive preference in terms of timely replacement, while younger pipes may be replaced in a later period. This means that, when $A_{0,i}$ consists of old and young pipes, old pipes are immediately replaced and reach directly the steady state in the sense that every consecutive replacement takes place with steps equal to the optimal replacement time t^*.

After the replacement, a pipe has an expected annual failure cost and replacement cost of V units. The question whether to replace a pipe of age j at the beginning of a horizon T or not reduces to the question whether its annual cost after replacement, $c^r + VT$, is lower than the cost of delaying the decision with one year, $c^r + c^f F_j + V(T-1)$. The general notation for the replacement decision is

$$c^f F_j > V. \tag{7}$$

While (7) holds, the expected failure cost $c^f F_j$ of a pipe with initial age j exceeds the long-term annual value V. Otherwise, the expected failure cost is incurred and the pipe becomes one year older, $j+1$, at which (7) is evaluated again.

3.2. *Optimal Number of Pipe Replacements with Budget Constraint*

The general notation for the replacement rule in (7) is valid for the case when the operator is not dependent on a budget. He can decide for each pipe, or alternatively for each pipe age, separately. In order to address the question how many pipes are replaced in each year with budget constraint under uncoordinated replacement and coordinated replacement, we introduce a budget constraint as in (4). Here it still holds that the oldest pipes are replaced first due to the increasing expected failure cost in age. This means that the replacement rule needs to be extended with a budget and a rule that says that in any period, the oldest pipe is replaced as long as the budget allows. This model may be solved using dynamic optimization, but with the presented model properties in (6)–(7), the model can be solved as in Algorithm 1. This pseudo code has as inputs the distribution of number of pipes A of

Algorithm 1 Pseudo code for optimal replacement decision R (in: A; out: R)

Funct: data, A vector
Find oldest existing age: $k = \max\{i | A_i > 0\}$
Replace all pipes with an expected failure cost that exceeds the replacement cost:
for $\forall t$ **do**
  while $c^f F_k > c^r$ **do**
    $R_k = A_k$
    $b = b - c^r R_k$
    $k = k - 1$
  end while
  $b = b - c^f \sum_{i=0}^{k} F_i A_i$
  if $b < 0$ **then**
    infeasible, budget too low
  end if
  With remaining budget b, replace as many pipes as possible for which the expected failure cost exceeds the annual steady state cost:
  while $c^f K_k > V$ and $b, k > 0$ **do**
    $R_k = \min\left\{A_k, \frac{b}{c^r - c^f F_k}\right\}$
    $b = b - (c^r - c^f F_k) R_k$
    $k = k - 1$
  end while
end for

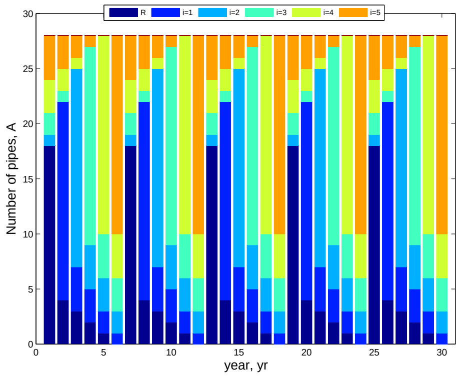

Fig. 4. Age composition under uncoordinated replacement and high budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 120 \times 10^3$ and $A_{0,i} = \{1, 2, \dots, 6, 7\}$.

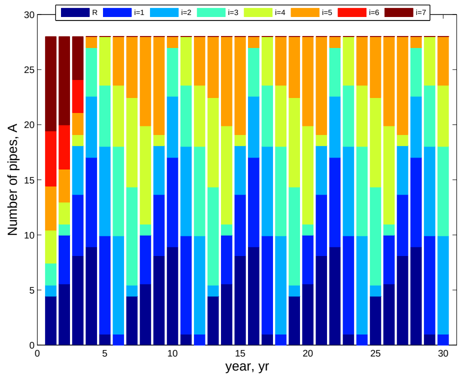

Fig. 5. Age composition under uncoordinated replacement and low budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 60 \times 10^3$ and $A_{0,i} = \{1, 2, \dots, 6, 7\}$.

age i and the budget b. The output is the replacement decision R in terms of the number of pipes of age i to be replaced. The algorithm starts with the selection of the oldest existing pipe age k. In the extreme case where there exists an age with an expected failure cost that is higher than the replacement cost, all of these pipes are replaced immediately. If there is no sufficient budget to do this and to incur the expected failure cost of the remaining pipes, the situation is infeasible. Otherwise, the budget is used to iteratively select the oldest pipes and to replace them until the budget is finished. The iteration works such that when pipes of age k are replaced and sufficient budget remains, the following pipe can be evaluated for replacement, which is the newest oldest pipe $k - 1$. With this algorithm uncoordinated replacement and coordinated replacement can be studied.

3.2.1. *Uncoordinated Replacement*

Under uncoordinated replacement different operators act without knowledge about each other's activities, such that the unit replacement cost c^r remains unchanged throughout the planning horizon. Figures 4 and 5 show simulations of the number of pipes A and age composition over a period of 30 years. The figures are based on inputs $c^f = 100 \times 10^3$ and $c^r = 5 \times 10^3$ and we initiate with $A_{0,i} = \{1, 2, \dots, 6, 7\}$, so that the total number of pipes is $A = 28$. Note that the purpose of this small artificial network of low ages is to illustrate the change in age composition over time. Figure 4 is based on an illustrative annual budget of $b = 120 \times 10^3$, which is reduced to $b = 60 \times 10^3$ in Fig. 5 to show the effect of the budget constraint on the replacement decision and age composition in each year. Applying (5), we find that in steady state the optimal replacement time is $t^* = 6$. This means that a specific replacement is repeated every 6 years, independently from the budget. For example, in Fig. 4, the budget is sufficiently high in the first year, $yr = 1$, to replace 18 pipes, $R = 18$. The remaining 10 pipes are of ages $i = 5, 4, 3, 2$, which indicates that the replaced pipes were of ages $i = 7, 6$. This operation is repeated every 6 years, in $yr = 7, 13, 19, 25$. Due to the more restrictive budget in Fig. 5, only $R = 4$ pipes

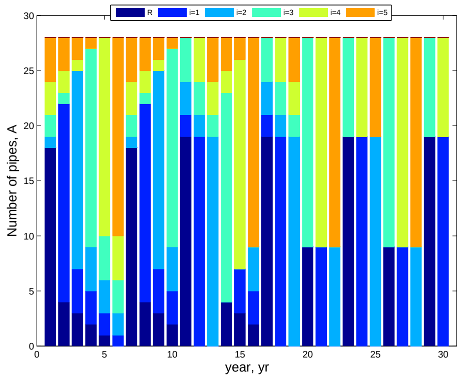

Fig. 6. Age composition under coordinated replacement and high budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 120 \times 10^3$ and $A_{0,i} = \{1, 2, \ldots, 6, 7\}$. In $yr = 11$ and $yr = 20$, the replacement cost c^r is reduced by $\gamma = 50\%$.

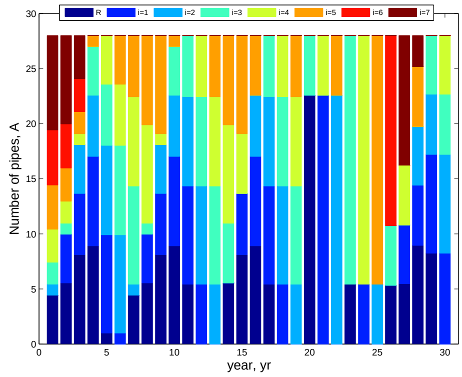

Fig. 7. Age composition under coordinated replacement and low budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 60 \times 10^3$ and $A_{0,i} = \{1, 2, \ldots, 6, 7\}$. In $yr = 11$ and $yr = 20$, the replacement cost c^r is reduced by $\gamma = 50\%$.

are replaced in the first year, leaving behind 14 pipes of ages $i = 7, \ldots, 2$. The effect of the low budget is not only observed in the extended transition to the steady state replacement time. Also in the remaining years, in steady state, relatively fewer pipes are replaced on an annual basis and each age is composed of more pipes. The replacement decision, however, still takes place every 6 years.

3.2.2. *Coordinated Replacement*

Under coordinated replacement, the water supply network operator is informed about planned road work by other operators. However, these plans are not anticipated by the operator, but are rather announced in the year that they take place. When this occurs, it is modelled as an unanticipated, instantaneous and temporary reduction in the replacement cost γc^r. In Fig. 6 we simulate the number of pipes A and age composition over a period of 30 years, with inputs $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$ and a high budget, $b = 120 \times 10^3$. In the years $yr = 11$ and $yr = 20$, the pipe replacement cost is reduced by $\gamma = 50\%$. From (5) we derive an optimal replacement time of $t^* = 6$, so that up to $yr = 10$ the replacement pattern is $R = 18$ in $yr = 1, 7$; $R = 4$ in $yr = 2, 8$; $R = 3$ in $yr = 3, 9$; and $R = 2$ in $yr = 4, 10$. Under unchanged conditions this would be followed by $R = 1$ in $yr = 5, 11$. However, this pattern is interrupted in $yr = 11$ due to planned road work and thus a 50% reduction in the replacement cost. Instead of replacing 1 pipe, the water supply network operator is induced to replace 19 pipes, $R = 19$. From the following year up to $yr = 19$, the optimal replacement time is back at $t^* = 6$, but with a new replacement pattern: the replacement decision of $R = 19$ in $yr = 11$ is repeated in $yr = 17$ and the zero replacement decision of $R = 0$ in $yr = 12, 13$ is repeated in $yr = 18, 19$. This pattern is interrupted again in $yr = 20$, due to the reduced replacement cost, resulting in a new pattern of $R = 9$ in $yr = 20, 26$; $R = 0$ in $yr = 21, 22, 27, 28$; and $R = 19$ in $yr = 23, 29$.

In Fig. 7 this exercise is repeated for a low budget of $b = 60 \times 10^3$. In the first 4 years the age composition is in transition to a steady state composition, which can be observed

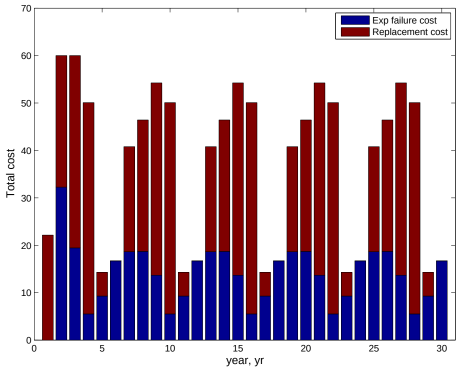

Fig. 8. Total cost under uncoordinated replacement and low budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 60 \times 10^3$ and $A_{0,i} = \{1, 2, \ldots, 6, 7\}$.

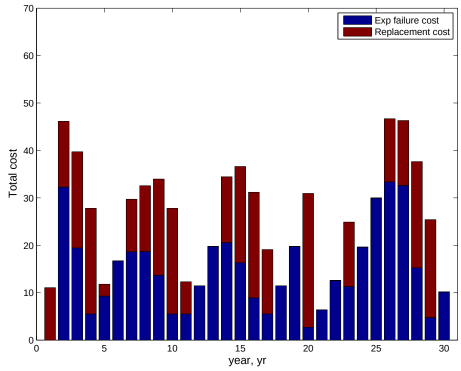

Fig. 9. Total cost under coordinated replacement and low budget. Simulated over 30 years; $c^f = 100 \times 10^3$, $c^r = 5 \times 10^3$, $b = 60 \times 10^3$ and $A_{0,i} = \{1, 2, \ldots, 6, 7\}$. In $yr = 11$ and $yr = 20$, the replacement cost c^r is reduced by $\gamma = 50\%$.

by relatively fewer replacements of older pipes of ages $i = 6, 7$. Nevertheless, a replacement pattern R is created that is repeated every 6 years. However, due to coordinated replacement, this pattern is disrupted in years $yr = 11, 20$. While with the high budget of $b = 120$ a replacement of 19 pipes is made in $yr = 11$, with the low budget of $b = 60$ only 5 pipes are replaced. This sets the pattern for the remaining years, until $yr = 19$, after which the operator faces another replacement cost reduction. The operator responds to this reduction in $yr = 20$ by replacing 23 pipes $R = 23$. Because of this large replacement, the operator only replaces one more time in the next 5 years. Only in $yr = 26$ a new pattern is created that should be repeated every 6 years.

3.2.3. *Costs*

In order to address the question what the effect of coordinated replacement and a budget constraint on costs is, we first compare total costs between uncoordinated and coordinated replacement based on a low budget. Figure 8 shows the simulated total cost including the expected failure cost $c^f F_i A_{t,i}$ and the replacement cost $c^r R_{t,i}$, summed over all ages. The figure is based on uncoordinated replacement and a budget of $b = 60 \times 10^3$. The total cost based on coordinated replacement and a budget of $b = 60 \times 10^3$ is shown in Fig. 9. It can be observed that in the years of coordinated replacement, in $yr = 11, 20$, the total cost is relatively higher. Considering uncoordinated replacement and the entire simulation period, the total expected number of pipe failures over all ages is 4.2 with a total expected failure cost of 413.6×10^3. The total number of pipe replacements is 140, which corresponds with a total replacement cost of 700×10^3. Together, this gives a total cost of 1113.6×10^3. Under coordinated replacement, the total cost reduces slightly to 1090.1×10^3. This is made up of a total expected failure cost of 438×10^3, with 4.4 expected failures, and a total replacement cost of 652.2×10^3, with 144 pipe replacements. This shows that coordinated replacement induces a 3% increase in pipe replacements at a

Table 1
Expected number of failures F_i, replacements $R_{t,i}$, expected failure costs $c^f F_i A_{t,i}$, replacement costs $c^r R_{t,i}$ and total costs $c^f F_i A_{t,i} + c^r R_{t,i}$, summed over the simulation period of 30 years and over all ages. Compared between uncoordinated and coordinated replacement, as well as between a low budget $b = 60 \times 10^3$ and a high budget $b = 120 \times 10^3$.

	Simulated over 30 years and all ages $\sum_{yr=1}^{30} \sum_{i}^{I}$	Low budget $b = 60 \times 10^3$	High budget $b = 120 \times 10^3$	Description
Uncoordinated replacement	F_i	4.2	4	exp. number of failures
	$R_{t,i}$	140	140	exp. number of replacements
	$c^f F_i A_{t,i}$	413.6×10^3	404×10^3	exp. failure cost
	$c^r R_{t,i}$	700×10^3	700×10^3	replacement cost
	$c^f F_i A_{t,i} + c^r R_{t,i}$	1113.6×10^3	1104×10^3	total cost
Coordinated replacement	F_i	4.4	3.6	exp. number of failures
	$R_{t,i}$	144	158	exp. number of replacements
	$c^f F_i A_{t,i}$	438×10^3	364.6×10^3	exp. failure cost
	$c^r R_{t,i}$	652.2×10^3	720×10^3	replacement cost
	$c^f F_i A_{t,i} + c^r R_{t,i}$	1090.1×10^3	1084.6×10^3	total cost

7% reduction in the total replacement cost, a 6% increase in the expected failure cost and a 2% reduction in the total cost.

In Table 1 we compare these results with expected failures, replacements and costs under a high budget of $b = 120$, summed over all years and all ages. Under a high budget, coordinated replacement leads to an increase in the total number of replacements and a lower expected number of failures. While this corresponds to 10% lower expected failure costs, the budget is sufficiently high to allow for 13% more replacements such that the total replacement cost increases with 3%. In the end, the total cost over the entire simulated period is reduced by 2%.

4. Discussion

Our results suggest that coordinated replacement induces the water supply network operator to increase the number of pipe replacements and to reduce the total cost. However, the budget constraint determines how this result is obtained. A number of differences can be found between a high budget and a low budget. First, while under a high budget the expected number of failures and failure costs reduce, a low budget restricts the replacement decision such that the total expected failure cost increases. This is explained by the increase in the expected failure cost in the period $yr = 21$ up to $yr = 30$, after the last year of coordinated replacement. Second, a high budget induces an increase in the number of replacements such that the replacement cost increases. This increase is offset by the reduction in the total expected failure cost. Therefore, with a 2% reduction in the total cost, for both a low budget and a high budget, coordinated replacement is economically preferred to uncoordinated replacement. This result, which is based on unanticipated coordination, agrees with Nafi and Kleiner (2010), who show savings based on anticipated coordination.

In Nafi and Kleiner (2010), however, savings are large, but this may be explained by the assumption that coordination is anticipated over multiple years.

Compared to a high budget, we further show that a low budget affects the transition to the steady state age composition in the sense that this period is extended. In steady state, a low budget leads to a replacement pattern of an annual replacement quantity ranging between 1 and 8, while this range is between 1 and 18 under a high budget. Nevertheless, the optimal replacement time is unaffected, because this timing is independent from the budget.

The reported results should be interpreted within the context of this model and considered parameter values. A number of simplifications can be identified that will be relaxed in future work. First, in this stylized model of a small artificial network the failure history of pipes is ignored. This may lead to pipes with poor failure histories being underrepresented in the data, affecting thereby parameter estimation. Nevertheless, the often absent pipe replacement data is well reported in Scheidegger (2011, 2013). Second, under coordinated replacement, a system-wide reduction in the replacement cost is incorporated. More realistically, this reduction is location specific and thus pipe specific, which would imply the introduction of a location index. Finally, for the purpose of illustration the results are based on an artificial network and artificial data. This numerical illustration may be verified with a real network.

5. Conclusions

Operators of underground water supply networks face challenges with pipe replacement decisions, because pipes are subject to increased failure rates as they age. Financial resources are often limited and so strategies need to be designed for water supply network operators to manage these pipe failure costs effectively. In addition, a policy maker may desire replacement activities of different underground infrastructure to be coordinated, in order to reduce the frequency of road work and social disruption. It remains unclear whether coordinated replacement is economically preferred to uncoordinated replacement and what the impact is of a budget constraint. We therefore investigated the optimal replacement time, replacement decision and total cost, including the expected failure cost and the replacement cost, for uncoordinated replacement and coordinated replacement and different budget constraints.

Our results suggest that coordinated replacement is economically preferred to uncoordinated replacement, both with a low budget and a high budget. With a low budget this is accomplished by means of more pipe replacements at a lower total replacement cost and in exchange for higher total expected failure costs. With a high budget the increase in pipe replacements is realized at a reduction in the total replacement costs, as well as at a reduction in the total expected failure costs. The combination of a higher budget and a reduction in the replacement cost in the years of coordinated replacement explains the trade-off between higher replacement costs and lower expected failure costs.

In future work, the stylized model will be extended to incorporate the location of a pipe, as well as the age. This will allow us to deal with coordinated replacement by means of a location specific and, therefore, a pipe specific reduction in the replacement cost. The application of the model to a real pipe network will give more insight into pipe replacement patterns.

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Appendix

Due to potential coordinated replacement activities, the replacement cost may go down and lead to a different replacement time. To understand the effect of a change in the replacement cost on the optimal replacement time, we are interested in the relative break-even cost for which the optimal replacement time t^* goes from time $t-1$ to t. At the break-even replacement cost both options $t-1$ and t are optimal. For replacement time $t = 1$ the annual total cost is $(c^f F_0 + c^r)/1$, for replacement time $t = 2$ the annual cost is $(c^f(F_0 + F_1) + c^r)/2$, for replacement time $t = 3$ this is $(c^f(F_0 + F_1 + F_2) + c^r)/3$, etc. For a relative break-even replacement cost, both options have the same annual cost, such that:

$$\frac{c^r + c^f \sum_{i=0}^{t-1} F_i}{t} = \frac{c^r + c^f \sum_{i=0}^{t} F_i}{t+1}. \tag{8}$$

Given failure cost c^f, the relative replacement cost c^r_{BE} at which we experience break-even, can be found by solving:

$$c^r_{\mathrm{BE}} = c^f \left(t \sum_{i=0}^{t} F_i - (t+1) \sum_{i=0}^{t-1} F_i \right). \tag{9}$$

If the actual replacement cost is below this break-even, $c^r < c^r_{BE}$, it is relatively inexpensive to replace a pipe and so replacement takes place every $t-1$ years. If the actual replacement cost exceeds the break-even replacement cost, $c^r > c^r_{BE}$, it is relatively expensive to replace a pipe so that replacement takes place less often, namely every t years. This can be verified along the following example. For replacement time $t = 2$ we evaluate going from $t = 1$ to $t = 2$. Here, replacement takes place every year at a replacement cost that falls below the break-even replacement cost and every 2 years at a replacement cost that exceeds the break-even replacement cost.

References

Chang, N.B., Hernandez, E.A. (2008). Optimal expansion strategy for a sewer system under uncertainty. *Environmental Modeling & Assessment*, 13(1), 93–113.

Hadzilacos, T., Kalles, D., Preston, N., Melbourne, P., Camarinopoulos, L., Eimermacher, M., Kallidromitis, V., Frondistou-Yannas, S., Saegrov, S. (2000). Utilnets: a water mains rehabilitation decision-support system. *Computers, Environment and Urban Systems*, 24(3), 215–232.

Jakaitiene, A., Žilinskas, A. (2010). Dynamic stochastic optimisation using endogenous gridpoints for consumption in Lithuania. In: *Proceedings of the 11th International Conference on Computer Systems and Technologies and Workshop for PhD Students in Computing on International Conference on Computer Systems and Technologies*, pp. 275–280. doi:110.1145/1839379.1839428.

Kim, J., Mays, L. (1994). Optimal rehabilitation model for water-distribution systems. *Journal of Water Resources Planning and Management*, 120(5), 674–692.

Kleiner, Y (2001). Scheduling inspection and renewal of large infrastructure assets. *Journal of Infrastructure Systems*, 7(4), 136–143.

Kleiner, Y., Rajani, B. (2001). Comprehensive review of structural deterioration of water mains: statistical models. *Urban Water*, 3(3), 131–150.

Kleiner, Y., Adams, B.J., Rogers, J.S. (1998). Long-term planning methodology for water distribution system rehabilitation. *Water Resources Research*, 34(8), 2039–2051.

Levišauskas, D., Galvanauskas, V., Simutis, R., Žilinskas, A., Žilinskas, J. (2006). Optimization of biomass production in fed-batch culture by feed and dilution control actions. *Information Technology and Control*, 35(4), 383–390.

Marinoni, O., Heyenga, S., Brigden, A., Archer, A., Higgins, A. (2012). Spending environmental expenditure more effectively: a case study from Brisbane, Australia. *Environmental Modeling & Assessment*, 17(3), 315–324.

Nafi, A., Kleiner, Y. (2010). Scheduling renewal of water pipes while considering adjacency of infrastructure works and economies of scale. *Journal of Water Resources Planning and Management*, 136(5), 519–530.

Papa, S., Casper, W., Moore, T. (2013). Securing wastewater facilities from accidental and intentional harm: a cost-benefit analysis. *International Journal of Critical Infrastructure Protection*, 6(2), 96–106.

Pelletier, G., Mailhot, A., Villeneuve, J. (2003). Modeling water pipe breaks—three case studies. *Journal of Water Resources Planning and Management*, 129(2), 115–123.

Rehan, R., Knight, M.A., Haas, C.T., Unger, A.J.A. (2011). Application of system dynamics for developing financially self-sustaining management policies for water and wastewater systems. *Water Research*, 45(16), 4737–4750.

Scheidegger, A., Hug, T., Rieckermann, J., Maurer, M. (2011). Network condition simulator for benchmarking sewer deterioration models. *Water Research* 45(16), 4983–4994.

Scheidegger, A., Scholten, L., Maurer, M., Reichert, P. (2013). Extension of pipe failure models to consider the absence of data from replaced pipes. *Water Research*, 47(11), 3696–3705.

Shamir, U., Howard, C. (1979). An analytic approach to scheduling pipe replacement. *Journal – American Water Works Association*, 71(5), 249–258.

Törn, A., Žilinskas, A. (1989). *Global Optimization*. *Lecture Notes in Computer Science*, Vol. 350. Springer, Berlin.

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Vamzdžių keitimas vandens tiekimo tinkle: koordinuotas ir nekoordinuotas keitimas bei biudžeto įtaka

Diana van DIJK, Eligius M.T. HENDRIX

Požeminių vandens tiekimo tinklų operatoriams kyla vamzdžių keitimo sprendimų iššūkių, nes senstant vamzdžiams avarijos dažnėja, o finansiniai ištekliai yra dažniausiai riboti. Tiriame optimalų keitimo laiką ir kiekį, kad tikėtini avarijų likvidavimo ir keitimo kaštai būtų minimizuoti ir tenkinantų biudžeto apribojimą, įtraukiant koordinuotą ir nekoordinuotą keitimą. Rezultatai rodo, kad koordinuotas keitimas parankesnis už nekoordinuotą dėl ekonominių priežasčių. Pasirinkimas priklauso nuo biudžeto dydžio, ar keitimų didinimas yra pakankamas sumažinti visus tikėtinus avarijų likvidavimo kaštus.