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Placement of Synchronized Measurements for Power System Observability

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***Abstract*—This paper presents a method for the use of synchronized measurements for complete observability of a power system. The placement of phasor measurement units (PMUs), utilizing time-synchronized measurements of voltage and current phasors, is studied in this paper. An integer quadratic programming approach is used to minimize the total number of PMUs required, and to maximize the measurement redundancy at the power system buses. Existing conventional measurements can also be accommodated in the proposed PMU placement method. Complete observability of the system is ensured under normal operating conditions as well as under the outage of a single transmission line or a single PMU. Simulation results on the IEEE 14-bus, 30-bus, 57-bus, and 118-bus test systems as well as on a 298-bus test system are presented in this paper.**

***Index Terms*—Integer quadratic programming, observability, optimal placement, phasor measurement units (PMUs), synchronized measurements.**

I. Introduction

MODERN-DAY power systems are being operated under heavily stressed conditions due to the ever-increasing demand for electricity and the operation in deregulated, competitive energy market conditions. A real-time wide-area monitoring, protection, and control (WAMPAC) system is therefore a critical necessity to enable the full utilization of the potential of the power system. Synchronized measurement technology (SMT) facilitates the realization of a WAMPAC by rendering the time-synchronized measurements from widely dispersed locations. Phasor measurement units (PMUs) are the most accurate and advanced instruments utilizing SMT available to the power system engineers and system operators [1]. The conventional SCADA-based state estimators cannot give a real-time picture of the power system due to the technical difficulties in synchronizing measurements from distant locations. The PMUs, when placed at a bus, can offer time-synchronized measurements of the voltage and current phasors at that bus [2].

Depending on the number of measurement channels and features, a PMU can be expensive. A suitable methodology is therefore needed to determine the optimal locations of the synchronized measurement devices so that the number of PMUs required to make the system completely observable is minimized. A power system is considered completely observable when all of the states in the system can be uniquely determined [3], [4]. The PMU placement methodology proposed in this paper ensures that the system is topologically observable during normal operating conditions as well as during the loss of a single transmission line or measurement unit.

The measurement placement problem, in general, is described as the problem of selecting locations to place measurements so that certain objectives and constraints are satisfied within a network. The problem has been around for a long time and has been examined under various research areas, such as in operational research, systems theory and control, and combinatorial optimization. In recent years, there has been significant research activity on the problem of finding the minimum number of PMUs and their optimal locations. In [5], a bisecting search method is implemented to find the minimum number of PMUs to make the system observable. The simulated annealing method is used to randomly choose the placement sets to test for observability at each step of the bisecting search. In [6], the authors use a simulated annealing technique in their graph-theoretic procedure to find the optimal PMU locations. The method is, however, limited by computational time and burden, even in the offline stage, as the size of the power system becomes large. In [7], a genetic algorithm (GA) is used to find the optimal PMU locations. The minimum number of PMUs needed to make the system observable is found by using a bus-ranking methodology. The PMU placement starts with one or more buses having maximum coverage. The main drawback of this approach, as pointed out in [5], is that it may not result in the minimum possible number of PMUs that can make the system observable. The authors in [8] use the condition number of the normalized measurement matrix as a criterion for selecting the candidate solutions, along with binary integer programming to select the PMU locations. The resulting solution, however, is not truly optimal, and the number of PMUs required is more than what is reported in other earlier works for standard test systems.

The PMU placement problem is similar to the set covering problem, which finds a subset of nodes with the minimum cardinality such that the whole graph is topologically observable [9]. The authors' approach in [10] and [11] is that of the set covering problem, where the integer programming is used to determine the minimum number of PMUs. However, the issue of measurement redundancy was not addressed. There can be more than one solution to the PMU placement problem with the same cost, but with different redundancy metrics. It is therefore important to choose the solution which results in the most desir-

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able distribution of measurement redundancy. In [12] and [13], the authors propose an exhaustive search-based methodology to determine the minimum number and optimal locations of PMUs for complete observability of the power system. Although the method gives the global optimal solution to the PMU placement problem, it becomes computationally intensive for large systems.

A natural extension of the set covering model is to combine two objectives in a quadratic function, formulating the quadratic set covering problem. In telecommunications, for instance, this problem may be formulated to maximize network capacity while having the minimum number of access points that cover an area [14]. In this paper, integer quadratic programming is used to achieve dual objectives: 1) to minimize the required number of PMUs and 2) to maximize the measurement redundancy. The constraints are formulated in such a way that the system remains observable as a single island even in the case of outage of a single transmission line or a PMU. Existing conventional measurements, such as power-injection measurements and power-flow measurements can be accommodated in the proposed PMU placement methodology. Further, the method allows for the inclusion of special requirements in the PMU placement strategy, such as specific redundancy levels at certain buses, and installation of PMUs in certain critical or preferred buses.

Section II gives the details of the PMU placement method, along with the formulation of the optimal PMU placement problem. The important steps of the proposed PMU placement methodology are illustrated with the help of a test system in Section III. Case studies and analysis of the results are given in Section IV, and Section V concludes this paper.

II. Optimal PMU Placement

When a PMU is placed at a bus, it can measure the voltage phasor at that bus, as well as at the buses at the other end of all the incident lines, using the measured current phasor and the known line parameters [12], [13]. It is assumed that the PMU has a sufficient number of channels to measure the current phasors through all branches incident to the bus at which it is placed.

It is to be noted here that the voltage phasors measured or estimated by the PMU are subjected to the errors in the measurement of voltage or current magnitudes and phase angles and the uncertainties in the transmission-line parameters [15]. The propagation of uncertainty in the voltage magnitude and phase angle along the transmission line can be computed by the use of the classical uncertainty propagation theory [16] or by using the random fuzzy variable approach [17], making use of the maximum measurement uncertainties provided by the manufacturer. The capability of PMUs to measure current phasors was examined so as to estimate the voltage phasors at some buses by using Kirchhoff's current law (KCL) (when applicable) [10], [11]. In the case of a power injection measurement at a bus, if the voltage phasors of all but one connected bus are known, the remaining one can be estimated by using KCL. However, the measurement uncertainties further propagate due to the use of KCL. In this paper, the use of current measurements by the PMUs to estimate voltage phasors is therefore limited only to the adjacent buses.

The first step in placing the PMUs is the identification of candidate locations. In an actual power system, there may be certain buses that are strategically important, such as a bus connected to a heavily loaded or economically important area, a bus anticipated to be a future expansion point, or a bus that already has a PMU installed. In such a case, the PMUs may be placed at the preferred buses. The rest of the buses are made observable by placing a minimum number of additional PMUs. The radial buses are excluded from the list of potential locations for placing a PMU because a PMU placed at a radial bus can measure the voltage phasors at that bus and only one additional bus that is connected to it, and a PMU placed at the bus connected to the radial bus can measure the voltage phasor of the radial bus by using the measurement of the current phasor through the radial line. Therefore, a PMU is preassigned to each bus connected to a radial bus. Preassigning PMUs to certain buses in this manner reduces the total number of possible combinations of PMU locations, thereby reducing the computational burden.

The elements of the binary connectivity matrix $\mathbf{A}$ for a power system, used in the formulation of the optimization problem, are defined as

$$\mathbf{A}(i,j) = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if bus } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The binary vector $\mathbf{x} \in \Re^n$ is defined as

$$x_i = \begin{cases} 1, & \text{if a PMU is placed at bus } i \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

and contains the PMU placement set.

The entries of the product $\mathbf{Ax}$ therefore represent the number of times a bus is observed by the PMU placement set defined by $\mathbf{x}$. The objective function $V(\mathbf{x})$ for optimization is formulated as in an integer quadratic program

$$V(\mathbf{x}) = \lambda(\mathbf{N} - \mathbf{Ax})^T \mathbf{R}(\mathbf{N} - \mathbf{Ax}) + \mathbf{x}^T \mathbf{Qx} \tag{3}$$

where $\lambda \in \Re$ is a weight, and $\mathbf{N} \in \Re^n$ is a vector representing the upper limits of the number of times that each bus can be observed by the PMU placement set $\mathbf{x}$. For the present case, the elements of $\mathbf{N}$ are equal to the number of incident lines to the corresponding bus plus one. The diagonal matrix $\mathbf{R} \in \Re^{n \times n}$ has entries r_{ii} representing the "significance" of each bus i, and allows for the allocation of varying significances to the buses. For example, a bus having critical facilities may be biased to attain a desired measurement redundancy before other buses. The diagonal matrix $\mathbf{Q} \in \Re^{n \times n}$ has entries q_{ii} allowing for the representation of varying installation costs of the PMUs at different buses. In the generic case, as assumed in this study, where all buses are equally significant and the PMU installation cost at all buses is the same, $\mathbf{Q}$ and $\mathbf{R}$ are equal to the identity matrix $\mathbf{I}^{n \times n}$.

The first part in (3) computes, for each bus in the system, the difference between the maximum possible number of times that the bus can be observed and the actual number of times it is observed by the PMU placement set $\mathbf{x}$. Minimization of this difference is therefore equivalent to maximizing the measurement redundancy. The measurement redundancy is defined as in [18]: the redundancy level of a measurement is equal to the number $(p-1)$ which corresponds to the smallest critical p-set to which the measurement belongs to. For instance, if the number of times a bus is observed by a PMU is increased by one, the measurement redundancy at that bus is also increased by one. The coefficient λ is used as a normalizing factor, such that $\lambda = (\mathbf{N}^T\mathbf{RN})^{-1}$. For varying installation costs, $\mathbf{x}^T\mathbf{Qx}$ [the second part of (3)] represents the total cost of PMU installation. In the generic case, when the installation costs of all the PMUs are assumed to be the same, $\mathbf{Q}$ is an identity matrix. Therefore, the minimization of $\mathbf{x}^T\mathbf{Qx}$ in this case is equivalent to minimizing the total number of PMUs in the system.

The normalization of the first part of (3) ensures that its value remains between 0 and 1, whereas the second part of (3) is an integer. Formulation of the optimization problem in this manner ensures that the minimization of the required number of PMUs is given higher priority, and the program does not increase the number of PMUs to increase the measurement redundancy. When the increase in measurement redundancy is more desirable than minimizing the number of PMUs, an alternate formulation is possible by using a weighted sum of the objective functions.

Equation (3) can be expanded as follows:

$$\begin{aligned} V(\mathbf{x}) &= \lambda\mathbf{N}^T\mathbf{RN} - 2\lambda\mathbf{N}^T\mathbf{RAx} \\ &\quad + \lambda\mathbf{x}^T\mathbf{A}^T\mathbf{RAx} + \mathbf{x}^T\mathbf{Qx} \\ &= \frac{1}{2}\mathbf{x}^T(2\lambda\mathbf{A}^T\mathbf{RA} + 2\mathbf{Q})\mathbf{x} \\ &\quad + (-2\lambda\mathbf{N}^T\mathbf{RA})\mathbf{x} + \lambda\mathbf{N}^T\mathbf{RN}. \end{aligned} \tag{4}$$

The optimization problem can therefore be formulated in an integer quadratic programming framework

$$\text{Minimize} \quad \frac{1}{2}\mathbf{x}^T\mathbf{Gx} + \mathbf{f}^T\mathbf{x} \tag{5}$$

$$\text{subject to} \quad \mathbf{Ax} \geq \mathbf{b} \tag{6}$$

where $\mathbf{G} = (2\lambda\mathbf{A}^T\mathbf{RA} + 2\mathbf{Q})$, $\mathbf{f} = (-2\lambda\mathbf{N}^T\mathbf{RA})^T$, and $\mathbf{b} = \mathbf{I}^{n\times 1}$.

The constant term $\lambda\mathbf{N}^T\mathbf{RN}$ on the right-hand side of (4) is not required to be included in the formulation of the optimization problem. Inequality (6) is the constraint for observability, which ensures that each bus in the system is observed at least once by the PMUs. If a measurement redundancy level of one or higher is desired for some or all of the system buses, the values of the elements in the vector $\mathbf{b}$ need to be increased accordingly. The objective function in (5) is minimized with respect to $\mathbf{x}$ using integer quadratic programming. The preassigned PMU locations, such as the buses connected to radial buses, are included by setting the corresponding elements in the vector $\mathbf{x}$ to 1.

A. Loss of a Single Transmission Line

The study of $N-1$ contingencies (i.e., a normal system minus one component) is widely practiced by utilities [19]. The optimal PMU placement methodology described here ensures that the system remains observable even in the case of outage of any single transmission line. When a line goes out, the rows of the connectivity matrix $\mathbf{A}$ corresponding to the terminal buses of the line need to be changed. For example, if the transmission line between buses i and j goes out, elements $\mathbf{A}(i,j)$ and $\mathbf{A}(j,i)$ in the original matrix $\mathbf{A}$ are set to zero. Denoting the ith and jth rows of the modified connectivity matrix by $\mathbf{a}_{\mathrm{i}}$ and $\mathbf{a}_{\mathrm{j}}$, respectively, it is necessary that the following constraint is satisfied to maintain observability under the outage of the line between buses i and j:

$$\begin{bmatrix} \mathbf{a}_i \\ \mathbf{a}_j \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{7}$$

A line outage table is constructed by including the lines, whose outage may affect the system. All of the radial lines are excluded from the branch outage table, since the outage of a radial line from an observable system is not expected to create any unobservable state in the remaining system. Radial links connected to radial subnetworks are also excluded from the branch outage table, since, after the outage of such a link, there is no way the system can be made completely observable without restoring the link. Let the set of row vectors in $\mathbf{A}$ that need to be modified due to the branch outages, one at a time, be denoted by $\mathbf{A}_1$. The set of additional constraints can then be written as

$$\mathbf{A}_1\mathbf{x} \geq \mathbf{b}_1 \tag{8}$$

where $\mathbf{b}_1 = \mathbf{I}^{2N_b\times 1}$ and N_{b} is the number of line outages considered.

B. Loss of a Single PMU

The proposed PMU placement method is designed to maintain complete observability even in the case of the outage of any single PMU. In general, a bus is observed by only one PMU by using a direct or a pseudomeasurement. The exceptions are cases, such as double lines between buses, where a bus may be observed more than once by the same PMU. A minimum redundancy level of one, therefore, ensures complete system observability, in general, for any single PMU outage. Buses with special cases are taken care of by setting the minimum redundancy levels accordingly. For example, in the case of a double line, the minimum redundancy level for the terminal buses is set at two. The constraint to ensure observability under single PMU outages in the absence of any such special cases is

$$\mathbf{Ax} \geq \mathbf{b}_2 \tag{9}$$

where $\mathbf{b}_2 = 2 * \mathbf{I}^{n\times 1}$.

C. Inclusion of Conventional Measurements

The PMU placement methodology discussed so far can make a system completely observable in the absence of any conventional measurement. However, since the PMU technology is relatively new, a more practical approach is to install the PMUs in an incremental fashion, and use the synchronized measurements in conjunction with the existing conventional measurements. There have been a number of approaches proposed in the literature for inclusion of PMUs into the existing conventional measurement system. Reference [20] proposes a method to install PMUs in the system to enhance the performance of the existing state estimator that uses data from the SCADA. The conceptual design of a "super calibrator" is described in [21], which recommends at least one PMU in each area or subnetwork to coordinate among individual state estimators in those areas.

The case studied here is the one where the power system has more than one island observable by conventional measurements. This situation may arise in practice as a result of the decision to replace some of the aging or malfunctioning conventional measurement units. The optimal PMU placement problem in this case is to find the optimal number and locations of the PMUs to make the system observable as a single island for normal operating conditions, as well as for the outage of a single transmission line or a single PMU. A numerical observability analysis is carried out to identify the observable islands in the system. A brief discussion of the numerical observability analysis technique proposed in [22] and used in this paper is presented to make the paper self-contained.

The gain matrix for real power measurements is given by

$$\mathbf{G} = \mathbf{H}^T\mathbf{H} \tag{10}$$

where $\mathbf{H}$ is the decoupled Jacobian of the real power measurements with respect to the bus voltage phase angles.

Using the well-known Gauss elimination technique for LU-decomposition, the gain matrix can be factorized as follows:

$$\mathbf{LU} = \mathbf{PG} \tag{11}$$

where $\mathbf{L}$ is a unitary lower triangular matrix, $\mathbf{U}$ is an upper triangular matrix, and $\mathbf{P}$ is the row permutation matrix.

The gain matrix will be singular even when the system is completely observable as a single island, since all of the buses, including the slack bus, are considered while formulating the gain matrix. In this case, the last diagonal element of $\mathbf{U}$ will be zero. In case zero pivots are encountered before the last diagonal element, the factorization process continues until the end, with zeros in the corresponding diagonal elements of the upper triangular matrix $\mathbf{U}$. Finally, the diagonal of $\mathbf{U}$ can be expressed as

$$\mathbf{D} = \mathbf{L}^{-1}\mathbf{G}(\mathbf{L}^{-1})^T \tag{12}$$

where $\mathbf{D}$ is a diagonal matrix containing zeros in the positions corresponding to zero pivots encountered during the LU-decomposition process of the gain matrix $\mathbf{G}$.

A test matrix $\mathbf{W}$ is now defined, which contains those rows of the matrix $\mathbf{L}^{-1}$, where the diagonal matrix $\mathbf{D}$ has zeros. For example, if the ith diagonal element of $\mathbf{D}$ is zero, the ith row of $\mathbf{L}^{-1}$ is included in $\mathbf{W}$. The unobservable branches are identified by using the matrix $\mathbf{C}$ that is defined as

$$\mathbf{C} = \mathbf{B}\mathbf{W}^T \tag{13}$$

where $\mathbf{B}$ is the bus to branch incident matrix defined as [4]

$$\mathbf{B}(i,j) = \begin{cases} 1, & \text{if bus } j \text{ is the sending end of branch } i \\ -1, & \text{if bus } j \text{ is the receiving end of branch } i \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

It can be shown that if at least one entry in a row of the matrix $\mathbf{C}$ is not zero, then the corresponding branch is unobservable. The observable islands in the system are obtained by removing the unobservable branches.

The objective of the work described in this paper is to find the minimal set of PMU locations that can make the system completely observable as a single island. It is to be noted that all buses inside an island found by the above process are observable within the island. To merge two observable islands, it is sufficient to provide a voltage phasor measurement that can be referred from both islands. To make the whole system observable under normal operating conditions, the proposed PMU placement strategy therefore ensures that at least one of the voltage phasors among the buses inside an island be measured by a PMU. In the form of a constraint for the optimization process, this can be expressed as

$$\mathbf{s}_k\mathbf{x} \geq 1, \quad \forall k = 1, \ldots, N_{\text{island}} \tag{15}$$

where N_{island} is the number of observable islands in the power system, $\mathbf{s}_k \in \Re^n$ is a vector representing the buses inside the kth island and the connected buses, and its elements are defined as follows:

$$\mathbf{s}_k(i) = \begin{cases} 1, & \text{if bus } i \text{ belongs to the } k\text{th observable island} \\ & \text{or it is connected to a bus inside the island} \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

To ensure complete observability under the outage of a single PMU, the constraints can be formulated as

$$\mathbf{s}_k\mathbf{x} \geq 2, \quad \forall k = 1, \ldots, N_{\text{island}}. \tag{17}$$

In the next step, the PMU placement methodology ensures that the system remains observable under the outage of a single transmission line. A list of branch outages to be considered is prepared beforehand. The radial lines and the links connected to radial subnetworks are excluded from this list due to the reasons described before. The outage of any branch other than these two types of branches inside an observable island does not create any new island. The branch outage table therefore consists of the unobservable branches outside the observable islands. Starting with the outage of the first branch in the list, the following steps are taken to ensure observability of the system under the outage of any single transmission line included in the branch outage table.

Step 1) Take a branch in the branch outage list out of the system.

Step 2) The following constraint is added to the optimization process to ensure observability of the system under the outage of the branch under consideration:

$$\mathbf{s}'_k\mathbf{x} \geq 1, \quad \forall k = 1, \ldots, N_{\text{island}} \tag{18}$$

where $\mathbf{s}'_k$ is the vector defined as in (16).

Step 3) Restore the branch to the system, go to step 1, and proceed with the next branch outage until the outages of all branches in the list, one at a time, are considered.

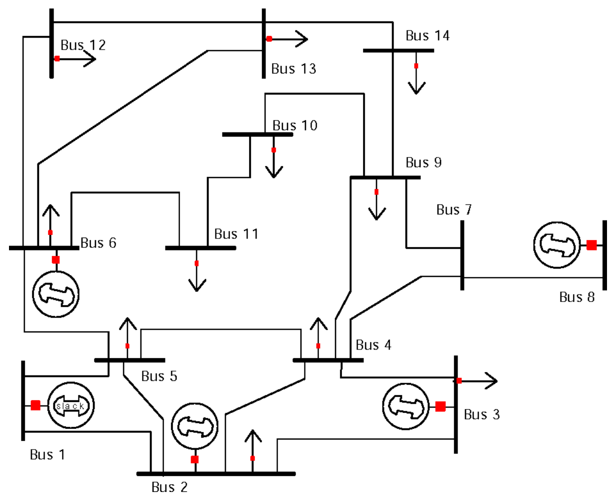

Fig. 1. IEEE 14-bus test system [24].

It is to be noted that the paper proposes a method of optimal placement of PMUs to ensure complete topological observability of a power system. A state estimation method, such as the least-squares estimator, should be used to estimate the states of the power system based on the measurements obtained from these PMUs. In the presence of conventional measurements, such as the asynchronous RTU-SCADA measurements, the least-square estimator will be nonlinear. If only synchronized measurements are used, the estimator will be linear [20]. The SCADA data are updated every 4–5 s. The PMU measurements can be synchronized with the conventional measurements by using the timestamps. Determining the best time at which synchronized measurements should be used with conventional measurements requires extensive system-specific case studies [23].

III. Formulation Example Using a Test System

The important steps in the formulation of the optimal PMU placement problem are illustrated with the help of the IEEE 14-bus test system [24], the single-line diagram of which is shown in Fig. 1.

A. Formulation Without Conventional Measurements

PMU placement for the 14-bus system in the absence of conventional measurements is illustrated in this section. The optimal PMU placements for the 14-bus system for 1) normal operating conditions, without maximizing the measurement redundancy; 2) normal operating conditions, maximizing the measurement redundancy; and 3) the outage of a single branch or a single PMU are shown in Table I. The TOMLAB optimization package [25] is used to carry out the optimization process. It uses a branch-and-cut algorithm for solving the integer quadratic program.

The first part in (3) tries to maximize the number of times a bus is observed by the PMU placement set. The second column in Table II shows the number of times the buses 1 to 14 in the 14-bus system are observed by the two different PMU placement sets. For the placement set $\{2,7,10,13\}$, only the number of PMUs is minimized, without maximizing the measurement redundancy. For the PMU placement set $\{2,6,7,9\}$, the number of PMUs is minimized and the measurement redundancy at the buses is maximized. Clearly, the latter one results in a more desirable distribution of measurement redundancy. For example, the number of times that buses 4, 5, and 7 are observed is more in the second case, compared to the first case.

TABLE I
Optimal Locations of PMUs for the IEEE 14-Bus Test System in the Absence of Conventional Measurements

System configuration	Optimal PMU locations
Normal operating conditions, without maximizing measurement redundancy	2, 7, 10, 13
Normal operating conditions, maximizing measurement redundancy	2, 6, 7, 9
Considering single branch or PMU outages	2, 4, 5, 6, 7, 8, 9, 11, 13

TABLE II
Effect of Maximization of PMU Measurement Redundancy on the 14-Bus Test System

PMU locations	Number of times each bus is observed
2, 7, 10, 13	1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1
2, 6, 7, 9	1, 1, 1, 3, 2, 1, 2, 1, 2, 1, 1, 1, 1, 1

The connectivity matrix for the system is given by

$$\mathbf{A} = \begin{bmatrix} 1\,1\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 1\,1\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,1\,1\,1\,0\,1\,0\,1\,0\,0\,0\,0\,0 \\ 1\,1\,0\,1\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,1\,1\,1\,0 \\ 0\,0\,0\,1\,0\,0\,1\,1\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0\,1\,0\,1\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1 \end{bmatrix}. \tag{19}$$

Additional constraints are included to ensure observability of the system under single-line outages. For example, if the transmission line between buses 2 and 3 is removed from service, $\mathbf{A}(2,3)$ and $\mathbf{A}(3,2)$ will be zero. The additional set of constraints in this case is

$$\begin{bmatrix} 1\,1\,0\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{20}$$

The observability of the system under the outage of a single PMU is ensured by setting the minimum redundancy level at one for all buses as shown in (9).

B. Formulation With Conventional Measurements

Actual power systems already have conventional measurements, such as power flow and power injection measurements. Therefore, the proposed method is applied to test systems containing conventional measurements. The optimal placement of PMUs is illustrated with the help of the IEEE 14-bus test system having conventional measurements at the locations shown in Fig. 2. The observable islands are $\{1,2,3,4,5,7,8,9\}$, $\{6\}$, $\{10\}$, $\{11\}$, $\{12\}$, $\{13\}$, and $\{14\}$, as shown in Fig. 2.

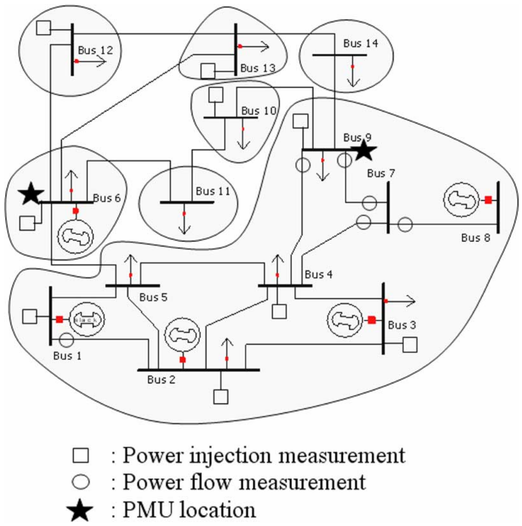

Fig. 2. Observable islands of the IEEE 14-bus test system with conventional measurements, and optimal PMU locations to make the system observable under normal operating conditions [22].

TABLE III
OPTIMAL LOCATIONS OF PMUS FOR THE IEEE 14-BUS TEST SYSTEM IN THE PRESENCE OF CONVENTIONAL MEASUREMENTS

System configuration	Optimal PMU locations
Normal operating conditions	6, 9
Considering single branch or PMU outages	6, 9, 10, 13

Under normal operating conditions, the following set of constraints needs to be satisfied to ensure complete observability of the system:

$$\begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \mathbf{s}_3 \\ \mathbf{s}_4 \\ \mathbf{s}_5 \\ \mathbf{s}_6 \\ \mathbf{s}_7 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1\,1\,0\,0\,0\,0\,1\,1\,1\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,1\,1\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,1\,1 \end{bmatrix} \mathbf{x} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}. \quad (21)$$

To ensure observability of the system under single PMU outages, 1's in the rightmost vector in (21) are replaced with 2's. Additional constraints shown in (18) are needed to ensure observability under single line outages. Table III shows the optimal PMU locations for complete observability for normal operating conditions as well as for the outage of a single PMU or transmission line, when conventional measurements are present in the system. The optimal locations of the PMUs to ensure observability under normal operating conditions are also shown in Fig. 2.

IV. CASE STUDIES

The proposed PMU placement method is applied to the IEEE 30-bus, 57-bus, 118-bus and 298-bus systems [24]. The buses of the 298-bus system are renumbered from 1 to 298 for ease of reference. Test results are reported first by assuming that the systems have no conventional measurements. The 298-bus system is then assumed to have a set of conventional measurements, and the test results are also reported for this system. The radial buses are eliminated from the potential PMU locations. Table IV shows the number of radial buses in each of the test systems. The computational burden is further reduced by pre-assigning PMUs to a bus connected to a radial bus in order to make all radial buses observable.

Table V shows the minimum number of PMUs required to make each test system observable under normal operating conditions. Table VI shows the required minimum number of PMUs to make the system observable under the outage of a single transmission line or a single PMU in the absence of any conventional measurements. Tables VII–X show the optimal PMU locations for the test systems as found by the integer quadratic programming method proposed in this paper.

The proposed method of PMU placement is also applied to the 298-bus test system in the presence of conventional measurements. The system is assumed to have flow measurements in 70% of its lines and injection measurements at 30% of its

TABLE IV
NUMBER OF RADIAL BUSES IN THE TEST SYSTEMS

Test systems	No. of radial buses
IEEE 30-bus	3
IEEE 57-bus	1
IEEE 118-bus	7
298-bus	67

TABLE V
MINIMUM NUMBER OF PMUS FOR OBSERVABILITY UNDER NORMAL OPERATING CONDITIONS IN THE ABSENCE OF CONVENTIONAL MEASUREMENTS

Test systems	Minimum number of PMUs
IEEE 30-bus	10
IEEE 57-bus	17
IEEE 118-bus	32
298-bus	86

TABLE VI
MINIMUM NUMBER OF PMUS FOR OBSERVABILITY UNDER THE OUTAGE OF A SINGLE TRANSMISSION LINE OR SINGLE PMU IN THE ABSENCE OF CONVENTIONAL MEASUREMENTS

Test systems	Minimum number of PMUs
IEEE 30-bus	21
IEEE 57-bus	33
IEEE 118-bus	68
298-bus	199

TABLE VII
OPTIMAL LOCATIONS OF PMUS FOR THE IEEE 30-BUS TEST SYSTEM

System configuration	Optimal PMU locations
Normal operating conditions	2, 4, 6, 9, 10, 12, 15, 19, 25, 27
Considering single branch or PMU outages	1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 15, 16, 18, 19, 22, 24, 25, 26, 27, 28, 29

TABLE VIII
OPTIMAL LOCATIONS OF PMUS FOR THE IEEE 57-BUS TEST SYSTEM

System configuration	Optimal PMU locations
Normal operating conditions	1, 4, 6, 9, 15, 20, 24, 25, 28, 32, 36, 38, 41, 47, 50, 53, 57
Considering single branch or PMU outages	1, 3, 4, 6, 9, 11, 12, 15, 19, 20, 22, 24, 25, 26, 28, 29, 30, 32, 33, 35, 36, 37, 38, 41, 45, 46, 47, 50, 51, 53, 54, 56, 57

TABLE IX
OPTIMAL LOCATIONS OF PMUS FOR THE IEEE 118-BUS TEST SYSTEM

System configuration	Optimal PMU locations
Normal operating conditions	3, 5, 9, 12, 15, 17, 21, 23, 28, 30, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 114
Considering single branch or PMU outages	1, 3, 5, 6, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 29, 30, 31, 32, 34, 35, 37, 40, 41, 44, 45, 46, 49, 51, 52, 54, 56, 57, 59, 61, 62, 64, 66, 68, 70, 71, 73, 75, 77, 79, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 116, 117, 118

TABLE X
OPTIMAL LOCATIONS OF PMUS FOR THE 298-BUS TEST SYSTEM IN THE ABSENCE OF CONVENTIONAL MEASUREMENTS

System configuration	Optimal PMU locations
Normal operating conditions	1, 2, 3, 11, 12, 15, 17, 19, 22, 23, 25, 27, 29, 33, 37, 38, 43, 48, 49, 53, 54, 55, 58, 59, 60, 62, 64, 65, 68, 71, 79, 83, 85, 86, 88, 89, 93, 95, 97, 101, 109, 111, 112, 113, 116, 118, 119, 122, 132, 133, 137, 143, 145, 152, 157, 163, 167, 173, 177, 183, 187, 189, 190, 193, 196, 202, 204, 208, 210, 211, 213, 216, 217, 219, 224, 225, 228, 265, 266, 267, 268, 270, 271, 272, 274, 292
Considering single branch or PMU outages	1, 2, 3, 5, 7, 8, 11, 12, 15, 16, 17, 19, 20, 22, 23, 25, 27, 29, 33, 34, 36, 37, 38, 39, 41, 43, 44, 46, 48, 49, 51, 53, 54, 55, 57, 58, 59, 60, 62, 64, 65, 68, 69, 71, 73, 76, 77, 78, 81, 82, 83, 85, 86, 88, 89, 90, 93, 94, 95, 96, 97, 99, 101, 103, 105, 109, 111, 112, 113, 115, 116, 118, 119, 122, 124, 125, 127, 132, 134, 135, 137, 143, 144, 145, 148, 149, 150, 151, 152, 155, 157, 160, 162, 163, 164, 167, 168, 170, 173, 175, 177, 179, 183, 184, 185, 189, 190, 192, 193, 194, 196, 198, 199, 201, 202, 203, 204, 206, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 296, 297, 298

TABLE XI
OPTIMAL LOCATIONS OF PMUS FOR THE 298-BUS TEST SYSTEM IN THE PRESENCE OF CONVENTIONAL MEASUREMENTS

System configuration	Optimal PMU locations
Normal operating conditions	19, 36, 64, 141, 268
Considering single branch or PMU outages	19, 23, 33, 36, 60, 64, 141, 144, 268, 292

buses. All of the conventional measurements are randomly distributed in the system. It was determined that by using the observability analysis technique described in Section II-C that the system consists of nine observable islands having 204, 3, 2, 2, 4, 73, 3, 5, and 2 buses inside the islands. The PMUs are placed so that the entire system becomes observable as a single island. Table XI shows the optimal PMU locations to ensure complete observability of the system under normal operating conditions as well as under the outage of a single transmission line or single PMU.

TABLE XII
CPU TIME REQUIRED TO FIND THE OPTIMAL PMU LOCATIONS

Test systems	Computational time	
	Normal operating conditions	Considering single branch or PMU outages
IEEE 14-bus without conventional measurements	3.09 s	9.72 s
IEEE 14-bus with conventional measurements	5.31 s	8.77 s
IEEE 30-bus	2.60 s	4.24 s
IEEE 57-bus	4.24 s	4.51 s
IEEE 118-bus	3.09 s	4.70 s
298-bus without conventional measurements	4.35 s	6.90 s
298-bus with conventional measurements	5 min 24 s	1 hr 40 min

Table XII shows the computational time requirements to find the optimal PMU locations for the case studies presented in this section. The simulations are carried out on an Intel Xeon 3.4-GHz CPU with 2-GB RAM.

As noted from Table XII, if no conventional measurements are considered, the computational time is minimal. The computational time is significant when conventional measurements are considered, especially if the single branch or PMU outages are taken into account. This is due to the additional computational time required for carrying out observability analysis with conventional measurements. The computational time, however, is not a serious issue since the PMU placement is a planning problem in nature.

In addition to the test results reported before, the proposed method of optimal PMU placement was applied to the New England 39-bus [26] and IEEE 24-bus test systems [24]. The minimum number of PMUs needed to make the system observable under normal operating conditions for these two systems and the IEEE 14-bus and IEEE 30-bus systems are the same as found in [13]. The PMU placement methodology proposed in this paper has the provision of including user-defined values of measurement redundancy at the buses. Further, as noted in Section II, it is possible to influence the placement result by assigning weights to model the significance of each bus or the varying installation costs at different buses. A desirable property of a measurement placement scheme is to avoid critical measurements, the outage of which makes the system unobservable. The optimal PMU placement considering single-branch or PMU outages, as shown in the simulation results in this section, improves the reliability of the state estimator by eliminating the occurrence of any critical measurements.

V. CONCLUSION

A methodology for the optimal placement of PMUs for rendering a power system topologically observable is proposed in this paper. An integer quadratic programming approach is used to determine the optimal locations of PMUs. The optimization process tries to attain dual objectives: 1) to minimize the number of PMUs needed to maintain complete observability of the system for normal operating conditions as well as for the outage of a transmission line or PMU and 2) to maximize the measurement redundancy at all buses in the system. The method was applied on IEEE test systems considering the outage of a single transmission line or a single PMU. The proposed method can be used to determine PMU locations when conventional measurements, such as line flows and power injection measurements, are available.

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