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Int. J. Electron. Commun. (AEÜ) 63 (2009) 116–122
International Journal of Electronics and Communications AEÜ
www.elsevier.de/aeue

Planar antenna pattern nulling using differential evolution algorithm

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Received 22 March 2007; accepted 12 November 2007

Abstract

Differential evolution algorithm is used for the pattern synthesis of planar antenna arrays with prescribed pattern nulls by position-only and position-amplitude optimization. The position-only optimization for a planar array allows null synthesis in any prescribed direction. For planar antenna array thinning it is necessary to use position-amplitude optimization for problems involving more than two nulls.

Keywords: Planar antenna arrays; Pattern nulling; Differential evolution

1. Introduction

Several global optimization techniques such as genetic algorithm, simulated annealing, modified touring ant colony optimization algorithm, and differential evolution algorithm are used in antenna array pattern nulling for linear antenna arrays [1–8]. Although there are many analytical and numerical techniques proposed for the null synthesis of planar antenna arrays [9–16], there is not so much in the literature regarding the application of global optimization techniques to planar antenna arrays [17–19]. One application of genetic algorithm to a 20 × 10 elements planar array is given in [17]. The author has obtained a planar array filled 54%. In [18] the time modulation approach is applied to planar antenna arrays with square lattices and circular boundaries to synthesize ultra-low sidelobe patterns. Differential evolution algorithm is used to suppress the sideband levels of the time modulated planar array. Differential evolution algorithm is applied to the minimization of sidelobe levels of planar arrays in [19]. Authors have optimized a 31-element linear array through differential evolution algorithm, and then used this linear array to build a 31 × 31 square planar array. This 31 × 31 square array is optimized again by using genetic algorithm. A two-step optimization process is used by the authors. Also it is emphasized that differential evolution algorithm and binary-coded genetic algorithm together are suitable in sidelobe level minimization of planar arrays.

In [1] genetic algorithm has been applied to the problem of array pattern nulling by element position perturbations. In [2] phase and position perturbations based on the genetic algorithm are used for pattern nulling of linear arrays. In [3] simulated annealing method is applied to the synthesis of unequally spaced linear arrays. Position-only optimization and position and weight optimizations are considered. In [4] modified touring ant colony optimization algorithm is applied to null steering of linear antenna arrays by controlling both the amplitude and phase of the array elements. Authors have shown that it is possible to obtain triple nulls having null depth levels deeper than −90 dB, and they have also shown that it is possible to obtain a broad null centered at a determined frequency having a null depth level of −65 dB. In [5] differential evolution algorithm is applied to the optimization of static excitation amplitudes of array

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1434-8411/$ - see front matter
doi:10.1016/j.aeue.2007.11.006

elements. It is shown that sidelobe levels of a moving phase center antenna array can be significantly lowered by this way. In [6] differential evolution algorithm is applied to linear array pattern synthesis with prescribed nulls. The only control parameters are array element excitation amplitudes. The authors have illustrated distinct features of the differential evolution algorithm. In [7] differential evolution algorithm is applied to uniform amplitude arrays with unequal spacing and equal phases (position-only synthesis) and with unequal spacing and unequal phases (position-phase synthesis). Authors have demonstrated that the position-phase synthesis results in reduced sidelobe levels compared to that of the position-only synthesis. In [8] a real coded genetic algorithm is used for the design of reconfigurable dual-beam linear isotropic antenna arrays. The only control parameters used are the phases of digital phase shifters. Dynamic range ratio may be taken as pre-fixed.

There are also analytical techniques such as [20–24] for the optimization of linear arrays. In [20] a linear array with arbitrarily distributed elements is discussed. Array factor is represented in terms of the sum of Bessel functions. In [21] an analytical approach to the synthesis of unequally spaced antenna arrays is described. This synthesis technique allows the determination of appropriate element spacings for a prescribed pattern by means of a Legendre transformation of the array factor. In [22] a perturbational procedure for reducing sidelobe levels by using nonuniform element spacings, while retaining uniform excitation is presented. It is shown that the sidelobes can be reduced in height to approximately $2/N$ times the main lobe level, where N is the number of elements. In [23] a method based on the use of Poisson's sum formula is presented. The author shows that this method is particularly suited for unequally spaced arrays with a large number of elements which are located on a line or on a curve. In [24] dynamic programming is applied to the minimization of sidelobe levels of unequally spaced linear arrays.

While some studies in the literature consider special planar geometries, some have a broader scope. In [9] a method for uniformly spaced planar arrays is presented that produces a Chebyshev pattern in any cross section with the same specified sidelobe level. Formulas for the determination of current amplitudes are given. In [10] it is shown that the radiation pattern of a hexagonal array with triangular arrangement of elements can be synthesized by using a pattern of linear array and that the resulting pattern has the same sidelobe level as the corresponding linear array pattern. In [11] the radiation pattern of a linear array and a hexagonal planar array is represented by Gegenbauer polynomials. It is shown that there is an optimum directivity for a specified sidelobe level. In [12] an array with a general triangular grid structure is considered and it is shown that this array has a grating lobe of -6 dB relative to the main lobe. Also, the necessary conditions to avoid grating lobes are presented. In [13] a method is developed for maximization of the gain of a planar array with a quadratic or triangular array lattice at a prescribed sidelobe level. In [14] a null synthesis procedure is given for uniformly spaced planar arrays. This synthesis procedure is based on a two-dimensional convolution process. A four-element canonical array is used as a building block for large planar arrays. In [15] a numerical search technique is given to minimize the maximum pattern sidelobe by phase-only adjustment of the element excitations. Results for linear arrays and circular planar arrays are presented. In [16] a general analytical technique for the synthesis of unequally spaced arrays with linear, planar, cylindrical or spherical geometry is presented. For planar arrays the element positions are determined by using Legendre transformation.

Differential evolution algorithm is also applied to electromagnetic inverse scattering problems [25,26]. In [25] differential evolution algorithm is applied to electromagnetic inverse scattering of multiple two-dimensional perfectly conducting objects. The author has demonstrated the search-ability, simplicity, robustness of the differential evolution algorithm and has shown that differential evolution algorithm outperforms the genetic algorithms in convergence performance. In [26] a novel evolution algorithm called dynamic differential evolution strategy is developed and applied to electromagnetic inverse scattering problems. This algorithm inherits the genetic operators from differential evolution strategy, but updates its population dynamically. The author has observed that dynamic differential evolution strategy outperforms differential evolution strategy in efficiency, robustness and memory requirements.

In planar antenna synthesis with using global optimization techniques nonconvergence by sticking in local maxima is a serious problem. In this work, it is shown that differential evolution algorithm can be used as an effective tool for the optimization of planar antenna arrays. It is observed that it is possible to synthesize nulls in prescribed directions by the position-only optimization of a planar array; on the other hand for planar array thinning problem it is possible to achieve satisfactory results for simple problems involving no more than 2 nulls.

2. Differential evolution algorithm

Differential evolution algorithm is a global search technique and is well documented in the literature [6,7,25–28]. In this paper, the notation of the differential evolution algorithm described in [28] is followed. After the specification of lower and upper bounds for parameters, the population is initialized. The population size is shown by Np, and the number of optimization parameters by D. The initial population is represented by $\mathbf{x}^{i,g}$. After initialization, differential mutation which adds a special vector difference to a third vector is applied. This process can be described as

$$\mathbf{v}^{i,g}=\mathbf{x}^{r0,g}+F\cdot(\mathbf{x}^{r1,g}-\mathbf{x}^{r2,g}), \quad i \neq r0 \neq r1 \neq r2, \qquad (1)$$

where $\mathbf{v}^{i,g}$ is the mutant vector, the superscript i is the population index which runs from 0 to $Np-1$, and the superscript g is the generation index of the parent population. $r0$ is called the base vector index and there are several algorithms for the selection of $r0$. $r1$ and $r2$ are the difference vector indices and are randomly selected. F is a real number between the limits (0, 1) and is called the scale factor. F controls the evolution rate of the population.

After mutation, each vector is crossed with a mutant vector to obtain the trial vector $\mathbf{u}^{i,g}$. The crossover operation can be defined as

$$(\mathbf{u}^{i,g})_j = \begin{cases} (\mathbf{v}^{i,g})_j, & \gamma \leqslant Cr, \\ (\mathbf{x}^{i,g})_j, & \text{otherwise,} \end{cases} \quad (2)$$

where the index j runs from 0 to $D-1$, $Cr \in [0,1]$ is the crossover probability, and $\gamma \in [0,1]$ is the output of a uniform random number generator.

After crossover operation, the selection process described as follows is realized

$$\mathbf{x}^{i,g+1} = \begin{cases} \mathbf{u}^{i,g} & \text{if } f(\mathbf{u}^{i,g}) \leqslant f(\mathbf{x}^{i,g}), \\ \mathbf{x}^{i,g} & \text{otherwise,} \end{cases} \quad (3)$$

where $f(\mathbf{x})$ is the objective function value. After the new population is generated, the processes mutation, crossover and selection are repeated until termination conditions are satisfied.

3. Problem statement

The geometry of a planar antenna array which is placed on the x–y plane is shown in Fig. 1. The array factor for this planar antenna array can be written as

$$\mathrm{AF}(\theta,\phi) = \sum_{i=1}^{N} \alpha_i \; \mathrm{e}^{j\beta_i} \mathrm{e}^{j2\pi\, \mathrm{d}x_i \sin\theta\cos\phi} \mathrm{e}^{j2\pi\, \mathrm{d}y_i \sin\theta\sin\phi}, \quad (4)$$

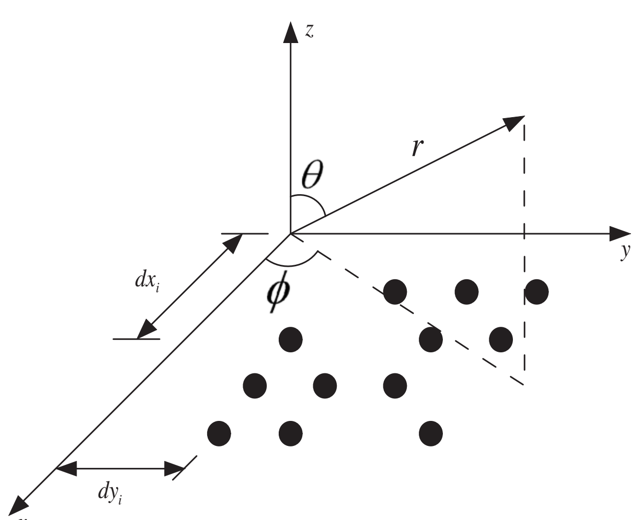

Fig. 1. Geometry of planar antenna array.

where N is the total number of antenna elements in the array, θ is the elevation angle with respect to z-axis, ϕ is the azimuth angle with respect to x-axis, α_i is the amplitude of the ith element current and β_i is the phase of the ith element current. dx_i and dy_i denote the distance between ith element and y-axis and the distance between ith element and x-axis in wavelength, respectively.

An arbitrarily shaped, non-symmetric planar antenna array placed on the x–y plane with uniform excitation and zero phase is considered as the first problem. The fitness function to be minimized can be defined as

$$f = \left[\left(\sum_{k=1}^{K} w_k \,\| \mathrm{AF}_k| - \mathrm{ND}_k|^2 \right) + w_d |d - d_{\min}|^2 + w_s |\mathrm{SLL} - \mathrm{SLL}_{\max}|^2 \right]^{1/2}, \quad (5)$$

where K is the number of desired nulls, AF_k is the value of the array factor for the kth direction to be suppressed and ND_k is the desired null depth for the kth null. For the calculation of AF_k, α_i is taken as 1 and β_i is taken as 0 in Eq. (4). The weight w_k $(k=1,\ldots,K)$ is set to zero (i.e., $w_k=0$) if the condition $|\mathrm{AF}_k| \leqslant \mathrm{ND}_k$ is satisfied. d and $d_{\min}$ are the minimum distance and the desired minimum distance between the array elements, respectively. If the condition $d > d_{\min}$ is satisfied then the weight w_d is set to zero. SLL and $\mathrm{SLL}_{\max}$ are sidelobe level and desired maximum sidelobe level for the overall pattern of the antenna array, respectively. The weight w_s is set to zero if the obtained sidelobe level SLL is smaller than the desired maximum sidelobe level $\mathrm{SLL}_{\max}$. All terms in Eq. (5) are numerical values. The positions of the antenna array elements are selected as the parameter vector to be optimized. The array elements are considered to be isotropic antennas and it is assumed that there is no coupling between them.

Array thinning problem for desired null positions can be modelled as a binary optimization problem. Differential evolution algorithm, besides being a heuristic global search method over continuous spaces, can be also used for binary optimization problems. As a second case, a symmetric rectangular lattice of uniformly excited, zero phased array elements is considered. The amplitude coefficients α_i are selected as the parameter vector and is tried to be optimized for desired specifications by changing them as 1's or 0's. Since the distances between array elements are fixed for this case, the weight w_d is taken as zero in Eq. (5).

As a third case, for the array thinning problem with unequal excitation and zero phase, the α_i vector is again selected as the parameter vector but this time α_i is allowed to change between $[0,\infty]$. Since this leads to non-normalized element weights, normalized element weights are obtained by a normalization process by using the maximum value of the α_i vector. Array elements which are unnecessary for obtaining the desired specifications are discarded. The same fitness function given in Eq. (5) is used with $w_d = 0$ for this case, too.

4. Numerical results

In this section, numerical results for three optimization problems mentioned above are presented. In the first example, an array of 36 isotropic elements with uniform excitation and zero phase is considered. The parameters of the differential evolution algorithm are taken as follows: scale factor $F = 0.2$, crossover probability $Cr = 0.95$ and population size $Np = 200$. For the selection of base vector $r0$, the algorithm “DE/best/1/bin with jitter” is used.

In the first problem three nulls at ($\theta = 40°$, $\phi = 30°$), ($\theta = 60°$, $\phi = 30°$) and ($\theta = 80°$, $\phi = 30°$) are required. Desired null depth level is set to -100 dB, desired minimum distance $d_{\min}$ between array elements is set to 0.2λ, desired maximum sidelobe level for the overall pattern $\mathrm{SLL}_{\max}$ is set to -13 dB, and coordinates of the array antennas are required to be between $-1.5\lambda \leqslant x, y \leqslant 1.5\lambda$. The weight coefficients are taken as $w_1 = w_2 = w_3 = 2$, $w_d = 1$, $w_s = 1$. The program is adjusted to stop whenever the fitness function is below the convergence rate of 10^{-6}. Optimized element positions are shown in Fig. 2 and θ pattern for $\phi = 30°$ is shown in Fig. 3. The minimum distance between array elements is obtained as $d_{\min} = 0.2\lambda$ and the maximum sidelobe level obtained for the overall pattern is -13.99 dB. As it is seen from Fig. 3 all nulls are below the level of -100 dB and also sidelobes on (θ, $\phi = 30°$) plane are below -38 dB. Positions of the array elements are given in Table 1 and three dimensional pattern is shown in Fig. 4. As it is seen from the figures, the desired criteria can be satisfied with position optimization, when the coupling between the array elements is neglected.

Because of the nature of the array factor defined in Eq. (4), and since isotropic elements are used in the array, the maxima of the pattern is formed at ($\theta = 0°$, ϕ) and elevation pattern will be symmetrical around $\theta = 0°$ over all (θ, ϕ) planes.

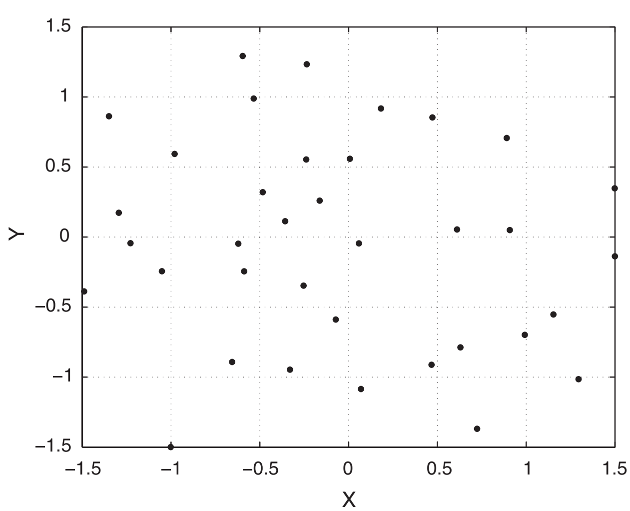

Fig. 2. Array geometry and optimized element positions in wavelength for 36 elements planar antenna array.

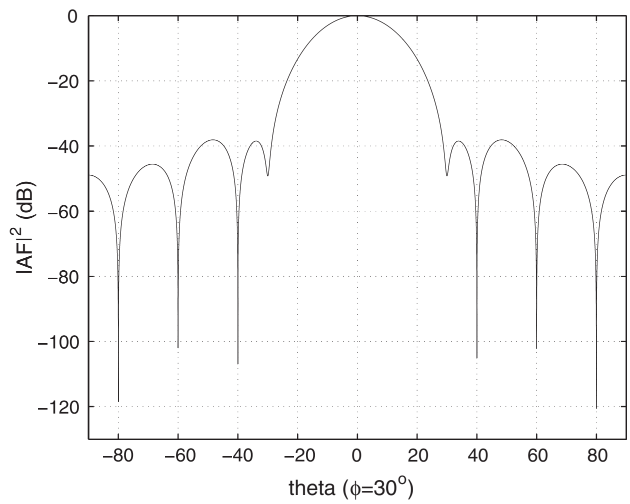

Fig. 3. Array factor pattern for position-only optimized 36 elements planar antenna array ($\phi = 30°$).

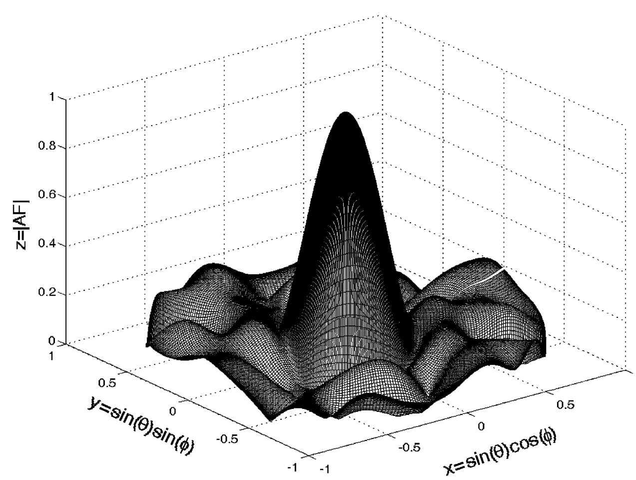

Fig. 4. Three dimensional array factor pattern for position-only optimized 36 elements planar antenna array.

As the second case, the problem of null synthesis with array thinning is considered. Array thinning means to decrease the number of array elements while maintaining the desired criteria. A planar array of 6 × 6 elements which is symmetrical with respect to the origin is placed on the x–y plane. The elements are assumed to be isotropic and the spacings between the elements to be half wavelength. In this scenario the amplitudes of the complex current terms defined in Eq. (4) are tried to be optimized as 1’s or 0’s. A 1 means the element is in its place, a 0 means the element is excluded from the array. Determining whether or not an element is in its place did not lead us to any satisfying result. It is found that for simple situations such as 1 null or 2 nulls satisfactory results can be obtained, but for a relatively complex

Table 1. Optimized element positions in wavelength for 36 elements planar antenna array

Element numbers	dx	dy
1	−1.228548	−0.044295
2	1.499621	−0.137850
3	1.498757	0.347449
4	1.295099	−1.014948
5	0.629959	−0.787883
6	−0.072184	−0.589418
7	0.057805	−0.045422
8	0.466707	−0.912209
9	−0.330208	−0.946456
10	−1.489828	−0.387969
11	0.992185	−0.698601
12	−0.596754	1.292325
13	−1.349878	0.862049
14	1.153535	−0.552605
15	−0.235783	1.233155
16	0.907704	0.049835
17	−0.655907	−0.891973
18	−1.051431	−0.244234
19	−0.483913	0.320251
20	−1.294604	0.173023
21	−1.001982	−1.499790
22	−0.534434	0.988765
23	−0.253684	−0.347173
24	0.723888	−1.369151
25	0.006951	0.558664
26	−0.979677	0.593612
27	0.610588	0.054267
28	−0.588339	−0.244767
29	0.471997	0.853465
30	−0.621586	−0.047520
31	0.182292	0.917360
32	−0.238581	0.553543
33	0.069721	−1.085107
34	−0.357232	0.112739
35	0.890559	0.706952
36	−0.162898	0.259529

Table 2. Element positions in wavelength and normalized current amplitudes for 36 elements planar antenna array

Element numbers	dx	dy	Normalized amplitudes
1	−1.25	−1.25	0.722226
2	−1.25	−0.75	0.463880
3	−1.25	−0.25	0.868827
4	−1.25	0.25	0.869559
5	−1.25	0.75	0
6	−1.25	1.25	0.562095
7	−0.75	−1.25	0
8	−0.75	−0.75	0.942381
9	−0.75	−0.25	0.491049
10	−0.75	0.25	0.753023
11	−0.75	0.75	0.575615
12	−0.75	1.25	0.552547
13	−0.25	−1.25	0.693205
14	−0.25	−0.75	0.736962
15	−0.25	−0.25	0.721790
16	−0.25	0.25	0.934772
17	−0.25	0.75	0.438767
18	−0.25	1.25	0.773491
19	0.25	−1.25	0.812589
20	0.25	−0.75	0.469307
21	0.25	−0.25	0.727699
22	0.25	0.25	0.910095
23	0.25	0.75	1.000000
24	0.25	1.25	0
25	0.75	−1.25	0.638290
26	0.75	−0.75	0
27	0.75	−0.25	0
28	0.75	0.25	0
29	0.75	0.75	0.430800
30	0.75	1.25	0.399979
31	1.25	−1.25	0
32	1.25	−0.75	0.940723
33	1.25	−0.25	0
34	1.25	0.25	0.243104
35	1.25	0.75	0
36	1.25	1.25	0

problem such as 3 or more nulls the algorithm did not converge to any acceptable value. This problem is also studied with a standard genetic algorithm and it is observed that the desired convergence could not be obtained in this case too [29].

As a remedy to this problem, array element amplitudes can be redefined as having integer values α_N, where α_N is an integer between $[0, \infty]$. Initial conditions of parameter set are taken as 1's or 0's. This new parameter space gives unnormalized integer amplitudes α_N, where some elements in the parameter set may have zero magnitude. Normalizing α_N values with the maximum of the parameter set gives normalized amplitudes. To simulate this situation, again, a 6×6 planar array placed symmetrically on the x–y plane is considered. Isotropic elements are placed as equally spaced with an inter element spacing of $d = \lambda/2$ in both x and y directions. Three nulls at (θ=40°, ϕ=30°), (θ=60°, ϕ=30°) and ($\theta = 80°$, $\phi = 30°$) are desired, same as in the first case. The algorithm "DE/target-to-best/1/bin" is used for the selection of $r0$. Other parameters for differential evolution algorithm are taken the same as for the first case. The weight coefficients are taken as $w_1 = w_2 = w_3 = 1$, $w_s = 1$. Desired null depth level is set to -100 dB, desired maximum sidelobe level for the overall pattern is set to -13 dB and again the program is adjusted to stop when the fitness function reaches below a convergence rate of 10^{-6}. The amplitudes for the array elements are given in Table 2, where a zero magnitude means this element is excluded from the array.

Positions of the array elements which are not excluded from the array are shown in Fig. 5, θ pattern for $\phi = 30°$ is shown in Fig. 6 and three-dimensional radiation pattern is shown in

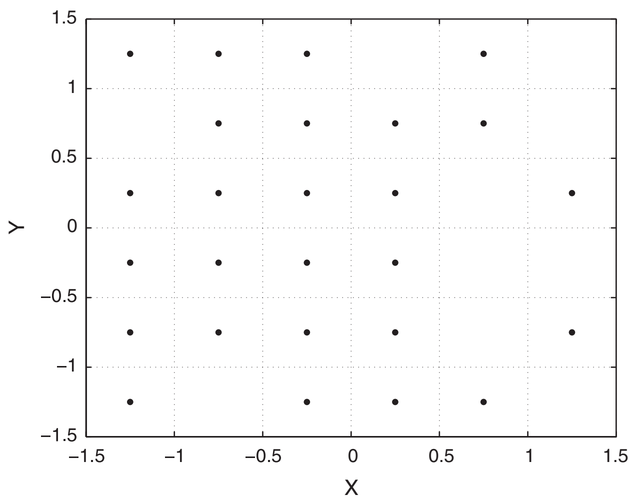

Fig. 5. Element positions in wavelength for 36 elements planar antenna array.

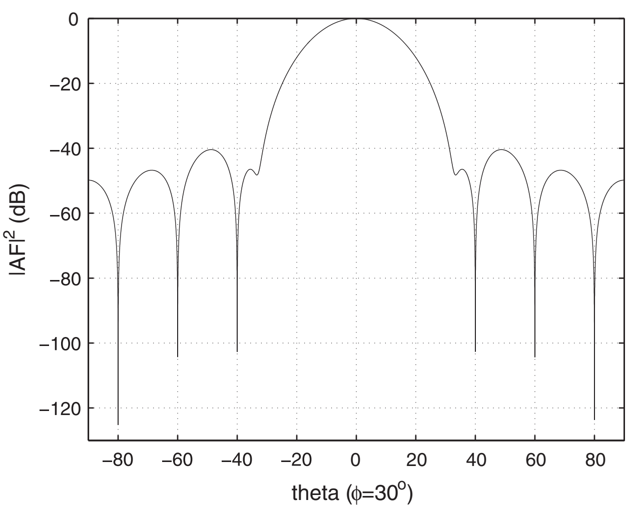

Fig. 6. Array factor pattern for thinned 36 elements planar antenna array ($\phi = 30°$).

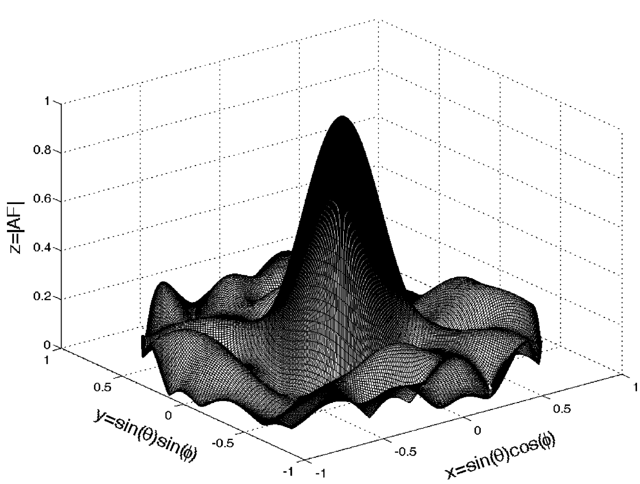

Fig. 7. Three dimensional array factor pattern for thinned 36 elements planar antenna array.

Fig. 7. As it is seen from Fig. 6 all nulls are below -100 dB and also sidelobes on $(\theta, \phi = 30°)$ plane are below -40 dB. The maximum sidelobe level obtained for the overall pattern is -13.59 dB. Dynamic range ratio is obtained as 4.11.

5. Conclusions

Numerical and iterative approaches based on the differential evolution algorithm for the pattern synthesis of planar arrays with prescribed pattern nulls by optimizing the position-only of array elements and thinning a planar array having a predefined shape by controlling the amplitudes are presented. It is observed that with differential evolution algorithm, the position-only optimization for a planar array which have elements initially distributed randomly on the x–y plane allows null synthesis in any prescribed direction. For the problem of thinning a planar array having a predefined shape with prescribed nulls, it is observed that it is possible to achieve an acceptable result for simple problems involving only 1 or 2 nulls. To remedy this, an amplitude controlling based numerical approach is suggested. Several simulations are realized to demonstrate the performances of the methods and it is seen that both methods are capable of synthesizing prescribed nulls. In position-only optimization the coupling effect between elements can be further reduced by increasing the desired minimum distance between array elements. The second method can also be implemented for more complex geometries and both methods can be implemented for non-isotropic elements.

Acknowledgements

The authors would like to thank to the editors and to the anonymous reviewers for their constructive comments.

References

[1] Tennant A, Dawoud MM, Anderson AP. Array pattern nulling by element position perturbations using a genetic algorithm. Electron Lett 1994;30:174–6.

[2] Liao WP, Chu FL. Array pattern nulling by phase and position perturbations with the use of the genetic algorithm. Microwave Opt Technol Lett 1997;15:251–6.

[3] Murino V, Trucco A, Regazzoni CS. Synthesis of unequally spaced arrays by simulated annealing. IEEE Trans Signal Process 1996;44:119–23.

[4] Karaboga D, Guney K, Akdagli A. Antenna array pattern nulling by controlling both amplitude and phase using modified touring ant colony optimization algorithm. Int J Electron 2004;91:241–51.

[5] Yang S, Gan YB, Qing A. Moving phase center antenna arrays with optimized static excitations. Microwave Opt Technol Lett 2003;38:83–5.

[6] Yang S, Gan YB, Qing A. Antenna-array pattern nulling using a differential evolution algorithm. Int. J RF Microwave CAE 2004;14:57–63.
[7] Kurup DG, Himdi M, Rydberg A. Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm. IEEE Trans Antennas Propagat 2003;51:2210–7.
[8] Mahanti GK, Chakraborty A, Das S. Design of fully digital controlled reconfigurable array antennas with fixed dynamic range ratio. J Electromagn Waves Appl 2007;21:97–106.
[9] Tseng FI, Cheng DK. Optimum scannable planar arrays with an invariant sidelobe level. Proc IEEE 1968;56:1771–8.
[10] Goto N. Pattern synthesis of hexagonal planar arrays. IEEE Trans Antennas Propagat 1972;20:479–81.
[11] Goto N. A synthesis of array antennas for high directivity and low sidelobes. IEEE Trans Antennas Propagat 1972;20: 427–31.
[12] Hsiao JK. Properties of a nonisosceles triangular grid planar phased array. IEEE Trans Antennas Propagat 1972;20: 415–21.
[13] Einarsson O. Optimization of planar arrays. IEEE Trans Antennas Propagat 1979;27:86–92.
[14] Laxpati SR. Planar array synthesis with prescribed pattern nulls. IEEE Trans Antennas Propagat 1982;30:1176–83.
[15] DeFord JF, Gandhi OP. Phase-only synthesis for minimum peak sidelobe patterns for linear and planar arrays. IEEE Trans Antennas Propagat 1988;36:191–201.
[16] Kumar BP, Branner GR. Generalized analytical technique for the synthesis of unequally spaced arrays with linear, planar, cylindrical or spherical geometry. IEEE Trans Antennas Propagat 2005;53:621–34.
[17] Haupt RL. Thinned arrays using genetic algorithms. IEEE Trans Antennas Propagat 1994;42:993–9.
[18] Yang S, Nie Z. Time modulated planar arrays with square lattices and circular boundaries. Int J Numer Model 2005;18:469–80.
[19] Alicano CR, Rosales DC, Rodriguez CB, Mendoza MP. Differential evolution algorithm applied to sidelobe level reduction on a planar array. Int J Electron Commun (AEU) 2007;61:286–90.
[20] Unz H. Linear arrays with arbitrarily distributed elements. IEEE Trans Antennas Propagat 1960;8:222–3.
[21] Kumar BP, Branner GR. Design of unequally spaced arrays for performance improvement. IEEE Trans Antennas Propagat 1999;47:511–23.
[22] Harrington RF. Sidelobe reduction by nonuniform element spacing. IEEE Trans Antennas Propagat 1961;9:187–92.
[23] Ishimaru A. Theory of unequally-spaced arrays. IEEE Trans Antennas Propagat 1962;11:691–702.
[24] Skolnik MI, Nemhauser G, Sherman III JW. Dynamic programming applied to unequally spaced arrays. IEEE Trans Antennas Propagat 1964;12:35–43.
[25] Qing A. Electromagnetic inverse scattering of multiple two-dimensional perfectly conducting objects by the differential evolution strategy. IEEE Trans Antennas Propagat 2003;51:1251–62.
[26] Qing A. Dynamic differential evolution strategy and applications in electromagnetic inverse scattering problems. IEEE Trans Geoscience Remote Sens 2006;44:116–25.
[27] Storn R, Price K. Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. J Global Optim 1997;11:341–59.
[28] Price KV, Storn RM, Lampinen JA. Differential evolution: a practical approach to global optimization. Berlin: Springer; 2005.
[29] Goldberg DE. Genetic algorithms in search, optimization and machine learning. Reading, MA: Addison-Wesley; 1989.

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