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Planetary oblateness and evolution of Earth flattening

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SUMMARY
A simple model of superficial hydrostatic equilibrium on a rotating isotropic self-gravitating sphere is employed to derive a planetary oblateness formula. The relation obtained describes oblateness on the basis of spin rate, polar radius and the mass of the planet only. An application to the oblate planets is made and the consistency with the most basic and known oblateness laws of geodesy is verified. By making the further hypothesis that the Earth volume and mass has stayed constant, one derives that the size of the terrestrial oblateness adaptation which has taken place during the last half a billion years is a contraction of 2 km at the equator an uplift of 4.1 km at the pole. For the more remote history it appears very likely that the central condensation of the Earth has decreased systematically over the past 3.5 billion years.

Key words: Earth history, oblateness, planetary science

1 INTRODUCTION

From the time of Isaac Newton to the present, the flattening of the Earth has been the subject of considerable interest and speculation. If one considers that all the planets from the Earth onwards are also oblate and that this is almost certainly since the very early days of the existence of the solar system, flattening still remains partly an unexplained fact. This state of affairs is hardly affected by our ignorance concerning the outermost planet Pluto for which no relevant data have been collected hitherto.

Our considerations on planetary oblateness bring us in contact with basic geodesy, planetary science and dynamics. They will lead us towards the evolution of the Earth oblateness over large palaeontological time scales. This is based upon the slow increase of the Length Of the Day (LOD) over the ages due to tidal friction. The idea that oblateness has changed due to a slow down of the Earth rotation seems inescapable if one considers that there should have been at least 650 solar days per year (i.e. a solar day of 13.5 hours) some 2000 million years BP, as claimed by Lambeck (1978).

We are certainly not the first to examine the change of the terrestrial oblateness figures (see Lambeck 1980, chapter 11). Nevertheless, the way oblateness has been linked to Earth rotation has—in successful theories—normally comprised inertia tensor elements or coefficients of the gravity potential development. In that context any definite representation of how oblateness has varied over the ages requires a statement of how oblateness and moments of inertia have changed together. This is an almost impossible task and, therefore, there is little commitment in the conclusions reached so far.

In an attempt to derive an oblateness relation independent from inertia and gravity potential terms, we have worked out a simple model of superficial hydrostatic equilibrium of a fluid layer on a self-gravitating body. The latter does not need to be rigid or spherical. It is only required that the interior body has a spherical gravity potential near its boundary. If the whole body is rotating with an angular velocity ω we will show that the height h of the equatorial bulge is

$$h = \frac{R_p^4 \omega^2}{GM}, \qquad (1)$$

where R_p is the polar radius, M the total mass of the sphere and G the universal constant of gravitation.

Formula (1) has to be seen against the background of well established laws of equilibrium which are known in geodesy. In the next section we will recall the two most basic independent relations linking flattening to inertia properties. Thereafter, we derive equation (1) in Section 3 and apply it to the planets in Section 4. In Section 5 we combine equation (1) with the flattening laws presented in the section hereafter. This leads to simple equations describing the inertias of planets. The spin axis inertia appears then to be a function of the total mass, the equatorial and the polar radius only. Application to the terrestrial data gives the same order of fit as we find when applying equation (1) to all oblate planets. In the last section we make an application to the Earth history. This requires a supplementary hypothesis and we propose to assume that the Earth volume and mass has not changed over the last half a billion years by an amount relevant for our considerations. In this way we derive the size of the adaptation of the Earth shape.

2 FLATTENING LAWS IN GEODESY

The successful laws in geodesy—linking the geometric flattening $f = h/R_e$ to spin rate and inertia properties of the Earth—are intimately connected with the name of Alexis Claude Clairaut and the idea of hydrostatic equilibrium. In the middle of the 18th century he derived a second order linear differential equation which connects the flattening f of any level surface of a spheroidal liquid rotating body in equilibrium, with the mean radius and density of that level surface. A solution of this equation given by Heiskanen & Moritz (1967) is

$$J_2 = \frac{C-A}{Mr_e^2} = \frac{2}{3}\left(1 - \frac{2}{5}\sqrt{\frac{5}{2}\frac{\omega^2 R_e}{g_e f} - 1}\right)\frac{C-A}{C}. \tag{2}$$

The new symbols we have introduced in equation (2) are: R_e the mean equatorial radius, the principal moments of inertia $A = B$ in the equatorial plane and C along the spin axis, g_e the mean gravitational acceleration at the equator and J_2 the 2,0-term in the spherical-harmonic expansion of the gravity potential of a planet.

Another relation often referred to as Clairaut's equation, reads

$$2f = 3J_2 + \frac{\omega^2 R_e}{g_e}. \tag{3}$$

It applies to the first order and is obtained by equating the polar and equatorial surface potential. Hence, equation (3) does not even imply that the outer surface is ellipsoidal. Rotational symmetry is sufficient for equation (3) to apply. An expansion of Clairaut's equation to a higher order of validity is known (see, e.g. Levallois 1970). For our purposes, the higher accuracy of that equation cannot be exploited as it requires coefficients of the Earth potential expansion higher than $J_{2,0}$, which we try to avoid.

Equations (2) and (3) show that the body as a whole is in hydrostatic equilibrium. The basic equilibrium properties of rotating planetary bodies seem to be exhausted by the two independent relations just presented. Any further independent equation describing f requires a supplementary or alternative model assumption, thereby leaving the grounds of obvious and basic dynamical principles. Levallois (1970) has reviewed a number of such causal theories, i.e. models which try to explain why the Earth is approximately a revolution ellipsoid with its particular properties. Such theories can be made either to fit reality by the selection of some free parameters, or, if there are no such parameters, they usually fail to provide satisfactory results. Amongst the more recent alternative theories, Gregori (1981) suggests that spinning planets should reach their indifferent equilibrium. The indifferent equilibrium principle does not lead to a hydrostatic equilibrium theory. It also fails to produce theoretical flattenings which approach reality. In another approach, Bobrov (1984) presents a theoretical study extending over 3.5 billion years of the whole history of the LOD. To establish the link between the spin rate and the angular momentum of the Earth he needs the principal moment of inertia along the spin axis. He assumes that the latter is constant together with the Love number k_2. Relying further on conventional means as presented in this section, Bobrov computes the hypothetical past flattenings of our planet. As his assumptions are purely intuitive, the results remain questionable.

The model we present in the next section is also of a causal and speculative nature. It agrees with the idea of hydrostatic equilibrium and—as is necessary in physics—its applicability can be established by means of verification. This verification has become possible by the availability of reasonable oblateness and spin rate information for the spinning planets.

3 A SUPERFICIAL EQUILIBRIUM MODEL

In order to highlight the merits and limitations of our model we introduce the hydrostatic principle we will apply to the outer layer of spinning planets in a very simplified example. Consider the plane cross-section of a vessel of length L. Let is represent a receptacle filled with a homogeneous incompressible liquid subject to a downward acceleration g and a perpendicular lateral acceleration a. The horizontal acceleration goes towards the right side, and the horizontal distance from the left is measured by a coordinate called l ($\leq L$). We introduce a horizontal reference line starting at the liquid surface at $l = 0$. At $l = l_0$ the integral over the forces created by the acceleration a amounts to the pressure $\rho a l_0$, if ρ is the density of the fluid. At the same location on the reference line the relative surface height h is h_0, where $h = 0$ at $l = 0$. The corresponding hydrostatic pressure is $\rho g h_0$ on the reference line. Thus, in equilibrium, we have $a l_0 = g h_0$. This remains true even if the bottom of the vessel is an inclined or irregular surface, provided no obstacle emerges over the liquid surface and runs fully across the vessel without being parallel to the left–right direction.

If we adopt this basic hydrostatic concept for the outer layer of a self-gravitating rotating body, we see that this implies:

(i) A sufficient thickness of the outer layer so that the centrifugal forces are unable to remove the layer completely at the rotation poles.

(ii) A relatively low density and limited oblateness of the outer layer in order to be able to neglect its self-gravitation.

(iii) an underlying body whose boundary consists of higher density material.

(iv) No limitation on the exact geometrical shape of the underlying body which can be an oblate spheroid with irregularities.

We further assume that the gravitational flattening $(g_e - g_p)/g_e$ can be neglected. Here, g_p represents the gravitational acceleration at the poles. This is certainly the most stringent limitation which may reduce the quality of the result for the planets with larger oblateness, namely Jupiter and Saturn. For both giant planets the mean density is low ($0.7\,\mathrm{g\,cm^{-3}}$ for Saturn and $1.3\,\mathrm{g\,cm^{-3}}$ for Jupiter) and central condensation is high. Their oblate layers will thus have a very low density, thereby mitigating their self-gravitating effect.

Let us now consider a rotating oblate planet whose polar radius is R. The latter corresponds to the spherical level surface of radius R. It will be the reference surface for the hydrostatic equilibrium of an outer incompressible liquid layer. We stress that this reference sphere may cross other layers of higher density without affecting the equilibrium equation. The main simplifying assumption states that the gravitational acceleration g at the reference surface is everywhere equal and oriented radially and, further, that the self-gravitation of the outer layer can be neglected.

The centrifugal force acting on a unit mass volume at the equator amounts to $R\omega^2\rho$, where ρ is now the density of the outer layer. At latitude φ this force will reduce by the factor $\cos\varphi$. This can be broken down in a tangential component

$$F_t = -\rho R\omega^2 \cos\varphi \sin\varphi$$

and a radial component

$$F_r = \rho\{R\omega^2 \cos^2\varphi - g\}$$

comprising the gravitational acceleration g. The tangential component is measured positive in the direction parallel to the positive spin axis. The radial force F_r subsists on the reference sphere after equilibrium is achieved. The change of F_r along a meridian, namely

$$\frac{\partial F_r}{\partial \phi} = -2\rho R\omega^2 \cos\varphi \sin\varphi,$$

will therefore translate into a differential pressure per unit length, which also induces a tangential force of the same magnitude. Hence, ρ on the reference sphere is subject to a tangential force $3F_t$ which can only be counteracted by a higher liquid level at lower latitudes.

The integral over all tangential forces acting at the reference sphere between the pole and a latitude Φ_0, namely

$$\int_0^{2\pi} d\lambda \int_{\Phi_0}^{\pi/2} 3F_t R^2 \cos\varphi \, d\varphi$$

must be equal to the force per length exerted by a liquid column on the reference sphere. The coordinate λ represents the longitude. The previous integral is equal to ρg times the surface of a conical frustrum of small radius $R\cos\Phi_0$, of large radius $(R+h_\varphi)\cos\Phi_0$ and of height $H = h_\varphi \sin\Phi_0$. Here, h_φ is the outer layer surface height in function of Φ_0 with respect to the reference sphere. Thus, we find

$$h_\varphi = \frac{R^2\omega^2}{g}\cos^2\Phi_0, \qquad (4)$$

where R is R_p and g can be approximated by GM/R_p^2. For $\Phi_0 = 0$ this yields equation (1).

Compared to Clairaut's equilibrium principle we may notice that equations (2) and (3) apply to self-gravitating rotating bodies of any oblateness. Equation (4) is not in contradiction with these laws as long as oblateness is small, i.e. the gravitation of the equatorial bulge influences the shape of the external surface to a negligible extend. We will be able to quantify this limitation in Section 5.

4 APPLICATION TO THE PLANET DATA

In Davies *et al.* (1980) we find the equatorial radius, the spin period and the flattening defined as $h_0/(R_p + h_0)$ for each planet. As for the Sun, the observable rotation period is also latitude dependent for Jupiter and Saturn. We propose to take both extreme values reported by Davies *et al.* (1980), when making a practical comparison between the observed and the theoretical oblateness.

The values of Davies *et al.* (1980) can no longer be considered as 'up to date' for Uranus and Neptune. For the former we have taken values from Elliot *et al.* (1981). The oblateness of Uranus quoted there is 0.024 ± 3 and we have opted for 0.027 as a compromise with the IAU value which is 0.03. The spin rate of Uranus has been measured accurately by the space probe *VOYAGER II* and has been taken from Ness *et al.* (1986). The figure of Neptune has been the subject of a recent campaign which is reported by French *et al.* (1985). The proposed best values given by French *et al.* (1985) have been used hereafter. The units employed in our comparison in Table 1 are km for the polar radius R_p, the observed equatorial increment h_0 and the theoretical increment h_t resulting from the application of equation (1); the spin period is expressed in ephemeris days of 86 4000 ephemeris seconds and the mass constant GM for each planet is given in $km^3 s^{-2}$. The percentage error is defined as $100(h_t - h_0)/h_0$, though this error criterion may give a somewhat severe impression for very low flattenings like those of the Earth and Mars.

Except for Mars, the overall impression provided by the results of the Table 1 is in agreement with our expectations. The errors are modest for the Earth, Uranus and Neptune, they are larger for the very oblate jovian planets. It may be interesting to note that equation (1) applied to h_0, ω and GM of the Earth yields an R_p which is 30 km smaller than the actual one. This is plausible if one considers that hydrostatic equilibrium is realised in the astenosphere and not in the lithosphere.

The problem encountered with Mars is not new, as the topographic flattening reported by Balmino (1981) is known to be $(6.12 + 0.4)\ 10^{-3}$. This corresponds to $R_E - R_p =$ 20.8 km. This is also quite different from the dynamic flattening which is derived from $J_{2,0}$ by using equation (3)

Table 1. Comparison of observed and computed planetary oblateness.

Planet	R_p	Flattening	Spin period	GM	h_0	h_t	% error
Earth	6356.76	0.0033528	0.9973	3.986×10^5	21.30	21.78	+1.9
Mars	3375.8	0.0051865	1.0260	4.283×10^4	17.60	15.23	−13.5
Jupiter	66770.8	0.0648088	0.4101	1.267×10^8	4627.2	4933	+6.6
			0.4137			4847	+4.8
Saturn	53542.7	0.1076209	0.4264	3.794×10^7	6457.3	6298	−2.5
			0.4375			5983	−7.4
Uranus	25412.9	0.0280	0.7204	5.794×10^6	732.1	733.5	+0.2
Neptune	24785.4	0.0191	0.7680	6.871×10^6	482.6	492.5	+2.0

and which corresponds closely to the IAU value of Table 1. Combining these results indicates that Mars has an oblateness which should correspond to a lower spin period, namely to 0.955 days if the dynamical flattening is entered into equation (1). This is conceivable if the martian lithosphere is rigidified to an extent not allowing a full oblateness adaptation to compensate for a potential de-spin due to tidal friction caused by the sun upon the planet.

To conclude this section, we can state that equation (1) seems satisfied to a good approximation for the planets with low oblateness, except Mars, and to an acceptable approximation for Jupiter and Saturn. Thus, there is sufficient evidence to claim that for a larger variation of ω equation (1) prescribes—within a few per cent—a planetary shape adaptation.

5 APPLICATION TO THE GEODETIC DATA

We can now combine equation (1) with equations (2) and (3). The only simplification we introduce is given by:

$$g_p = \frac{GM}{R_p^2} \qquad g_e = \frac{GM}{R_e^2}. \tag{5}$$

Substituting equation (1) in equation (2) yields

$$C_0 = \frac{2}{3}\left[1 - \frac{2}{5}\sqrt{\frac{5}{2}\left(\frac{R_e}{R_p}\right)^4 - 1}\right], \tag{6}$$

where C_0 is the coefficient of the polar moment of inertia equal to C/MR_e^2. In the limit $R_e = R_p$, the value of C_0 becomes 0.3400 for all planets. For it is not possible that C_0 becomes zero or negative, the applicability of equation (6) ceases completely if R_e tends to $(2.9)^{1/4}R_p$. This corresponds to a flattening of 0.305 and means that for say $f > 0.2$, the assumptions leading to equation (1) conflict with the hydrostatic equilibrium law expressed by equation (2).

From equation (3) we deduce that

$$J_2 = \frac{\omega^2(2R_p^4 - R_e^4)}{3GMR_e}. \tag{7}$$

It will be noticed that the accuracy of equation (7) is questionable, because it expresses J_2 as the difference of two very large numbers. Assume that R_0 is the mean radius of a planet and let ΔR_i be an inaccuracy on R_p or R_e. Especially for the jovian planets, the radius data of the liquid or solid boundary could not yet be observed directly, which leaves some room for a non-zero ΔR_i. The error magnification due to ΔR_i in equation (7) is proportional to R_0^2, while in equation (6) it is only proportional to R_0^{-1}. Hence, equation (6) should be a rather reliable formula if f stays well below 0.2.

For all oblate planets, Table 2 gives J_2 values derived from equation (7) (the model value) together with the accurate value as known from the gravitational potential determination (observed). Table 2 also gives C_0 obtained from equation (6) (the model value) and C_0 derived from equation (2) (the reference value). The latter corresponds to the values found in the literature, because equation (2) is the common way to find the spin inertia coefficient.

The comparison of J_2 observed with J_2 model confirms the limited value of equation (7). The comparison of model and reference C_0 yields—except for the outlayer Mars—a surprise. All model inertia coefficients suggest a lower central condensation than the reference C_0 values do. It seems likely that the difference noted for Jupiter is significant, but for the rest it is difficult to decide whether the systematic differences are due to the imperfection of our model or not. The reasonable validity of equation (6) also suggests that central condensation of planets is essentially a matter of hydrostatic equilibrium and only indirectly a matter of cosmogenesis.

Table 2. Comparative table of J_2 and C_0 obtained from equations (6) and (7) with J_2 and C_0 obtained otherwise.

Planet	J_2 model	J_2 observed	C_0 model	C_0 reference
Earth	0.00112	0.001083	0.338	0.331
Mars	0.00146	0.001965	0.336	0.373
Jupiter	0.0156	0.014750	0.284	0.251
Saturn	0.0141	0.027000	0.244	0.232
Uranus	0.00818	0.003354	0.317	0.308
Neptune	0.0060	(0.0035–0.0046)	0.324	0.313

The terrestrial parameters fit the model rather well. Also, the ratio $(C - A)/C$ is—according to equations (6) and (7)—equal to 1/300.4, thereby deviating by 1.7 per cent from the empirical value. Therefore, we may conclude that equation (1) is consistent with the basic geodetic properties of our planet within a few per cent.

As a consequence, equations (1) and (6) are well suited to make statements about h and C_0 for planets for which no better data are available, and for spin rates of the Earth very different from the value today. In such a context errors of a few percent can easily be tolerated.

The previous formulae on their own are not sufficient to study the shape of the Earth. When we take ω as an independent variable, then not only h, A_0 and C_0 are unknown, but also R_p or $R_e = R_p + h$. Therefore, a further condition is required. We propose to call 'palaeogeodesy' the speculative theories providing a full set of equations which allow to compute shapes of the Earth for remote palaeontological times. In the next section we give our view upon that question.

6 PALAEOGEODESY

If we consider equation (6) for the special case of our planet, it appears that C_0 varies very little with changes below 10 km in either R_e or R_p. In other words, the mass density distribution of the Earth is hardly affected if the oblateness varies within palaeontologically relevant limits. Hence, it is reasonable to assume that the tellurian volume is a constant, or

$$R_e^2 R_p = V_0 = 2.5859 \times 10^{11}\ \text{km}^3. \tag{8}$$

This hypothesis is also consistent with the presumed incompressibility of the surface layer. We may further assume that GM has not changed noticeably over the ages, and then equation (8) leads to

$$R_p^3 = \frac{-1 + \sqrt{1 + 8V_0W}}{4W}, \tag{9}$$

where $W = \omega^2/GM$.

An important question is related to the neutral latitude below which the geoid apparently contracts and above which it apparently dilatates upon a major despin. If r is the position vector for a point on the oblate terrestrial surface at latitude φ, then R can be approximated by

$$R^2 = R_e^2 \cos^2 \varphi + R_p^2 \sin^2 \varphi \tag{10}$$

under the assumption that the oblateness is small and that the Earth can also be considered to be a revolution ellipsoid for the remote past. The neutral latitude φ_0 is obtained from $dR = 0$. By equation (8) we know that $-R_e\, dR_p = 2R_p\, dR_e$ and thus we find

$$tg\varphi_0 = \frac{\pm R_e}{\sqrt{2}\, R_p}. \tag{11}$$

Today φ_0 is at the latitude of ±35.36° and in the limit $R_e = R_p$ it becomes ±35.26°.

We can now attempt to quantize the terrestrial oblateness change. The secured fossil evidence which allows the LOD to be estimated does not go beyond 450 million yr BP. From a fit made to the available LOD estimates presented by Lambeck (1978), we deduce that there were 412 days in a terrestrial year 0.5 billion yr ago. By application of equation (9) to this value it is found that the polar radius has increased by 4.09 km while R_e has decreased by only 2.04 km.

At a first glance we may believe that the oblateness adaptation leads to an obvious scenario. It could look as follows: upon Earth de-spin, hydrostatic pressure on the oceanic masses at low latitude decreases. Thereby the seas will immediately seek their new equilibrium. The continents emerging out of the oceans will adapt with some delay. Their prime and immediate role is to disturb the old symmetry by impeding that equal water volumes move to the poles at all longitudes. The principal moments of inertia will then change their location. Pole wandering follows. We can further guess that isostatic equilibrium requires a global uplifting of the continents as well as an ocean bottom subsidence for all areas between the neutral latitude and the poles. The reverse should occur at low latitudes.

The speed of such a scenario implies a pole length adaptation dR_p of some 16.4 cm in 20000 yr. This rate of change is vanishing small compared to ocean level changes each time on ice-age sweeps over our planet. In the last 20000 yr we went through changes of at least 100 m in sea level in most regions of the Earth. Hence, the steady oblateness change may be quite difficult to isolate amongst the other geodetic phenomena, except if crustal adaptation is heavily delayed and occurs in bursts, a possibility which may be worth considering.

If we try to go further than half a billion years BP we can only rely on two relatively well-established facts. First, we can assume that the Earth must have reached its hydrostatic equilibrium already in the very early days of its history. Secondly, the Earth must have lost angular momentum due to tidal friction—also without moon and without oceans—from the beginning onwards. The latter is not necessarily equal to saying that the spin rate has decayed from the beginning, as moment of inertia changes may also absorb the angular momentum loss. Nevertheless, there is some general consensus to assume that the spin rate has probably decreased since 3–3.5 billion yr, perhaps with some parallel change of the spin moment of inertia. If we now add the hypothesis that the Earth mass has not changed noticeably in the last 3 billion yr, then we can derive that the central condensation of the Earth has decreased, i.e. C_0 in equation (6) has increased. This follows from the fact that despin implies a decrease of flattening f. By making the differential df, the definition of a flattening reduction reads:

$$R_p\, dR_e - R_e\, dR_p < 0.$$

If we take the differential of equation (6) we find that dC_0 is always proportional to $-df$.

This is quite important because it implies that outward migration must have reduced the density of the Earth core since say 3 billion yr. This conclusion is inescapable if C_0 has increased while R_e has decreased.

7 CONCLUSION

We have been able to show that the spinning planets have an oblateness which largely originates from the behaviour of their outer non-gaseous layer. Combining the flattening formula describing the aforementioned fact with two independent oblateness formulae—going directly or indirectly back to Clairaut—allows us to find an approximate law for the inertia of spinning planets. This law indicates that principal inertia coefficients along the rotation axis are confined in a narrow range of values and thereby the mass density distribution of the spinning planets must be similar if considered on a relative scale.

By adding the hypothesis that the Earth volume and mass has been constant over the last half a billion yr, we have found that the terrestrial radius at the equator must have decreased by 2.04 km while the polar radius grew by 4.09 km.

Considering a time span of up to 3.5 billion yr reasonable evidence that the central condensation of the Earth has decreased systematically provided the Earth mass has remained the same.

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