

Lecture slides for  
*Automated Planning: Theory and Practice*

# **Chapter 17**

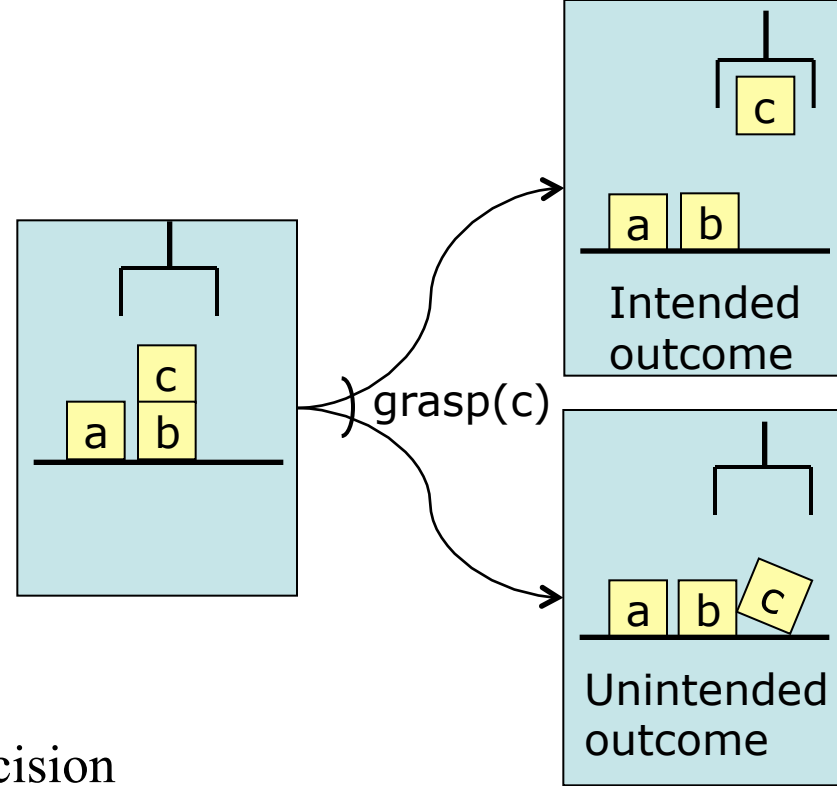
## **Planning Based on Model Checking**

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University of Maryland

1:19 PM February 29, 2012

# Motivation

- Actions with multiple possible outcomes
  - ◆ Action failures
    - » e.g., gripper drops its load
  - ◆ Exogenous events
    - » e.g., road closed
- *Nondeterministic systems* are like Markov Decision Processes (MDPs), but without probabilities attached to the outcomes
  - ◆ Useful if accurate probabilities aren't available, or if probability calculations would introduce inaccuracies

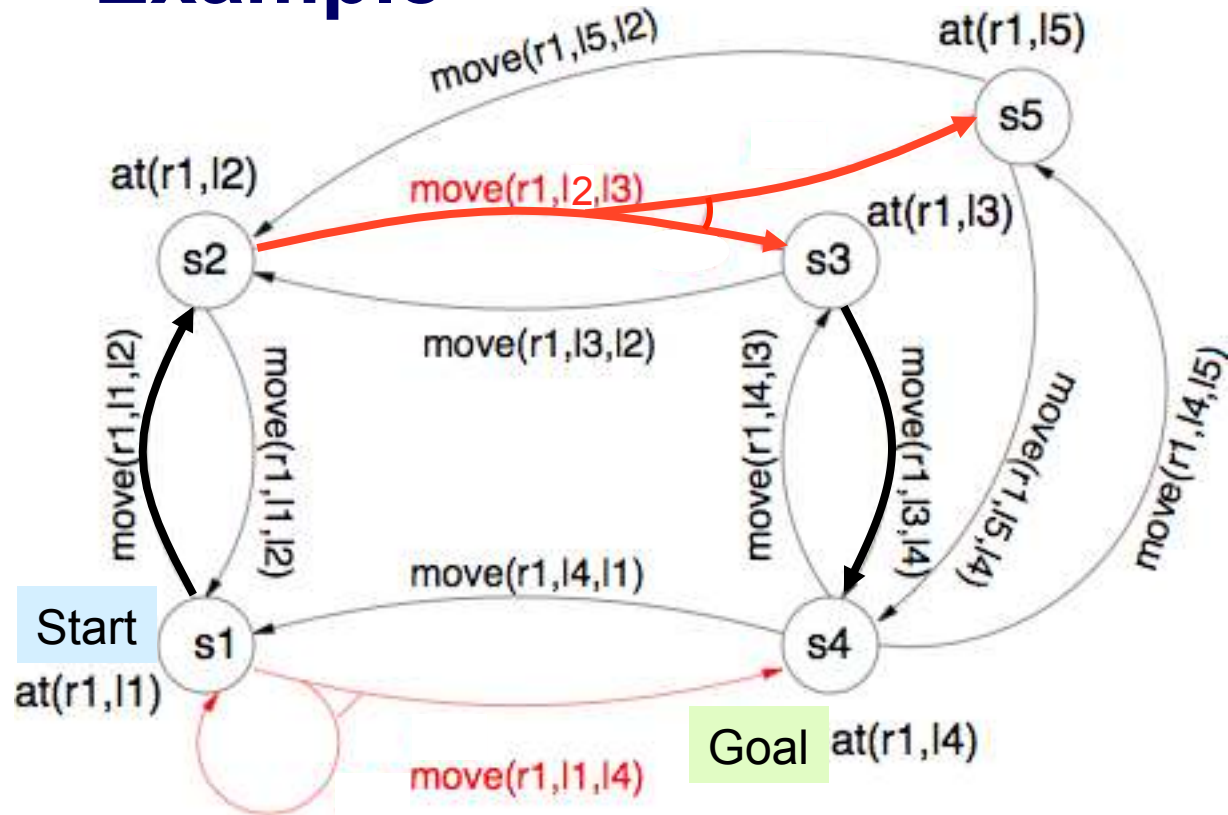


# Nondeterministic Systems

- *Nondeterministic system*: a triple  $\Sigma = (S, A, \gamma)$ 
  - ◆  $S =$  finite set of states
  - ◆  $A =$  finite set of actions
  - ◆  $\gamma: S \times A \rightarrow 2^S$
- Like in the previous chapter, the book doesn't commit to any particular representation
  - ◆ It only deals with the underlying semantics
  - ◆ Draw the state-transition graph explicitly
- Like in the previous chapter, a policy is a function from states into actions
  - ◆  $\pi: S \rightarrow A$
- Notation:  $S_\pi = \{s \mid (s, a) \in \pi\}$ 
  - ◆ In some algorithms, we'll temporarily have *nondeterministic policies*
    - » Ambiguous: multiple actions for some states
  - ◆  $\pi: S \rightarrow 2^A$ , or equivalently,  $\pi \subseteq S \times A$
  - ◆ We'll always make these policies deterministic before the algorithm terminates

# Example

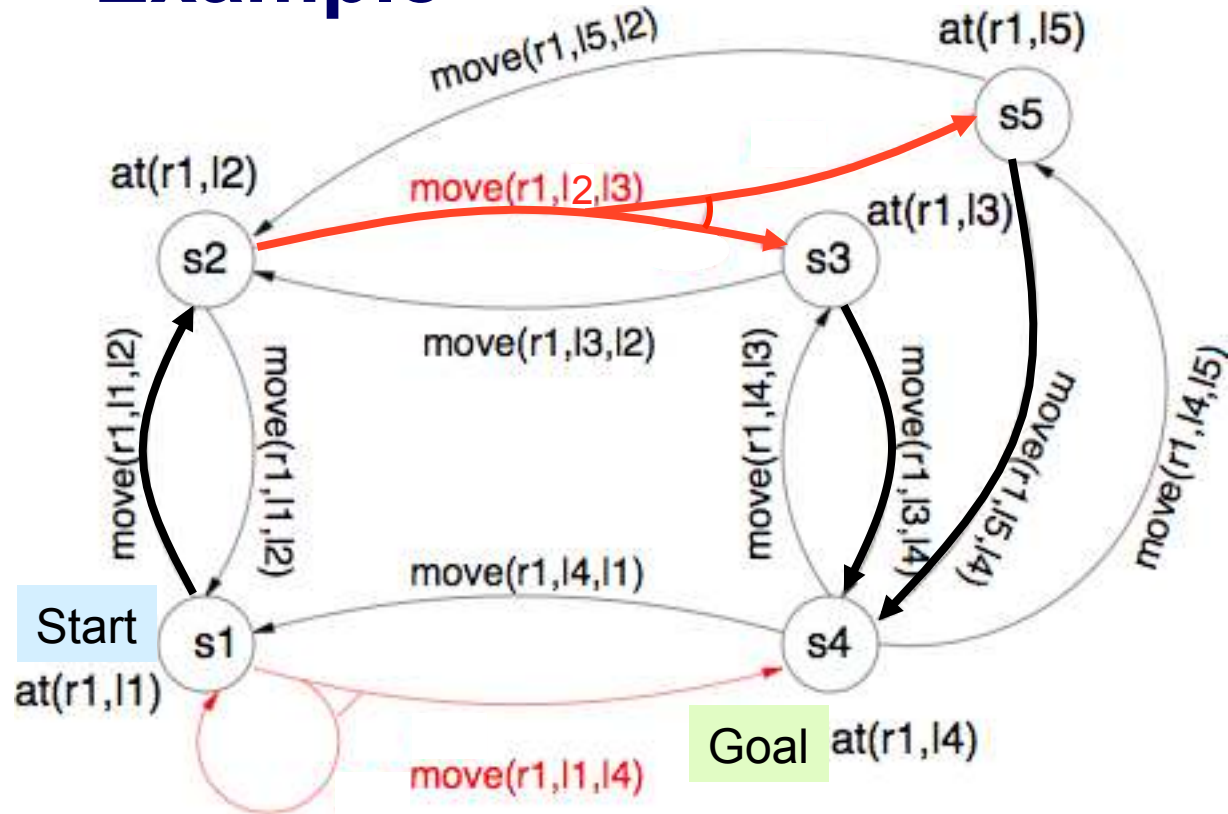
- Robot r1 starts at location l1
- Objective is to get r1 to location l4



- $\pi_1 = \{(s1, \text{move}(r1, l1, l2)), (s2, \text{move}(r1, l2, l3)), (s3, \text{move}(r1, l3, l4))\}$
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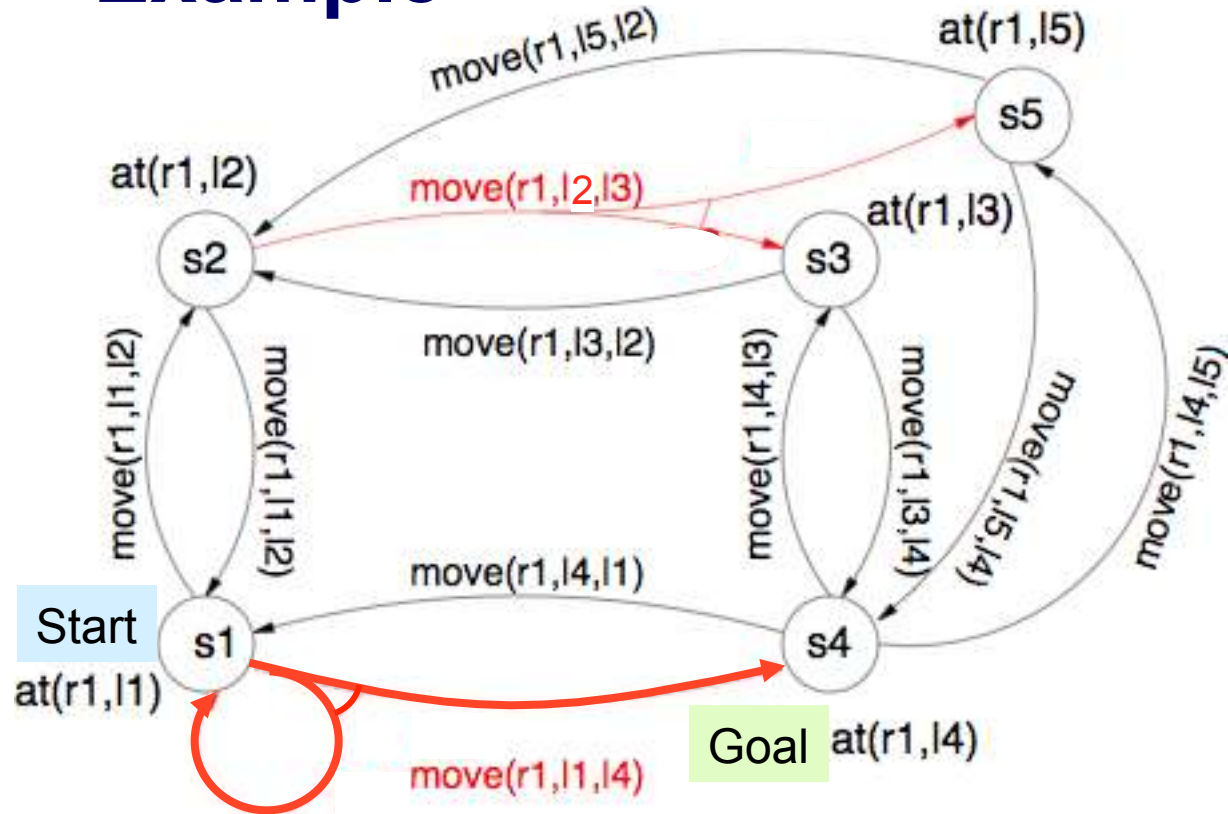
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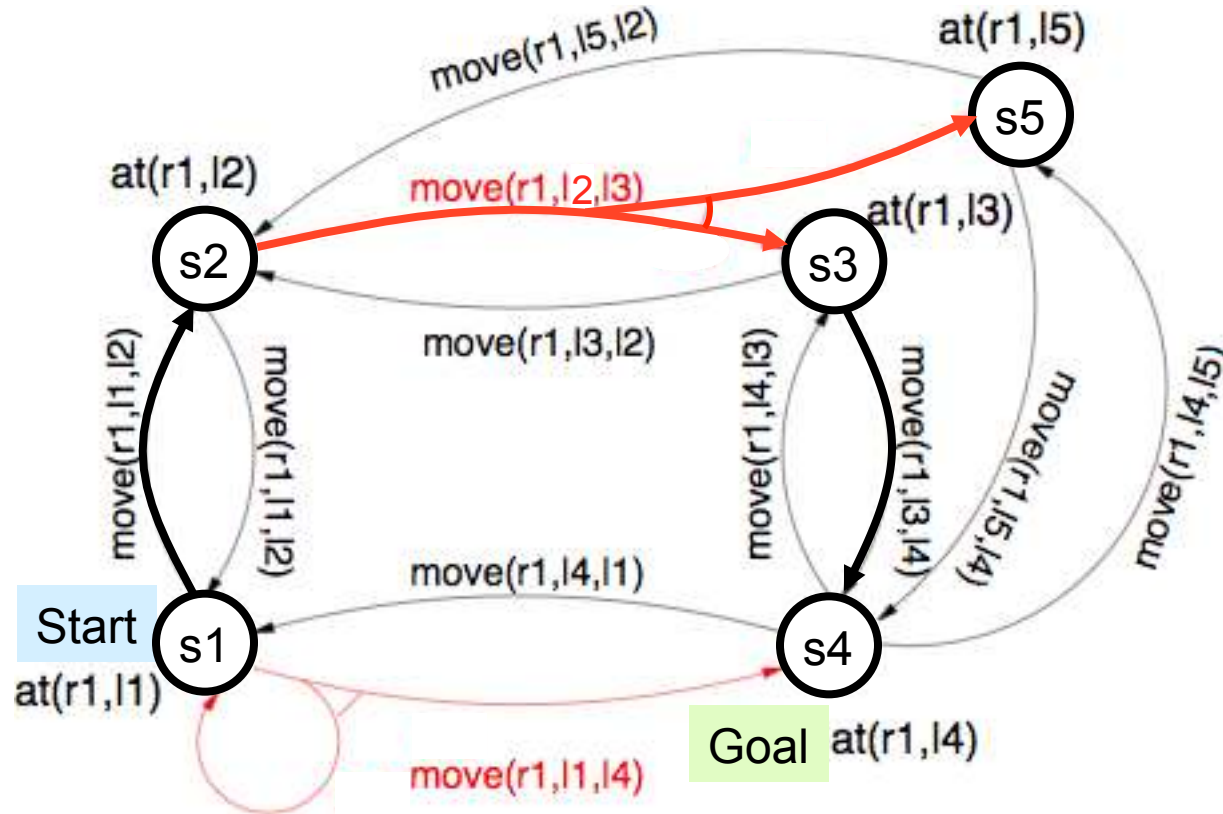
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- Execution structure for a policy  $\pi$ :
  - ◆ The graph of all of  $\pi$ 's execution paths
- Notation:  $\Sigma_\pi = (Q, T)$ 
  - ◆  $Q \subseteq S$
  - ◆  $T \subseteq S \times S$

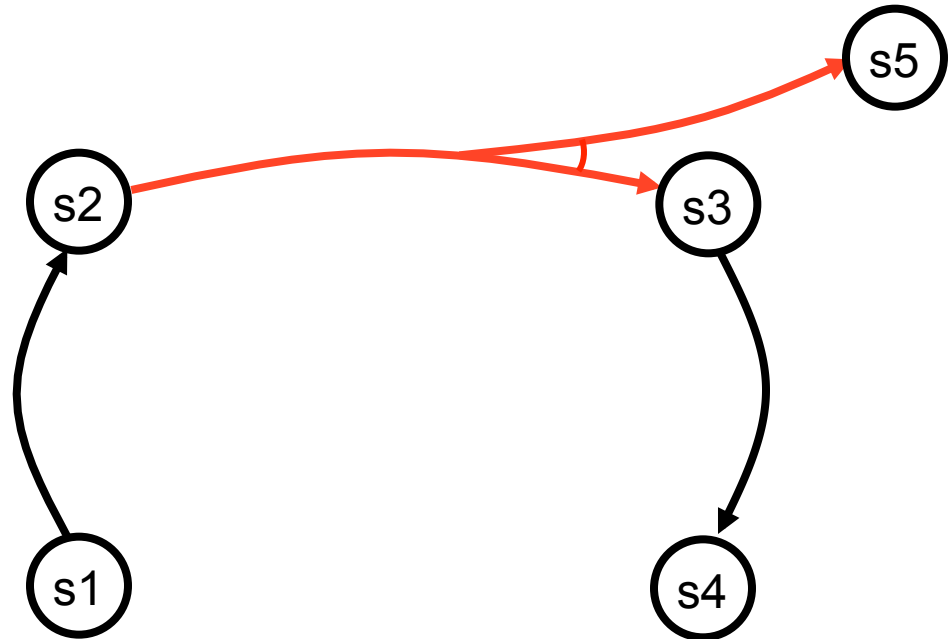


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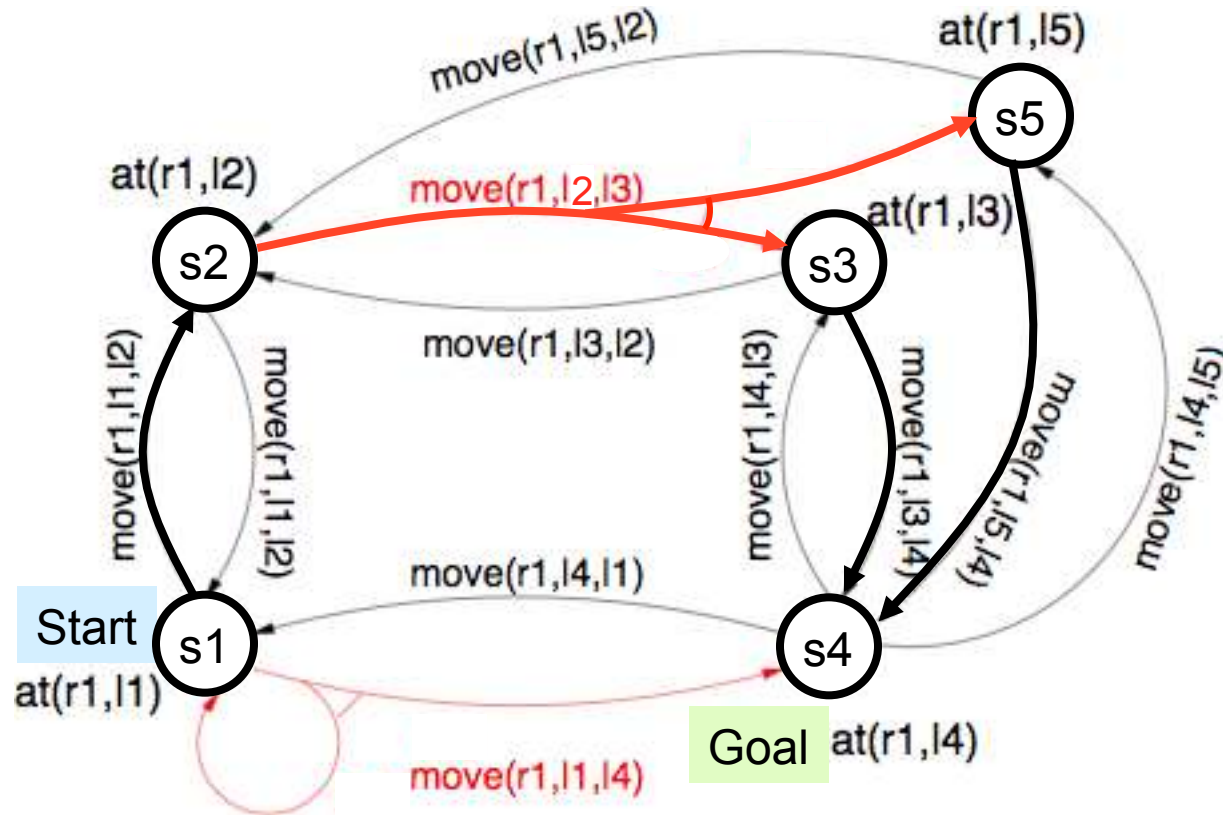


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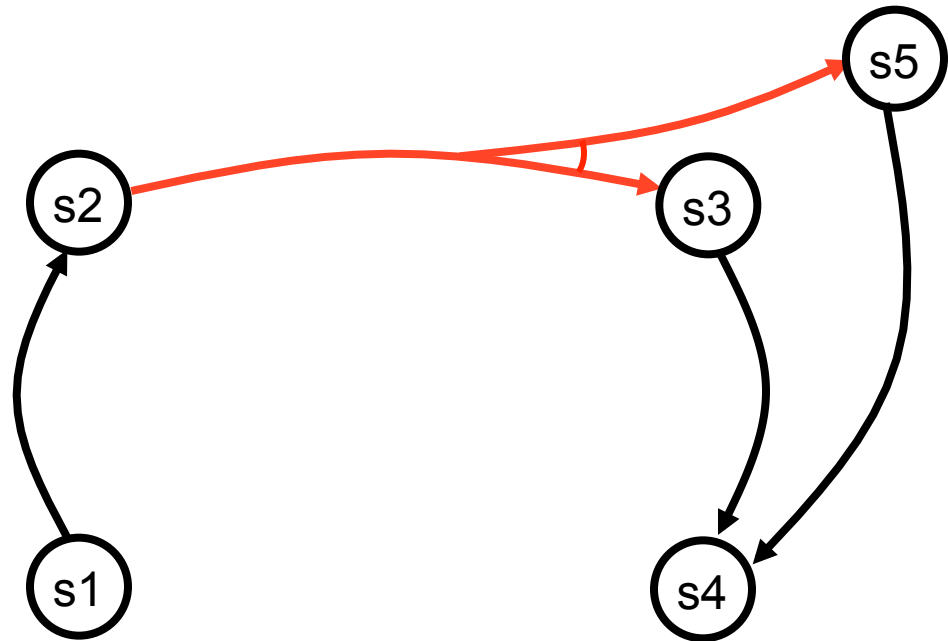
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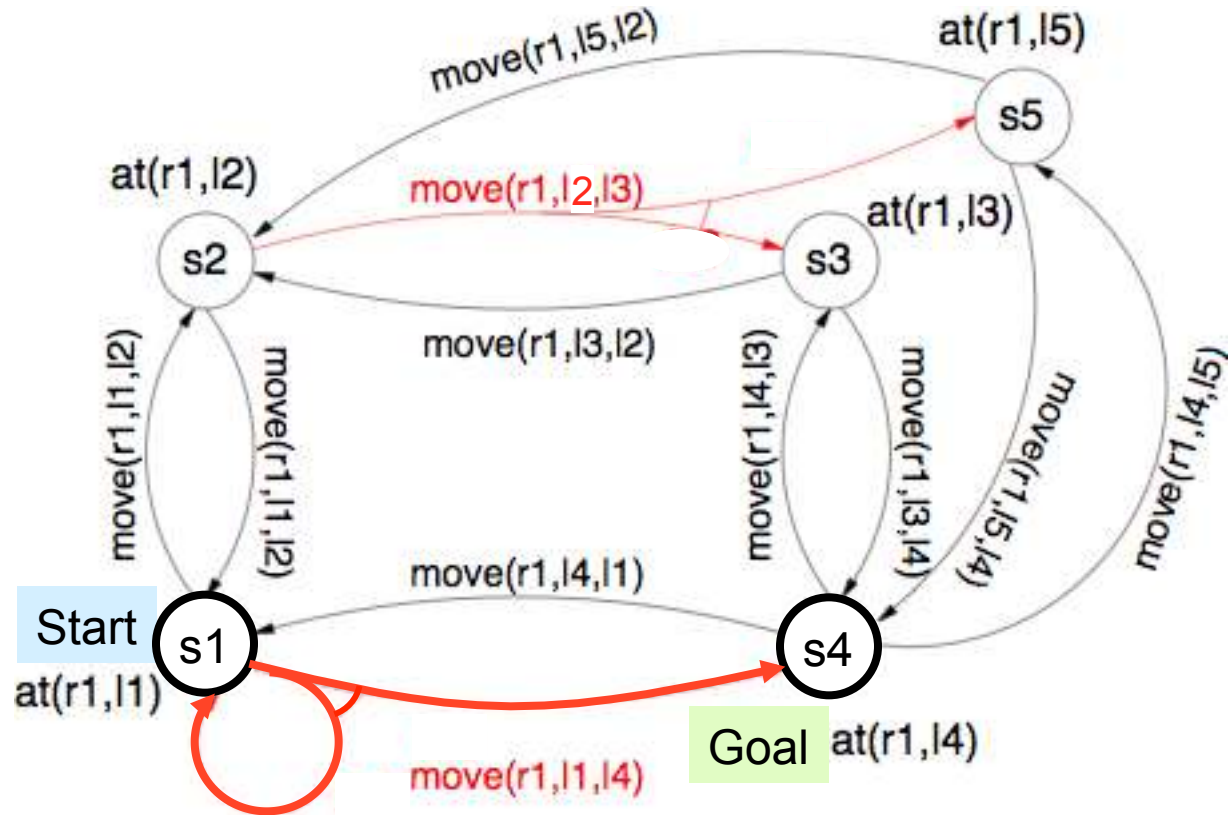
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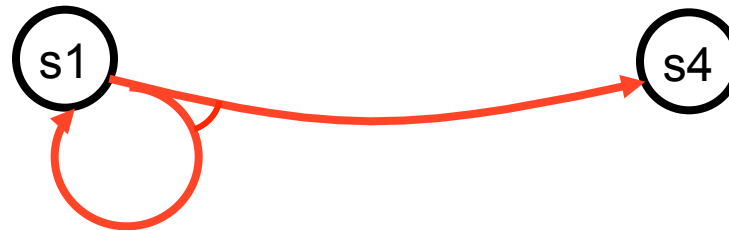
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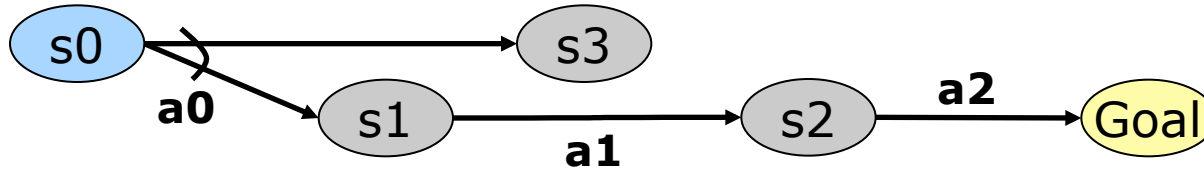
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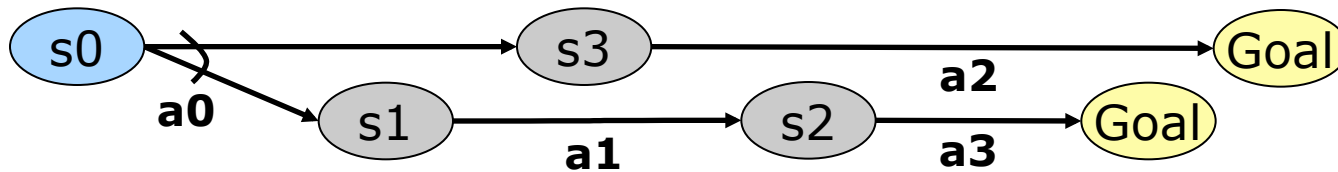
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# Types of Solutions

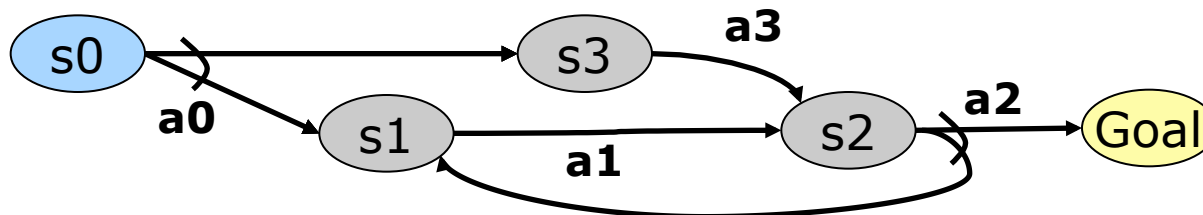
- **Weak solution:** at least one execution path reaches a goal



- **Strong solution:** every execution path reaches a goal



- **Strong-cyclic solution:** every *fair* execution path reaches a goal
  - ◆ Don't stay in a cycle forever if there's a state-transition out of it



# Finding Strong Solutions

- Backward breadth-first search
- $\text{StrongPreImg}(S)$   
 $= \{(s, a) : \gamma(s, a) \neq \emptyset, \gamma(s, a) \subseteq S\}$ 
  - ◆ all state-action pairs for which all of the successors are in  $S$
- $\text{PruneStates}(\pi, S)$   
 $= \{(s, a) \in \pi : s \notin S\}$ 
  - ◆  $S$  is the set of states we've already solved
  - ◆ keep only the state-action pairs for other states
- $\text{MkDet}(\pi')$ 
  - ◆  $\pi'$  is a policy that may be nondeterministic
  - ◆ remove some state-action pairs if necessary, to get a deterministic policy

$\text{Strong-Plan}(P)$

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \not\subseteq (S_g \cup S_{\pi'})$  do

$\text{PreImage} \leftarrow \text{StrongPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(\text{PreImage}, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

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if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return( $\text{MkDet}(\pi')$ )

else return(failure)

end

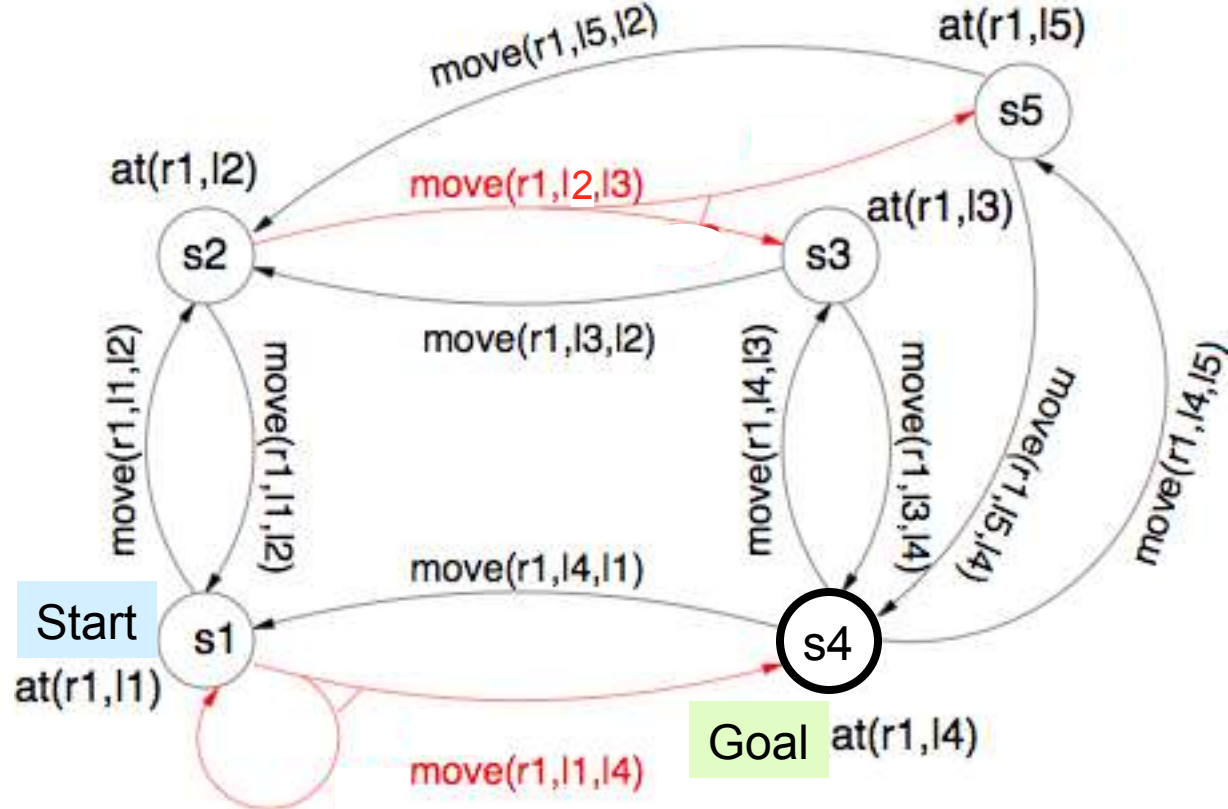
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$\pi = \text{failure}$

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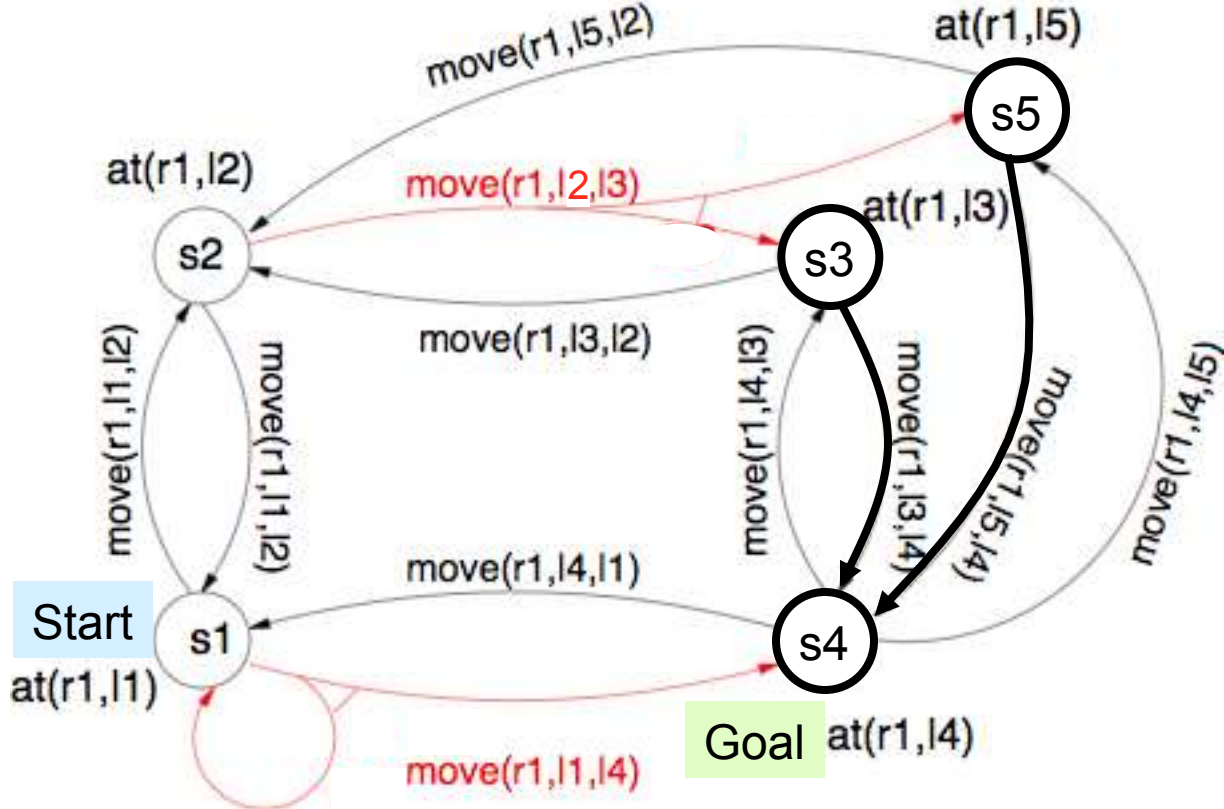
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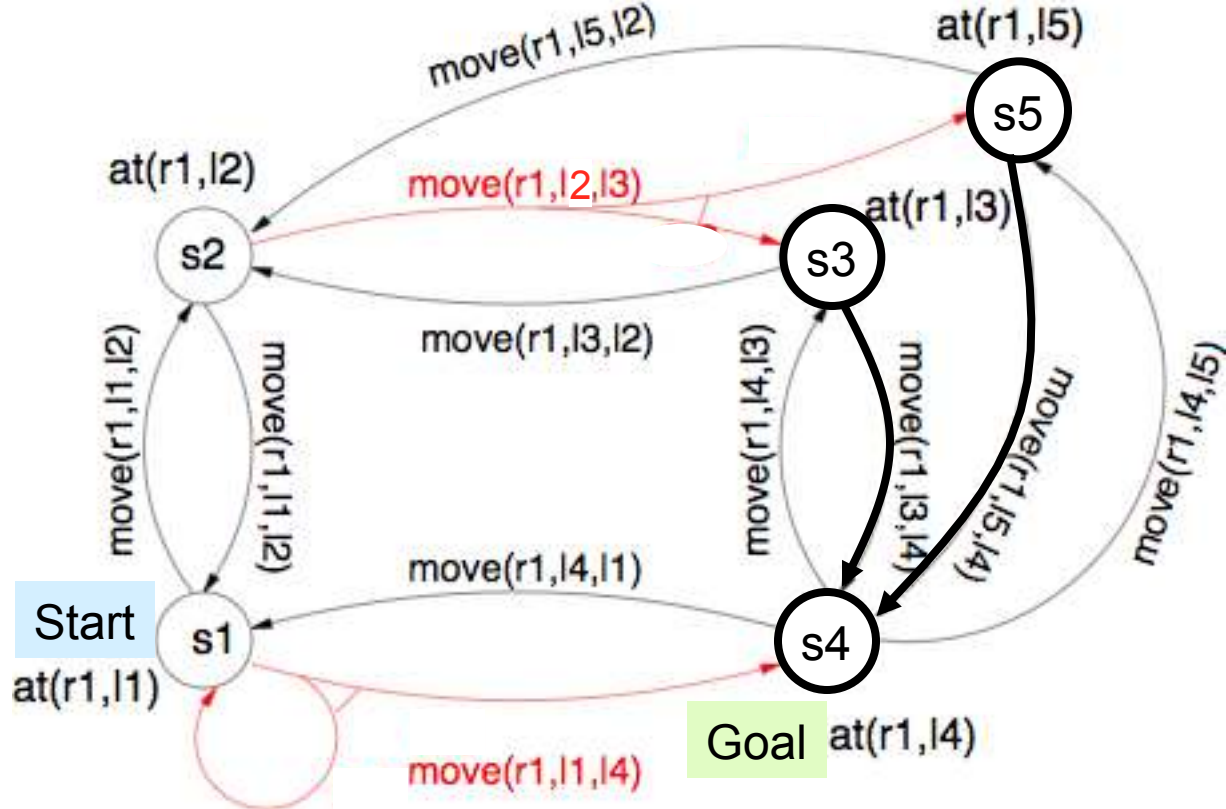
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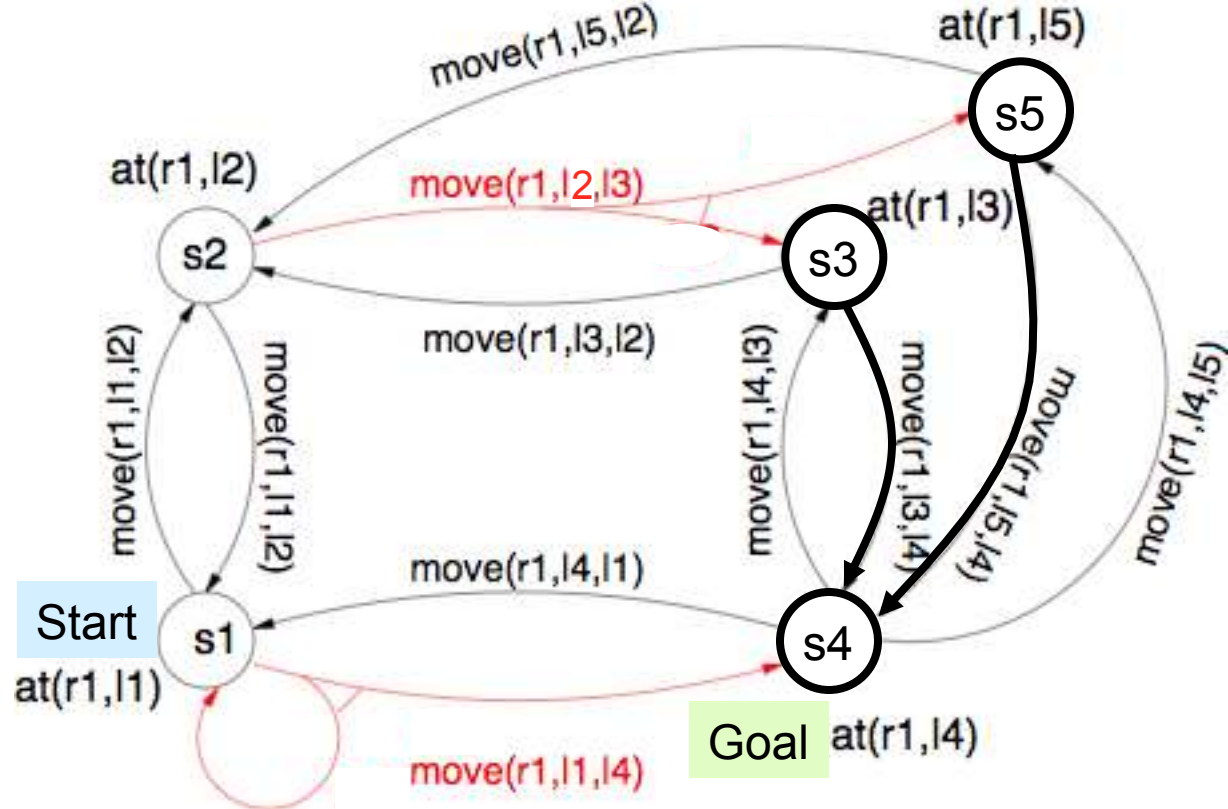
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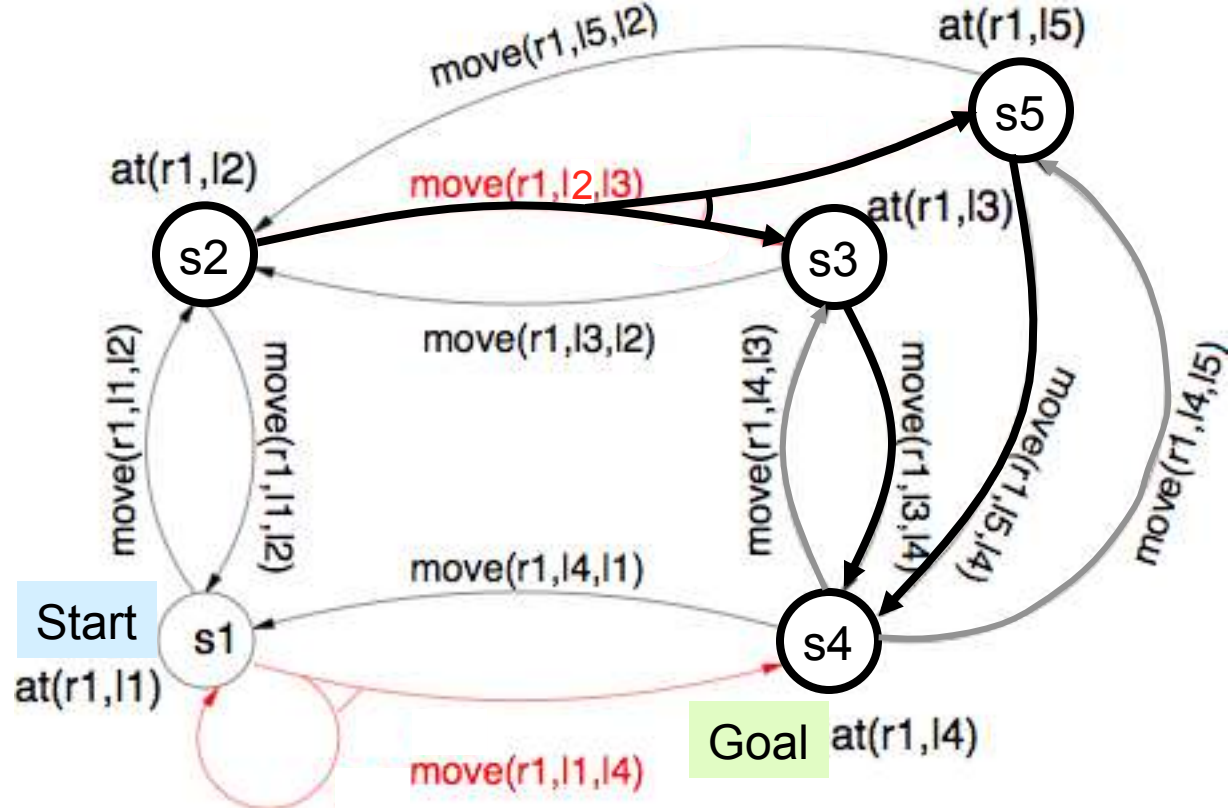
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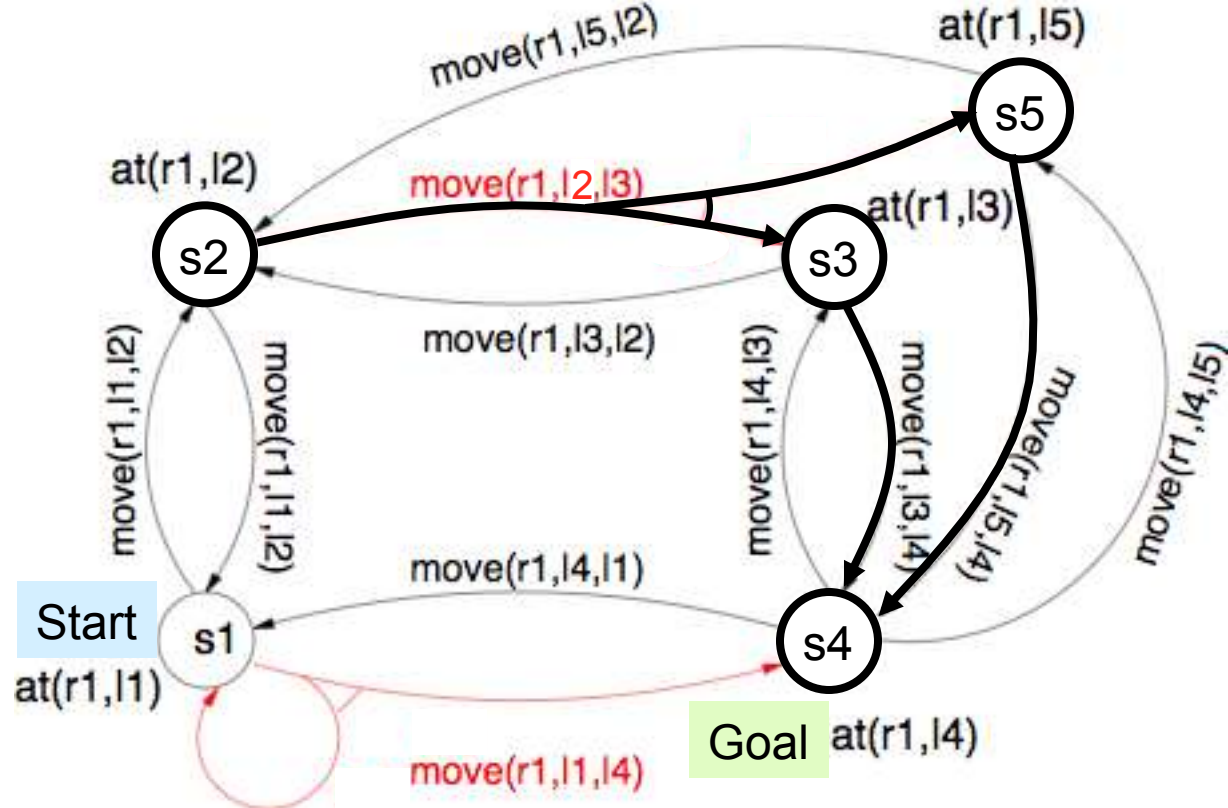
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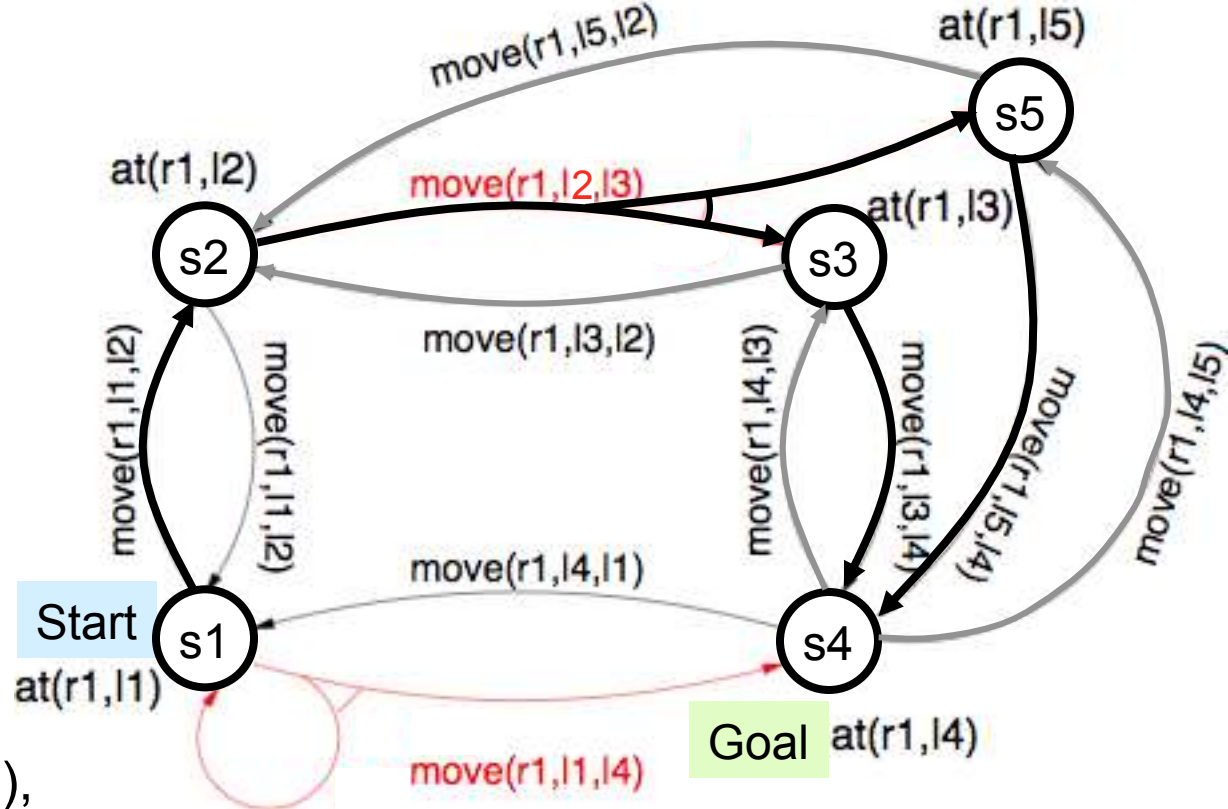
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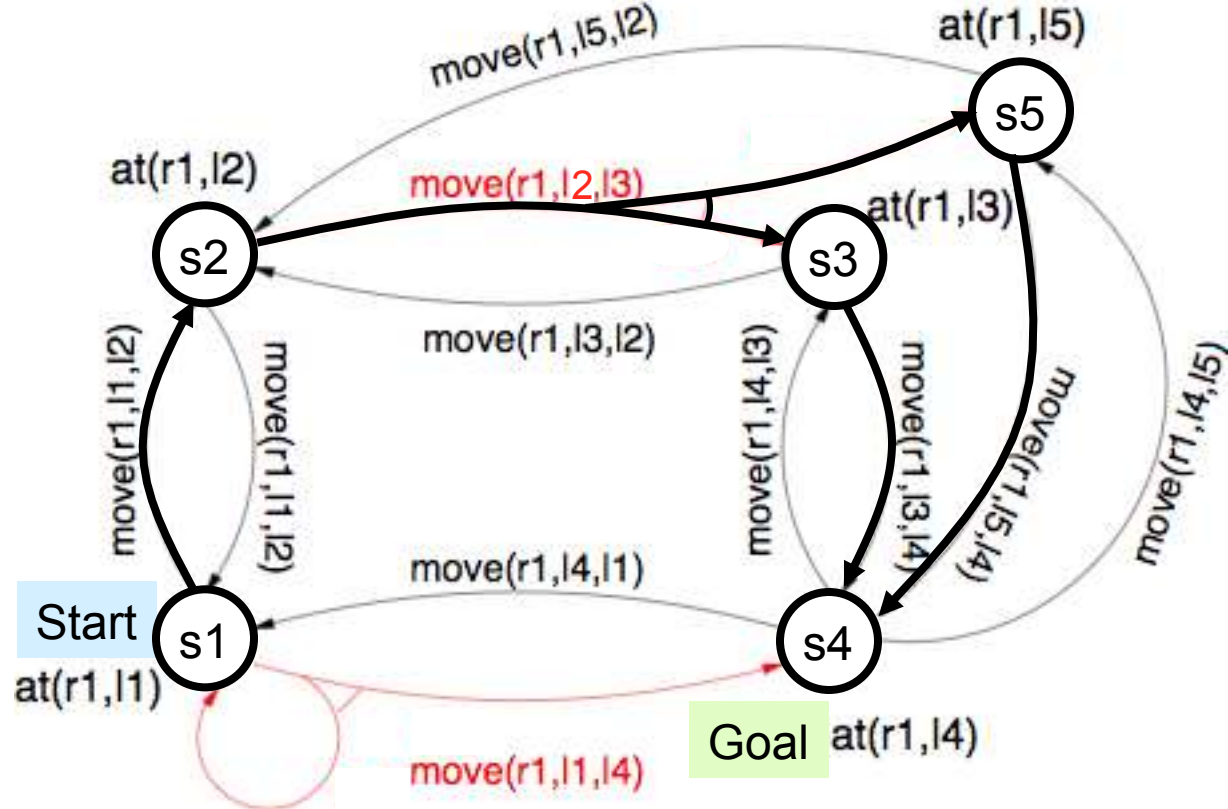
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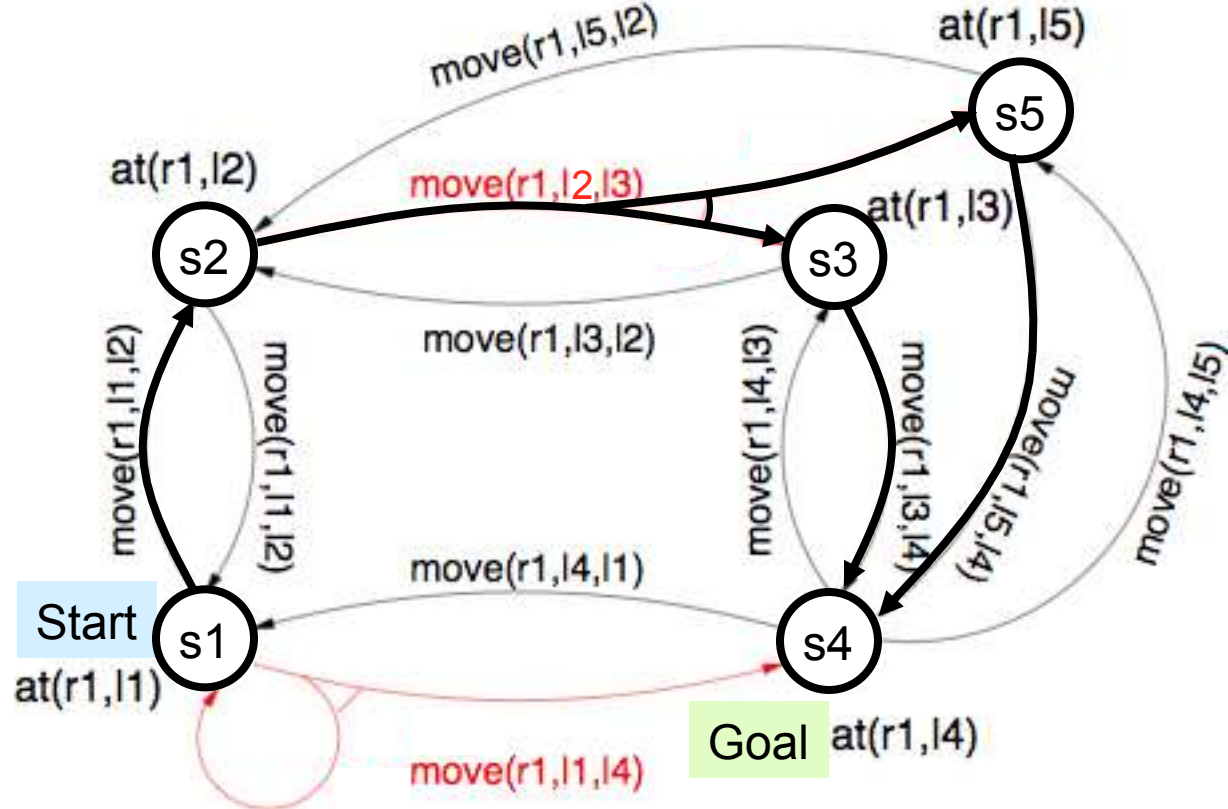
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$S_{\pi'} = \{s1, s2, s3, s5\}$

$S_g \cup S_{\pi'} = \{s1, s2, s3, s4, s5\}$

$S_0 \subseteq S_g \cup S_{\pi'}$

$\text{MkDet}(\pi') = \pi'$



Strong-Plan( $P$ )

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \not\subseteq (S_g \cup S_{\pi'})$  do

$PreImage \leftarrow \text{StrongPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(PreImage, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

$\pi' \leftarrow \pi' \cup \pi''$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return( $\text{MkDet}(\pi')$ )

else return(failure)

# Finding Weak Solutions

- Weak-Plan is just like Strong-Plan except for this:
- $\text{WeakPreImg}(S) = \{(s, a) : \gamma(s, a) \text{ i } S \neq \emptyset\}$ 
  - ◆ at least one successor is in  $S$

Weak-Plan( $P$ )

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \not\subseteq (S_g \cup S_{\pi'})$  do

$\text{PreImage} \leftarrow \text{WeakPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(\text{PreImage}, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

$\pi' \leftarrow \pi' \cup \pi''$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return( $\text{MkDet}(\pi')$ )

else return(failure)

end

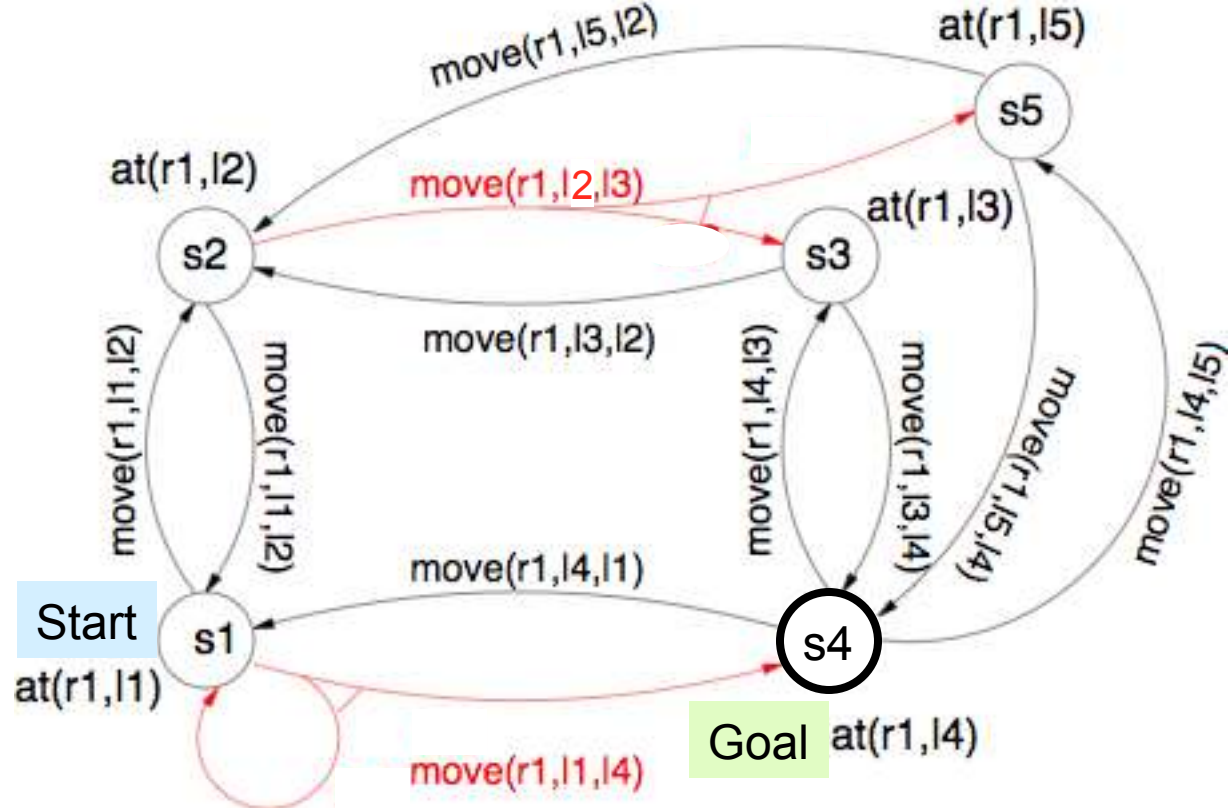
# Example

$\pi = \text{failure}$

$\pi' = \emptyset$

$S_{\pi'} = \emptyset$

$S_g \cup S_{\pi'} = \{s4\}$



Weak-Plan( $P$ )

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \not\subseteq (S_g \cup S_{\pi'})$  do

$PreImage \leftarrow \text{WeakPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(PreImage, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

$\pi' \leftarrow \pi' \cup \pi''$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return( $\text{MkDet}(\pi')$ )

else return(failure)

end

# Example

$\pi = \text{failure}$

$\pi' = \emptyset$

$S_{\pi'} = \emptyset$

$S_g \cup S_{\pi'} = \{s4\}$

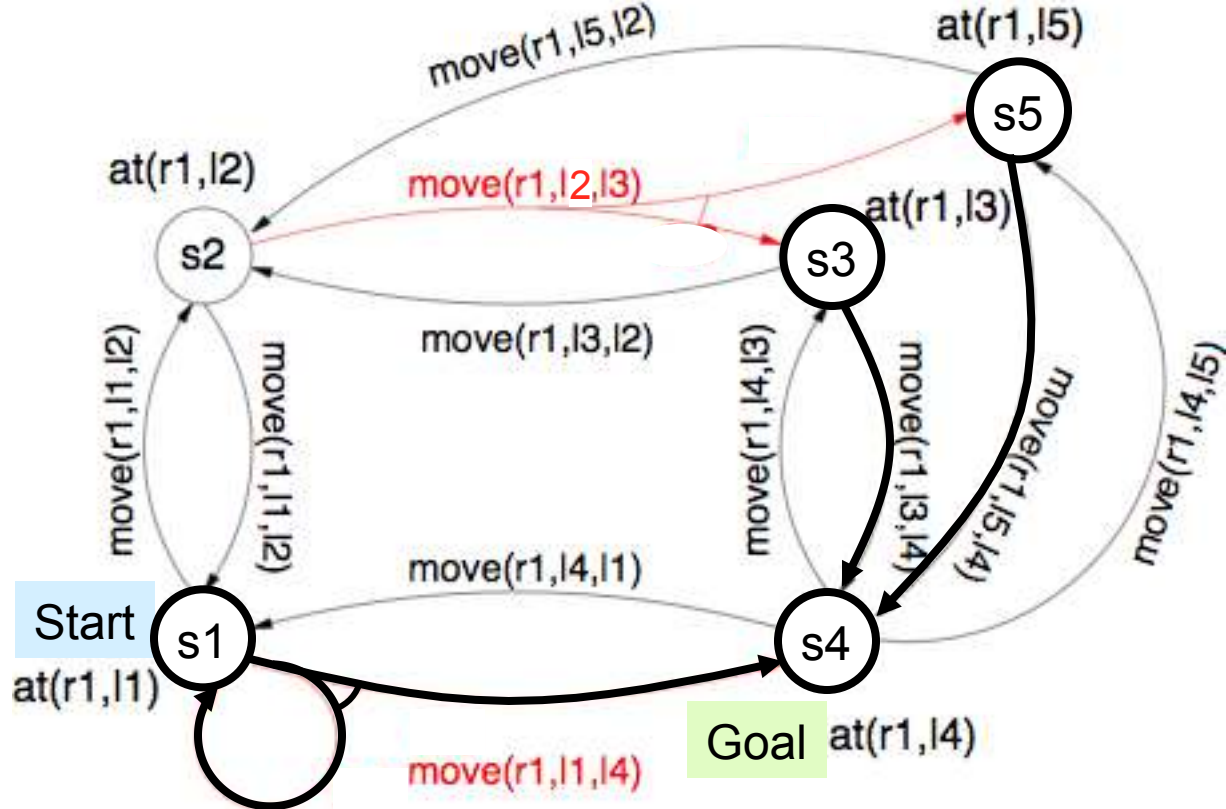
$\pi'' = \text{PreImage} =$

$\{(s1, \text{move}(r1, l1, l4)),$   
 $(s3, \text{move}(r1, l3, l4)),$   
 $(s5, \text{move}(r1, l5, l4))\}$

$\pi \leftarrow \pi' = \emptyset$

$\pi' \leftarrow \pi' \cup \pi'' =$

$\{(s1, \text{move}(r1, l1, l4)),$   
 $(s3, \text{move}(r1, l3, l4)),$   
 $(s5, \text{move}(r1, l5, l4))\}$



Weak-Plan( $P$ )

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \notin (S_g \cup S_{\pi'})$  do

$\text{PreImage} \leftarrow \text{WeakPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(\text{PreImage}, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

$\pi' \leftarrow \pi' \cup \pi''$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return( $\text{MkDet}(\pi')$ )

else return(failure)

end



# Example

$$\pi = \emptyset$$

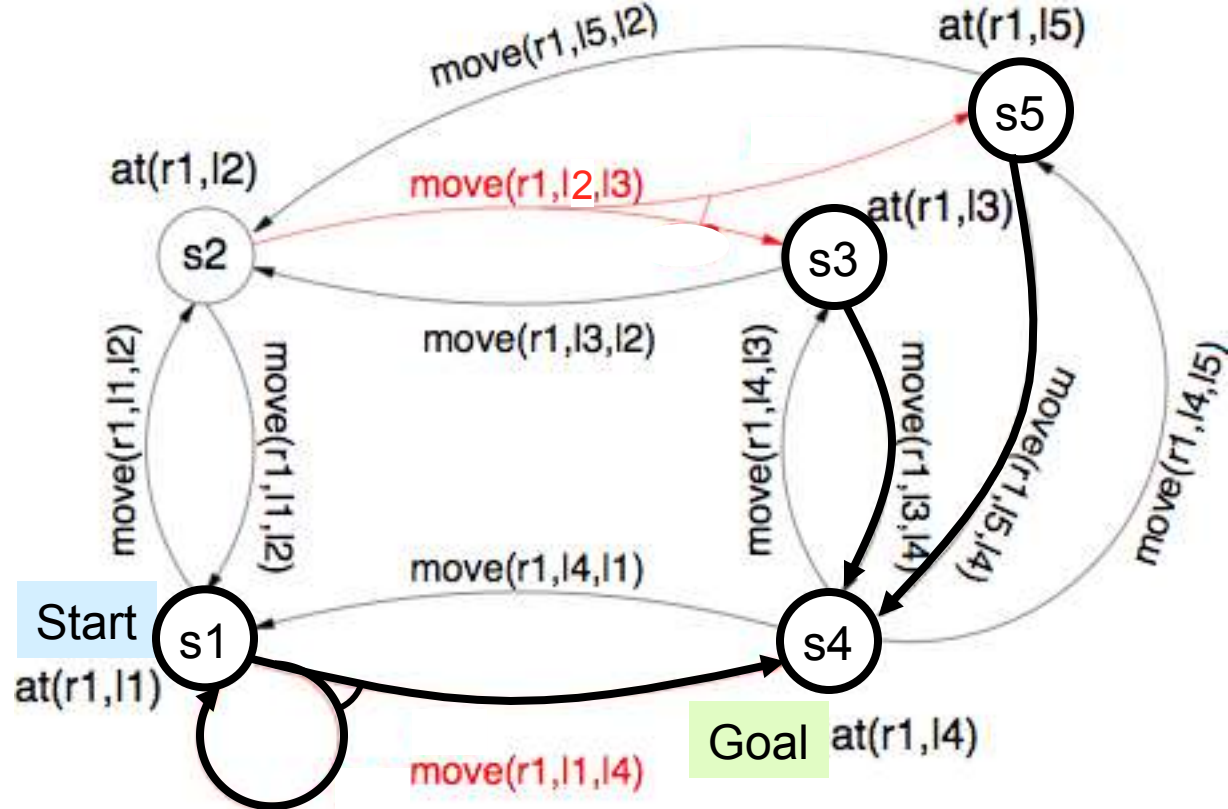
$$\pi' = \{(s1, \text{move}(r1, l1, l4)), \\ (s3, \text{move}(r1, l3, l4)), \\ (s5, \text{move}(r1, l5, l4))\}$$

$$S_{\pi'} = \{s1, s3, s5\}$$

$$S_g \cup S_{\pi'} = \{s1, s3, s4, s5\}$$

$$S_0 \subseteq S_g \cup S_{\pi'}$$

$$\text{MkDet}(\pi') = \pi'$$



## Weak-Plan(P)

$\pi \leftarrow \text{failure}; \pi' \leftarrow \emptyset$

While  $\pi' \neq \pi$  and  $S_0 \not\subseteq (S_g \cup S_{\pi'})$  do

$PreImage \leftarrow \text{WeakPreImg}(S_g \cup S_{\pi'})$

$\pi'' \leftarrow \text{PruneStates}(PreImage, S_g \cup S_{\pi'})$

$\pi \leftarrow \pi'$

$\pi' \leftarrow \pi' \cup \pi''$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$  then return(MkDet( $\pi'$ ))

else return(failure)

# Finding Strong-Cyclic Solutions

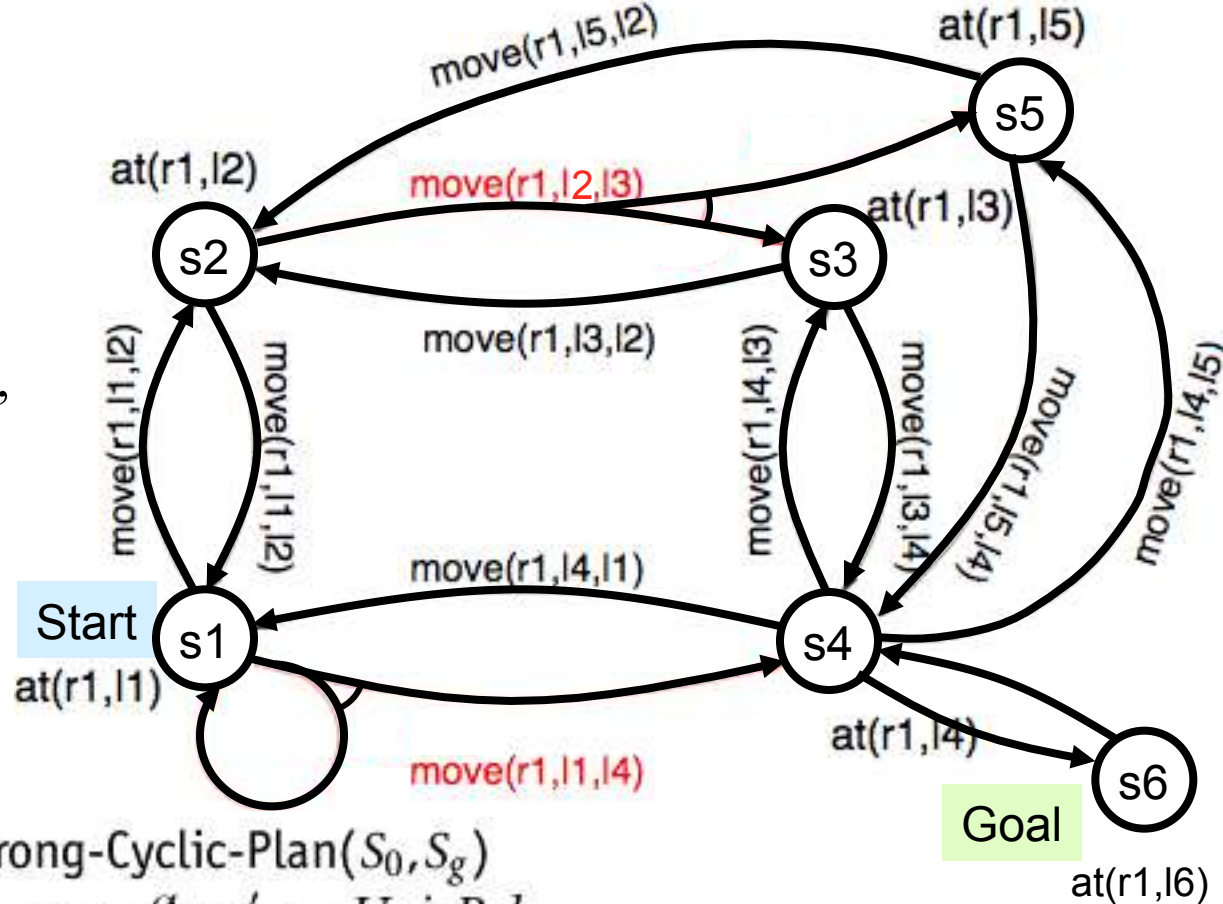
- Begin with a “universal policy”  $\pi'$  that contains *all* state-action pairs
- Repeatedly, eliminate state-action pairs that take us to bad states
  - ◆ PruneOutgoing removes state-action pairs that go to states not in  $S_g \cup S_{\pi}$ 
    - »  $\text{PruneOutgoing}(\pi, S) = \pi - \{(s, a) \in \pi : \gamma(s, a) \subseteq S \cup S_{\pi}\}$
  - ◆ PruneUnconnected removes states from which it is impossible to get to  $S_g$ 
    - » Start with  $\pi' = \emptyset$ , compute fixpoint of  $\pi' \leftarrow \pi \cap \text{WeakPrelmg}(S_g \cup S_{\pi'})$

```
Strong-Cyclic-Plan( $S_0, S_g$ )
   $\pi \leftarrow \emptyset$ ;  $\pi' \leftarrow \text{UnivPol}$ 
  while  $\pi' \neq \pi$  do
     $\pi \leftarrow \pi'$ 
     $\pi' \leftarrow \text{PruneUnconnected}(\text{PruneOutGoing}(\pi', S_g), S_g)$ 
  if  $S_0 \subseteq (S_g \cup S_{\pi'})$ 
    then return( $\text{MkDet}(\text{RemoveNonProgress}(\pi', S_g))$ )
    else return(failure)
end
```

# Finding Strong-Cyclic Solutions

Once the policy stops changing,

- If it's not a solution, return failure
- RemoveNonProgress removes state-action pairs that don't go toward the goal
  - ◆ implement as backward search from the goal
- MkDet makes sure there's only one action for each state



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutGoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

else return(failure)

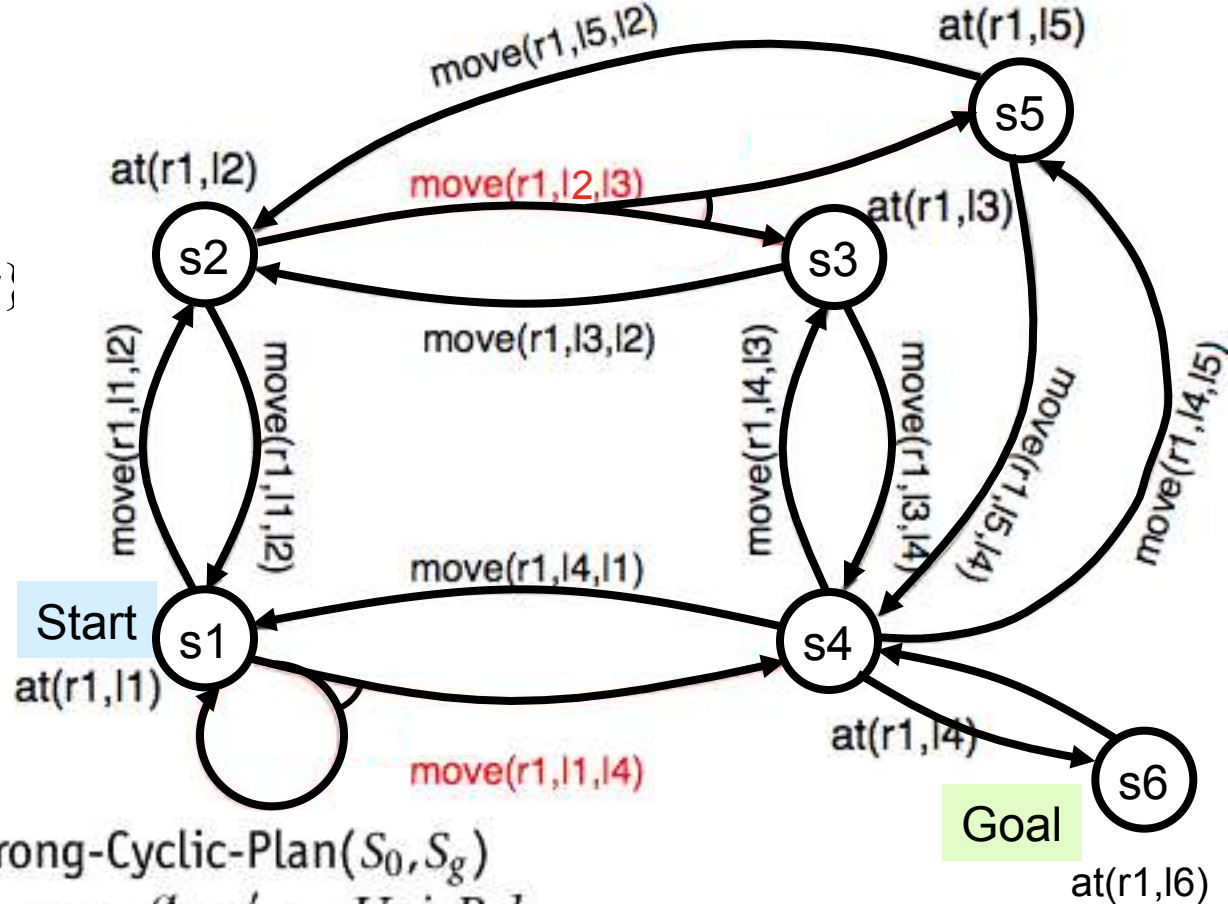
end



# Example 1

$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutGoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

    else return(failure)

end

# Example 1

$\pi \leftarrow \emptyset$

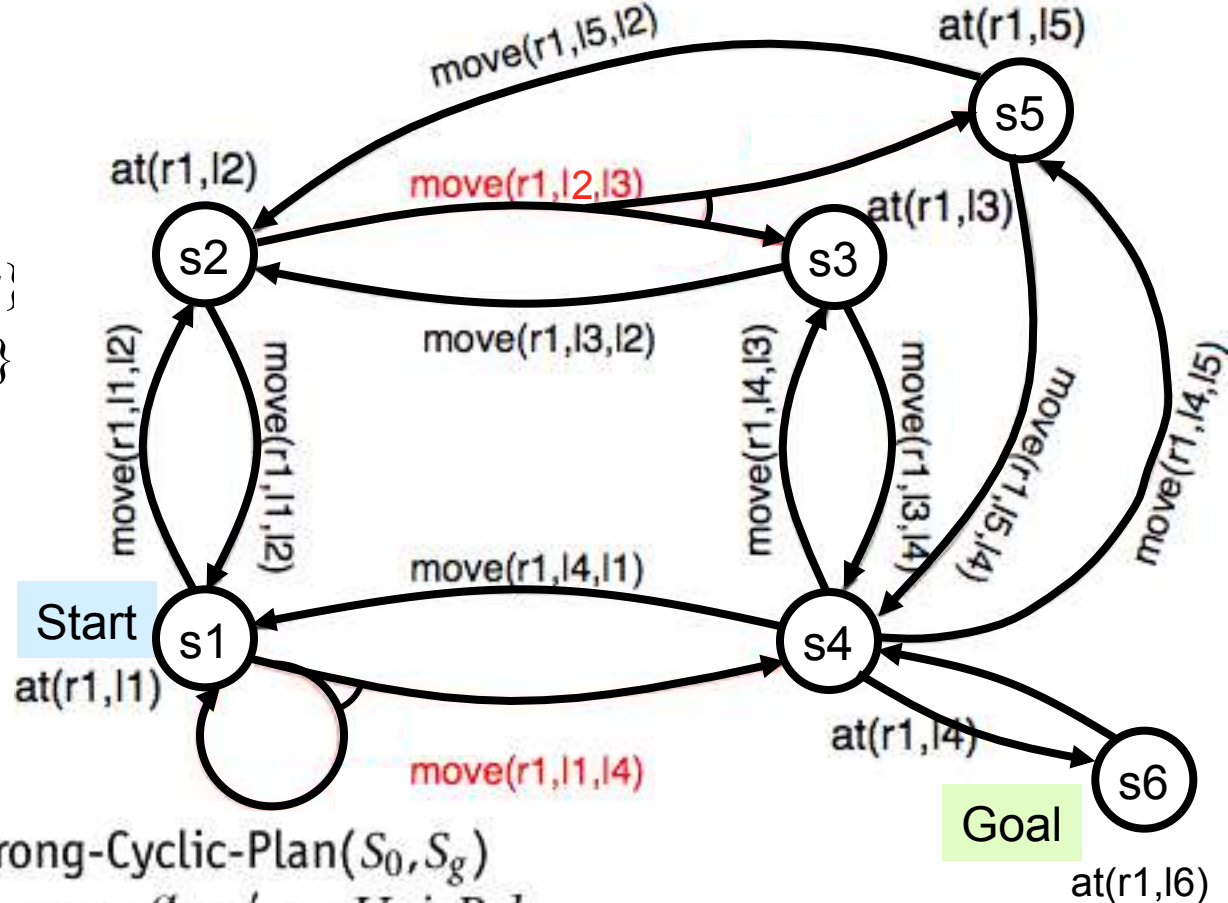
$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\text{PruneOutgoing}(\pi', S_g) = \pi'$

$\text{PruneUnconnected}(\pi', S_g) = \pi'$

$\text{RemoveNonProgress}(\pi') = ?$



$\text{Strong-Cyclic-Plan}(S_0, S_g)$

$\pi \leftarrow \emptyset; \pi' \leftarrow \text{UnivPol}$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow \text{PruneUnconnected}(\text{PruneOutgoing}(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

    else return(failure)

end

# Example 1

$\pi \leftarrow \emptyset$

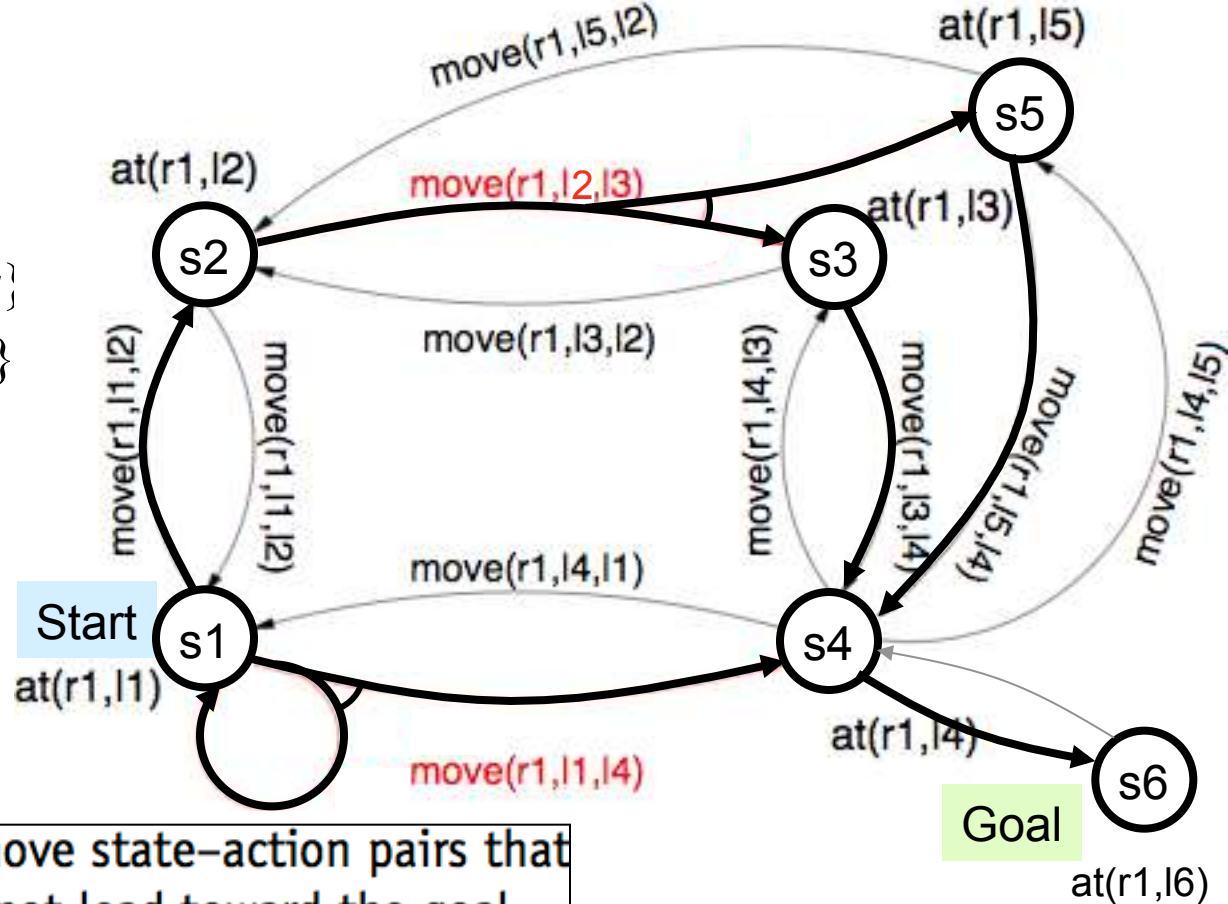
$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\text{PruneOutgoing}(\pi', S_g) = \pi'$

$\text{PruneUnconnected}(\pi', S_g) = \pi'$

$\text{RemoveNonProgress}(\pi') =$   
*as shown*



$\text{RemoveNonProgress}(\pi, S_g)$  ;; remove state-action pairs that  
;; do not lead toward the goal

$\pi^* \leftarrow \emptyset$

repeat

$\text{PreImage} \leftarrow \pi \cap \text{WeakPreImg}(S_g \cup S_{\pi^*})$

$\pi_{old}^* \leftarrow \pi^*$

$\pi^* \leftarrow \pi^* \cup \text{PruneStates}(\text{PreImage}, S_g \cup S_{\pi^*})$

until  $\pi_{old}^* = \pi^*$

return( $\pi^*$ )

end

$\text{PruneStates}(\pi, S) = \{(s, a) \in \pi \mid s \notin S\}$

# Example 1

$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\text{PruneOutgoing}(\pi', S_g) = \pi'$

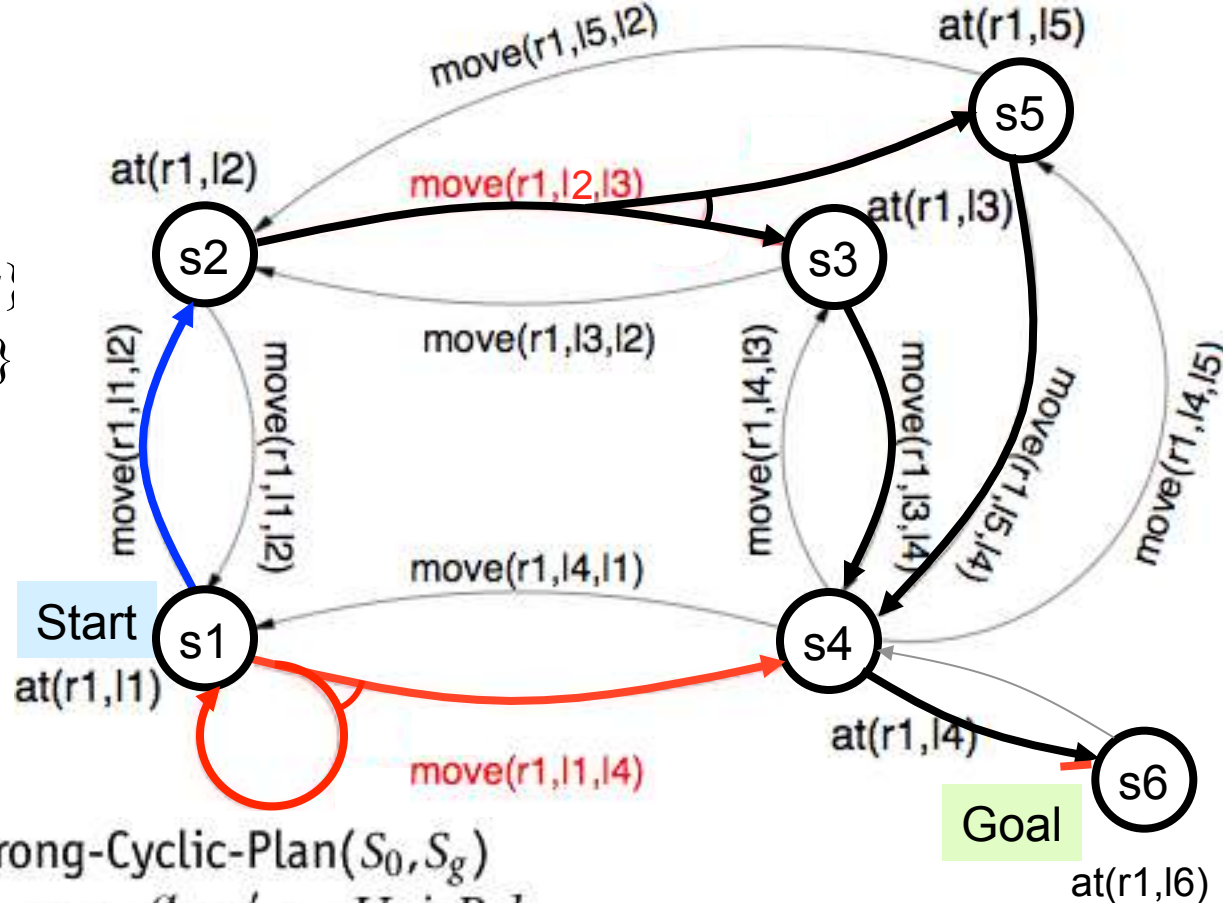
$\text{PruneUnconnected}(\pi', S_g) = \pi'$

$\text{RemoveNonProgress}(\pi') =$   
*as shown*

$\text{MkDet}(\dots) = \text{either}$

$\{(s1, \text{move}(r1, l1, l4)),$   
 $(s2, \text{move}(r1, l2, l3)),$   
 $(s3, \text{move}(r1, l3, l4)),$   
 $(s4, \text{move}(r1, l4, l6)),$   
 $(s5, \text{move}(r1, l5, l4))\}$

or  $\{(s1, \text{move}(r1, l1, l2)),$   
 $(s2, \text{move}(r1, l2, l3)),$   
 $(s3, \text{move}(r1, l3, l4)),$   
 $(s4, \text{move}(r1, l4, l6)),$   
 $(s5, \text{move}(r1, l5, l4))\}$



$\text{Strong-Cyclic-Plan}(S_0, S_g)$

$\pi \leftarrow \emptyset; \pi' \leftarrow \text{UnivPol}$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow \text{PruneUnconnected}(\text{PruneOutgoing}(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

then return( $\text{MkDet}(\text{RemoveNonProgress}(\pi', S_g))$ )

else return(failure)

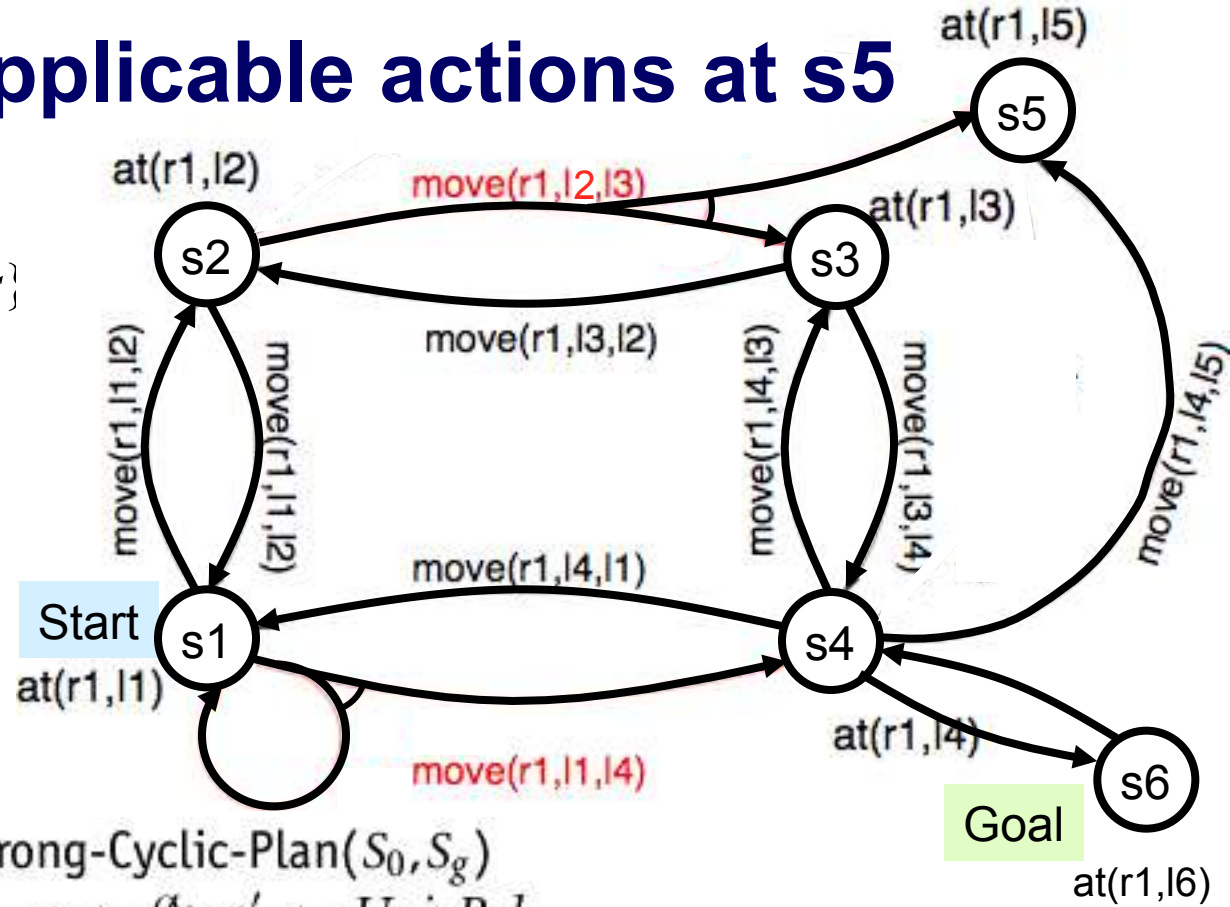
end



# Example 2: no applicable actions at s5

$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutGoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

    else return(failure)

end

# Example 2: no applicable actions at s5

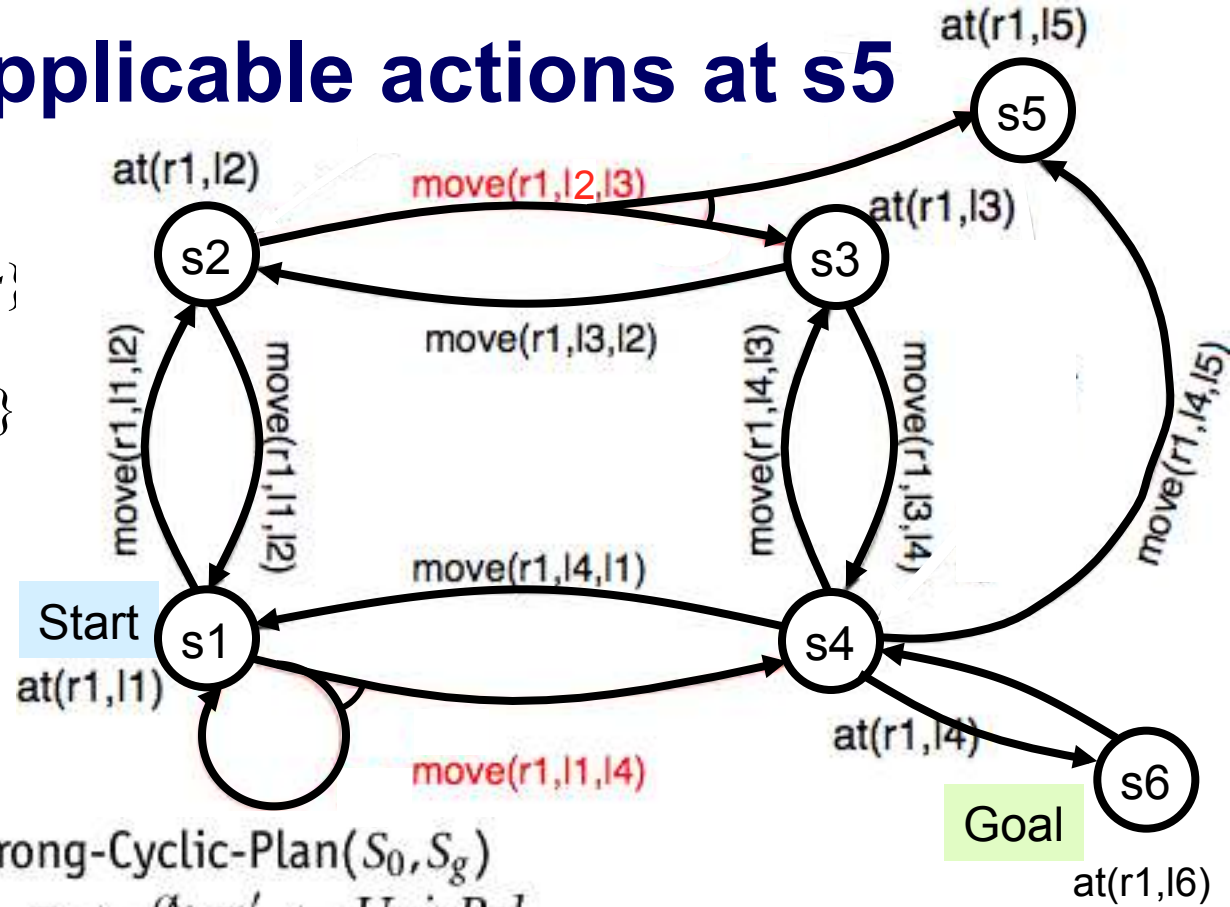
$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

PruneOutgoing( $\pi', S_g$ )

= ...



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutgoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

    else return(failure)

end

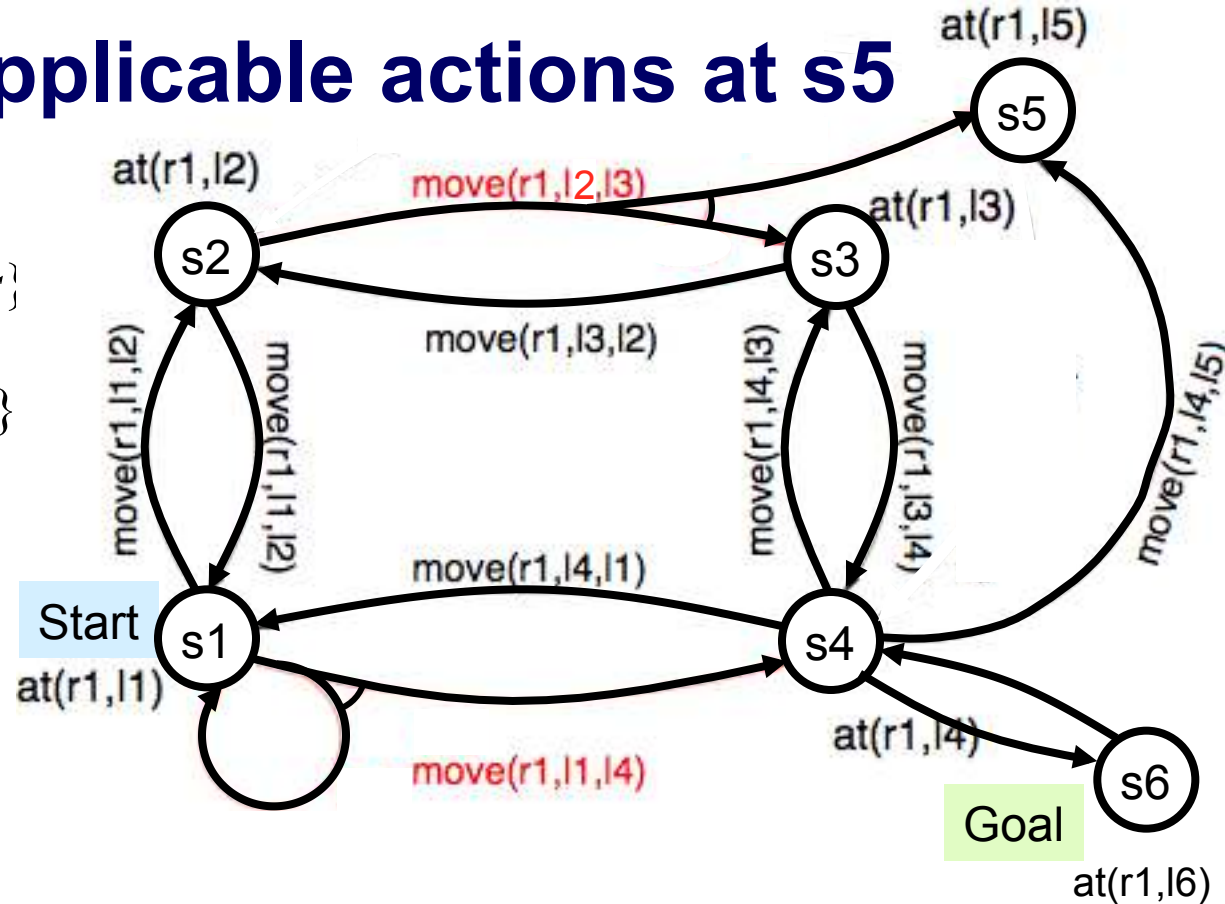
# Example 2: no applicable actions at s5

$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\text{PruneOutgoing}(\pi', S_g) = \pi$



$\text{PruneOutgoing}(\pi, S_g) ;;$  removes outgoing state-action pairs

$\pi' \leftarrow \pi - \text{ComputeOutgoing}(\pi, S_g \cup S_\pi)$

return( $\pi'$ )

end

$\text{ComputeOutgoing}(\pi, S) = \{(s, a) \in \pi \mid \gamma(s, a) \notin S\}$



# Example 2: no applicable actions at s5

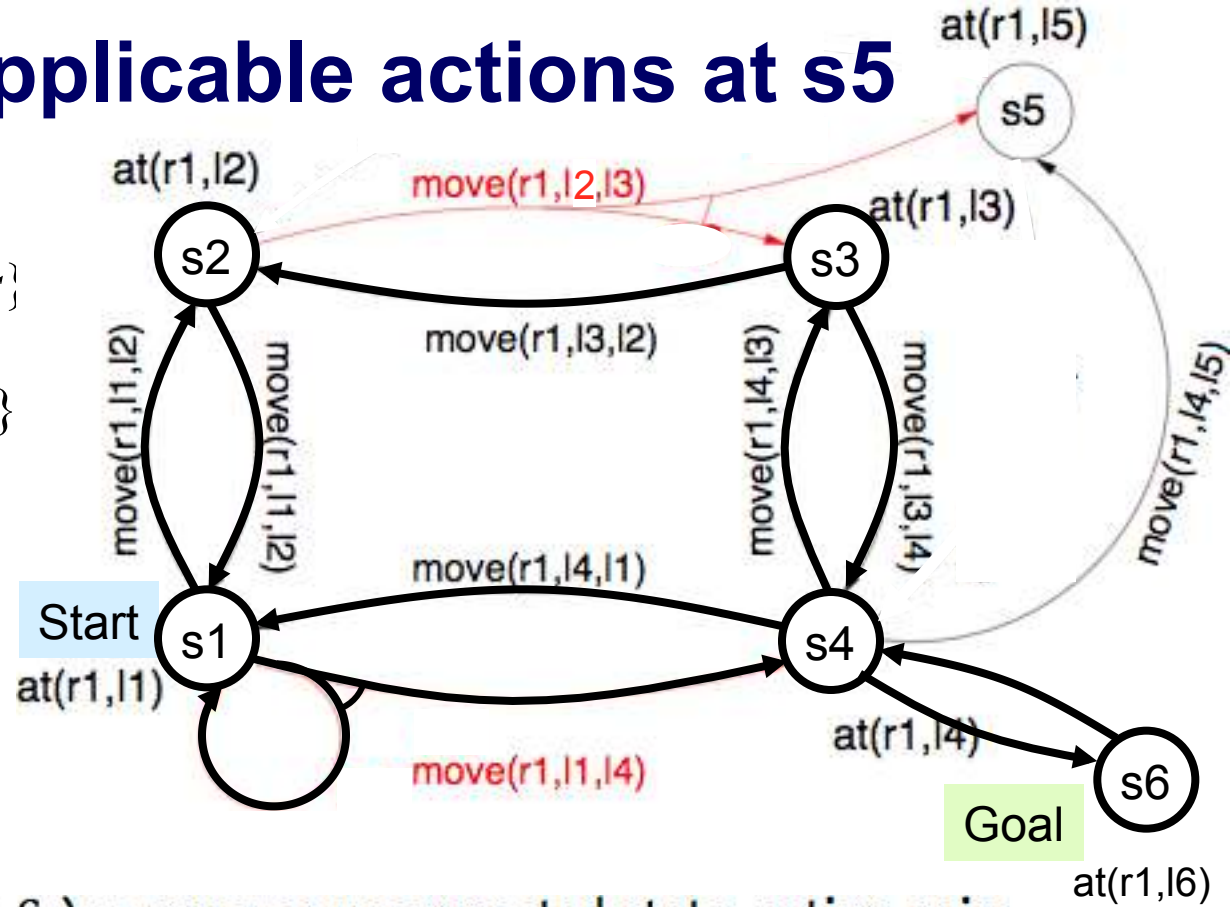
$\pi \leftarrow \emptyset$

$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

PruneOutgoing( $\pi', S_g$ ) =  $\pi'$

PruneUnconnected( $\pi', S_g$ )  
= as shown



PruneUnconnected( $\pi, S_g$ ) ;; removes unconnected state-action pairs

$\pi' \leftarrow \emptyset$

repeat

$\pi'' \leftarrow \pi'$

$\pi' \leftarrow \pi \cap \text{WeakPreImg}(S_g \cup S_{\pi'})$

until  $\pi'' = \pi'$

return( $\pi'$ ) end

# Example 2: no applicable actions at s5

$\pi \leftarrow \emptyset$

~~$\pi' \leftarrow \{(s,a) : a \text{ is applicable to } s\}$~~

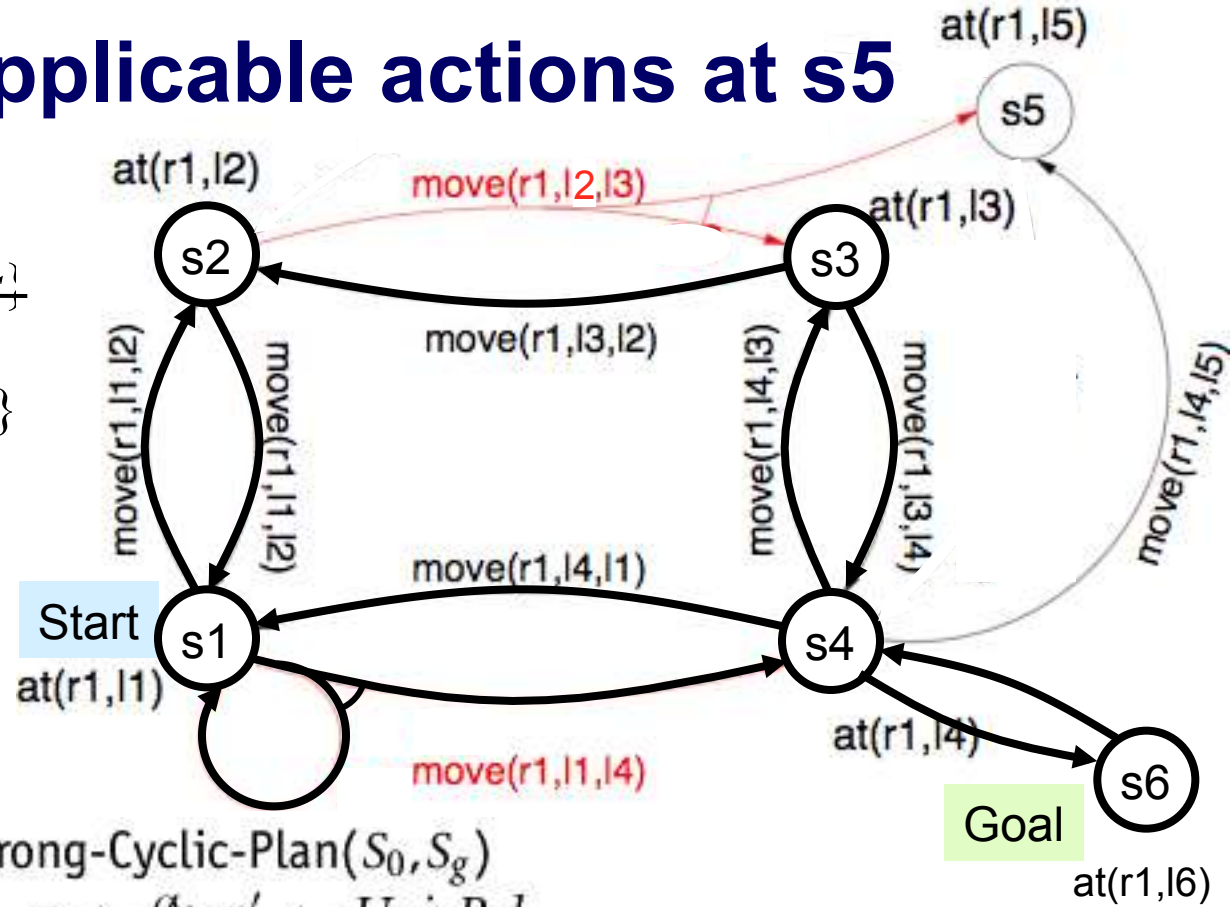
$\pi \leftarrow \{(s,a) : a \text{ is applicable to } s\}$

PruneOutgoing( $\pi', S_g$ ) =  $\pi'$

PruneUnconnected( $\pi', S_g$ )

= as shown

$\pi' \leftarrow$  as shown



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutgoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return(MkDet(RemoveNonProgress( $\pi', S_g$ )))

    else return(failure)

end

# Example 2: no applicable actions at s5

$\pi' \leftarrow \text{as hown}$

$\pi \leftarrow \pi'$

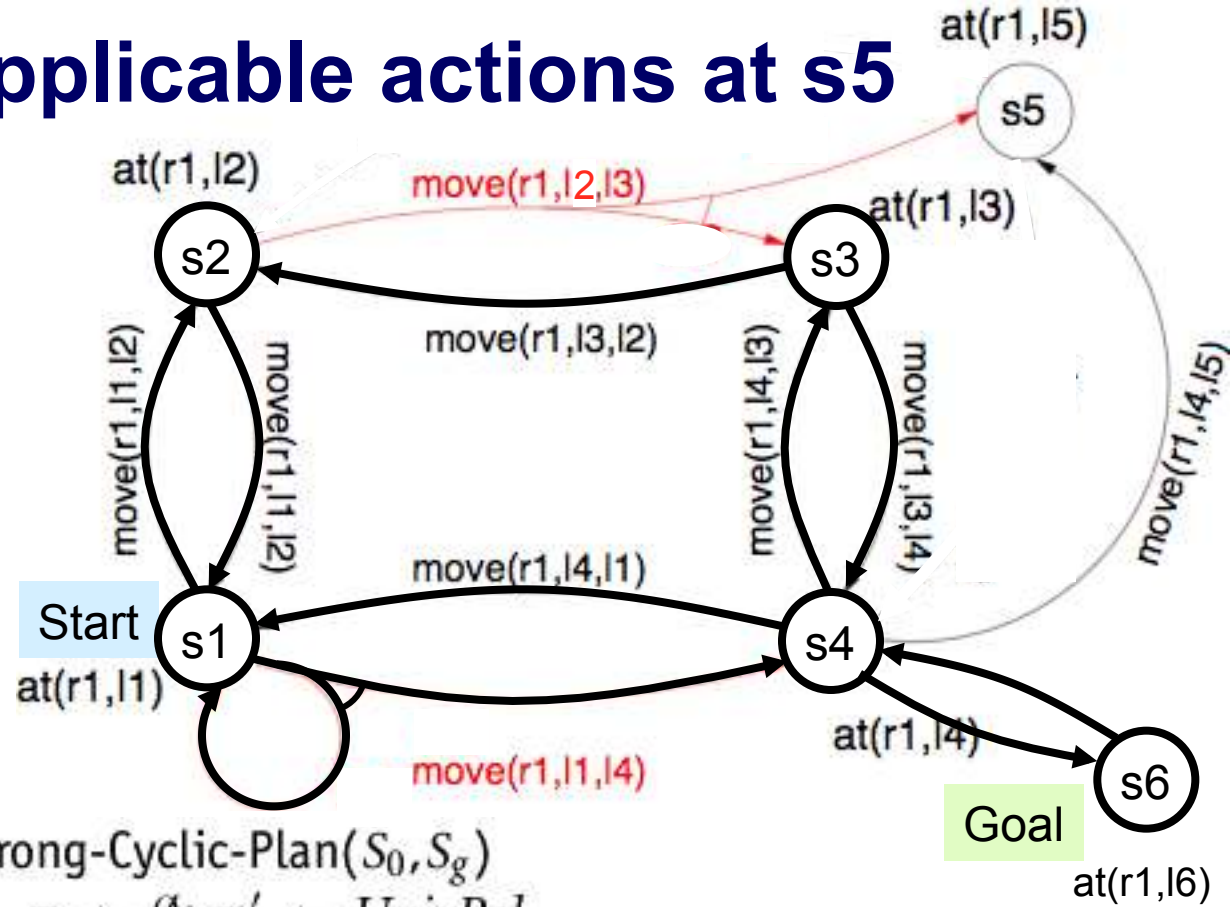
$\text{PruneOutgoing}(\pi', S_g) = \pi'$

$\text{PruneUnconnected}(\pi', S_g) = \pi'$

so  $\pi = \pi'$

$\text{RemoveNonProgress}(\pi') =$

...



$\text{Strong-Cyclic-Plan}(S_0, S_g)$

$\pi \leftarrow \emptyset; \pi' \leftarrow \text{UnivPol}$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow \text{PruneUnconnected}(\text{PruneOutgoing}(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

    then return( $\text{MkDet}(\text{RemoveNonProgress}(\pi', S_g))$ )

    else return(failure)

end

# Example 2: no applicable actions at s5

~~$\pi' \leftarrow shown$~~

$\pi \leftarrow \pi'$

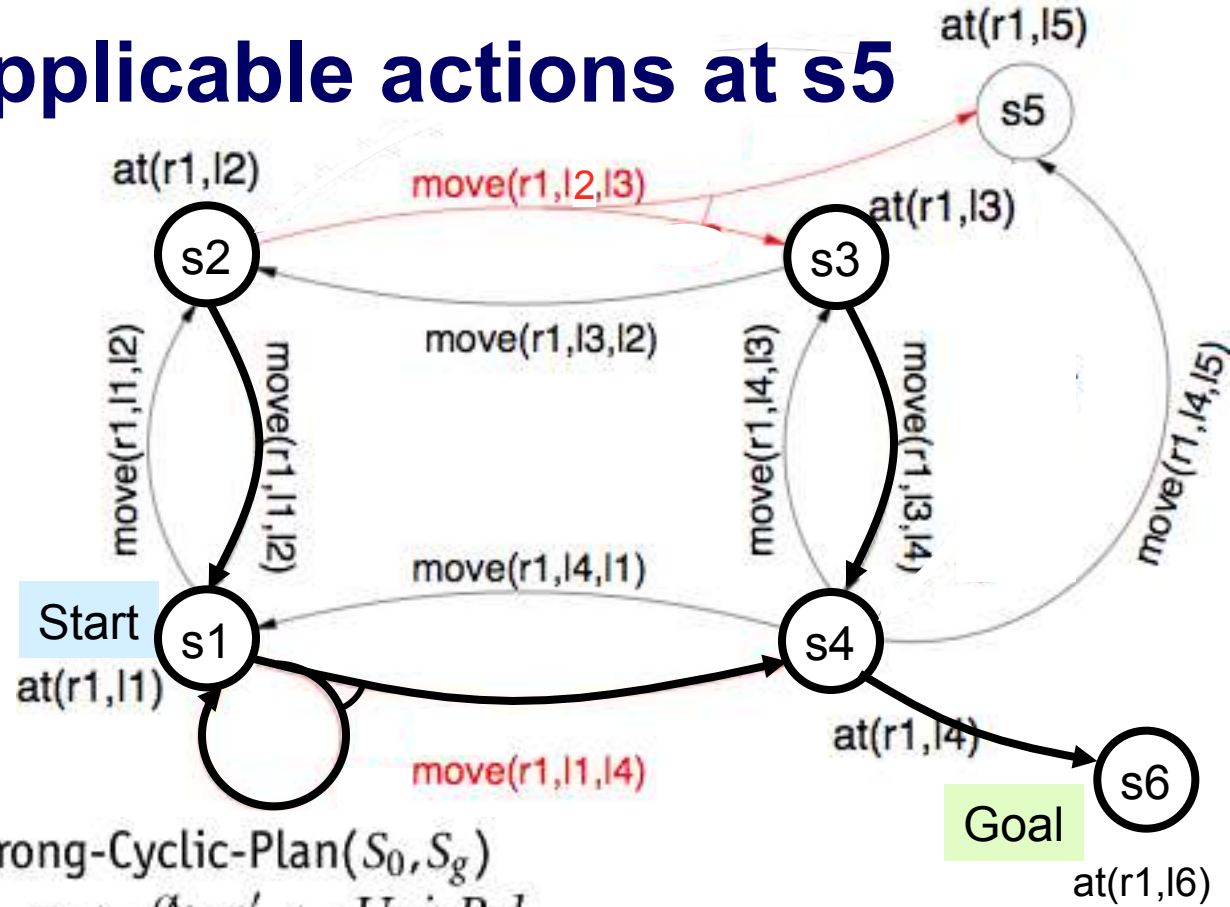
$PruneOutgoing(\pi', S_g) = \pi'$

$PruneUnconnected(\pi'', S_g) = \pi'$

so  $\pi = \pi'$

$RemoveNonProgress(\pi') =$   
as shown

$MkDet(shown) =$   
no change



Strong-Cyclic-Plan( $S_0, S_g$ )

$\pi \leftarrow \emptyset; \pi' \leftarrow UnivPol$

while  $\pi' \neq \pi$  do

$\pi \leftarrow \pi'$

$\pi' \leftarrow PruneUnconnected(PruneOutgoing(\pi', S_g), S_g)$

if  $S_0 \subseteq (S_g \cup S_{\pi'})$

then return( $MkDet(RemoveNonProgress(\pi', S_g))$ )

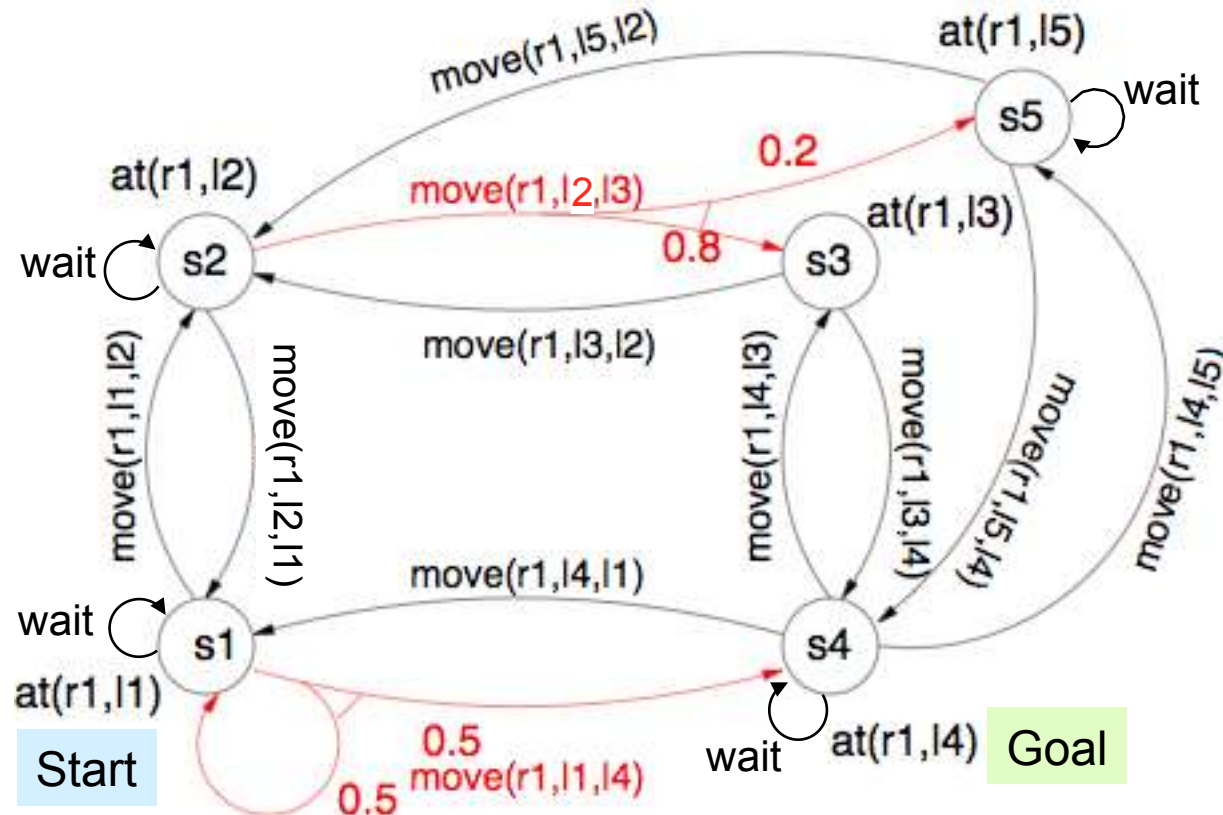
else return(failure)

end



# Planning for Extended Goals

- Here, “extended” means *temporally extended*
  - ◆ Constraints that apply to some sequence of states
- Examples:
  - ◆ want to move to l3, and then to l5
  - ◆ want to keep going back and forth between l3 and l5





# Planning for Extended Goals

- *Context*: the internal state of the controller
- *Plan*:  $(C, c_0, act, ctxt)$ 
  - ◆  $C$  = a set of execution contexts
  - ◆  $c_0$  is the initial context
  - ◆  $act: S \times C \rightarrow A$
  - ◆  $ctxt: S \times C \times S \rightarrow C$
- Sections 17.3 extends the ideas in Sections 17.1 and 17.2 to deal with extended goals
  - ◆ We'll skip the details