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978-0-521-85107-7 - Plasma Physics and Fusion Energy  
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## PLASMA PHYSICS AND FUSION ENERGY

There has been an increase in worldwide interest in fusion research over the last decade due to the recognition that a large number of new, environmentally attractive, sustainable energy sources will be needed during the next century to meet the ever increasing demand for electrical energy. This has led to an international agreement to build a large, \$4 billion, reactor-scale device known as the “International Thermonuclear Experimental Reactor” (ITER).

*Plasma Physics and Fusion Energy* is based on a series of lecture notes from graduate courses in plasma physics and fusion energy at MIT. It begins with an overview of world energy needs, current methods of energy generation, and the potential role that fusion may play in the future. It covers energy issues such as fusion power production, power balance, and the design of a simple fusion reactor before discussing the basic plasma physics issues facing the development of fusion power – macroscopic equilibrium and stability, transport, and heating.

This book will be of interest to graduate students and researchers in the field of applied physics and nuclear engineering. A large number of problems accumulated over two decades of teaching are included to aid understanding.

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CAMBRIDGE UNIVERSITY PRESS  
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo  
Cambridge University Press  
The Edinburgh Building, Cambridge CB2 8RU, UK  
Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9780521851077](http://www.cambridge.org/9780521851077)  
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First published 2007

Printed in the United Kingdom at the University Press, Cambridge  
*A catalog record for this publication is available from the British Library*

ISBN 13 978 0 521 85107 7 hardback

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For Karen

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## Preface

*Plasma Physics and Fusion Energy* is a textbook about plasma physics, although it is plasma physics with a mission – magnetic fusion energy. The goal is to provide a broad, yet rigorous, overview of the plasma physics necessary to achieve the half century dream of fusion energy.

The pedagogical approach taken here fits comfortably within an Applied Physics or Nuclear Science and Engineering Department. The choice of material, the order in which it is presented, and the fact that there is a coherent storyline that always keeps the energy end goal in sight is characteristic of such applied departments. Specifically, the book starts with the design of a simple fusion reactor based on nuclear physics principles, power balance, and some basic engineering constraints. A major point, not appreciated even by many in the field, is that virtually no plasma physics is required for the basic design. However, one of the crucial outputs of the design is a set of demands that must be satisfied by the plasma in order for magnetic fusion energy to be viable. Specifically, the design mandates certain values of the pressure, temperature, magnetic field, and the geometry of the plasma. This defines the plasma parameter regime at the outset. It is then the job of plasma physicists to discover ways to meet these objectives, which separate naturally into the problems of macroscopic equilibrium and stability, transport, and heating. The focus on fusion energy thereby motivates the structure of the entire book – how can we, the plasma physics community, discover ways to make the plasma perform to achieve the energy mission.

Why write such a book now? Fusion research has increased worldwide over the last several years because of the internationally recognized pressure to develop new reliable energy sources. With the recently signed agreement to build the next generation International Thermonuclear Experimental Reactor (ITER), I anticipate a substantial increase in interest on the part of new students and young scientists to join the fusion program. While fusion still has a long way to go before becoming a commercially viable source of energy, the advent of ITER enhances the already existing worldwide interest and excitement in plasma physics and fusion research. The incredibly challenging science and engineering problems coupled with the dream of an energy system characterized by unlimited fuel, near environmental perfection, and economical competitiveness are still big draws to new students and researchers.

Who is the intended audience? This textbook is aimed at seniors, first year graduate students, and new scientists joining the field. In general, the style of presentation includes in depth physical explanations aimed at developing physical intuition. It also includes many detailed derivations to clarify some of the mathematical mysteries of plasma physics. The book should thus be reasonably straightforward for newcomers to fusion to read in a stand alone fashion. There is also an extensive set of homework problems developed over two decades of teaching the subject at MIT.

With more explanations and detailed derivations something must give or else the book would become excessively long. The answer is to carefully select the material covered. In deciding how to choose which material to include and not to include, there are clearly tough decisions to be made. I have made these choices based on the idea of providing newcomers with a good first pass at understanding all the essential issues of magnetic fusion energy. Consequently, the material included is largely focused on the plasma physics mandated by fusion energy, which for a first pass is most easily described by macroscopic fluid models.

As to what is not included, there is very little discussion of fusion engineering. There is also very little discussion of plasma kinetic theory (e.g. the Vlasov equation and the Fokker–Planck equation). Somewhat surprisingly to me, it was not until the next-to-last chapter in the book that I first actually needed any of the detailed results of kinetic theory (i.e., the collisionless damping rates of RF heating and current drive), which I then derived using a simple, intuitive single-particle analysis. The point is that the first time through, the best way to develop an overall understanding of all the issues involved, with particular emphasis on self-consistent integration of the plasma physics, is to focus on macroscopic fluid models which are more easily tied to physical intuition and experimental reality. Ideally, a follow-on study based on kinetic theory would be the next logical step to master fusion plasma physics. In such a study, many of the topics described here would be analyzed at the more advanced level marking the present state of the art in fusion research.

As is clear from the length of the book, it would take a two semester course to cover the entire material in detail. However, a cohesive one semester course can also be easily constructed by picking and choosing from among the many topics covered. In terms of prerequisites, my assumption is that readers will have a solid foundation in undergraduate physics and mathematics. The specific requirements include: (1) mathematics up to partial differential equations, (2) mechanics, (3) basic fluid dynamics, and (4) electromagnetic theory (i.e., electrostatics, magnetostatics, and wave propagation). Experience has shown that an undergraduate degree in physics or most engineering disciplines provides satisfactory preparation.

In the end it is my hope that the book will help educate the next generation of fusion researchers, an important goal in view of the international decision to build ITER, the world's first reactor-scale, burning plasma experiment.

## Acknowledgements

The material for this book has evolved over many years of research and teaching. Many friends, colleagues, and students, too numerous to mention, have contributed in a significant way to my knowledge of the field, making this book possible. I acknowledge my deep appreciation for their collaboration, cooperation, and comraderie.

A number people at MIT also deserve special thanks. Bob Granetz, Ian Hutchinson, Ron Parker, and Abhay Ram have also all taught the subject upon which the book is based. I am grateful to them for sharing their notes and experiences with me.

Many colleagues at MIT have also been kind enough to read chapters of the book and provide me with me valuable feedback. I would like to thank Paul Bonoli, Leslie Bromberg, Peter Catto, Jan Egedal, Martin Greenwald, Jay Kesner, Jesus Ramos, and John Wright for their efforts. Other MIT colleagues gave generously of their time by means of intensive discussions. My appreciation to Darin Ernst, Joe Minervini, Kim Molvig, Miklos Porkolab, and Steve Scott.

A number of friends and colleagues from the general fusion community also read sections of the manuscript and provided me with valuable comments, particularly with respect to Chapter 13, which describes many present day fusion concepts. I would like to acknowledge help from Dan Barnes and Dick Siemon (the FRC), Riccardo Betti and Dale Meade (the tokamak and fusion reactors), Alan Boozer and Hutch Neilson (the stellarator), Bick Hooper (the spheromak), Martin Peng (the spherical tokamak), and John Sarff (the RFP).

Special thanks to my colleague Don Spong for producing the striking illustration appearing on the cover of the book.

As one might expect, preparing a manuscript is an ambitious task. I am extremely grateful to a cadre of MIT graduate students (many of them now full-time researchers) for their help in preparing the figures. My thanks to Joan Decker, Eric Edlund, Nathan Howard, Alex Ince-Cushman, Scott Mahar, and Vincent Tang. Special thanks to Vincent Tang who proof-read the entire manuscript for content and style. My assistant Liz Parmelee also provided invaluable administrative and organizational support during the entire preparation of the manuscript.

The team at Cambridge University Press has been a great help in publishing the manuscript, from the initial agreement to write the book to the final production. Thanks to

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*Acknowledgements*

Simon Capelin (publishing director), Lindsay Barnes (assistant editor), Dan Dunlavey (production editor), Emma Pearce (production editor) and Maureen Storey (copy editor).

Last, but most certainly not least, I would like to thank my wife Karen for her unending support and encouragement while I prepared the manuscript. She was also kind enough to proofread a large fraction of the text for which I am most grateful.

## Units

Throughout the textbook standard MKS units are used. The one exception is the temperature. It is now common practice in the field of fusion plasma physics to absorb Boltzmann's constant  $k$  into the temperature so that the combination  $kT$  always appears as  $T$ ; that is,  $kT \rightarrow T$ , where  $T$  has the units of energy (joules).

There are also a number of relationships expressed in "practical" units, which unless otherwise specified, are given by

Number density	$n$	$10^{20} \text{ m}^{-3}$
Temperature	$T$	keV
Pressure	$p$	atmospheres
Magnetic field	$B$	tesla
Current	$I$	megamperes
Minor radius	$a$	m
Major radius	$R$	m
Confinement time	$\tau_E$	s