Plastic Analysis and Design of Steel Plate Shear Walls

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Abstract: A revised procedure for the design of steel plate shear walls is proposed. In this procedure the thickness of the infill plate is found using equations that are derived from the plastic analysis of the strip model, which is an accepted model for the representation of steel plate shear walls. Comparisons of experimentally obtained ultimate strengths of steel plate shear walls and those predicted by plastic analysis are given and reasonable agreement is observed. Fundamental plastic collapse mechanisms for several, more complex, wall configurations are also given. Additionally, an existing codified procedure for the design of steel plate walls is reviewed and a section of this procedure which could lead to designs with less-than-expected ultimate strength is identified. It is shown that the proposed procedure eliminates this possibility without changing the other valid sections of the current procedure.

DOI: 10.1061/(ASCE)0733-9445(2003)129:11(1448)

CE Database subject headings: Plastic analysis; Design; Shear walls; Plates; Steel.

Introduction

Steel plate shear walls (SPSW) have sometimes been used as the lateral load resisting system in buildings. Until the 1980s, the design limit state for SPSW in North America was out-of-plane buckling of the infill plates. This led engineers to design heavily stiffened plates that offered little economic advantage over reinforced concrete shear walls. However, as Basler (1961) demonstrated for plate girder webs, the post-buckling tension field action of steel plate shear walls can provide substantial strength, stiffness, and ductility. The idea of utilizing the postbuckling strength of steel plate shear walls was first formulated by Thorburn et al. (1983) and verified experimentally by Timler and Kulak (1983). Studies performed to evaluate the strength, ductility, and hysteretic behavior of such SPSW designed with unstiffened infill plates demonstrated their significant energy dissipation capabilities (Timler and Kulak 1983) and substantial economic advantages (Timler 1998).

Additional research on unstiffened steel plate shear walls has investigated the effect of simple versus rigid beam-to-column connections on the overall behavior (Caccese et al. 1993), the dynamic response of steel plate shear walls (Sabouri-Ghomi and Roberts 1992; Rezia 1999), the behavior of light-gauge steel plate shear walls (Berman and Bruneau 2003), the effects of holes in the infill plates (Roberts and Sabouri-Ghomi 1992), and the effects of bolted versus welded infill connections, as well as other practical considerations, by Elgaaly (1998). Furthermore, finite element modeling of unstiffened steel plate shear walls has been investigated in some of the aforementioned papers as well as by Elgaaly et al. (1993) and Driver et al. (1997).

Recent research on the postbuckling strength of plate girder webs using finite element analysis (Marsh et al. 1988; Lee and Yoo 1998; Roberts and Shahabian 2001) also provided insight into the postbuckling behavior and bending-shear interaction of steel plate shear walls having web width-to-thickness ratios comparable to typical plate girders (i.e., 350 or less). However, for the type of SPSWs considered in this paper (for example, with a bay width of 4 m, infill thickness of 5 mm, and resulting width-to-thickness ratio of 800), important differences with plate girder behavior are known to exist (Thorburn et al. 1983). This is not only due to the large width-to-thickness ratios, but also because the ‘‘flanges’’ of steel plate shear walls are columns, typically wide flange shapes, with significantly more bending stiffness than the typical flanges of a plate girder.

At the time of this writing, there are no U.S. specifications or codes addressing the design of steel plate shear walls. The 2001 Canadian standard, CAN/CSA S16-01 (CSA 2001), now incorporates mandatory clauses on the design of steel plate shear walls; these are reviewed briefly in the next section. One of the models recommended to represent steel plate shear walls, which was developed by Thorburn et al. (1983) and named the strip model, is recognized for providing reliable assessments of their ultimate strength. In this paper, using this strip model as a basis, the use of plastic analysis as an alternative for the design of steel plate shear walls is investigated. Fundamental plastic collapse mechanisms are described for single story and multistory SPSW with either simple or rigid beam-to-column connections. Ultimate strengths predicted from these collapse mechanisms are compared with experimental results by others, and used to assess the CAN/CSA S16-01 design procedure.

Analysis and Design of Steel Plate Shear Walls—CAN/CSA S16-01

The CAN/CSA S16-01 seismic design process for steel plate shear walls follows the selection of a lateral load resisting system (i.e., shear walls with rigid or flexible beam-to-column connections), calculation of the appropriate design base shear, and dis-

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Note. Associate Editor: Sherif El-Tawil. Discussion open until April 1, 2004. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on March 18, 2002; approved on February 5, 2003. This paper is part of the ***Journal of Structural Engineering***, Vol. 129, No. 11, November 1, 2003. ©ASCE, ISSN 0733-9445/2003/11-1448–1456/$18.00.

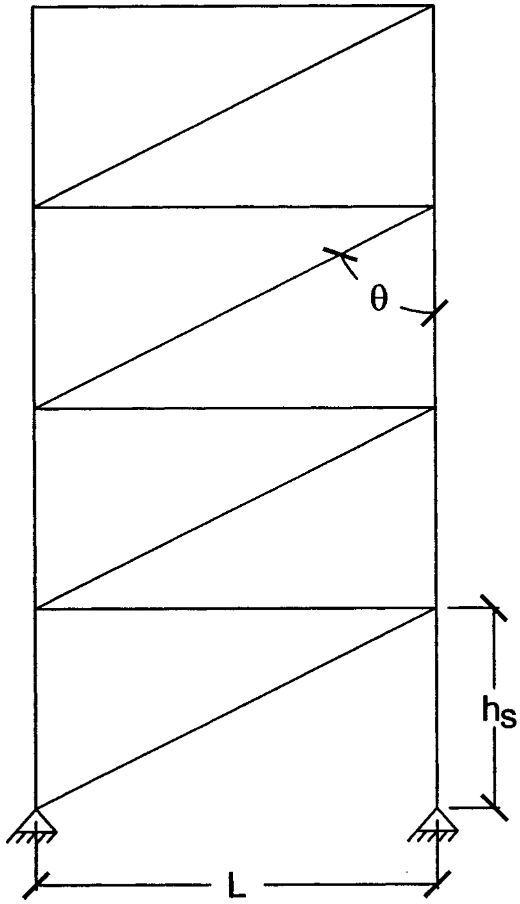

Fig. 1. Equivalent story brace model

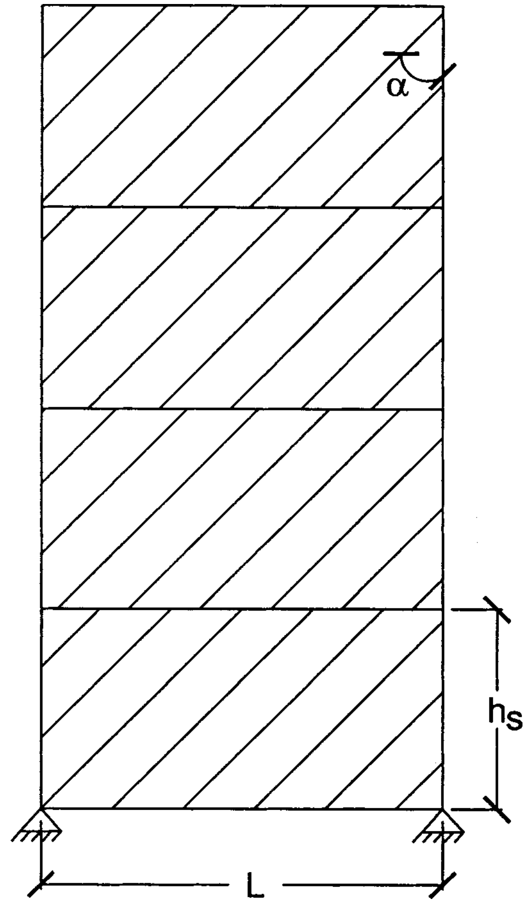

Fig. 2. Detailed strip model

tribution of that base shear along the building height by the usual methods described in building codes. Preliminary sizing of members is done using a model that treats the plate at each story as a single pin-ended brace (known as the equivalent story brace model) that runs along the diagonal of the bay (Fig. 1). From the area of the story brace, A, determined from that analysis, an equivalent plate thickness can be calculated using the following equation based on an elastic strain energy formulation (Thorburn et al. 1983):

$$t=\frac{2A\sin\theta\sin 2\theta}{L\sin^2 2\alpha} \tag{1}$$

where θ = angle between the vertical axis and the equivalent diagonal brace; L = bay width; and α = angle of inclination of the principal tensile stresses in the infill plate measured from vertical, which is given by

$$\tan^4\alpha=\frac{1+\dfrac{tL}{2A_c}}{1+th_s\left(\dfrac{1}{A_b}+\dfrac{h_s^3}{360I_cL}\right)} \tag{2}$$

where t = thickness of the plate; A_c and I_c = respectively, the cross-sectional area and moment of inertia of the bounding column; h_s = story height; and A_b = beam cross-sectional area (Timler and Kulak 1983). CAN/CSA S16-01 also provides the following equation to ensure that a satisfactory minimum moment of inertia is used for columns in steel plate shear walls to prevent excessive deformation leading to premature buckling under the pulling action of the plates (derived from Kuhn et al. 1952)

$$I_c\geqslant\frac{0.00307th_s^4}{L} \tag{3}$$

Once the above requirements have been satisfied, a more refined model, known as the strip or multistrip model, that represents the plates as a series of inclined tension members or strips (Fig. 2) is required for the analysis of steel plate shear walls [with α as calculated by Eq. (2)]. Through comparison with experimental results, the adequacy of the strip model to predict the ultimate capacity of SPSW has been verified in several studies. Fig. 3, adapted from Driver et al. (1997), is one example of this verification.

A minimum of ten strips is required at each story to adequately model the wall. Each strip is assigned an area equal to the plate thickness times the tributary width of the strip. Drifts obtained from the elastic analysis of the multistrip model are then amplified by factors prescribed by the applicable building code to account for inelastic action and then checked against allowable drift limits. For SPSW having rigid beam-to-column connections, CAN/CSA S16-01 also requires that a capacity design be conducted to prevent damage to the bounding columns of the wall. Due to practical considerations, infill thicknesses may be larger than necessary to resist the seismic loads, therefore, capacity design is required to insure a ductile failure mode (i.e., infill yielding prior to column buckling). To achieve this, the moments and axial forces (obtained from an elastic analysis) in these columns are magnified by a factor B, defined as the ratio of the probable

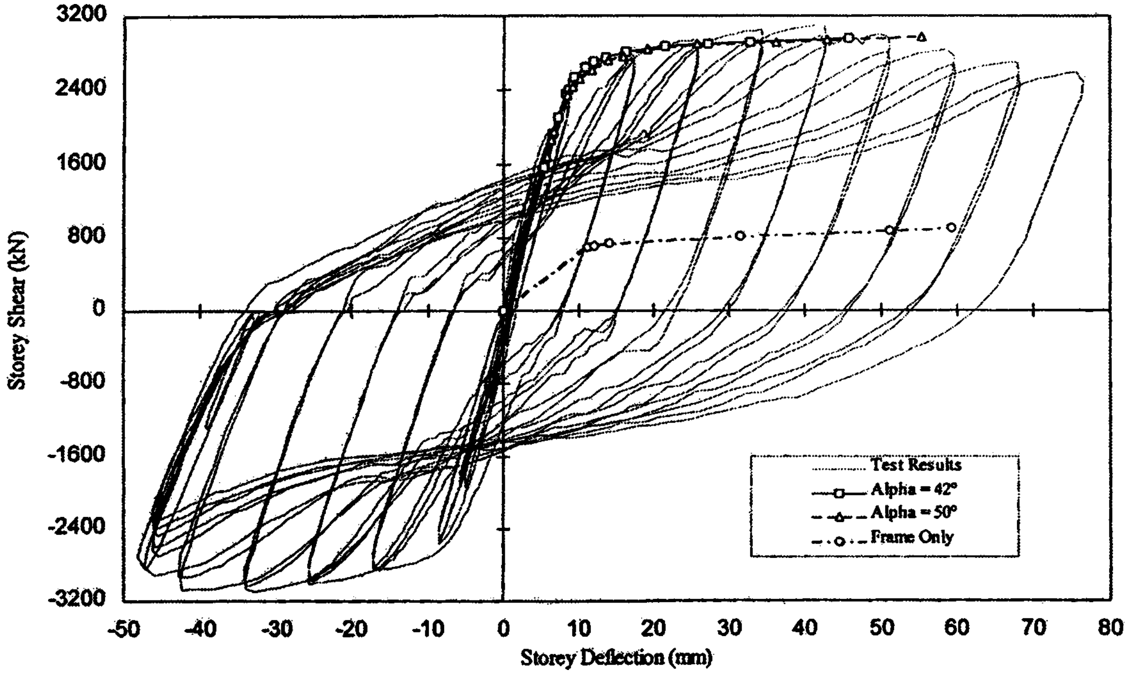

Fig. 3. Comparison between experimental results and strip model (Adapted from Driver et al. 1997)

shear resistance at the base of the wall for the supplied plate thickness, to the factored lateral force at the base of the wall obtained from the calculated seismic load. The probable resistance of the wall (V_{re}) is given by

$$V_{re} = 0.5 R_y F_y t L \sin 2\alpha \quad (4)$$

where R_y = ratio of the expected (mean) steel yield stress to the design yield stress (specified as 1.1 for A572 Gr. 50 steel); F_y = design yield stress of the plate; and all other parameters have been defined previously. Note that B need not be greater than the ratio of the ultimate elastic base shear to the yield base shear, which is the ductility factor R_μ specified as 5.0 by CAN/CSA S16-01. Column axial loads are found from the overturning moment BM_f, where M_f is the factored overturning moment at the bottom of the wall. Local column moments from tension field action of the plates, as determined from the elastic analysis, are also amplified by B. If a nonlinear pushover analysis is carried out, these corrections need not be done and more accurate values for the column axial forces and moments can be obtained. Since pushover capabilities are becoming more common in structural analysis software, this is also a viable option.

Plastic Analysis of Steel Plate Shear Walls—Single Story Frames

In this section, plastic analysis of the strip model is used to develop equations for the ultimate capacity of different types of steel plate shear walls. In cases where general equations depend on actual member sizes and strengths, procedures are presented to determine the necessary equations. In the following section the results of these analyses are used to develop a simple, consistent method for determining the preliminary plate sizes for steel plate shear walls.

Equilibrium Method—Simple Beam-to-Column Connections

First the ultimate strength of a single story steel plate shear wall having simple beam-to-column connections is found using the equilibrium method of plastic analysis. Consider Fig. 4, which shows a single story SPSW in a frame with pin-ended beams. The model is divided into three zones. Zones 1 and 3 contain strips which run from a column to a beam while Zone 2 contains any strips that connect from the top beam to the bottom beam. Note that the strip spacing in the direction perpendicular to the strip is s, with the first and last (upper left and lower right) strips located at $s/2$ from the closest beam-to-column connections. The distance d_i from the beam-to-column connection to each strip is measured (again, perpendicularly to the strip) from the upper-left beam-to-column connection in Zones 1 and 2, and in Zone 3 from the lower right connection.

From Figs. 5(a and b) the applied story shear for Zone 1 strips can be found in terms of the strip force F_{st}, the distance d_i, and the story height h_s, in two steps. First, summation of moments about Point C [bottom of the left column in Fig. 5(a)] gives, $V_1 h_s = R_{by} L$ (where L is the bay width and R_{by} is the vertical

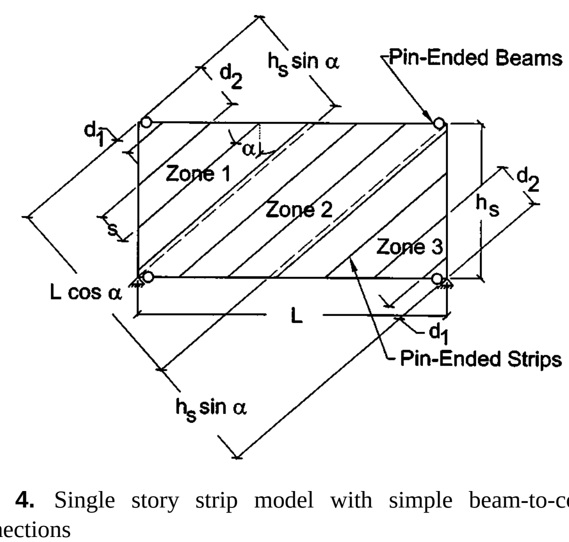

Fig. 4. Single story strip model with simple beam-to-column connections

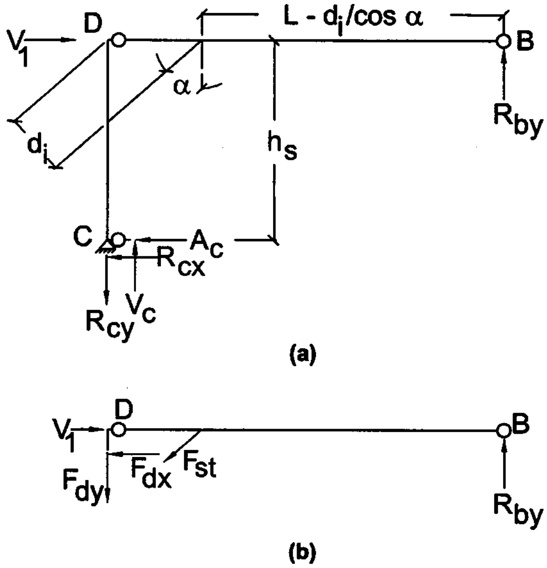

Fig. 5. Zone 1 free body diagram

support reaction at B). Then looking at the free body diagram of the beam alone [Fig. 5(b)] and taking moments about Point D, one obtains $R_{\text{by}}L = F_{\text{st}}d_i$. Therefore, the story shear due to the forces in Zone 1 strips can be written as

$$V_1 = \sum_{i=1}^{l} \frac{F_{\text{st}}d_i}{h_s} \tag{5}$$

where l = number of strips in Zone 1.

For Zone 2, by taking moments about Point C again [Fig. 6(a)], one obtains $V_2h_s = R_{\text{by}}L + F_{\text{st}}(\sin\alpha)h_s - F_{\text{st}}d_j$. From the free body diagram of the beam [Fig. 6(b)] and taking moments about Point D, one finds $R_{\text{by}}L = F_{\text{st}}d_j$. The story shear due to the forces in Zone 2 strips is therefore

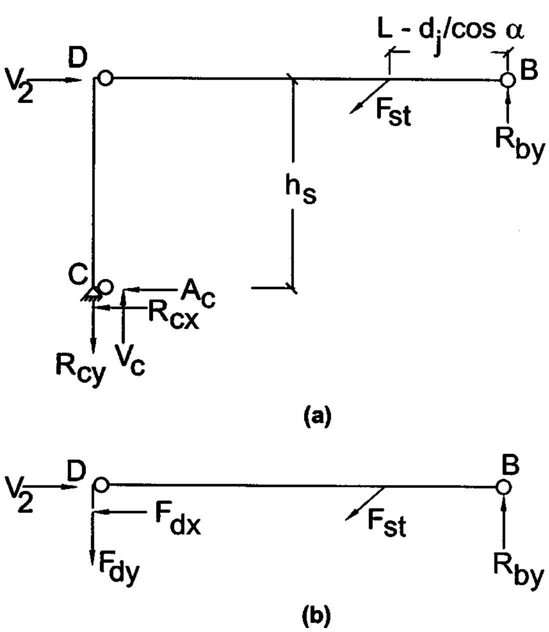

Fig. 6. Zone 2 free body diagram

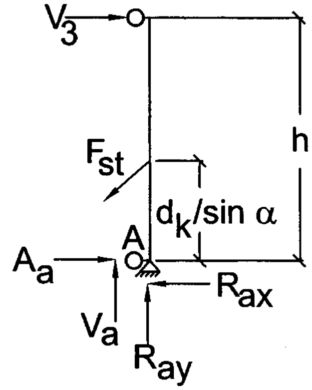

Fig. 7. Zone 3 free body diagram

$$V_2 = \sum_{j=1}^{m} F_{\text{st}} \sin\alpha \tag{6}$$

where m = number of strips in Zone 2.

Finally, taking moments about Point A in Fig. 7 one finds that the contribution to the story shear from Zone 3 strips can be written as

$$V_3 = \sum_{k=1}^{n} \frac{d_kF_{\text{st}}}{h_s} \tag{7}$$

where n = number of strips in Zone 3.

Now if it is assumed that there are n strips in each of Zones 1 and 3 (due to equal strip spacing) the contributions from each zone can be combined to give

$$V = \sum_{i=1}^{l} \frac{d_iF_{\text{st}}}{h_s} + \sum_{j=1}^{m} F_{\text{st}} \sin\alpha + \sum_{i=1}^{n} \frac{d_iF_{\text{st}}}{h_s} \tag{8}$$

From the geometry in Fig. 4 it can be shown that

$$l = n = \frac{h \sin\alpha}{s} \quad \text{and} \quad m = \frac{L \cos\alpha - h \sin\alpha}{s} \tag{9}$$

Keeping in mind that the upper left and lower right strips are $s/2$ from the closest beam-to-column connections, the summation over d_i can be written as

$$\sum_{i=1}^{n} d_i = \frac{n^2 s}{2} \tag{10}$$

Substituting Eqs. (9) and (10) into Eq. (8), recognizing the fact that the strip force (F_{st}) is equal to the strip area (st) times the plate yield stress in tension (F_y) and using the trigonometric identity $[(1/2)\sin 2\alpha = \cos\alpha \sin\alpha]$ the story shear strength can be expressed as

$$V = \frac{1}{2} F_y tL \sin 2\alpha \tag{11}$$

Note that this equation is identical to the one used to calculate the probable shear resistance of a SPSW in the CAN/CSA S16-01 procedure [Eq. (4)], without the material factor R_y.

Kinematic Method—Simple Beam-to-Column Connections

The same result can be obtained more directly using the kinematic method of plastic analysis. Consider the same frame with

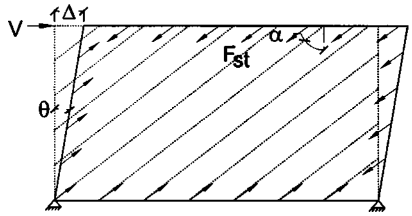

Fig. 8. Single story kinematic collapse mechanism

inclined strips shown in Fig. 4. When the shear force V displaces the top beam by a value Δ sufficient to yield all the strips, the external work done is equal to $V\Delta$ (see Fig. 8). If the beams and columns are assumed to remain elastic, their contribution to the internal work may be neglected when compared to the internal work done by the strips, hence, the internal work is $(n_b A_{st} F_y \sin\alpha)\Delta$, where n_b is the number of strips anchored to the top beam. This result can be obtained by the product of the yield force times the yield displacement of the strips, but for simplicity it can also be found using the horizontal and vertical components of these values. Note that the horizontal components of the yield forces of the strips on the columns cancel (the forces on the left column do negative internal work and the forces on the right column do positive internal work) and the vertical components of all the yield forces do no internal work because there is no vertical deflection. Therefore, the only internal work done is by the horizontal components of the strip yield forces anchored to the top beam. Equating the external and internal work gives

$$V = n_b F_{\text{st}} \sin\alpha \tag{12}$$

Using the geometry shown in Fig. 4, $n_b = (L\cos\alpha)/s$ and the strip force F_{st} is again $F_y ts$. Substituting these into Eq. (12) and knowing $(1/2)\sin 2\alpha = \cos\alpha \sin\alpha$, the resulting base shear relationship is again

$$V = \frac{1}{2} F_y t L \sin 2\alpha \tag{13}$$

Kinematic Method—Rigid Beam-to-Column Connections

In single story steel plate shear walls having rigid beam-to-column connections (as opposed to simple connections), plastic hinges also need to form in the boundary frame to produce a collapse mechanism. The corresponding additional internal work is $4M_p\ \theta$, where $\theta = \Delta/h_s$, is the story displacement over the story height, and M_p is the smaller of the plastic moment capacity of the beams M_{pb}, or columns M_{pc} (for most single-story frames that are wider than tall, if the beams have sufficient strength and stiffness to anchor the tension field, plastic hinges will typically form at the top and bottom of the columns and not in the beams). The ultimate strength of a single-story steel plate shear wall in a moment frame with plastic hinges in the columns becomes

$$V = \frac{1}{2} F_y t L \sin 2\alpha + \frac{4M_{pc}}{h_s} \tag{14}$$

In a design process, failure to account for the additional strength provided by the beams or columns results in a larger plate thicknesses than necessary, this would translate into lower ductility demands in the walls and frame members, and could therefore be considered to be a conservative approach. However, capacity design of the beams and columns must still be performed to insure that a ductile failure mode will be achieved (i.e., plate yielding prior to columns or beams developing plastic hinges).

Plastic Analysis of Steel Plate Shear Walls—Multistory Frames

For multistory SPSWs with pin-ended beams, plastic analysis can also be used to predict the ultimate capacity. The purpose here is not to present closed-form solutions for all possible failure mechanisms, but to identify some key plastic mechanisms that should be considered in estimating the ultimate capacity of a steel plate shear wall. These could be used to define a desirable failure mode in a capacity design perspective, or to prevent an undesirable failure mode, as well as complement traditional design approaches.

In soft-story plastic mechanisms [Fig. 9(a)], the plastic hinges that would form in the columns at the mechanism level could be included in the plastic analysis. Calculating and equating the internal and external work, the following general expression could be used for soft-story i in which all flexural hinges develop in columns:

$$\sum_{j=i}^{n_s} V_j = \frac{1}{2} F_y t_i L \sin 2\alpha + \frac{4M_{\text{pci}}}{h_{si}} \tag{15}$$

where V_j = applied lateral forces above the soft-story i; t_i = plate thickness at the soft-story; M_{pci} = plastic moment capacity of the columns at the soft-story; h_{si} = height of the soft-story; and n_s = total number of stories. Note that only the applied lateral forces above the soft-story do external work and they all move the same distance (Δ). The internal work is done only by the strips on the soft-story itself and by column hinges forming at the top and bottom of the soft-story. Using the above equation, the possibility of a soft-story mechanism should be checked at every story in which there is a significant change in plate thickness or column size. Additionally, the soft-story mechanism is independent of the beam connection type (simple or rigid) because hinges must form in the columns, not the beams.

A second (and more desirable) possible collapse mechanism involves uniform yielding of the plates over every story [Fig. 9(b)]. For this mechanism, each applied lateral force V_i moves a distance $\Delta_i = \theta h_i$, and does external work equal to $V_i \theta h_i$, where h_i is the elevation of the ith story. The internal work is done by the strips of each story yielding. It is important to note that the strip forces acting on the bottom of a story beam do positive internal work and the strip forces acting on top of the same beam do negative internal work. Therefore, the internal work at any story i is equal to the work done by strip yield forces along the bottom of the story beam minus the work done by strip yield forces on the top of the same beam. This indicates that in order for every plate at every story to contribute to the internal work, the plate thicknesses would have to vary at each story in direct proportion to the demands from the applied lateral forces. Even with this in mind, this mechanism provides insight into the capacity and failure mechanism of the wall. The general equation for the ultimate strength of a multistory SPSW with simple beam-to-column connections and this plastic mechanism (equating the internal and external work) is

$$\sum_{i=1}^{n_s} V_i h_i = \sum_{i=1}^{n_s} \frac{1}{2} F_y (t_i - t_{i+1}) L h \sin 2\alpha \tag{16}$$

where h_i = ith story elevation; n_s = total number of stories; and t_i = thickness of the plate on the ith story.

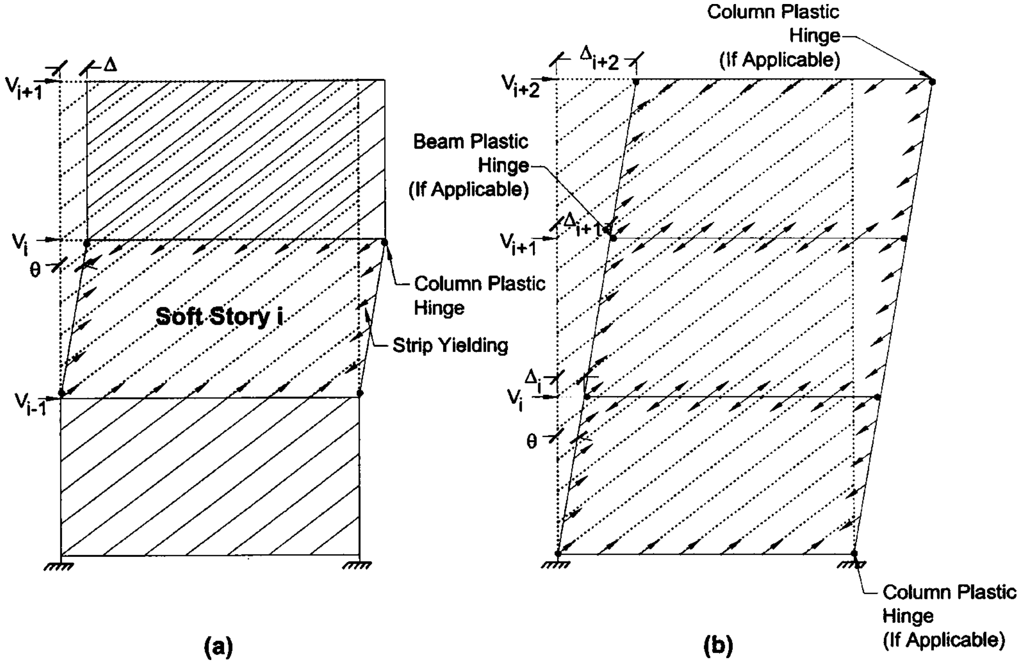

Fig. 9. Examples of collapse mechanisms for multistory steel plate shear walls

The ultimate strength of SPSW having rigid beam-to-column connections capable of developing the beam's plastic moment, can also be calculated following the same kinematic approach. The resulting general equation for the uniform yielding mechanism can be written as

$$\sum_{i=1}^{n_s} V_i h_i = 2M_{\mathrm{pc1}} + 2M_{\mathrm{pc}n} + \sum_{i=1}^{n_s-1} M_{\mathrm{pb}i} + \sum_{i=1}^{n_s} \frac{1}{2} F_y L h_i (t_i - t_{i+1}) \sin 2\alpha \qquad (17)$$

where M_{pc1} = first story column plastic moment; $M_{\mathrm{pc}n}$ = top story column plastic moment; $M_{\mathrm{pb}i}$ = plastic moment of the ith story beam, and the rest of the terms were previously defined. Note that it is assumed that column hinges will form instead of beam hinges at the roof and base levels. Sizable beams are usually required at these two locations to anchor the tension field forces from the wall plate and hence plastic hinges typically develop in columns there. However, this may not be the case for certain wall aspect ratios, and the engineer is cautioned to use judgement, as M_{pc1} and $M_{\mathrm{pc}n}$ may have to be replaced by M_{pb1} and $M_{\mathrm{pb}n}$ in some instances, where M_{pb1} and $M_{\mathrm{pb}n}$ are the plastic moment capacities of the base and roof beams, respectively. Furthermore, note that hinges were assumed to develop in beams at all other levels, which is usually the case as small beams are required there in well proportioned SPSW.

After examining the results of several different pushover analyses for such multistory SPSW (using a single three-story frame geometry, with arbitrarily selected beams, columns, and plate thicknesses), it has been observed that the actual failure mechanism is typically somewhere between a soft-story mechanism and uniform yielding of the plates on all stories. Finding the actual failure mechanism is difficult by hand, therefore, a computerized pushover analysis should be used. However, the mechanisms described above will provide a rough estimate of the ultimate capacity. They will also provide some insight as to whether a soft story is likely to develop (by comparing the ultimate capacity found from the soft story mechanism with that of the uniform yielding mechanism).

Comparison with Experimental Results

To validate ultimate strengths predicted by Eqs. (13) and/or (14) for the plastic analysis of single story frames with either simple or rigid beam-to-column connections, a comparison is made with results obtained experimentally by others (Table 1). The experimental results given for multistory specimens are either those for the first story shear (in the case of Driver et al. 1997) or they are the total base shear in cases where loading was applied to the top story only (Caccese et al. 1993; Elgaaly 1998). Furthermore, no results are given for tests on SPSWs that had openings. As shown in Table 1, on average Eq. (13) predicts an ultimate load capacity for steel plate shear walls with true pin or semirigid beam-to-column connections that is 5.9% below the experimentally obtained values. Eq. (14) gives predictions for ultimate capacity of steel plate shear walls with rigid beam-to-column connections that are on average 17% above the experimentally obtained values, however, that equation assumes a fully developed frame mechanism which was not reported in any of the cited tests. Note that Cases 6 and 10, included in Table 1 for completeness, were not included in the averages because their ultimate failure was due to column instability or problems with the test setup. Hence, the equations derived from plastic analysis of the strip model are generally conservative for calculating the expected ultimate strength of steel plate shear walls.

Impact of Design Procedure on Ultimate Strength of Steel Plate Shear Walls

CAN/CSA S16-01 Approach

The procedure given for preliminary sizing of plates in CAN/CSA S16-01 is simple but results in designs that may not be consistent

Table 1. Comparison of Experimental and Calculated Ultimate Strengths

Case	Researcher	Specimen identification	Number of stories	h (mm)	L (mm)	t (mm)	F_y (Mpa)	α (°)	V_{uexp} (kN)	V_{upred} (kN) Eq. (13)	V_{upred} (kN) Eq. (14)	% Error for Eq. (13)
				(i) Simple (physical pin) beam-to-column connections								
1	Timler and Kulak (1983)	—[c]	1	2,500	3,750	5	270.8	42.7	2698	2,530.9	—[c]	−6.2
2	Roberts and Sabouri-Ghomi	SW2	1	370	370	0.83	219	45.0	35.1	33.6	—[c]	−4.2
3	(1992)	SW3	1	370	370	1.23	152	45.0	38.2	34.6	—[c]	−9.5
4		SW14	1	370	450	0.83	219	45.0	44.5	40.9	—[c]	−8.1
5		SW15	1	370	450	1.23	152	45.0	45.3	42.1	—[c]	−7.1
				(ii) Semirigid beam-to-column connections (web-angle or other)								
6	Elgaaly (1998)	SWT11[b]	2	1,118	1,380	2.28	239	41.5	370	373.1	—[c]	0.85
7		SWT15	2	1,118	1,380	2.28	239	41.3	426	372.9	—[c]	−12
8	Caccese et al. (1993)	S22	3	838	1,244	0.76	256	42.2	142	120.4	—[c]	−15
9		S14	3	838	1,244	1.9	332	40.2	356	386.8	—[c]	8.64
				(iii) Rigid beam-to-column connections								
10	Lubell et al. (2000)	SPSW1[a]	1	900	900	1.5	320	36.9	210	207.3	261.4	—[c]
11		SPSW2	1	900	900	1.5	320	36.9	260	207.3	261.4	—[c]
12	Driver et al. (1997)	—[c]	4	1,927	3,050	4.8	355.4	41.1	3080	2,577.7	3886.4	—[c]

[a]Testing stopped due to failure of lateral bracing.
[b]Testing stopped due column buckling.
[c]Not applicable.

with the demands implicit in the seismic force modification factor *R*. The transition from the equivalent story brace model (used for preliminary proportioning and to select the amount of steel in the infill plates) to the multistrip model (used for final analysis) may change the ultimate capacity and shape of the pushover curve for the structure being designed, while the *R*-factor is not revised.

In the equivalent story brace model, the ultimate capacity of the wall is only a function of the brace area, yield stress, and the bay geometry (aspect ratio). The story shear can be used to size the equivalent brace for each story by using simple statics (recall Fig. 1). Then Eq. (1) can be employed to relate the brace area to the plate thickness which, along with the strip spacing, yields the strip area for the detailed strip model. However, these two models will not produce the same ultimate capacity unless the aspect ratio of the bay is 1:1. To demonstrate this consider a single story SPSW (with simple beam-to-column connections) as shown in Fig. 4. Let the aspect ratio of the bay be equal to the bay width over the story height. Using the same design base shear, beam, and column sizes (selected to remain elastic for all cases), the area of the equivalent story braces were found for several aspect ratios. From these, the plate thicknesses were found as described above and the detailed strip models were developed using Eq. (2) to find the angle of inclination for the strips.

Pushover analyses of all resulting SPSW were then conducted and the resulting ultimate strengths of the various walls, designed to resist the same applied lateral loads, were compared. Fig. 10 shows a plot of the base shear (normalized by dividing out the design base shear used to find the area of the equivalent story brace) versus percent story drift for several SPSW of different aspect ratios, obtained from pushover analyses of the strip models and equivalent story brace models. The resulting ultimate capacity of the strip model is below the capacity of the equivalent story brace model for all aspect ratios, except 1:1 for which it is the same. The difference between the capacity of the strip model and equivalent story brace model increases as the aspect ratio further deviates from 1.0. Fig. 11 shows how the difference between the strip model capacity and the equivalent story brace model capacity changes with the aspect ratio of the bay. At an aspect ratio of 2:1 (or 1:2 since the results are symmetric in that sense) the strip model is only able to carry 80% of the base shear for which it should have been designed.

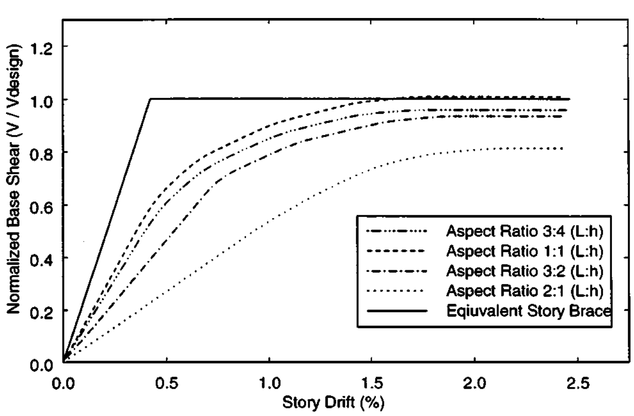

Fig. 10. Pushover curves for different aspect ratios

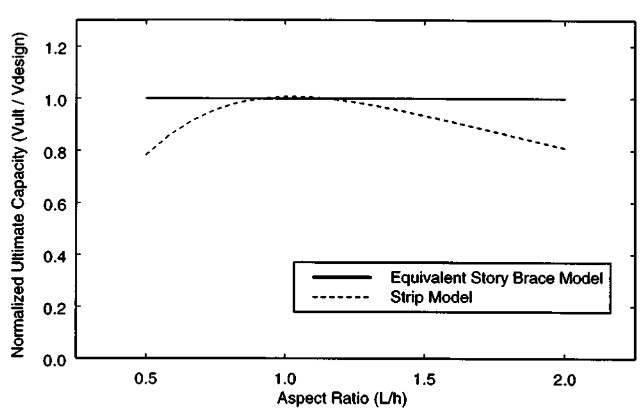

Fig. 11. Variation of ultimate capacity with aspect ratio

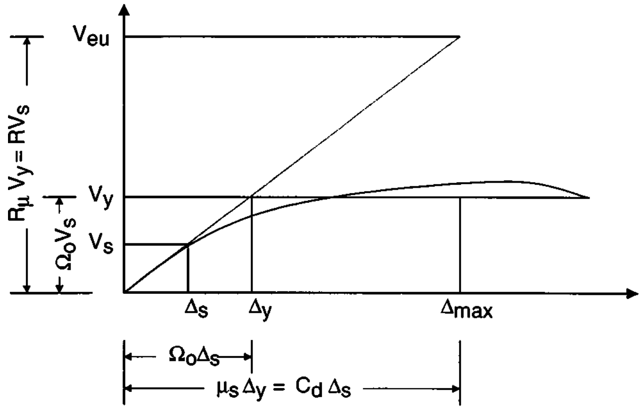

Fig. 12. Generic pushover curve

Plastic Analysis

Using the results of the plastic analyses described previously, the infill plates of steel plate shear walls can be sized to consistently achieve the desired ultimate strength. The procedure is simple, even for a multistory SPSW, and neglecting the contribution of plastic hinges in beams and columns will always give a conservative design in the case of rigid beam-to-column connections. The proposed procedure requires the designer to

1. Calculate the design base shear, and distribute it along the height of the building as described by the applicable building code;
2. Use the following equation to calculate the minimum plate thicknesses required for each story:

$$t = \frac{2V_s\Omega_s}{F_y L \sin 2\alpha} \tag{18}$$

   where Ω_s = system overstrength described below and V_s = design story shear found using the equivalent lateral force method;
3. Develop the strip model for computer (elastic) analysis using Eq. (2) to calculate the angle of inclination of the strips;
4. Design beams and columns according to capacity design principles (to insure the utmost ductility) or other rational methods using plate thicknesses specified (in case those exceed the minimum required for practical reasons); and
5. Check story drifts against allowable values from the applicable building code.

Note that Eq. (18) is identical to Eq. (13) but modified to account for the proper relationship between the equivalent lateral force procedure and R, the seismic force modification factor. Fig. 12 is the generic pushover curve used to define R, where R_μ is the ductility factor, V_y is (for practical purposes) the fully yielded base shear, Ω_o is the overstrength factor, V_s is the design base shear, Δ_s is the displacement at the design base shear, Δ_y is the displacement at the yield base shear, V_{eu} is the ultimate elastic base shear, $\Delta_{\max}$ is the displacement at the ultimate elastic base shear, μ_s is the displacement ductility factor, and C_d is the elastic displacement amplification factor. Note the distinction between V_y (the yield base shear) and V_s (the design base shear). Because Eq. (13) was obtained from plastic analysis of the strip model, it gives the maximum strength achieved at the peak of the pushover curve, which is V_y. Therefore, to account for this and to use the calculated design base shear V_s from the equivalent lateral force procedure with Eq. (13) to size the infill plates of a steel plate shear wall, V_s must be amplified by the overstrength factor. In this particular case the majority of the overstrength factor comes from the system overstrength factor.

Using the concepts presented in FEMA 369 (FEMA 2001), the R factor can be expressed as

$$R = R_\mu\Omega_o = R_\mu\Omega_D\Omega_M\Omega_S \tag{19}$$

where Ω_D = design overstrength factor; Ω_M = material factor; and Ω_S = system overstrength factor. Although the quantification of Ω_D, Ω_M, and Ω_S, are to be determined by panels of experts in code committees, the following observations are submitted for consideration. According to FEMA 369 (FEMA 2001) these overstrength factors may be thought of as follows:

- Ω_D is to account for overstrength resulting from the design procedure. This would occur in drift controlled designs and designs in which architectural considerations may result in overstrength. For most low to medium rise SPSW, this is unlikely to be an issue and Ω_D could be considered as low as 1.0.
- Ω_M accounts for overstrength due to strength reduction factors used in load and resistance factor design, strain hardening, and the ratio of mean to specified yield stress. In this particular case R_y already accounts for the ratio of mean to specified yield stress in the capacity design approach and there is no strength reduction factor involved in the sizing of the infill plates. Furthermore, simple calculations would show that strain hardening would begin to develop at drifts of approximately $0.02h$, the typical drift limit, assuming a strain of 0.01 before strain hardening. Therefore, Ω_M may be taken as low as 1.0 as well.
- Ω_S accounts for the difference between the ultimate lateral load and the load at first significant yielding. Based on pushover results, the system overstrength Ω_S appears to vary between 1.1 and 1.5 depending on aspect ratio.

Using these definitions, it appears that only the system overstrength factor needs to be used to amplify the design base shear (or reduce the plastic capacity) in order to use Eq. (13) for the design of infill plates for steel plate shear walls. Therefore, Eq. (18) is recommended, with values for the system overstrength factor taken between 1.1 and 1.5, the actual system overstrength factor can be obtained from a pushover analysis, or conservatively used as 1.5.

Incidentally, on the basis of the work presented above, the CAN/CSA S16-01 procedure should be modified to eliminate the possibility of designs having less-than-expected ultimate strength. For this purpose, Eq. (1) could be rewritten as

$$t = \frac{2A\beta \sin\theta \sin 2\theta}{L \sin^2 2\alpha} \tag{20}$$

where β = correction factor obtained by calibrating the equivalent story brace model to the plastic analysis results presented in this paper. Setting Eq. (20) equal to Eq. (18) and solving for β gives

$$\beta = \frac{\Omega_s \sin 2\alpha}{\sin 2\theta} \tag{21}$$

Conclusions

The CAN/CSA S16-01 recommended procedure for the analysis and design of steel plate shear walls has been reviewed and instances where this procedure can lead to unconservative designs with lower than expected ultimate capacity have been identified. Plastic collapse mechanisms for single and multistory SPSW with simple and rigid beam-to-column connections have been investigated and simple equations that capture the ultimate strength of SPSW have been developed and compared with experimental results reported by others with reasonable agreement. Using the results of these plastic analyses a new procedure for the sizing of the infill plates has been proposed. The proposed procedure allows the engineer to control the ultimate failure mechanism of the SPSW, and directly accounts for structural overstrength.

Acknowledgments

This work was supported in part by the Earthquake Engineering Research Centers Program of the National Science Foundation under Award No. ECC-9701471 to the Multidisciplinary Center for Earthquake Engineering Research. However, any opinions, findings, conclusions, and recommendations presented in this paper are those of the writers and do not necessarily reflect the views of the sponsors.

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