

Platform Competition with Endogenous Homing*

Thomas D. Jeitschko

Department of Economics

Michigan State University

E-mail: jeitschk@msu.edu

Mark J. Tremblay[†]

Department of Economics

Michigan State University

E-mail: trembl22@msu.edu

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Abstract

We develop a model for two-sided markets with consumers and producers, who interact through a platform. Typical settings for the model are the market for smartphones with phone users, app producers, and smartphone operating systems; or the video game market with game players, video game producers, and video game consoles. Only consumers who purchase the platform can access content from the producers. Consumers are heterogeneous in their gains from the producer side; and producers are heterogeneous in their costs of bringing apps to the platform. We consider competition between two platforms that allows consumers and firms to optimize with respect to how they home, i.e. we allow both individual consumers and individual producers to multi-home or single-home depending on whether it is optimal based on their type and the pricing policies of the two platforms. This leads to multiple equilibrium allocations of consumers and firms — all of which are seen in existing markets. We then find conditions under which a monopoly platform generates higher surplus than the two competing platforms.

Keywords: two-sided markets, platforms, platform competition, multi-homing, single-homing, endogenous homing decisions, network effects, smartphones, video games and game consoles.

JEL Classifications: L14, L22, D40, L13.

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[†]Corresponding Author: 486 W. Circle Drive, 110 Marshall-Adams Hall, East Lansing, MI 48824.

1 Introduction

Two thirds of U.S. households own a video game console, and the average time spent by gamers on their consoles is eight hours a week—the equivalent of a full work day. While this is a considerable amount of time, it pales in comparison to college students who spend as much as eight to ten hours on their phones *every day*—amounting to half their waking hours. Nearly 85 percent of 18–29 year olds in the U.S. have smartphones, and Nielson data show a 65 percent increase in time spent using apps by Android and iPhone users over the last two years, with 18–44 year olds using close to thirty different apps each month; and these trends are not confined to the U.S., they are present in Europe and Asia as well.¹

Video game consoles and smartphones are two platforms that have grown in importance in many people’s lives. Widespread access to the internet provides many additional opportunities to connect through platforms: passengers arrange for ride-sharing opportunities through Uber and Lyft, consumers make purchases at eBay or Amazon’s marketplace, and do-it-yourselfers exchange tips and ideas. Some buyers consume while on the platform: Google users search on Google, Facebook users visit friends’ pages, and LinkedIn users exchange information with their connections. Video game users play games on their gaming console, and smartphone users download and use apps—including apps that grant access to other platforms, such as the apps for Google, Facebook, or LinkedIn.

The market structures in which platforms offer their services vary considerably. For example, Google and Facebook have at times been characterized as near-monopolists who provide their services to users for free; whereas smartphones and video game consoles are concentrated markets with competing platforms that price well above zero. In this paper we consider a model of platform competition in which consumers and firms endogenously

¹The data on gaming come from ESRP (2010), those on college students’ phone usage from Roberts et al. (2014), and the data on prevalence of smartphones at Smith (2012). The Nielson data is at Nielsen (2014a,b).

choose which and how many platforms to join. We show that in equilibrium different allocations of consumers and firms emerge, mirroring the configurations found on many platforms, including those for smartphones and game consoles.

The literature on platforms can be traced back to work on markets where network externalities are prominent (e.g., Katz and Shapiro (1985) and Economides (1996)). In the case of platforms, markets are two-sided and the network effects carry over across the platform from one side of the market to the other; see, e.g., Evans (2003), Ellison and Fudenberg (2003), Rochet and Tirole (2003, 2006), Armstrong (2006); and sometimes also within one side of the platform, e.g., Deltas and Jeitschko (2007). While much of the literature considers monopoly platforms, competition between platforms is recognized to be an important characteristic that shapes these markets; and this is the focus of our paper.

Armstrong (2006) has proved to be seminal with his modeling choices playing a role in much of the literature on platforms. Armstrong’s competing platforms are horizontally differentiated—with each platform being located at the terminal point of a Hotelling line on both of the sides of the market. The Hotelling line implies that platforms are inversely valued by consumers; that is, the more someone likes one platform’s design and features (in absolute terms), the less that person will care for a rival’s platform (also in absolute terms). In many platform markets, and in particular for smartphones and video game consoles, differentiation of this type appears to be uncommon. Thus, Bresnahan et al. (2014), e.g., note that consumers do care about the different apps that are available on different platforms, however consumers do not experience significant platform differentiation in using different platforms. We capture this in our model by assuming that platforms differ only potentially in their pricing and in the availability of agents on the other side with whom agents wish to match.

The availability of agents on the other side of the platform is critical in determining the equilibrium, as agents must choose which platforms to join and potential coordination

issues within and across the two sides of the platform may arise. Caillaud and Jullien (2003) assume that with platform competition coordination favors the incumbent platform; otherwise platforms may fail to gain a critical mass, i.e. “fail to launch.” They argue this solves the “chicken and the egg” problem of each side’s action depending on the other side’s action. Hagiu (2006) shows the chicken and the egg problem does not occur when sides join platforms sequentially; and Jullien (2011) investigates this further over a broader class of multi-sided markets. Armstrong considers two different settings: one in which agents patronize only one of two platforms (single-home) and another in which members on the firm side join both platforms (multi-home) and members on the consumer side single-home. As the allocation decisions are assumed exogenously, coordination concerns are greatly alleviated. Lee (2013) investigates the video game market, and shows that Xbox was able to enter the video game market because exclusive contracts with game developers allowed Microsoft to overcome the coordination issue. In general, the role of beliefs and information play an important role in determining the equilibria as examined by Hałaburda and Yehezkel (2013), who show how multi-homing alleviates coordination issues tied to asymmetric information, Hagiu and Hałaburda (2014), who consider ‘passive’ price expectations on one side in contrast to complete information about prices on the second side, and Gabszewicz and Wauthy (2014), who also consider active and passive beliefs in determining platform allocations.

Weyl (2010) circumvents coordination issues by proposing the use of insulating tariffs. With an insulating tariff, the price that a platform charges on one side of the market depends on the number of agents on the other side—doing so resolves the failure to launch and multiple equilibrium issues. White and Weyl (2013) extend this model with insulating tariffs to the case where there exist multiple competing platforms.²

²Both models are very general, allowing for rich heterogeneity. However, they also find that more heterogeneity of agents can lead to issues with the platform’s profit maximization problem. Deltas and Jeitschko (2007) come to a similar conclusion even with linear pricing, noting that due to feedback effects, standard first order conditions are often neither necessary nor sufficient in many settings.

Our focus is on homing decisions in light of linear prices, as are standard in most two-sided markets, including the markets for smartphones and game consoles. One of the features of many of these markets is that there are a mix of single-homers and multi-homers. In general, these allocation decisions should be a part of the equilibrium allocation. One of the early papers that allows for endogenous homing is Rochet and Tirole (2003), where buyers and sellers are heterogeneous and they gain utility from a matched transaction that comes at a cost to the platform. The platform sets the transaction prices, which is an example of usage based pricing were the platform only sets transaction fees. However, this is not how platforms behave in the market for smartphones or video game consoles were the platform sets the price for the device—the membership fee. We focus on both usage and membership benefits for consumers, but only membership is priced by the platform. We abstract away from the transactions that occur between consumers and firms.³ This relates more closely to markets for smartphones and video game consoles.

There is also a nascent literature on the role of multi-homing in markets in which advertisers use media platforms to reach consumers (see, e.g., Ambrus et al. (2014), Anderson et al. (2013), Athey et al. (2014)). This literature also finds that previous models that assume single-homing frequently miss important aspects of competition and some previous results are reversed when accounting for the fact that consumers access multiple sources of media and can therefore be reached by advertisers through multiple channels.

In allowing for endogenous homing decisions, we find two allocation equilibria that are of particular interest. In one, all consumers single-home, whereas all firms multi-home. This is the common allocation observed in the market for smartphones. Indeed, Bresnahan et al. (2014) find that the practice of multi-homing by app producers—that is their simultaneous

³Thus, the benefits that accrue to consumers from interacting with firms—apps and games—can be viewed as being net of prices paid to firms. Reisinger (2014) generalizes the model in Armstrong (2006) and considers tariff-pricing with heterogeneous trading. See Tremblay (2014) for a more detailed analysis of pricing across the platform in a framework that is more similar to our current setting.

presence on competing platforms—insures against a tipping in the market that would concentrate all economic activity on a single firm’s platform. The other equilibrium allocation of particular interest to us is one in which there is a mix of single-homing and multi-homing on both sides. This is the division found in the market for game consoles (Lee (2013)).

After presenting the model in Section 2, we first consider all possible equilibrium allocations that arise for arbitrary platform prices and then derive the optimal pricing strategies in light of the possible equilibrium allocations and the implications for profits (Section 3). Equilibrium configurations are generally not unique, but we compare possible configurations to a monopoly benchmark in Section 4 by making functional form assumptions that allow us to give closed form solutions for welfare. Finally, there is a brief conclusion in Section 5, followed by an appendix that contains the proofs of all the formal findings.

2 The Model

Two groups of agents can benefit from interaction, but require an intermediary in order to do so. The benefits from the interaction to an agent in one group depends on the number of agents of the other group that are made available through the intermediary.⁴ This intermediary—the platform—charges agents in each group a price to participate on the platform and in exchange brings these groups together. We consider two platforms, indexed by $X \in \{A, B\}$.⁵

⁴Thus, we consider platforms with indirect network effects, i.e. one side benefits from participation on the other side of the platform. The model readily generalizes to include direct network effects where one side of the market cares about the total participation on its own side.

⁵The model can readily be generalized to an arbitrary number of platforms.

2.1 Platforms

Agents on each side of the platform are described by continuous variables. Agents on Side 1 are consumers or buyers, and agents on Side 2 are firms or sellers. The number of consumers that join Platform X is $n_1^X \in [0, \bar{N}_1]$, and the number of firms on Platform X is $n_2^X \in [0, \bar{N}_2]$.

The cost to the platform of accommodating an agent on side $i \in \{1, 2\}$ who joins the platform is $f_i \geq 0$, and there are no fixed costs. Platform X has profits of

$$\Pi^X = n_1^X(p_1^X - f_1) + n_2^X(p_2^X - f_2), \quad (1)$$

where p_i^X is the price that platform X charges to the agents on side i .

Once we discuss consumers and firms it will be clear that platforms are homogeneous in that their abilities to provide utility from agents' participation is the same. The only differences between the two platforms therefore arise solely either due to different prices being charged, $p_i^A \leq p_i^B$, or due to different participation rates on the opposing side, $n_i^A \leq n_i^B$, which affect the attractiveness of a platform.

2.2 Side 1: Consumers

Consumers on Side 1 are indexed by $\tau \in [0, \bar{N}_1]$. The utility for a consumer of type τ from joining Platform X is

$$u_1^X(\tau) = v + \alpha_1(\tau) \cdot n_2^X - p_1^X. \quad (2)$$

Here $v \geq 0$ is the membership value every consumer receives from joining the platform. This is the stand alone utility of being a member of the platform that one gets even if no firms join the platform. Note that it is possible for $v = 0$, but for smartphones and video game consoles $v > 0$. For smartphones v is the utility from using a smartphone as a phone, including the preloaded features, and for video game consoles v is the utility from

using the console to watch Blu-ray discs. Consumers are homogeneous in their membership benefit to the platform; so v does not depend on consumer type τ ; and because platforms are homogeneous, the stand-alone value of joining a platform is the same regardless of which platform is joined.

Consumers are heterogeneous in their marginal benefit from firms. The network effect or the marginal benefit to a consumer of type τ for an additional firm on the platform is constant and given by $\alpha_1(\tau)$; and the number of firms that join the platform is n_2^X . We focus on the case when network effects are positive so $\alpha_1(\tau) \geq 0$ for all τ , where $\alpha_1(\cdot)$ is a decreasing, twice continuously differentiable function. Since $\alpha_1(\tau)$ is decreasing, it orders consumers by their marginal benefits. Consumers whose type τ is close to zero have marginal benefits that are high relative to those consumers whose type is located far from zero.

The platform knows v and $\alpha_1(\cdot)$ but cannot distinguish the individual values for each consumer τ . Thus, it cannot price discriminate between consumers, so the price or membership fee that consumers pay the platform is a uniform price given by p_1^X .

With there being two platforms in the market, consumers and firms can either join a single platform (single-home) or join multiple platforms (multi-home). A consumer who multi-homes has utility

$$u_1^{AB}(\tau) = (1 + \delta)v + \alpha_1(\tau) \cdot N_2 - p_1^A - p_1^B. \quad (3)$$

Notice that if a consumer participates on two platforms the intrinsic benefit from membership to the second platform diminishes by $\delta \in [0, 1]$ so that the total stand-alone membership benefit from the two platforms is $(1 + \delta)v$. If $\delta = 0$, then there is no additional membership benefit from joining the second platform, and when $\delta = 1$ the membership benefit is unaffected by being a member of another platform.⁶

⁶Depreciation in network benefits, α_1 , is also a possibility, see, e.g., Ambrus et al. (2014).

Apart from the positive membership value of being on a second platform, the main gain to joining a second platform is access to additional firms. Letting n_2^m denote the number of multi-homing firms, a consumer that multi-homes has access to $N_2 := n_2^A + n_2^B - n_2^m$ distinct firms: these are all the firms that join at least one platform. The above utility function implies a multi-homing firm provides only a one-time gain to a consumer that multi-homes. Having a firm available on both platforms to which the consumer has access provides no added benefit.

2.3 Side 2: Firms

On the other side of the platform, Side 2, are firms that are indexed by $\theta \in [0, \bar{N}_2]$. A firm's payoff from joining Platform X is

$$u_2^X(\theta) = \alpha_2 \cdot n_1^X - c\theta - p_2^X. \quad (4)$$

The marginal benefit firms receive from an additional consumer on the platform is α_2 —which is the same for all firms, so firms' marginal benefits for an additional consumer are homogeneous across firm type. The logic here is that an additional consumer will (in expectation) shift the demand curve for a firm's app up in the same way for all firms. The assumption we are making here is that each consumer sees firm products—their app, or game—as homogeneous, but consumers differ in their preferences, resulting in different willingness to purchase apps and games.

Firms incur a cost of $c > 0$ to join the platform. This cost reflects development and synchronization costs associated with programming and formatting their product to fit the platform. Firms are heterogeneous with respect to their development and synchronization costs. Firms with type θ close to zero have lower costs compared to those with higher θ .

The platform knows the firm's profit structure but cannot identify firms individually;

hence, it cannot price discriminate between firms and the price or membership fee the firms pay to the platform is given by p_2^X for all firms.

A firm that multi-homes has payoff

$$u_2^{AB}(\theta) = \alpha_2 \cdot N_1 - (1 + \sigma)c\theta - p_2^A - p_2^B, \quad (5)$$

where $N_1 := n_1^A + n_1^B - n_1^m$ is the number of distinct consumers to which the firm gains access; these are all the consumers that join at least one platform. As noted above, when a firm's product is available to the multi-homing consumer on both platforms, a consumer will only purchase the product at most once. Therefore a firm only cares about the number of distinct consumers that are available to it through the platforms.

When a firm participates on two platforms its development and synchronization cost for joining the second platform diminishes by $\sigma \in [0, 1]$. Thus, σ represents the amount of scale economies that exist in synchronizing an app or game to a platform. If $\sigma = 1$ then there are no economies of scale and as σ decreases, there exists economies of scale.⁷

2.4 Allocation Decisions

By allowing consumers and firms to make homing decisions there are potentially many allocations that can occur for a given set of prices. Agents' beliefs about the allocation decision on the opposite side of the platform play a critical role in determining their membership decisions. We make a few basic assumptions about agents' beliefs and allocation decisions that are particularly salient in our context. We then determine all possible equilibrium allocations of consumers and firms, for arbitrary prices, that are consistent with these minimal

⁷The relative lack of scale economies played a role in providing an app for Facebook in the tablet market. For some time the app 'Friendly for Facebook' was used by Facebook users because Facebook itself had not developed an app for the tablet. It was rumored that Apple later 'assisted' Facebook in developing the official Facebook app.

assumptions.

First, we require a tie-breaking rule for the case when agents have beliefs such that they are indifferent between joining Platform A or B .

Assumption 1 (Tie-Breaking Rule). *If, for given beliefs about the allocation on the opposite side and for given prices, an agent (consumer or firm) is indifferent between joining platforms A and B , i.e. $u_i^A = u_i^B > 0$, then the agent either multi-homes, or chooses to join one of the platforms with equal probability.*

Assumption 1 implies that there is no intrinsic preferences of one platform over the other by consumers or firms. This means that if it is optimal for a consumer or firm to single-home then it will single-home on Platform A or B with equal probability whenever the expected utility they obtain on either platform for given prices and allocations on the other side is the same.

Note that Assumption 1 does not rule out a clustering of agents on one platform due to, say, successful ‘viral marketing’ efforts, because the assumption only addresses agent behavior for a given set of prices and beliefs that the agent has. A successful marketing campaign is successful precisely in affecting (tipping/skewing) beliefs to achieve the desired outcome.

We also preclude dis-coordinated allocation configurations in which despite having worse (i.e., higher) prices a platform corners the market on the firm side.

Assumption 2 (No Dis-Coordination). *If $p_i^X \leq p_i^Y, \forall i$ then $n_2^X \neq 0$.*

Assumption 2 states that a platform that offers a price advantage on at least one side and is no worse than its rival in terms of the price it charges on the other side will attract at least some firms. Note in particular, however, that Assumption 2 says nothing about the equilibrium allocation of consumers. And, importantly, it says nothing about consumers or

firms for the case that one platform has a lower price on one side, but the rival platform has the lower price on the other side.

As indicated, the assumption pertains only to minimum participation of the firms—rather than guaranteeing minimum participation by consumers. The rationale for this is twofold. First, a platform can always attract some consumers when it prices sufficiently low, because the platform offers a stand-alone value to consumers; and second, in the contexts we have in mind it is reasonable to assume that firms are aware of pricing on both sides of the platform, whereas consumers are likely to only observe prices on Side 1. Hence, firms are able to observe any price-advantages regardless of the side on which they are offered (see Hagiu and Hałaburda (2014)).

Lastly, we include the standard equilibrium requirement that all agents' beliefs about allocations are consistent with equilibrium actions taken by agents on the other side. That is, when an agent makes a participation decision based on an expectation of the number of agents on the other side, then in equilibrium this expectation must coincide with the actual decisions of the agents on the other side so that expectations are correct.⁸

Our basic assumptions are used to characterize the set of all allocations that are possible in equilibrium for any arbitrarily given price constellations. To be sure, these generally do not generate unique equilibrium configurations. However, there is enough structure in order to derive meaningful pricing strategies for the platforms that yield clear equilibrium implications.

3 Equilibrium

The sequence of play is as follows: first the platforms simultaneously (and non-cooperatively) choose consumer and firm prices, p_i^X for $X = A, B$ and $i = 1, 2$. Then consumers and

⁸Jullien and Pavan (2014) further investigate responses by platforms to imperfect information within a two-sided market, however their main focus is restricted to single-homing consumers.

firms simultaneously choose whether and which platforms to join, yielding n_i^X and $N_i = n_i^A + n_i^B - n_i^m$, $i = 1, 2$.

Using backward induction we first investigate the allocation subgame that obtains between consumers and firms in joining platforms for given prices charged by the platforms; and then we determine the equilibria for the entire game by considering price competition between the two platforms, in light of the profits obtained in the allocation subgame equilibrium.

For simplicity, we focus exclusively on the cases when prices are sufficiently low for at least some participation to exist. Constellations in which a platform prices itself out of the market are easily derived, but are merely a distraction as they do not arise in any of the pricing equilibrium configurations of the entire game. An implication of this is that the total participation of agents on each side across both platforms is positive, $N_i > 0$ for $i = 1, 2$.

3.1 The Allocation Equilibrium for Arbitrary Prices

The following lemma is useful in proving the equilibrium allocations, but it is also instructive in its own right. The lemma states that in equilibrium all participating firms either join only one and the same platform, or each platform attracts the same measure of firms, which happens if and only if both platforms generate the same firm payoff.

Lemma 1. *In equilibrium $n_2^X \in \{0, n_2^Y, N_2\}$, $Y \neq X$ whenever $p_2^X, p_2^Y \geq 0$; and $n_2^A = n_2^B$ if and only if $u_2^A(\theta) = u_2^B(\theta)$ for all $\theta \in [0, N_2]$.*

We now consider the equilibrium allocations that occur for given prices. The first allocation equilibrium concerns the case where platforms choose symmetric pricing strategies.

Proposition 1 (Allocations under Symmetric Pricing). *If $p_i^X = p_i^Y = p_i$ then $n_i^X = n_i^Y = n_i$, $i = 1, 2$, and $X, Y = A, B$.*

Moreover, we have the following allocations of firms and consumers. The set of multi-homing consumers is given by $\tau \in [0, n_1^m]$, and the set of single-homing consumers is given by $\tau \in [n_1^m, N_1]$, with

$$n_1^m = \alpha_1^{-1} \left(\frac{p_1 - \delta v}{n_2 - n_2^m} \right) \quad \text{and} \quad N_1 = \alpha_1^{-1} \left(\frac{p_1 - v}{n_2} \right). \quad (6)$$

The set of multi-homing firms is given by $\theta \in [0, n_2^m]$, and the set of single-homing firms is given by $\theta \in (n_2^m, N_2]$, with

$$n_2^m = \min \left\{ \frac{\alpha_2 \cdot N_1 - 2p_2}{(1 + \sigma)c}, \frac{\alpha_2 \cdot (n_1 - n_1^m) - p_2}{\sigma c} \right\} \quad \text{and} \quad N_2 = \frac{\alpha_2 n_1 - p_2}{c}. \quad (7)$$

For each side of the market at least one set is non-empty, and it is possible for both sets to be non-empty; as a result there exist multiple equilibrium allocations.

Proposition 1 says that when platforms set equal prices, the platforms split both sides of the market equally. However, this equal division does not determine the extent to which consumers and firms multi-home in equilibrium. In fact, the allocation of one side of the market depends on the allocation on the other side; and this results in the possibility of multiple equilibrium allocations—depending on the platform prices chosen.

Consider first consumers. Consumers always obtain an added benefit from joining a second platform, namely, δv . Hence, if prices to consumers are low enough, $p_1 < \delta v$, then all consumers join both platforms: $n_1^m = N_1 = \bar{N}_1$.⁹ For consumer prices above this threshold, but still below the stand-alone utility from a single platform membership, $\delta v < p_1 < v$, all consumers will join one platform, $N_1 = \bar{N}_1$; but whether any consumers join a second platform (multi-home) depends on whether firms multi-home. In particular, if the number of multi-homing firms is large ($n_2 - n_2^m$ is small), then consumers have access to many firms

⁹Note that since $\alpha(\cdot)$ is positive and decreasing, so is $\alpha^{-1}(\cdot)$ and therefore when $p_1 - \delta v < 0$ the corner solution obtains in which $n_1^m = N_1 = \bar{N}_1$.

when joining the first platform and so the number of multi-homing consumers is small, or even zero. For even higher consumer prices, consumers with large values of τ even refrain from joining a single platform, $N_1 < \bar{N}_1$.

Unlike consumers, firms do not obtain a stand-alone benefit from joining a platform. However, they experience scale economies in production when joining a second platform. This implies that a firm will multi-home only when the marginal gain from joining a second platform and the total payoff from being on two platforms are both positive. And hence, the set of multi-homing firms depends on the number of consumers that multi-home. If all consumers multi-home then $n_1^m = n_1$ and no firm will multi-home, provided that $p_2 > 0$. Second, the firms that choose to multi-home instead of single-home are the firms with sufficiently low synchronization costs, θ close to zero. As the synchronization cost gets larger the marginal cost for joining another platform becomes larger than the marginal gain from having access to additional consumers. Hence, for firms with higher synchronization costs, θ farther from zero, it becomes too costly to join more than one platform. Thus, a firm is more likely to multi-home if it faces a lower synchronization cost to join a platform.

Note finally that if few consumers multi-home (n_2^m is small) and there are strong scale economies (small σ), then it is possible that no firms single-home and all firms multi-home.

We now determine the allocations that occur with unequal price constellations.

Proposition 2 (Allocations with Price-Undercutting). *If $p_i^Y \leq p_i^X$ with at least one strict inequity then there exists a unique allocation equilibrium. In this equilibrium $n_i^Y = N_i$, $i = 1, 2$, with $n_1^X = n_1^m > 0$ only when $p_1^X \leq \delta v$ and $n_2^X = n_2^m > 0$ only when $p_2^X < 0$.*

So Proposition 2 shows that when one platform has better prices (at least one better price, and the other price no worse), then all agents—consumers and firms alike—will join the platform with the price advantage. Whether agents also join the second platform (and, thus, multi-home) depends on the prices on the second platform. Consumers will join the

second platform only if the price is below their marginal stand-alone benefit from joining a second platform, $p_1^X < \delta v$, because they already have access to all active firms through the first platform so that any firm presence on the second platform is of no value to consumers. Similarly, firms will only join the second platform if they are paid to do so, $p_2^X < 0$, because they already have complete market access to all consumers on the other platform.

Lastly, consider the case when prices are unequal and neither platform has a clear price advantage.

Proposition 3 (Allocations under Orthogonal Pricing). *If $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ for $X \neq Y \in \{A, B\}$ then the following are possible equilibrium allocations:*

- $n_1^Y = N_1 > n_1^X = n_1^m > 0$ with $n_2^X = n_2^Y$,
- $n_1^Y = N_1$ and $n_1^X = 0$, with $n_2^Y = N_2$ and $n_2^m = n_2^X > 0$ only when $p_2^X < 0$. This only exists when $p_1^X > \delta v$.
- $n_2^X = N_2$ and $n_2^m = n_2^Y > 0$ only when $p_2^Y < 0$ with
 - $n_1^X = N_1$ and $n_1^Y = 0$ when $p_1^Y > v$,
 - $n_1^X, n_1^Y > 0$ with no multi-homing when $v \geq p_1^Y > \delta v$, and
 - $n_1^Y = N_1$ with $n_1^X > 0$ multi-homers when $\delta v \geq p_1^Y$.

When prices are unequal and neither platform has the lower price on both sides of the market there are three possible ways consumers and firms can divide themselves onto the two platforms—these depend on the relative magnitude of prices, but are not mutually exclusive. Thus, in this case it is possible to have multiple equilibrium allocations.

In the first two cases listed in the proposition, all consumers join the platform that has the better consumer price ($n_1^Y = N_1$). Beyond that, in the first case platforms capture an

equal number of firms, which comes about either due to some consumers multi-homing and the difference in the price on the firm side is large, or because firms are being subsidized.

In the second case listed, if the platform with the higher price for consumers doesn't price low enough to capture consumers seeking the marginal stand-alone benefit of the second platform, i.e., $p_1^X \not\leq \delta v$, then no consumers will multi-home, and firms only multi-home when they are subsidized to do so, $p_2^X < 0$.

The third possibility differs markedly from the other two in that all firms congregate on the platform with the higher consumer price. All consumers will join the firms when the platform with the better consumer price is not attractive enough to make the stand-alone value worth capturing, $p_1^Y > v$. However, consumers who have little value for apps, will switch to the otherwise empty platform in order to capture the stand-alone value when $v \geq p_1^Y > \delta v$; and when prices are even lower, then all consumers will join this platform—many of whom will also remain members of the other platform and thus multi-home.

3.2 Equilibria of the Pricing Game

In the allocation configurations a recurring theme was whether a platform sets prices low enough to attract consumers merely for the stand-alone value. This pricing decision often plays a special role in determining whether consumers multi-home. In particular, if a platform sets $p_1^X < \delta v$, then it is sure to capture all consumers—regardless of all other prices and homing decisions. In light of this, when determining the platforms' pricing decisions it is important to consider the relationship between the cost of providing service to a consumer and the consumer's marginal stand-alone value for the second platform, i.e., $f_1 \gtrless \delta v$.

For many products the membership benefit depreciates almost to zero when a consumer multi-homes, $\delta \approx 0$. This implies the marginal cost of accommodating an additional consumer on the platform is greater than the additional membership benefit from joining another

platform, even for small f_1 . In the smartphone case, the membership benefit is the ability to make calls and use the phone's preloaded features. Since most phones have similar preloaded features, this implies a δ close to zero and any additional benefit would not overcome the cost of producing the additional phone. We first investigate this case by assuming that $f_1 \geq \delta v$, so the cost of attracting a consumer who has already joined the rival platform exceeds the platform's stand-alone value to the consumer. As a result, platforms compete primarily for single-homers, rather than trying to attract multi-homers. This leads to fierce price-competition resulting in the Bertrand Paradox.

There are potentially three allocations of consumers and firms in equilibrium.

Theorem 1 (Strong Competition; $f_1 \geq \delta v$). *The unique equilibrium prices are $p_1^A = p_1^B = f_1$ and $p_2^A = p_2^B = f_2$ so that $\Pi^A = \Pi^B = 0$.*

There exists at least one and possibly as many as three types of equilibrium allocations:

- I. All consumers single-home and all firms multi-home: $n_1^m = 0$, $n_2^m = N_2$. This is always an equilibrium.*
- II. A mix of multi-homing and single-homing consumers with multi-homing and single-homing firms: $n_i^m \in [0, N_i]$.*
- III. All firms single-home and many, potentially all, consumers multi-home: $n_2^m = 0$, $n_1^m \in (0, N_1]$. When $f_2 > 0$ then $v = 0$ is sufficient for this equilibrium to exist and when $f_2 = 0$ it requires $v = 0$.*

Allocation I mirrors the two-sided market for smartphones. Almost all consumers single-home, they own only one phone; and almost all firms multi-home, while the vast majority of apps are available on all types of smartphones.

Allocation II resembles current allocations seen in many two-sided markets, including those for game consoles: For video game platforms, there exist consumers who multi-home—

buying several game consoles—and others that single-home; and there exists game designers whose games are available across platforms, i.e., they multi-home, while others are exclusively available on only one system, i.e., they single-home. Allocations I and II are of particular interest to us and we discuss them in greater detail in Section 4.

Allocation III is best characterized when considering the sufficient condition of $v = 0$. An example of this type of configuration is seen with the ride sharing companies Uber and Lyft. These are platforms that connect drivers (i.e., firms) with passengers seeking transportation (consumers). Drivers offer their services through one ride sharing company (i.e., they single-home); whereas many customers seeking rides use both companies and compare availability (i.e., they multi-home). Since there is no benefit from linking to a ride sharing company that has no drivers $v = 0$.¹⁰

We now turn to the second case, where $f_1 < \delta v$. In this case a platform can charge a consumer price of $p_1^X = \delta v > f_1$ and guarantee itself positive profits since consumers will either single-home on platform X or if a consumer is already on platform $Y \neq X$ then they will be willing to multi-home even absent any firms on platform X . Hence, both platforms are guaranteed profits and, in equilibrium, all consumers $\tau \in [0, \bar{N}_1]$ join at least one platform.

Furthermore, in this case failure to launch issues are more generally precluded, since both platforms are able to establish themselves on the consumer side of the market. Thus, platforms no longer have to set prices equal to marginal costs and therefore earn positive profits.

Theorem 2 (Weak Competition; $f_1 < \delta v$). *There exists a unique symmetric equilibrium with $p_1^A = p_1^B = \delta v$ and $p_2^A = p_2^B = f_2$. All consumers multi-home, $n_1^m = \bar{N}_1$, and firms that join a platform single-home on each platform with equal probability, $n_2^A = n_2^B$, $n_2^m = 0$. The*

¹⁰Another example are antique malls with many individual stalls each rented out to individual antiques dealers (i.e., firms), and consumers who visit the mall to browse the individual stalls. Vendors sell their antiques in only one mall (single-home), yet consumers browse at different malls (multi-home). There is no benefit from going to a vacant antique mall so $v = 0$.

platforms receive positive profits: $\Pi^A = \Pi^B = \bar{N}_1(\delta v - f_1) > 0$.

Thus, if a platform has significantly high retained membership benefit for consumers when they multi-home, then competing platforms can avoid the Bertrand Paradox on the consumer side of the market and make positive profits. It is straightforward to show that this result generalizes to more than two platforms who compete in prices.

Corollary 1. *Platforms obtain the same market shares in equilibrium: $n_1^A = n_1^B$ and $n_2^A = n_2^B$.*

Given that platform competition leads to symmetric pricing, the corollary follows directly from Proposition 1. Nevertheless, it is worth further discussion. When two platforms charge equal prices then in expectation we have each platform capturing half of the single-homers. This is not far off from the market for smartphones in which established providers have similar pricing strategies and also similar market shares (see Bresnahan et al. (2014)), as well as the market for games in which after successful entry systems became similar competitors (see Lee (2013)).

4 Monopoly versus Strong Competition

Does strong competition between two platforms result in higher welfare when compared to a monopoly platform? The answer is not readily apparent. On the one hand competition results in lower prices and additional stand-alone membership benefits to consumers who multi-home. On the other hand, however, competition can also destroy network surplus and creates more synchronization costs for firms who multi-home.

We investigate welfare implications of platform competition by comparing our model to the case of monopoly. This shows the trade-off between the benefits of price competition among two platforms with those of greater network effects of a monopoly platform. We

consider the case of strong competition, and show that even in this case the monopoly equilibrium may welfare-dominate, despite consumers not being restricted to unit demands and despite the Bertrand Paradox occurring.

To obtain closed form solutions and welfare we assume that $\alpha_1(\cdot)$ is linear, $\alpha_1(\tau) = a - b\tau$, which implies that τ is distributed uniformly on $[0, a/b]$, so the number of potential consumers is $\bar{N}_1 = \frac{a}{b}$. To simplify calculations, we further assume $v = f_1$ and $f_2 = 0$ (which implies the case of strong competition since $f_1 = v > \delta v$).¹¹

4.1 Monopoly

The additional functional form assumptions do not affect firm utility, and we assume \bar{N}_2 is sufficiently large so that the platform can always attract more app producers. That is, there exists many potential app producers, many of which end up not developing an app because their synchronization costs are too high.¹²

With $\alpha_1(\tau) = a - b\tau$, the highest marginal benefit any consumer has (namely a consumer of type $\tau = 0$) for firm participation is a . This implies that if the firms' constant marginal valuation of consumer participation exceeds that of consumers, $\alpha_2 > a \geq \alpha_1(\tau)$, then the platform strategy is to attract as many consumers as possible in order to make the platform as valuable as possible to firms. In turn, this allows the platform to extract a larger surplus from firms than was the cost of attracting consumers. Hence, whenever $\alpha_2 \geq a$ a corner solution is obtained in which the platform prices consumer participation such that all consumers join. In contrast, when $\alpha_2 \leq a$ an interior equilibrium emerges, in which some consumers do

¹¹These assumptions are not that critical in the analysis and they make computations straightforward: We are assuming the membership benefit consumers receive is approximately the marginal cost for adding an additional consumer and that adding an additional firm is costless to the platform. This simplification can be explained in the market for smartphones and video game consoles where both the marginal cost to produce the platform and the membership gains consumers receive are positive, we are assuming relatively close, and the cost to platforms to add an additional app or video game is nearly costless.

¹²Alternatively, assuming a limited number of app producers, one can consider these restricting the number of apps which they would provide when faced with increasing development costs for apps.

not join the platform. We consider the two cases in turn, starting with the latter case, the interior solution.

For given prices, the participation decisions are implied by the marginal agents on both sides being indifferent between participation and opting out, given the participation decision on the opposite side of the platform. Thus, on the consumer side $u_1(\tau = N_1) \equiv 0$ in conjunction with our functional form assumptions yield $p_1 = v + (a - bN_1) \cdot N_2$ (see 2). And on the firm side $u_2(\theta = N_2) \equiv 0$ yields $p_2 = \alpha_2 \cdot N_1 - cN_2$ (see 4).

Using these relations between participation and prices in conjunction with the platform's profit function (1), the monopolist's objective is to chose the implied participation levels, N_1 and N_2 to maximize

$$\Pi^M = N_1(v + (a - bN_1) \cdot N_2 - f_1) + N_2(\alpha_2 \cdot N_1 - cN_2 - f_2). \quad (8)$$

The second order conditions hold for this problem and it is straightforward to show that for the interior equilibrium the prices and allocations are

$$p_1^{M_I} = v + \frac{1}{16bc}(a + \alpha_2)^2(a - \alpha_2), \quad p_2^{M_I} = \frac{1}{8b}(a + \alpha_2)(3\alpha_2 - a), \quad \text{and} \quad (9)$$

$$N_1^{M_I} = \frac{1}{2b}(a + \alpha_2), \quad N_2^{M_I} = \frac{1}{8bc}(a + \alpha_2)^2; \quad (10)$$

where M_I is a mnemonic that denotes the interior monopoly solution.

There are two things to notice in this equilibrium. First, recall the usual monopoly problem with linear inverse demand $P = a - bQ$ and marginal cost equal to zero, yielding the monopoly output of $Q^M = \frac{a}{2b}$. Now notice that $N_1^{M_I} > Q^M$, so in equilibrium, a monopoly platform will price to have more consumers than a traditional (one-sided) monopolist. This is because the added consumers generate additional surplus on the platform which can then partly be extracted through the firm price.

Second, for a similar reason, firm price can be negative, $p_2^{M_I} \leq 0$. Firms are subsidized to join the platform when $a > 3\alpha_2$. Intuitively this means that if adding firms generates a significantly larger amount of surplus for consumers than consumers generate for firms, then the total surplus on the firm side is less important. The platform will subsidize firms allowing for a greater generation and subsequent extraction of surplus on the consumer side.

Given equilibrium prices, the number of consumers, and the number of firms, we calculate platform profits, consumer, firm, and total surplus.

$$\Pi^{M_I} = N_1(p_1^{M_I} - f_1) + N_2(p_2^{M_I} - f_2) = \frac{(a + \alpha_2)^4}{64b^2c}, \quad (11)$$

$$CS^{M_I} = \int_0^{N_1^{M_I}} (v + \alpha_1(\tau)N_2^{M_I} - p_1^{M_I}) d\tau = \frac{(a + \alpha_2)^4}{64b^2c}, \quad (12)$$

$$FS^{M_I} = \int_0^{N_2^{M_I}} (\alpha_2 N_1^{M_I} - c\theta - p_2^{M_I}) d\theta = \frac{(a + \alpha_2)^4}{128b^2c}, \quad (13)$$

$$W^{M_I} = \frac{5(a + \alpha_2)^4}{128b^2c}. \quad (14)$$

Consider now the corner solution (denoted by C) which occurs when $\alpha_2 \geq a$. Recall that the monopolist will attract all consumers to the platform in order to generate the maximum firm surplus. This requires that $p_1^{M_C} = v$. Given this price, the platform maximizes profits with respect to N_2 with $p_2 = p_2(N_1 = a/b, N_2) = \alpha_2 \frac{a}{b} - cN_2$. This yields

$$p_1^{M_C} = v, \quad p_2^{M_C} = \frac{a\alpha_2}{2b}, \quad \text{and} \quad (15)$$

$$N_1^{M_C} = \frac{a}{b} = \bar{N}_1, \quad N_2^{M_C} = \frac{a\alpha_2}{2bc}. \quad (16)$$

Given the corner solution, we calculate welfare.

$$\Pi^{M_C} = N_1(p_1^{M_C} - f_1) + N_2(p_2^{M_C} - f_2) = N_1 \times 0 + N_2 p_2^{M_C} = \frac{a^2 \alpha_2^2}{4b^2 c}, \quad (17)$$

$$CS^{M_C} = \int_0^{N_1^{M_C}} (v + \alpha_1(\tau) N_2^{M_C} - p_1^{M_C}) d\tau = \frac{a^3 \alpha_2}{4b^2 c}, \quad (18)$$

$$FS^{M_C} = \int_0^{N_2^{M_C}} (\alpha_2 N_1^{M_C} - c\theta - p_2^{M_C}) d\theta = \frac{a^2 \alpha_2^2}{8b^2 c}, \quad (19)$$

$$W^{M_C} = \frac{a^2 \alpha_2}{8b^2 c} (3\alpha_2 + 2a). \quad (20)$$

4.2 Welfare Comparison

Consider now competing platforms. Because we are dealing with the case of strong competition, Theorem 1 holds and so $p_1^A = p_1^B = f_1 = v \geq 0$ and $p_2^A = p_2^B = f_2 = 0$. Moreover, from Theorem 1 we know that there can be up to three distinct allocations of consumers and firms in equilibrium. The ones of interest to us are Allocation I, in which all consumers single-home and all firms multi-home—the common allocation with smartphones; and Allocation II with its mix of single- and multi-homing on both sides—as is observed with game consoles.

In Allocation I all consumers single-home and all firms multi-home; we have $n_1^m = 0$ and $n_2^A = n_2^B = n_2^m = N_2$. Given the prices from Theorem 1 in conjunction with our functional form assumptions, $n_1 := n_1^A = n_1^B = \frac{1}{2} \bar{N}_1 = \frac{1}{2} \frac{a}{b}$ and $n_2 := n_2^A = n_2^B = n_2^m = \frac{\alpha_2}{(1+\sigma)c} \frac{a}{b}$;

resulting in

$$\Pi^A = \Pi^B = 0, \quad (21)$$

$$CS^{AI} = \int_0^{a/b} (a - b\tau) \frac{2\alpha_2 \cdot n_1}{(1 + \sigma)c} d\tau = \frac{a^3 \alpha_2}{2(1 + \sigma)cb^2}, \quad (22)$$

$$FS^{AI} = \int_0^{n_2} (\alpha_2 \bar{N}_1 - 2c\theta - 2p_2) d\theta = \frac{a^2 \alpha_2^2}{2(1 + \sigma)cb^2}, \quad (23)$$

$$W^{AI} = \frac{a^2 \alpha_2}{2(1 + \sigma)cb^2} (\alpha_2 + a); \quad (24)$$

where the superscript AI denotes Allocation I.

From this follows:

Theorem 3 (Allocation I v. Monopoly). *Whenever $\alpha_2 \geq a$ all consumers join a platform regardless of the market structure; and there exists $\sigma^C := \frac{a+2\alpha_2}{3a+2\alpha_2} \in (0, 1)$ such that for all $\sigma \geq \sigma^C$ the monopoly generates more welfare than competition.*

When $\alpha_2 < a$ competition serves all consumer types, whereas the monopoly limits consumer participation; and yet there exists $\sigma^I := \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1 < 1$ such that for all $\sigma \geq \sigma^I$ the monopoly generates more welfare than competition.

Notice that competition between two platforms always leads to all consumer types being included in the market, $N_1^{AI} = \bar{N}_1$, whereas a monopoly excludes some when their benefit derived from firms is not so large (the interior solution), $N_1^I < \bar{N}_1$. Despite the greater market coverage when platforms compete, whenever $\sigma^I < 0$ the monopoly generates higher surplus independent of the level of scale economies that firms experience when they join a second platform.

In general, scale economies are an important factor in determining the welfare comparison between monopoly and Allocation I of competing platforms. Thus, regardless of whether there is a corner or an interior solution for the monopoly platform, if there are no scale economies, $\sigma = 1$, then the benefits of pooled networking on the monopoly platform always generate

sufficiently more welfare than the lower prices and added consumer benefit from stand-alone values generated by competing platform, and therefore the monopoly welfare-dominates competition.

This last result is tied to the fact that in the two-platform equilibrium of Allocation I all firms that join the market actually end up multi-homing—incurring the added expense of synchronizing their app to the second platform at cost $\sigma c\theta$. This is not the case, however, when considering Allocation II, where some firms may single-home.

Turning to the comparison between monopoly and Allocation II under competition, note from Theorem 1 that a type-II Allocation may not exist. Indeed, it only occurs when network effects are sufficiently strong, viz., $a > \sqrt{\frac{(1-\delta)v}{8bc\alpha_2}}$.

Theorem 4 (Allocation II v. Monopoly). *If there are no added costs to a firm of making its product compatible with a second platform ($\sigma = 0$), then competition always generates more total surplus; but regardless of whether the monopoly has an interior or a corner solution, there exists mixed allocations such that the welfare from the competitive mixed allocation equilibrium is greater than the welfare with the monopoly platform; even when there are no scale-economies ($\sigma = 1$).*

Thus, when under competition the mixed allocation emerges in equilibrium then sufficiently strong scale economies (small enough σ) will assure greater welfare from competition than in monopoly, because firms are able to cheaply multi-home. However, Theorem 4 also makes clear that the converse need not hold. That is, even when it is costly for firms to multi-home in terms production and synchronization costs ($\sigma = 1$), competition can generate greater welfare in the mixed allocation.

Taken together, Theorems 3 and 4 show that when scale economies are small (large σ) monopoly is always preferred to competition, unless under competition a mixed allocation emerges in which not all firms multi-home. Notice also that for the case of an interior

monopoly solution and parameters such that $\sigma^I < 0$, then for sufficiently large scale effects (σ small) competition generates less welfare than monopoly if with competition all consumers single-home, but competition generates more welfare if a mixed-homing equilibrium emerges, because multi-homing is not expensive and it increases network effects.

Three more points are worth making here. First, we assumed $v = f_1$. For $v > f_1$ similar arguments to those above apply and welfare results resemble what we have presented here. However, when $v < f_1$ platform membership becomes less attractive, especially for those with large values of τ . As a result, a monopoly platform's market coverage is likely to be smaller than that occurring in competition and so competition is more likely to generate higher welfare than monopoly.

Second, we are assuming that marginal returns for the other side of the platform are constant. In many instances consumers have decreasing marginal returns from the number of apps available on their smartphone, and similarly the value of additional games declines as enjoying each game requires an investment in time. Overall decreasing marginal valuations reduces surplus for both market structures. However, there are no price effects in competition (prices are already competed to the lowest levels) whereas the monopoly has less incentive to facilitate entry of marginal agents and so the change in prices further destroys surplus (see Tremblay (2014)).

Finally, we assumed homogeneous platforms. If platforms are quite different, then the standard issues surrounding trade-offs between increased consumer surplus due to greater differentiation and choice on the one hand, and higher prices due to dampened competition on the other hand add complexity to the analysis.

5 Conclusion

This paper contributes to the literature on platforms and two-sided markets by considering competition between two platforms where agents on both sides of the market, consumers and firms, are heterogeneous. Consumers and firms choose whether to single-home or multi-home, leading to multiple equilibrium allocations that mirror constellations observed in many markets. Competition leads to similar pricing and market shares in equilibrium, but homing decisions can vary.

In one equilibrium allocation all consumers single-home, whereas all firms multi-home. This equilibrium configuration always exists and it mirrors what is seen in the market for smartphones: virtually all consumers use only one smartphone, and almost all apps are available across smartphone providers.

When network effects are strong enough, another type of equilibrium allocation emerges in which there is a mix of multi-homing and single-homing on both sides of the platforms. This is the constellation that is found in the market for video game consoles. While many consumers have only one console, serious gamers often have more than one system; and while some games are available across providing platforms, others are exclusive to one system.

The model admits welfare comparisons across levels of competition; a monopoly platform and two competing platforms. We find that unless there exists sufficient economies of scale in synchronization costs from multi-homing by firms, a monopoly platform leads to greater welfare than two competing platforms when all consumers single-home and all firms multi-home. In other words, the benefits of competition only come to bear when firms experience sufficient scale economies when multi-homing. However, when network effects are inherently strong and an equilibrium allocation results in a mix between single- and multi-homing, then competition can increase welfare even when there are no scale economies in production. Thus, the endogenous homing decisions that determine the allocation that occurs in equilibrium

play a key role in the welfare analysis of platform competition.

Appendix of Proofs

Proof of Lemma 1 Note that if for some θ , $u_2^A(\theta) \lesseqgtr u_2^B(\theta)$, then this holds for all θ ; and because $u_2(\cdot)$ is linear, there exist three mutually exclusive and exhaustive relations in comparing $u_2^A(\theta)$ to $u_2^B(\theta)$, for all θ , which are covered by the following two cases:

1. $u_2^X(\theta) > u_2^Y(\theta)$ which implies that firms that join a platform will only join Platform X : $n_2^X = N_2$ and $n_2^Y = 0$; $X, Y = A, B$; $X \neq Y$ for all $p_2^X, p_2^Y \geq 0$.
2. $u_2^A(\theta) = u_2^B(\theta)$ for all θ which by Assumption 1 implies $n_2^A = n_2^B$ for all $p_2^X, p_2^Y \geq 0$.

Thus, $n_2^X \in \{0, n_2^Y, N_2\}$ for $X = A, B$ and $Y \neq X$ for all $p_2^X, p_2^Y \geq 0$.

As for $n_2^A = n_2^B$, the ‘only if’ follows directly from Assumption 1; and for the ‘if’ part, notice from above that $n_2^A = n_2^B$ can only occur when $u_2^A(\theta) = u_2^B(\theta)$. \square

Proof of Proposition 1 If $p_i^X = p_i^Y$ then by Assumption 2 $n_2^X, n_2^Y > 0$. By Lemma 1 $n_2^X = n_2^Y$ in the allocation equilibrium. This implies $u_1^X(\tau) - u_1^Y(\tau) = 0, \forall \tau$. By Assumption 1 then $n_1^X = n_1^Y$. Thus, $n_i^X = n_i^Y$ is the unique allocation equilibrium.

Consider the homing decisions. All τ with $u_1^{AB}(\tau) > u_1^X(\tau)$ and $u_1^{AB}(\tau) > 0$ multi-home. Equations (2) and (3) imply $0 < \delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1$ and $0 < v + \alpha_1(\tau)N_2 - p_1$. However, $\delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1 \leq v + \alpha_1(\tau)N_2 - p_1$, so the first equation implies the second. So all $\tau < \alpha_1^{-1} \left(\frac{p_1 - \delta v}{n_2^Y - n_2^m} \right)$ multi-home.

All τ with $u_1^X(\tau) > u_1^{AB}(\tau)$ and $u_1^X(\tau) > 0$ single-home. Equations (2) and (3) now imply $0 > \delta v + \alpha_1(\tau)(n_2^Y - n_2^m) - p_1$ and $0 < v + \alpha_1(\tau)n_2^X - p_1$ and, if $p_1 > \delta v$ then for all $\delta < 1$ the set of single-homing consumers is nonempty since $v + \alpha_1(\tau)n_2^X > \delta v + \alpha_1(\tau)(n_2^Y - n_2^m)$. Thus, the set of single-homing consumers is $\tau \in \left[\alpha_1^{-1} \left(\frac{p_1 - \delta v}{n_2^Y - n_2^m} \right), \alpha_1^{-1} \left(\frac{p_1 - v}{n_2^X} \right) \right]$.

All θ with $u_2^{AB}(\theta) > u_2^X(\theta)$ and $u_2^{AB}(\theta) > 0$ multi-homing. Equations (4) and (5) now imply $\theta < \frac{\alpha_2(N_1)-2p_2}{(1+\sigma)c}$ and $\theta < \frac{\alpha_2(n_1^Y-n_1^m)-p_2}{\sigma c}$. Which inequality dominates depends on the parameters. Thus, the set of firms who multi-home is $\theta < \min \left\{ \frac{\alpha_2 \cdot (n_1^X+n_1^Y-n_1^m)-2p_2}{(1+\sigma)c}, \frac{\alpha_2 \cdot (n_1^Y-n_1^m)-p_2}{\sigma c} \right\}$.

All θ with $u_2^X(\theta) > u_2^{AB}(\theta)$ and $u_2^X(\theta) > 0$ single-homing. Equations (4) and (5) imply this occurs when $\theta \geq n_2^m$ and when $\theta \leq \frac{\alpha_2 n_1^X - p_2}{c}$. Thus, the total number of firms that single-home on platforms is $\theta \in \left[n_2^m, \frac{\alpha_2 n_1^X - p_2}{c} \right]$. \square

Proof of Proposition 2 If $p_1^X = p_1^Y$ and $p_2^X > p_2^Y$ then by Assumption 2 $n_2^Y > 0$. By Lemma 1 there are two possible allocations in equilibrium, $n_2^Y = n_2^X$ and $n_2^Y = N_2$ for $p_2^X \geq 0$.

1. Suppose $n_2^Y = n_2^X$. This with $p_2^Y < p_2^X$ and the Lemma implies $n_1^Y < n_1^X$. For consumer, $n_2^Y = n_2^X$ implies $u_1^Y(\tau) - u_1^X(\tau) = p_1^X - p_1^Y = 0$ for all τ since $p_1^Y = p_1^X$. By Assumption 1, this implies $n_1^Y = n_1^X$, a contradiction. Thus, $n_2^Y = n_2^X$ is not possible.
2. Suppose $n_2^Y = N_2$. For consumers, $u_1^Y(\tau) - u_1^X(\tau) = \alpha_1(\tau) \cdot N_2 + p_1^X - p_1^Y > 0$ for all τ . Thus, $n_1^Y = N_1$. For firms, $u_2^Y(\theta) - u_2^X(\theta) = \alpha_2 \cdot N_1 - p_2^Y + p_2^X > 0$ for all θ .

Thus, $n_2^Y = N_2$ and $n_1^Y = N_1$ is the unique allocation equilibrium.

Arguments for the allocations prescribed in $p_1^X > p_1^Y$ with $p_2^X > p_2^Y$ and $p_1^X > p_1^Y$ and $p_2^X = p_2^Y$ follow similarly. \square

Proof of Proposition 3 If $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ then by Lemma 1 there are three possible allocations in equilibrium, $n_2^Y = n_2^X$, $n_2^Y = N_2$, and $n_2^Y = 0$ for $p_2^X, p_2^Y \geq 0$.

1. Suppose $n_2^Y = n_2^X$. For consumers, $n_2^Y = n_2^X$ implies $u_1^X(\tau) - u_1^Y(\tau) = p_1^Y - p_1^X < 0$ for all τ since $p_1^X > p_1^Y$. This implies $n_1^Y = N_1$ and $n_1^X \geq 0$ depending on prices some consumers may join X and multi-home. For firms, the Lemma implies $u_2^X(\theta) = u_2^Y(\theta)$ for all θ . This implies $n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \geq 0$. Thus, if $n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} \geq 0$ holds then $n_2^Y = n_2^X$, $n_1^Y = N_1$, and $n_1^X \geq 0$ is a possible equilibrium allocation.

2. Suppose $n_2^Y = N_2$. For consumers, $u_1^Y(\tau) - u_1^X(\tau) = \alpha_1(\tau) \cdot N_2 - p_1^Y + p_1^X > 0$ for all τ . Thus, $n_1^Y = N_1$. Since all single-homing consumers and all firms join platform Y , consumers will join platform X and multi-home when $u_1^X(\tau) > u_1^{AB}(\tau)$. This occurs when $\delta v \geq p_1^X$. If $\delta v \geq p_1^X$ then all consumers join platform X , otherwise none will, so either $n_1^X = 0$ or $n_1^X = N_1$. For firms, the Lemma implies $u_2^X(\theta) < u_2^Y(\theta)$ for all θ . This implies $n_1^X < N_1 - \frac{p_2^Y - p_2^X}{\alpha_2} < N_1$. Thus, the only allocation equilibrium that can exist is $n_2^Y = N_2$, $n_1^X = 0$, and $n_1^Y = N_1$ with $\delta v < p_1^X$.
3. Suppose $n_2^Y = 0$. Since $n_2^Y = 0$ it must be that $n_2^X = N_2$ for $p_2^X, p_2^Y \geq 0$. If $p_1^Y > v$ then consumers that join a platform will join only platform X , so $n_1^X = N_1$ and $n_1^Y = 0$. When $v \geq p_1^Y > \delta v$ we have $u_1^X(\tau) - u_1^Y(\tau) = \alpha_1(\tau) \cdot N_2 - p_1^X + p_1^Y \geq 0$. Thus there exists τ' such that consumers $\tau \in [0, \tau']$ join Platform X and consumers $\tau \in (\tau', N_1]$ join Platform Y . This implies $n_1^X, n_1^Y > 0$ with no multi-homing. Lastly, when $\delta v \geq p_1^Y$ we have $n_1^Y = N_1$. For firms, since $n_2^Y = 0$ it must be that $u_2^X(\theta) > u_2^Y(\theta)$ for all θ . This implies $n_1^X > N_1 - \frac{p_2^Y - p_2^X}{\alpha_2}$ join platform X and multi-home. Thus, this allocation equilibrium is possible and for all price levels of p_1^Y .

Thus, with these prices we have three possible allocation equilibria. □

Proof of Theorem 1 Given these prices it is clear that both platforms make zero profits.

When $p_i^X \geq p_i^Y$, $i = 1, 2$, with at least one inequality being strict. If $\Pi^Y > 0$ then Platform X will undercut its prices. If $\Pi^Y = 0$ then it will increase its price but still undercut Platform X 's prices.

When the platforms set equal prices on both sides of the platforms then $p_i^X = p_i^Y = f_i$ is the only equal price constellation where neither platform has an incentive to deviate.

When $p_1^X > p_1^Y$ and $p_2^X < p_2^Y$ there are three possible allocations we must check from Proposition 3. Some equations used below are from the proof of Proposition 3.

1. When $n_2^X = n_2^Y = n_2$, $n_1^Y = N_1$, and $n_1^X = N_1 - \frac{p_2^Y - p_2^X}{\alpha_2}$. This is an equilibrium when

$\Pi^X = \Pi^Y$ since otherwise the lower profit platform will deviate. Thus, $n_2(p_2^Y - p_2^X) = N_1(p_1^Y - f_1) - (N_1 - \frac{p_2^Y - p_2^X}{\alpha_2})(p_1^X - f_1)$. However, both platforms have an incentive to deviate by raising their lower price to just undercutting the other platforms price on that side of the market. Thus, this cannot be an equilibrium.

2. When $n_1^Y = N_1$ and $n_2^Y = N_2$. In this case when $\Pi^X \geq 0$ it must be that $\Pi^Y > \Pi^X \geq 0$. Thus, Platform X always has an incentive to deviate. This allocation cannot be an equilibrium.

3. When $n_2^Y = 0$ and $n_2^X = N_2$. If $p_1^Y > \delta v$ then $n_1^Y = 0$ and either $\Pi^X > 0$ and Platform Y has an incentive to deviate or $\Pi^X \leq 0$ and Platform X has an incentive to deviate. If $v \geq p_1^Y > \delta v$ then $n_1^X, n_1^Y > 0$ with no multi-homing consumers. However, for all $\Pi^X \geq \Pi^Y$ both platforms have an incentive to raise their lower price to just less than the other platforms price on that side. Thus, this cannot be an equilibrium.

If $\delta v \geq p_1^Y$ then $n_1^Y = N_1$ and $n_1^X > N_1 - \frac{p_2^Y - p_2^X}{\alpha_2}$ and Platform X always has an incentive to increase its price p_2^X to just below p_2^Y . This allocation cannot be an equilibrium.

Thus, the unique set of prices that occurs in equilibrium is $p_1^A = p_1^B = f_1$ and $p_2^A = p_2^B = f_2$.

We now show the equilibrium allocations for general symmetric prices p_1 and p_2 .

Allocation I: Allocations (7) imply all firms multi-home when all consumers single-home, since $n_2^m > N_2$ i.e. $[n_2^m, N_2]$ is empty when $n_1^m = 0$. Furthermore, when $n_2^m = n_2^A = n_2^B$, allocation (6) implies no consumer multi-homes. Hence, all consumers single-home if and only if all firms multi-home. Thus, the allocation where all firms multi-home and all consumers single-home is a Nash Equilibrium.

Allocation II: Since $p_1 > \delta v$, allocation (6) implies the set of multi-homing consumers is non-empty when the number of multi-homing firms is not too large. Let $x \in [0, 1]$ be the percent of consumers who multi-home of those n_1^X who join platform X so that in expectation

$n_1^m = xn_1^X = xn_1^Y$. This implies $N_1 = (2-x)n_1^X$ since $N_1 = n_1^X + n_1^Y - n_1^m$ and in expectation $n_1^X = n_1^Y$. From the Allocation *I* $x > 0$ occurs when not all of the firms are multi-homing. This occurs when $\min \left\{ \frac{\alpha_2 \cdot (2-x)n_1^X - 2p_2}{(1+\sigma)c}, \frac{\alpha_2 \cdot (1-x)n_1^Y - p_2}{\sigma c} \right\} < \frac{\alpha_2 n_1^X - p_2}{c}$.

In the remainder of this proof we assume $\sigma = 1$, no economies of scale. Using allocation (7) there exists x^m such that for $x > x^m$ no firm will multi-home. Allocation (7) implies $0 = \alpha_2(1-x^m)n_1^Y - p_2$. Thus, $x^m = 1 - \frac{p_2}{\alpha_2 n_1^Y}$. And for all $x > x^m$ no firm multi-homes. Note, $p_2 < \alpha_2 n_1^Y$ since otherwise the market collapses, hence $x^m \in (0, 1)$.

If $0 < x < x^m$ then some firms will single-home and some firms will multi-home. Allocation (7) implies $n_2^m = \frac{\alpha_2(1-x)n_1^Y - p_2}{c}$ and allocation (7) implies $n_2^Y = (1/2)(N_2 + n_2^m) = (1/2c)[\alpha_2(2-x)n_1^X - 2p_2]$. Similarly, allocation (6) defines the number of multi-homing consumers: $0 = \delta v + \alpha_1(n_1^m)(n_2^Y - n_2^m) - p_1$; using this equation and the equations for n_2^m , n_2^Y , and $n_1^m = xn_1^X = xn_1^Y$ we can characterize x by:

$$\begin{aligned} 0 &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2(2-x)n_1^X - 2p_2 - 2\alpha_2(1-x)n_1^Y + 2p_2] - p_1 \\ &= \delta v + \alpha_1(xn_1^X)(1/2c)[\alpha_2 \cdot xn_1^X] - p_1, \end{aligned} \quad (25)$$

Furthermore, allocation (6) defines N_1 , the number of consumers on Platform *X*: $0 = v + \alpha_1(N_1)n_2^X - p_1$. Thus we have:

$$0 = v + \alpha_1(N_1)n_2^X - p_1 = v + \alpha_1((2-x)n_1^X)(1/2c)(\alpha_2 \cdot (2-x)n_1^X - 2p_2) - p_1. \quad (26)$$

Thus, we have two equations (25) and (26) and two unknowns, x and n_1^X . If the solution is $x \in (0, x^m)$ then we have a Nash Equilibrium. Note, this equilibrium does not exist when $x \notin (0, x^m)$.

Allocation III: Allocation (7) implies all firms single-home when the number of multi-homing consumers is relatively large, $n_1^Y \leq n_1^m + p_2/\alpha_2$. If $p_2 = 0$, then this holds when all

consumers multi-home. By allocation (6), this will only be an equilibrium when $v = 0$. If $p_2 > 0$, then allocation (6) implies there exists an equilibrium where all firms single-home and a large portion of consumers multi-home given prices such that $N_1 - n_1^m = \alpha_1^{-1} \left(\frac{p_1 - v}{n_2^X} \right) - \alpha_1^{-1} \left(\frac{p_1 - \delta v}{n_2^X} \right) \leq \frac{2p_2}{\alpha_2}$.

Thus, there exists at least one and potentially three allocations that occur in equilibrium with unique equilibrium prices $p_1^X = p_1^Y = f_1$ and $p_2^X = p_2^Y = f_2$. \square

Proof of Theorem 2 Prices must be set equally in equilibrium follows as in Theorem 1. The only price constellation where neither platform has an incentive to deviate is $p_1^X = p_1^Y = \delta v$ and $p_2^X = p_2^Y = f_2$. At any $p_1 < \delta v$ both platforms will increase their price. If $p_1 > \delta v$ then both platforms will undercut. Similarly for any $p_2 \neq f_2$. The resulting profits are $\Pi^X = n_1^X(p_1^X - f_1) + n_2^X(p_2^X - f_2) = \bar{N}_1(\delta v - f_1) > 0$. \square

Proof of Theorem 3 A monopoly corner solution occurs when $\alpha_2 \geq a$. Using the welfare equations (20) and (24), $W^{AI} < W^{MC}$ occurs when $\sigma > \frac{a+2\alpha_2}{3a+2\alpha_2}$. A monopoly interior solution occurs when $\alpha_2 < a$. Using welfare equations (14) and (24), $W^{AI} > W^{MI}$ occurs when $\sigma > \frac{64\alpha_2 a^2}{5(a+\alpha_2)^3} - 1$. \square

Proof of Theorem 4 We first show that Allocation *II* in Theorem 1 exists when $\frac{b(1-\delta)vc}{a^2\alpha_2} \in (0, \frac{1}{8})$: Equations (25) and (26) imply we have two equations and two unknowns, x and n_1^A . Solving these equations implies x is implicitly defined by: $t \equiv \frac{b(1-\delta)vc}{a^2\alpha_2} = \frac{(1-x)x}{(2-x)^2}$. This implies $0 = (1+t)x^2 - (1+4t)x + 4t$. Solving for x as a function of t and using the quadratic formula such that $x \in (0, 1)$ implies we must have $t \in (0, \frac{1}{8})$.

Consider now the Theorem. When $x = 1/2$, equations (25) and (26) imply half of firms and a third of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly interior solution if and only if $0 > 135a^4 - 484a^3\alpha_2 - 150a^2\alpha_2^2 + 540a\alpha_2^3 + 135\alpha_2^4$. This occurs when $\alpha_2 \in [h \cdot a, g \cdot a]$ where g and h are irrational numbers with $g \approx .8274$ and $h \approx .2768$. However, the welfare for $x = 1/2$ is never greater than the

monopoly corner solution.

When $x = .9$, equations (25) and (26) imply a tenth of firms and (8/11)s of consumers will multi-home. The welfare from this allocation is greater than the welfare from the monopoly corner solution for all $\alpha_2 \geq a$ since $3.3388\alpha_2 + 3.5823a > 3\alpha_2 + 2a$. \square

References

- Ambrus, A., Calvano, E., and Reisinger, M. (2014). Either or both competition: A “two sided” theory of advertising with overlapping viewerships. *Working Paper*.
- Anderson, S. P., Foros, Ø., and Kind, H. J. (2013). Competition for advertisers and for viewers in media markets. *Working Paper*.
- Armstrong, M. (2006). Competition in two-sided markets. *RAND Journal of Economics*, 37(3):668–91.
- Athey, S., Calvano, E., and Gans, J. S. (2014). The impact of the internet on advertising markets for news media. *Working Paper*.
- Bresnahan, T., Orsini, J., and Yin, P.-L. (2014). Platform choice by mobile app developers. *Mobile Innovation Group Working Paper*.
- Caillaud, B. and Jullien, B. (2003). Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics*, 34(2):309–28.
- Deltas, G. and Jeitschko, T. (2007). Auction hosting site pricing and market equilibrium with endogenous bidder and seller participation. *International Journal of Industrial Organization*, 25:1190–1212.
- Economides, N. (1996). The economics of networks. *International Journal of Industrial Organization*, 14(6):673–99.

- Ellison, G. and Fudenberg, D. (2003). Knife-edge or plateau: When do market models tip? *Quarterly Journal of Economics*, 118(4):1249–78.
- ESRP (2010). How much do you know about video games? Entertainment Software Rating Board. Web. <http://www.esrb.org/about/video-game-industry-statistics.jsp>.
- Evans, D. S. (2003). The antitrust economics of multi-sided platform markets. *Yale Journal on Regulation*, 20(2):325–81.
- Gabszewicz, J. and Wauthy, X. (2014). Vertical product differentiation and two-sided markets. *Economics Letters*, 123:58–61.
- Hagiu, A. (2006). Pricing and commitment by two-sided platforms. *RAND Journal of Economics*, 37(3):720–37.
- Hagiu, A. and Hałaburda, H. (2014). Information and two-sided platform profits. *International Journal of Industrial Organization*, 34C:25–35.
- Hałaburda, H. and Yehezkel, Y. (2013). Platform competition under asymmetric information. *American Economic Journal: Microeconomics*, 5(3):22–68.
- Jullien, B. (2011). Competition in multi-sided markets: Divide and conquer. *American Economic Journal: Microeconomics*, 3(4):186–219.
- Jullien, B. and Pavan, A. (2014). Platform pricing under dispersed information. *Working Paper*.
- Katz, M. L. and Shapiro, C. (1985). Network externalities, competition, and compatibility. *American Economic Review*, 75(3):424–40.
- Lee, R. (2013). Vertical integration and exclusivity in platform and two-sided markets. *American Economic Review*, 103(7):2960–3000.

- Nielsen (2014a). How smartphones are changing consumers' daily routines around the globe. <http://www.nielsen.com/content/corporate/us/en/insights/news/2014/how-smartphones-are-changing-consumers-daily-routines-around-the-globe.html>.
- Nielsen (2014b). Smartphones: So many apps, so much time. Web. <http://www.nielsen.com/us/en/insights/news/2014/smartphones-so-many-apps--so-much-time.html>.
- Reisinger, M. (2014). Two-part tariff competition between two-sided platforms. *European Economic Review*, 68:168–180.
- Roberts, J. A., Yaya, L. H. P., and Manolis, C. (2014). The invisible addiction: Cell-phone activities and addiction among male and female college students. *Journal of Behavioral Addictions*.
- Rochet, J.-C. and Tirole, J. (2003). Platform competition in two-sided markets. *Journal of the European Economic Association*, 1(4):990–1029.
- Rochet, J.-C. and Tirole, J. (2006). Two-sided markets: A progress report. *RAND Journal of Economics*, 37(3):645–67.
- Smith, A. (2012). The best and worst of mobile connectivity. Pew Research Internet Project. <http://www.pewinternet.org/2012/11/30/the-best-and-worst-of-mobile-connectivity/>.
- Tremblay, M. J. (2014). Vertical relationships between platforms and providers. *Working Paper*.
- Weyl, E. G. (2010). A price theory of multi-sided platforms. *American Economic Review*, 100:1642–72.
- White, A. and Weyl, E. G. (2013). Insulated platform competition. *Working Paper*.