



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-LEP-RF/85-41

POINT CHARGE PASSING A RESONATOR WITH BEAM TUBES

H. Henke

Abstract

Charged particle beams interact with the surroundings because of the co-travelling electromagnetic fields. This interaction limits the beam. One of the most interesting cases is a point charge travelling with arbitrary velocity along the axis of a cylindrical resonator with two semi-infinite tubes. The structure consists of two regions: the tube region, where the fields are given by the source fields plus a continuous spectrum of waveguide modes, and the resonator region with a discrete set of modes. Matching the fields at the common interface yields a system of linear equations for the expansion coefficients. The occurring integrals contain fractions of the Bessel function and its derivative, which were expanded into algebraic series of Bessel function zeros, and are solved by means of the residuum calculus. Numerical results are given for the coupling impedance between charge and structure and for the energy loss of a Gaussian charge distribution.

Geneva, Switzerland  
November 1985

## POINT CHARGE PASSING A RESONATOR WITH BEAM TUBES

H. Henke, CERN, 1211 Geneva 23, Switzerland

Abstract

Charged particle beams interact with the surroundings because of the co-travelling electromagnetic fields. This interaction limits the beam. One of the most interesting cases is a point charge travelling with arbitrary velocity along the axis of a cylindrical resonator with two semi-infinite tubes. The structure consists of two regions: the tube region, where the fields are given by the source fields plus a continuous spectrum of waveguide modes, and the resonator region with a discrete set of modes. Matching the fields at the common interface yields a system of linear equations for the expansion coefficients. The occurring integrals contain fractions of the Bessel function and its derivative, which were expanded into algebraic series of Bessel function zeros, and are solved by means of the residuum calculus. Numerical results are given for the coupling impedance between charge and structure and for the energy loss of a Gaussian charge distribution.

1. Introduction

Beams of charged particles, as in travelling wave tubes, klystrons and accelerators, interact with the surroundings by reason of the co-travelling electromagnetic fields. Modern high-current accelerators and storage rings, which have tightly bunched beams with peak currents of about 1 KA and bunch lengths between several millimetres and centimetres, are especially limited by this interaction. The only way to reduce this interaction is to make the vacuum vessel of the beam as smooth as possible. But even the best design needs some non-smooth components like flanges, transitions, resonators etc.,

where the homogeneous fields of the beam will be scattered and act back on the beam. These fields are called wake fields. They are functions of space and time. If the beam energy is sufficiently high, the motion of the particles changes only slowly compared to the time it takes to pass a certain component. Then the beam can be taken to be stiff and only the fields integrated over the total passage are of interest. They are called wake potentials and are a function of the position only. Their Fourier transforms are the coupling impedances between the beam and a specific component.

The present paper treats analytically one of the most interesting arrangements: a point charge travelling with arbitrary velocity  $v$  along the axis of a cylindrical resonator with two semi-infinite tubes (Fig. 1). Depending on the dimensions the structure represents either an accelerating or a decelerating resonator, a gap of a vacuum flange or one undulation of a bellows. The cylindrical wall of the resonator,  $\rho = b$ ,  $-q < z < q$ , can also be removed thus making the charge pass a gap and radiating into an infinite radial line.

Considerable effort has already been put into the solution of similar problems. Most of the papers treat closed cylindrical resonators or two parallel plates so that a set of axial eigenmodes with time-dependent amplitudes allows for a solution. In case of a resonator with beam pipes the energy lost by radiation into the pipes has been estimated<sup>1)</sup> or calculated by using a high-frequency diffraction solution<sup>2,3)</sup>. A class of problems related in some way are the infinite periodic structures. Here the Floquet theorem permits the decomposition of the fields in space-harmonics thus easing either the field matching<sup>4)</sup> or the fulfilment of the boundary conditions on a smooth wall<sup>5,6,7)</sup>. All these methods suffer from quite strong restrictions, especially those for the infinite periodic structures which do not give the radiation loss into the pipes in case of a finite number of elements.

Certainly the most versatile tool is a purely numerical method<sup>8)</sup> if one is looking for a specific result. But on the other hand it does not allow approximate solutions or general parameter dependences to be found.

The method used in this paper is field matching. In the tube region,  $0 < \rho < a$ , the fields consist of the source fields plus a continuous spectrum of waveguide modes. In the resonator region,  $-g < z < g$ ,  $a < \rho < b$ , an infinite, discrete set of radially forward and backward travelling modes is used. In the case of the open radial line only forward travelling waves exist. At the common interface the fields have to be matched yielding an inhomogeneous set of linear equations for the expansion coefficients of the resonator region. While the match is being done, integrals have to be solved containing ratios of the Bessel function and its derivative. These ratios are expanded into series of algebraic terms of Bessel function zeros, which then can be integrated by means of the residuum calculus. Poles of the integrands were shifted so that waves propagating in the tubes always travel away from the resonator region. The coupling impedance is then purely imaginary below the pipe cut-off and complex above, the real part reflecting the radiation loss.

## 2. Fields in the Different Regions

Referring to Fig. 1 we separate the whole region into two sub-regions: region I called the tube region and region II called the resonator region. The general solution in region I is given by a particular solution of the inhomogeneous equations (source fields) plus the solution of the homogeneous equations represented by a continuous spectrum of waveguide modes. In the resonator region, delimited to  $-g \leq z \leq g$ , the eigenvalues in  $z$  direction are fixed and hence one gets a discrete set of modes.

Cylindrical coordinates  $\rho, \phi, z$  are used and all fields are Fourier transformed, i.e. they are proportional to  $\exp(j\omega t)$  which is omitted throughout the paper.

The charge  $Q$  travels with constant velocity  $v$  along the axis representing a current density

$$\vec{j} = \frac{Qv}{2\pi\rho} \delta(\rho)\delta(z-vt)\vec{e}_z \quad (1)$$

With  $\vec{j}$  one can easily derive the source fields in an ideal conducting infinite tube of radius  $a$ . One way, for instance, is to subject Maxwell's equations to a Fourier transform in  $t$  and  $z$  and to a Hankel transform in  $\rho$ . Solving the resulting equation for the source and source-free case in free space and superimposing both solutions so that  $E_z = 0$  on the tube wall yields the source fields in region I

$$E_z^S = j \frac{QZ_0}{2\pi} \frac{k}{\beta^2 \gamma^2} [K_0(\bar{\rho}) - K_0(\bar{a})I_0(\bar{\rho})/I_0(\bar{a})] e^{-jkz/\beta}$$

$$H_\phi^S = \frac{Q}{2\pi\gamma} \left| \frac{k}{\beta} \right| [K_1(\bar{\rho}) + K_0(\bar{a})I_1(\bar{\rho})/I_0(\bar{a})] e^{-jkz/\beta}$$
(2)

where

$Z_0 = \sqrt{\mu_0/\epsilon_0}$	free space impedance
$k = \omega/c_0$	$c_0$ velocity of light
$\beta = v/c_0$	
$\gamma^2 = 1/(1-\beta^2)$	relativistic factor
$I, K$	modified Bessel functions
$\bar{a} = \frac{a}{\gamma} \left  \frac{k}{\beta} \right $	$\bar{\rho} = \frac{\rho}{\gamma} \left  \frac{k}{\beta} \right $

For the source-free fields no boundary conditions exist in region I. The region is unbounded in  $z$  direction and for  $\rho = a$  the fields are unknown. The only requirement is regularity at  $\rho = 0$ . Thus, one has to sum over all radial ( $K$ ) or longitudinal ( $k_z$ ) wave numbers. We choose  $k_z$ , yielding

$$E_z^I = \int_{-\infty}^{\infty} F(k_z) \frac{J_0(K\rho)}{J_0(Ka)} e^{-jk_z z} dk_z$$

$$Z_0 H_\phi^I = -jk \int_{-\infty}^{\infty} F(k_z) \frac{J_0'(K\rho)}{KJ_0'(Ka)} e^{-jk_z z} dk_z, \quad K = \sqrt{k^2 - k_z^2}$$
(3)

In region II the boundary conditions are

$$E_\phi^{II}(z = \pm g) = 0, \quad E_z^{II}(\rho = b) = 0$$
(4)

resulting in a set of modes

$$E_z^{II} = \sum_{0,2} A_n \frac{R_0(K_n \rho)}{R_0(K_n a)} \cos k_{zn} z + \sum_{1,3} B_n \frac{R_0(K_n \rho)}{R_0(K_n a)} \sin k_{zn} z \quad (5)$$

$$Z_0 H_\phi^{II} = -jk \sum_{0,2} A_n \frac{R_0'(K_n \rho)}{K_n R_0(K_n a)} \cos k_{zn} z - jk \sum_{1,3} B_n \frac{R_0'(K_n \rho)}{K_n R_0(K_n a)} \sin k_{zn} z$$

with  $R_0(K_n \rho) = H_0^{(1)}(K_n b) H_0^{(2)}(K_n \rho) - H_0^{(2)}(K_n b) H_0^{(1)}(K_n \rho)$

$$K_n = \begin{cases} \sqrt{k^2 - k_{zn}^2} & \text{for } k > k_{zn} \\ -j \sqrt{k_{zn}^2 - k^2} & \text{for } k < k_{zn} \end{cases}, \quad k_{zn} = n\pi/2g$$

$H^{(1)}, H^{(2)}$  Bessel functions of the third kind.

In the case of the radiation problem, when region II is an open radial line, only the first condition of equation (4) exists. This means the waves are travelling outwards and

$$R_0(K_n \rho) = H_0^{(2)}(K_n \rho) \quad (6)$$

### 3. Field matching

In region I the complete solution is given by the source fields [equation (2)] and source-free fields [equation (3)] whereas in II we only take source-free fields [equation (5)]. The expansion coefficients  $A_n$ ,  $B_n$  and the amplitude function  $F(k_z)$ , all unknown, can now be determined by matching the tangential fields at the interface  $\rho = a$ . Since

$$E_z^S + E_z^I = \begin{cases} E_z^{II} & \text{for } -g < z < g, \rho = a \\ 0 & \text{for } |z| > g \end{cases} \quad (7)$$

we expand the  $E^I$  fields in terms of the coefficients  $A_n$ ,  $B_n$ ,

$$\begin{aligned}
 2\pi F(k_z) &= \sum_{0,2} A_n \int_{-g}^g \cos k_{zn} z e^{jkz} dz + \sum_{1,3} B_n \int_{-g}^g \sin k_{zn} z e^{jkz} dz = \\
 &= 2k_z \sin k_z g \sum_{0,2} (-)^{\frac{n}{2}} \frac{A_n}{k_z^2 - k_{zn}^2} - j2k_z \cos k_z g \sum_{1,3} (-)^{\frac{n-1}{2}} \frac{B_n}{k_z^2 - k_{zn}^2} \quad (8)
 \end{aligned}$$

Substituting  $F(k_z)$  into (3), one obtains for the tangential magnetic field in region I

$$Z_0 H_{\phi}^I = -j \frac{k}{\pi} \sum_{0,2} (-)^{\frac{n}{2}} A_n I s_n - \frac{k}{\pi} \sum_{1,3} (-)^{\frac{n-1}{2}} B_n I c_n \quad (9)$$

where the integrals

$$\begin{Bmatrix} I c_n \\ I s_n \end{Bmatrix} = \int_{-\infty}^{\infty} \begin{Bmatrix} \cos k_z g \\ \sin k_z g \end{Bmatrix} \frac{k_z}{k_z^2 - k_{zn}^2} \frac{J_0'(k)}{K J_0(k)} e^{-jk_z z} dk_z$$

are solved in the appendix. Here and in the following all quantities with a length dimension are normalized with respect to  $a$ .

The tangential magnetic field is undetermined on the tube wall and has to be continuous at the interface

$$H_{\phi}^S + H_{\phi}^I = H^{II} \quad \text{for} \quad -g \leq z \leq g, \quad \rho = a \quad (10)$$

which requires the expansion of  $H_{\phi}^I$  in terms of the eigen-functions of  $H_{\phi}^{II}$ , e.i. multiplying (10) with  $\cos k_{zp} z$  or  $\sin k_{zp} z$  and integrating over  $-g \leq z \leq g$ . This yields after substitution of (2), (5), (9) into (10)

$$\begin{aligned}
 -jk \frac{R_0'(K_p)}{K_p R_0(K_p)} A_p (1 + \delta_p^o) g &= \frac{QZ_0}{2\pi} \frac{1}{I_0(\bar{a})} \int_{-g}^g \cos k_{zp} z e^{-jkz/\beta} dz - \\
 &- j \frac{k}{\pi} \sum_{0,2} \frac{n}{2} A_n \int_{-g}^g I_s n \cos k_{zp} z dz, \quad p = 0, 2, \dots
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 -jk \frac{R_0'(K_p)}{K_p R_0(K_p)} B_p g &= \frac{QZ_0}{2\pi} \frac{1}{I_0(\bar{a})} \int_{-g}^g \sin k_{zp} z e^{-jkz/\beta} dz - \\
 &- \frac{k}{\pi} \sum_{1,3} \frac{n-1}{2} B_n \int_{-g}^g I_c n \sin k_{zp} z dz, \quad p = 1, 3, \dots
 \end{aligned}$$

or together with (A3)

$$\begin{aligned}
 (1 + \delta_p^o) \frac{g}{K_p} \frac{J_0'(K_p)}{J_0(K_p)} - \frac{R_0'(K_p)}{R_0(K_p)} A_p + j2 \sum_{0,2} \frac{n+p}{2} C_{pn} A_n &= \\
 = -\frac{j}{\pi} (-)^{\frac{p}{2}} QZ_0 \frac{\beta}{I_0(\bar{a})} \frac{\sin(kg/\beta)}{k^2 - (k_{zp}\beta)^2}, \quad p = 0, 2, \dots
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 \frac{g}{K_p} \frac{J_0'(K_p)}{J_0(K_p)} - \frac{R_0'(K_p)}{R_0(K_p)} B_p - j2 \sum_{1,3} \frac{n+p}{2} S_{pn} B_n &= \\
 = \frac{1}{\pi} (-)^{\frac{p-1}{2}} QZ_0 \frac{\beta}{I_0(\bar{a})} \frac{\cos(kg/\beta)}{k^2 - (k_{zp}\beta)^2}, \quad p = 1, 3, \dots
 \end{aligned}$$

where



$$\left\{ \begin{array}{l} C_{pn} \\ S_{pn} \end{array} \right\} = \sum_{s=1}^S \frac{\beta_s (1 \mp \exp(-j2\beta_s g))}{(K_p^2 - j_{0s}^2)(K_n^2 - j_{0s}^2)} - j \sum_{s=S+1}^{\infty} \frac{\alpha_s (1 \mp \exp(-2\alpha_s g))}{(K_p^2 - j_{0s}^2)(K_n^2 - j_{0s}^2)}$$

$Z_0, k, \beta, \bar{a}$  are defined in equation (2);  $K_n, k_{zn}, R_0$  in equation (5), and  $\alpha_s, \beta_s$  in equation (A1).

Equation (12) is an inhomogeneous system of linear equations which determines the coefficients  $A_n, B_n$  and thus the fields in region II through equation (5) and in region I through equation (9).

#### 4. Derivation of coupling impedance and energy loss

The wake potential is defined as the integrated decelerating voltage acting on a probing particle which travels with the same velocity as the exciting charge a distance  $\Delta z = \tau v$  behind

$$w(\tau) = -\frac{1}{Q} \int_{-\infty}^{\infty} E_z(\rho, t = z/v + \tau) dz \quad (13)$$

Taking the Fourier transform of (13) and of  $E_z$  one finds

$$\begin{aligned} QW(\omega) &= - \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\tau E_z(t = z/v + \tau) e^{-j\omega\tau} = \\ &= - \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\Omega e^{j\Omega z/v} E_z(\Omega) \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau e^{j(\Omega - \omega)\tau} \end{aligned}$$

where the last term on the right side is equal to the  $\delta$  function. Thus one obtains

$$QW(\omega) = - \int_{-\infty}^{\infty} E_z(\omega) e^{j\omega z/v} dz = - \int_{-\infty}^{\infty} E_z(\omega) e^{jkz/\beta} dz \quad (14)$$

In equation (14)  $E_z$  consists of  $E_z^S$  and  $E_z^I$ . The integral over  $E_z^S$  gives the static repelling voltage between the exciting and the probing charge. It is identical to that in an infinite homogeneous tube and has no physical meaning here. The voltage we are interested in is that coming from the fields scattered by the resonator, i.e. the integral over  $E_z^I$  only:

$$QW(\omega) = - \int_{-\infty}^{\infty} dk_z F(k_z) \frac{J_0(K\rho)}{J_0(Ka)} \int_{-\infty}^{\infty} dz e^{j(k/\beta - k_z)z} = - 2\pi \frac{I_0(\bar{\rho})}{I_0(a)} F\left(\frac{k}{\beta}\right) \quad (15)$$

Now, substituting (8) into (15), we obtain

$$W(\omega) = - 2 \frac{k\beta}{Q} \frac{I_0(\bar{\rho})}{I_0(\bar{a})} \left[ \sin(kg/\beta) \sum_{0,2} (-)^{\frac{n}{2}} \frac{A_n}{k^2 - (k_{zn}\beta)^2} - \right. \\ \left. - j \cos(kg/\beta) \sum_{1,3} (-)^{\frac{n-1}{2}} \frac{B_n}{k^2 - (k_{zn}\beta)^2} \right] \quad (16)$$

The Fourier transform of the wake potential equals the ratio of voltage over current, both taken as time and spatial ( $z$  coordinate) Fourier components. It is the impedance seen by a unit current in  $z$  direction varying as  $\exp j(\omega t - kz/\beta)$ .

The energy loss of a real charge distribution (finite length) is given by

$$\Delta u = \frac{1}{2\pi} \int_{-\infty}^{\infty} W(\omega) |I(\omega)|^2 d\omega = \frac{1}{\pi} \int_0^{\infty} |I(\omega)|^2 \text{Re}W(\omega) d\omega \quad (17)$$

where  $I(\omega)$  is the Fourier transform of the current. For instance a Gaussian charge distribution with rms value  $\sigma$  and velocity  $v$  experiences an energy loss of

$$\Delta u_{\text{Gauss}} = \frac{Q^2}{\pi} \int_0^{\infty} e^{-(\omega\sigma/v)^2} \text{Re}W(\omega) d\omega \quad (18)$$

## 5. Numerical results

For some special cases such as short gap length of low frequency approximate formulae can be derived for the coefficients  $A_n$ ,  $B_n$ . In general, however, the system of linear equations (12) has to be solved. For this purpose a computer code ICYRP (Impedance of Cylindrical Resonator with Pipes) has been written which truncates the system to a finite size in order to solve for the coefficients.

Some results obtained by ICYRP are presented in the following. Fig. 2 shows the impedance (Fourier transformed wake potential) of a radial line for different energies of the particle beam. Due to the modified Bessel function in equ. (12), the impedance decays with frequency faster for low than for high energy beams. From the impedance point of view a beam starts to have high energy from somewhere around  $\gamma = 2ka$  on or at least  $\gamma = 3$ . This means for higher energies the impedance does not change any more. The notches in  $\text{Re}(Z)$  or the rapid changes in  $\text{Im}(Z)$  happen at the cut-off frequencies,  $ka = n\pi a/2g$ ,  $n = 0, 1, 2, \dots$ , of the radial line. Note that the lowest mode,  $n = 0$ , is a TEM-mode with vanishing cut-off. Thus the radiation and hence the losses into the radial line already start at low frequencies. In Fig. 3 the energy loss is given for a Gaussian shaped charge distribution as a function of the gap width.

In a closed resonator (Fig. 1a) the situation is quite different. At firstly, radiation can occur only into the pipes with a lowest cut-off frequency of  $ka = j_{01} = 2.405$ . Below,  $\text{Re}(Z)$  is zero. Secondly, there exist resonances. Figures 4 and 5 show the impedance of a 350 MHz resonator as it is used for instance in particle accelerators. Below  $ka = j_{01}$  the impedance is purely reactive. The phase jumps occur at the resonance frequencies of loss-free modes captured in the resonator. Since the pipes represent only small perturbations these modes are very close to the modes of a closed cylindrical resonator, a so-called pill-box (see Table I). Above cut-off (Fig. 5) the impedance is complex. The real part represents the radiation loss into the pipes. Again resonant modes exist. For frequencies only slightly higher than cut-off the mode is well trapped with small losses and high Q-values. With increasing frequency the losses increase exponentially and the Q's degrade very rapidly. The energy loss for short Gaussian bunches can be taken from Fig. 3. If the time light needs to travel from one corner to the outer wall and then to the other corner is longer than the

charge needs to pass the gap, the charge does not see the resonator wall and experiences the same loss as in a radial line configuration. This means charge distributions of total length smaller than  $2(\beta\sqrt{d^2+g^2}-g)$  (here 392 mm) cannot distinguish between the resonator and radial line structure.

TABLE I : Resonant modes of an open/closed cylindrical resonator  
( $a = 50$  mm,  $2g = 198$  mm,  $d = 278$  mm).

$$\text{Closed resonator } (ka)_{ns} = \sqrt{(n\pi a/2g)^2 + (j_{0s}a/b)^2}$$

n	s	$(ka)_{ns}$	ka open resonator (Fig. 4)
0	1	0.367	0.367
0	2	0.842	0.842
1	1	0.874	0.875
1	2	1.157	1.165
0	3	1.319	1.335
1	3	1.539	1.557
2	1	1.629	1.629
2	2	1.796	1.797
0	4	1.798	1.815
1	4	1.965	1.989
2	3	2.063	2.075
0	5	2.276	2.285
2	4	2.398	2.401
1	5	2.410	2.409

A last example is a very small resonator (Fig. 6). It corresponds to one undulation of a bellows used as compensation element in the vacuum pipe of an accelerator. Apparently the impedance consists of three different parts. A broad band resonator with resonance around  $ka = 13$  and a Q of about 5. This corresponds to a resonance on a radial line of length  $d \approx \lambda/4$  which is heavily damped by radiation into the pipes. In series to the broad band impedance one finds several high Q resonators, in this case four, belonging to well trapped modes in the region of the cross-section enlargement. Finally, there is some sort of a notch filter in parallel to these resonant circuits. The notches are located exactly at  $ka = j_{0s}$ ,  $s = 1, 2$ , the cut-off frequencies of the pipes. They are due to the fact that for  $2g \ll \lambda$  only the z-independent mode is excited in the resonator region and consequently the waves with  $k_z^2 = k^2 - j_{0s}^2 = 0$  fulfill the boundary condition in the pipe region. Otherwise spoken there are no secondary fields excited at cut-off.

APPENDIX

In order to solve the integrals in equation (9), we normalize all quantities with respect to  $a$ , i.e.  $a \rightarrow 1$ . The ratio Bessel function derivative over Bessel function can be expanded in an algebraic series over Bessel function zeros [see for instance (9)]

$$\frac{J_0'(K)}{KJ_0(K)} = -2 \sum_{s=1}^{\infty} \frac{1}{k_z^2 - b_s^2} \quad (A1)$$

with

$$b_s = \begin{cases} \beta_s = \sqrt{k^2 - j_{0s}^2} & , \quad k > j_{0s} & , \quad s \leq S \\ j\alpha_s = j \sqrt{j_{0s}^2 - k^2} & , \quad k < j_{0s} & , \quad s > S \end{cases}$$

With (A1) the integrals  $I_{c_n}$ ,  $I_{s_n}$  can be written as

$$\begin{aligned} \begin{Bmatrix} I_{c_n} \\ jI_{s_n} \end{Bmatrix} &= - \sum_s \left[ \int_{-\infty}^{\infty} \frac{k_z \exp[j(g-z)k_z]}{(k_z^2 - k_{zn}^2)(k_z^2 - b_s^2)} dk_z \pm \int_{-\infty}^{\infty} \frac{k_z \exp[-j(g+z)k_z]}{(k_z^2 - k_{zn}^2)(k_z^2 - b_s^2)} dk_z \right] \\ &= - \sum_s \{ I_{ns}^- \pm I_{ns}^+ \} \end{aligned} \quad (A2)$$

The integrand in  $I^-$  converges in the upper/lower half plane for  $z < g$  and  $z > g$  respectively whereas the integrand in  $I^+$  converges in the upper/lower half plane for  $z < -g$  and  $z > -g$ . The matching of the magnetic field requires  $H_{\phi}^I$  in the interval  $-g \leq z \leq g$  [see equation (10)]. Therefore we need  $I^-$  in the upper and  $I^+$  in the lower half plane. In the complex plane the location of the poles  $\pm b_s$  has to be chosen so that the waves excited at the tube ports of the resonator are damped and travel away from the resonator. This gives the distribution of poles and integration paths shown in Fig. 7.

The solutions of the integrals (A2) are, for  $-g \leq z \leq g$ ,

$$\begin{aligned}
 \begin{Bmatrix} I_{c_n} \\ jI_{s_n} \end{Bmatrix} &= j\pi \frac{J'_0(K_n)}{K_n J_0(K_n)} \begin{Bmatrix} (-)^{(n-1)/2} \sin k_{zn} z \\ (-)^{n/2} \cos k_{zn} z \end{Bmatrix} + 2\pi \sum_{s=1}^S \frac{\exp(-j\beta_s g)}{K_n^2 - j_0^2 s} \begin{Bmatrix} \sin \beta_s z \\ -j \cos \beta_s z \end{Bmatrix} \\
 &- j2\pi \sum_{s=1}^{\infty} \frac{\exp(-\alpha_s g)}{K_n^2 - j_0^2 s} \begin{Bmatrix} \sinh \alpha_s z \\ \cosh \alpha_s z \end{Bmatrix} \tag{A3}
 \end{aligned}$$

REFERENCES

1. O.A. Kolpakov and V.I. Kotov, "Radiation of a charge passing a cylindrical resonator", Sov. Phys.-Tech. Phys., vol. 9, no. 8, pp. 1072-1074, Feb. 1965.
2. A. Faltens, "Radiation by a relativistic particle passing through a cavity", Symposium on Electron Ring Accelerators, WC/ERA-28, p. 363, Berkeley Feb. 1968.
3. J.D. Lawson, "Radiation from a ring charge passing through a resonator", Rutherford Lab. report RHEL/M 144, April 1968, unpublished.
4. E. Keil, "On the energy loss of a charged ring passing a corrugated cylindrical waveguide", Pro. VII Int. Conf. High Energy Acc., Vol. II, pp. 551-560, Yerevan, USSR, 1969.
5. M. Chatard-Moulin and A. Papiernik, "Energy losses of an electron bunch moving along the axis of a circular waveguide with periodically perturbed walls", IEEE Trans. Nucl. Sci., NS-26, no. 3, pp. 3523-3525, June 1979.
6. R.K. Cooper, S. Krinsky and P.L. Morton, "Transverse wake force in periodically varying waveguide", Particle Accelerators, vol. 12, pp. 1-12, 1982.
7. S. Krinsky and R. Gluckstern, "Higher order terms in a perturbation expansion for the impedance in a corrugated waveguide", IEEE Trans. Nucl. Sci., NS-28, no. 3, pp. 2621-2623, June 1981.
8. T. Weiland, "Transverse beam cavity interaction", Nucl. Instr. Meth., vol. 212, pp. 13-21, 1983.
9. H. Buchholz, "Bemerkungen zu einer Entwicklungsformel aus der Theorie der Zylinderfunktionen", Z. angew. Math. Mech., Bd. 25/27, Nr. 8/9, pp. 245-252, 1947.

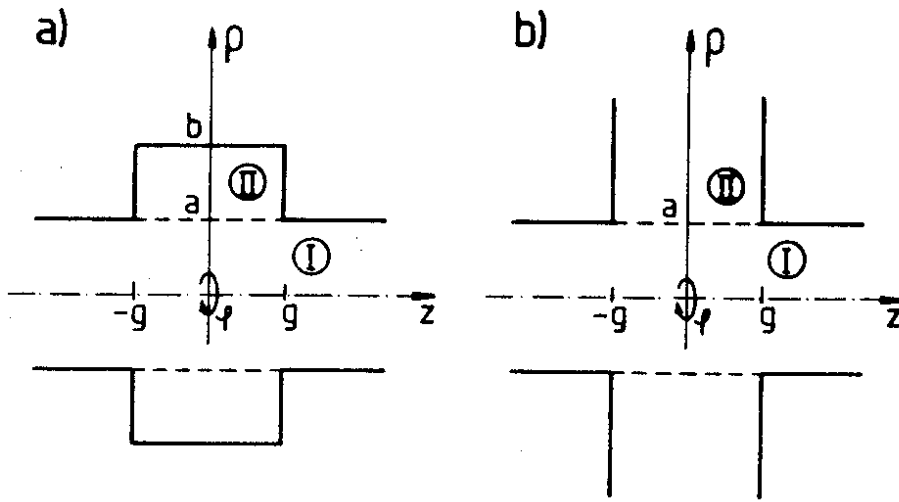


Fig. 1. Geometry of a) cylindrical resonator and b) radial line with pipes

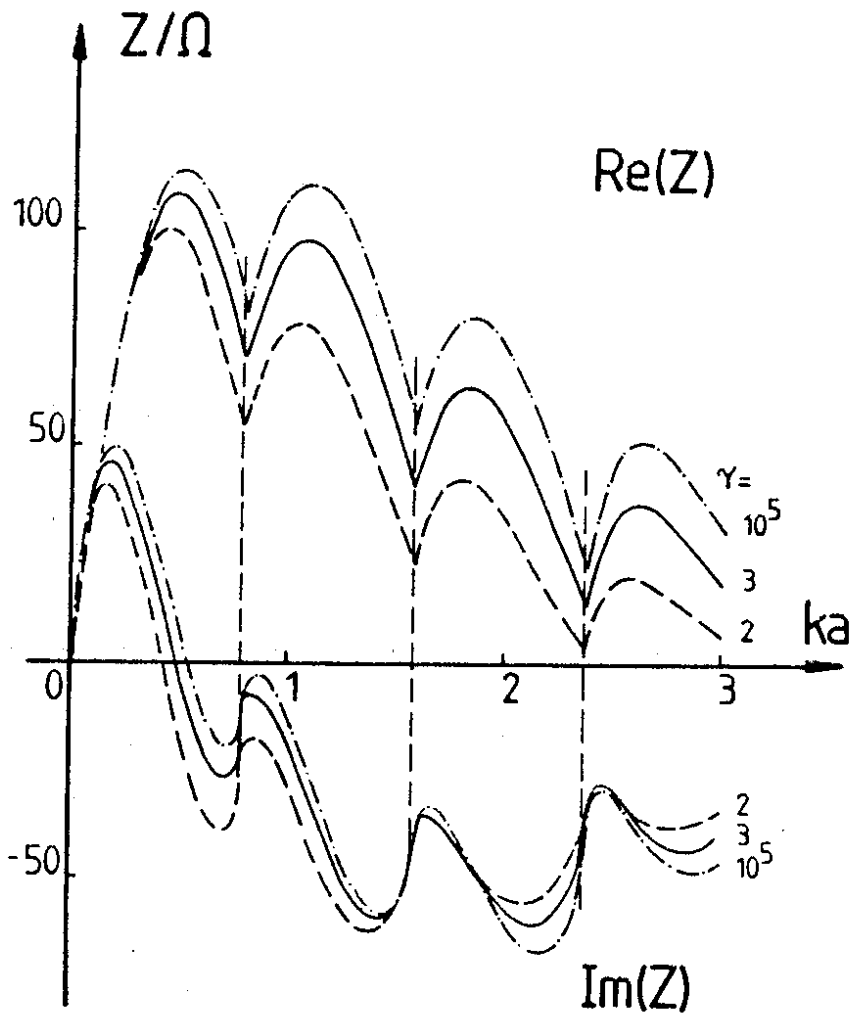


Fig. 2. Impedance of the structure in Fig 1b.  $g/a = 1.98$ ,  $\gamma$  relativistic factor



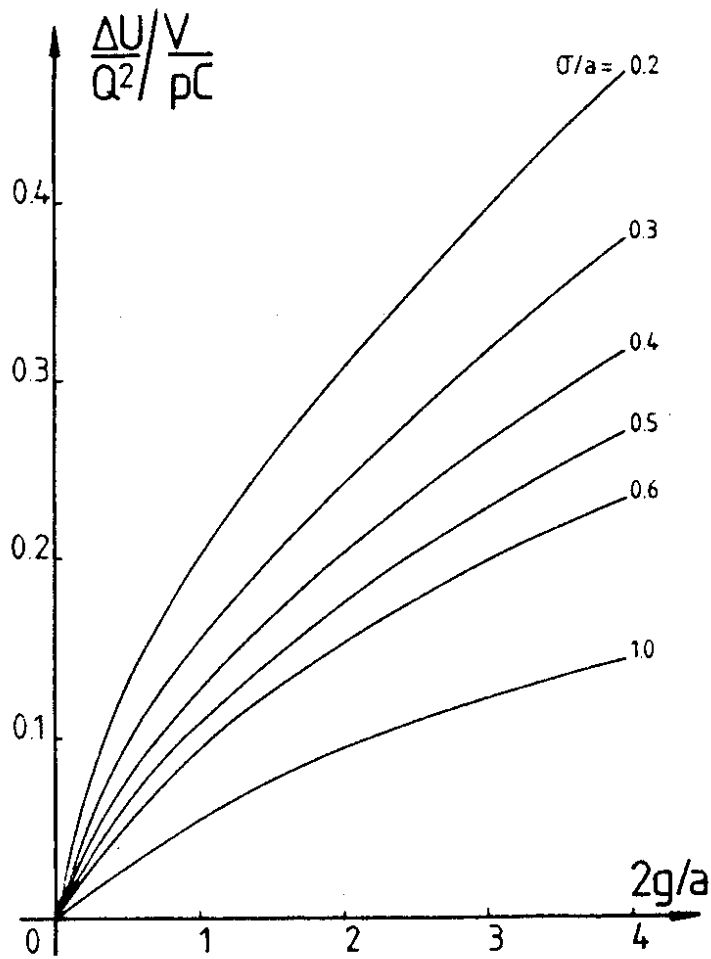


Fig. 3. Energy of a Gaussian shaped unit charge which passed the structure of Fig. 1b with light velocity.  $\sigma$  standard deviation of the distribution.

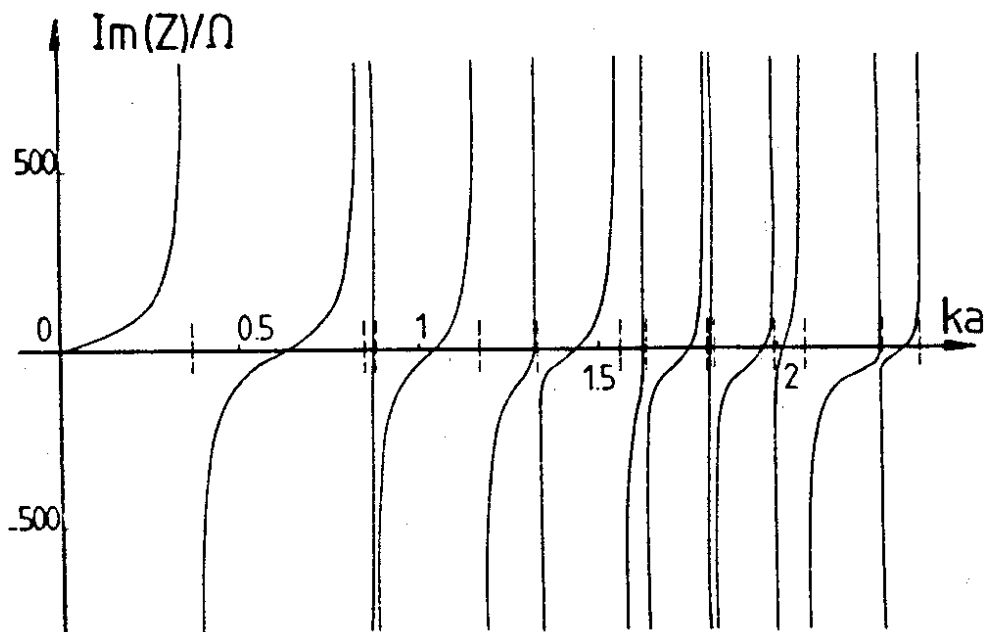


Fig. 4. Reactive impedance of the structure in Fig. 1a below the pipe cut-off  $ka = j_{01}$ .  $g/a = 1.98$ ,  $d/a = 5.56$ , beam velocity  $c_0$

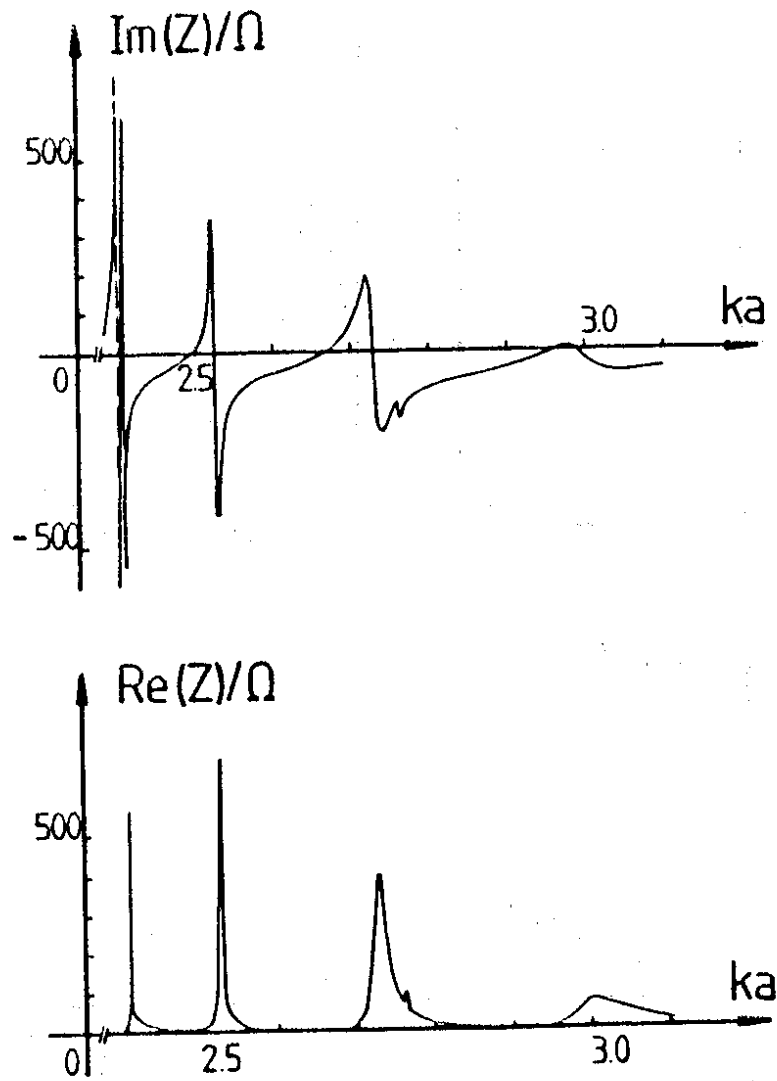


Fig. 5. Impedance of the structure in Fig. 1a above the pipe cut-off  $ka = j_{01}$ .  $g/a = 1.98$ ,  $d/a = 5.56$ , beam velocity  $c_0$

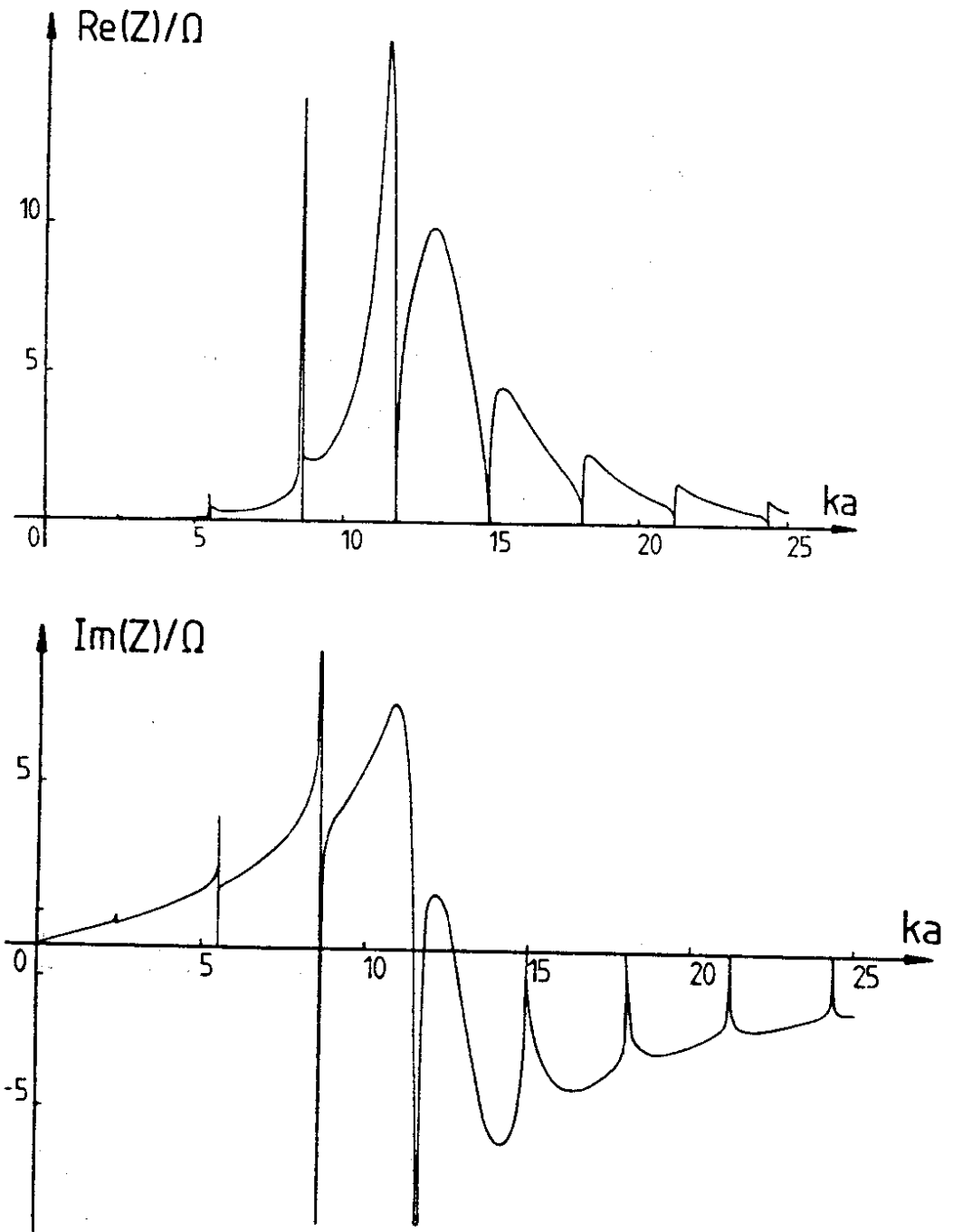


Fig. 6. Impedance of the structure in Fig. 1a.  $g/a = 0.025$ ,  $d/a = 0.1$ , beam velocity  $c_0$

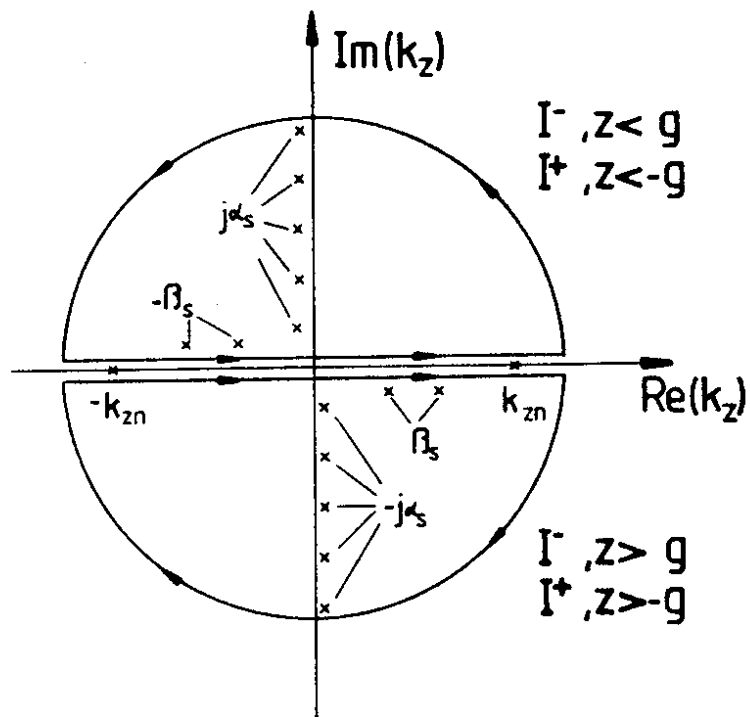


Fig. 7 Integration path and poles in the  $k_z$ -plane