Point Symmetric Ribbon Patterns using a Hexagonal Motif from M.C. Escher

David A. Reimann
Department of Mathematics and Computer Science • Albion College
Albion, Michigan, 49224, USA
dreimann@albion.edu

Abstract

M.C. Escher explored patterns created from simple motifs carved from geometric shapes such as a square or a hexagon. Rather than creating patterns having translational planar symmetry, this paper describes how point symmetric patterns can be created using Escher's hexagonal motif and gives several examples. Creating a point symmetric pattern from decorated regular hexagonal tiles depends on the symmetry groups of the hexagonal tessellation and the available tile decorations. One can use the hexagonal motif created by Escher to make point symmetric tessellations having four symmetry types.

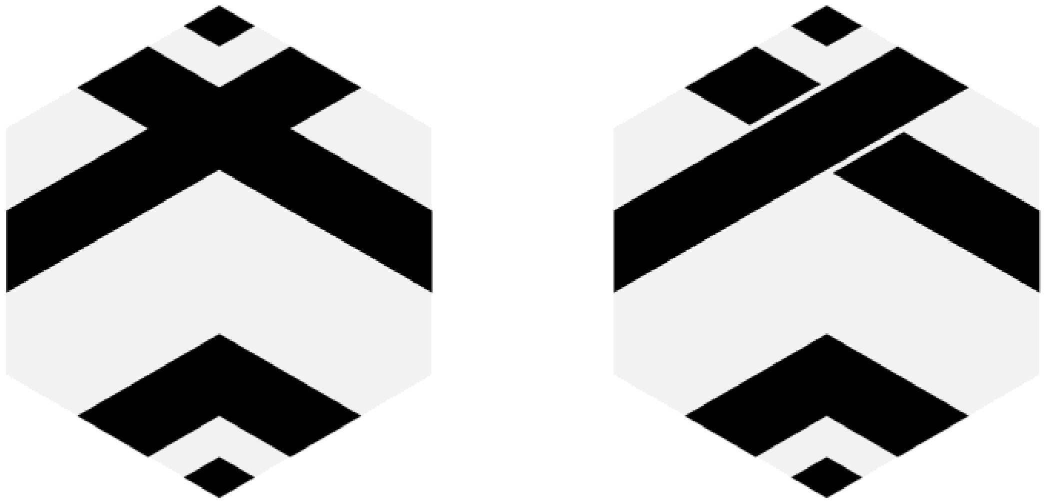
Introduction

M.C. Escher explored patterns created from simple motifs carved from square stamps during the years 1938–1943 [1, 5]. Escher referred to this as the "Potato Game" [2], which he played with his children, as the stamps could be easily carved from potatoes. He preserved in a notebook tessellations using several motifs from plain to yarn-like in appearance. He then created repeating patterns using a 2×2 arrangement of rotations and reflections of the base pattern. Many of these patterns Escher placed together in a ringbinder [3]. These patterns mostly show simple tessellations having translational planar symmetry. One pattern Escher created with the yarn motif has C_2 (cyclic of order 2) point symmetric pattern; it has D_2 (dihedral of order 4) symmetry when one ignores the over/under crossings. M.C. Escher's son George commented "…of course, the game can be extended to triangular and hexagonal grids" [2]. Schattschneider shows an example motif M.C. Escher carved on a hexagon [5]. The pattern and a variant are shown in Figure 1. However, no ribbon patterns using the hexagonal motif are shown; they are only mentioned.

One of the challenges in using non-square stamps is they do not tessellate as easily as a square. With a simple square, one can form a 2×2 block that is also a square, and thus can be used to easily tessellate the plane. Rather than creating patterns having translational planar symmetry, this paper describes how point symmetric patterns can be created using Escher's hexagonal motif.

Methods and Results

One can simply tessellate the plane using the motif placed at random. An example tessellation is shown in Figure 2. The author has previously studied point symmetric patterns on a regular hexagonal tessellation [4]. Creating a point symmetric pattern from decorated regular hexagonal tiles depends on the symmetry groups of the hexagonal tessellation and the available tile decorations. The subgroup structure of the three dihedral groups found in the hexagonal tessellation ($D_6, D_3,$ and D_2) give all possible symmetry groups. Here the D_6 point group is centered at a hexagon center, the D_3 point group is centered at a vertex, and the D_2 point group is centered at an edge midpoint. While there are a total of 27 subgroups, there are only 14 geometrically unique symmetry patterns for the hexagonal tessellation. Furthermore, some of these symmetry patterns



A B

Figure 1 : *A hexagonal stamp designed by Escher. Each side is trisected, then bands that are one third of the side length wide connect pairs of edges. Two regions near two opposite vertices are also decorated as shown. Figure A shows Escher's original motif and B shows the motif modified to give the appearance of an over/under crossing of two of the bands. The mirror image of the modified motif will also be used.*

require tiles having a particular symmetry. The ability to create a given pattern therefore depends on the symmetry type of the tile used.

The original Escher motif contains D_1 symmetry and the modified Escher motif has no symmetry (C_1). One can create point symmetric patterns having symmetry types $D_6 : D_1$, $D_3 : D_3$, and C_1 using the original motif and symmetry types $D_3 : C_3$, $D_2 : C_2$, and C_1 using the modified motif. Here the notation $G : H$ indicates G is the parent group (D_6, D_3, D_2) and H is a particular subgroup of G. Examples for the symmetric patterns are shown in Figure 3.

Discussion

The hexagonal motif created by Escher can be used to make point symmetric tessellations having four symmetry types: D_1, D_3, C_3, and C_2. One can also create patterns having no symmetry (C_1). Point symmetric patterns that are bounded, such as those created in this manner, have visual appeal similar to patterns with planar symmetry. Escher's modified motif gives rise to patterns similar to those previously shown by the author [4].

Acknowledgments

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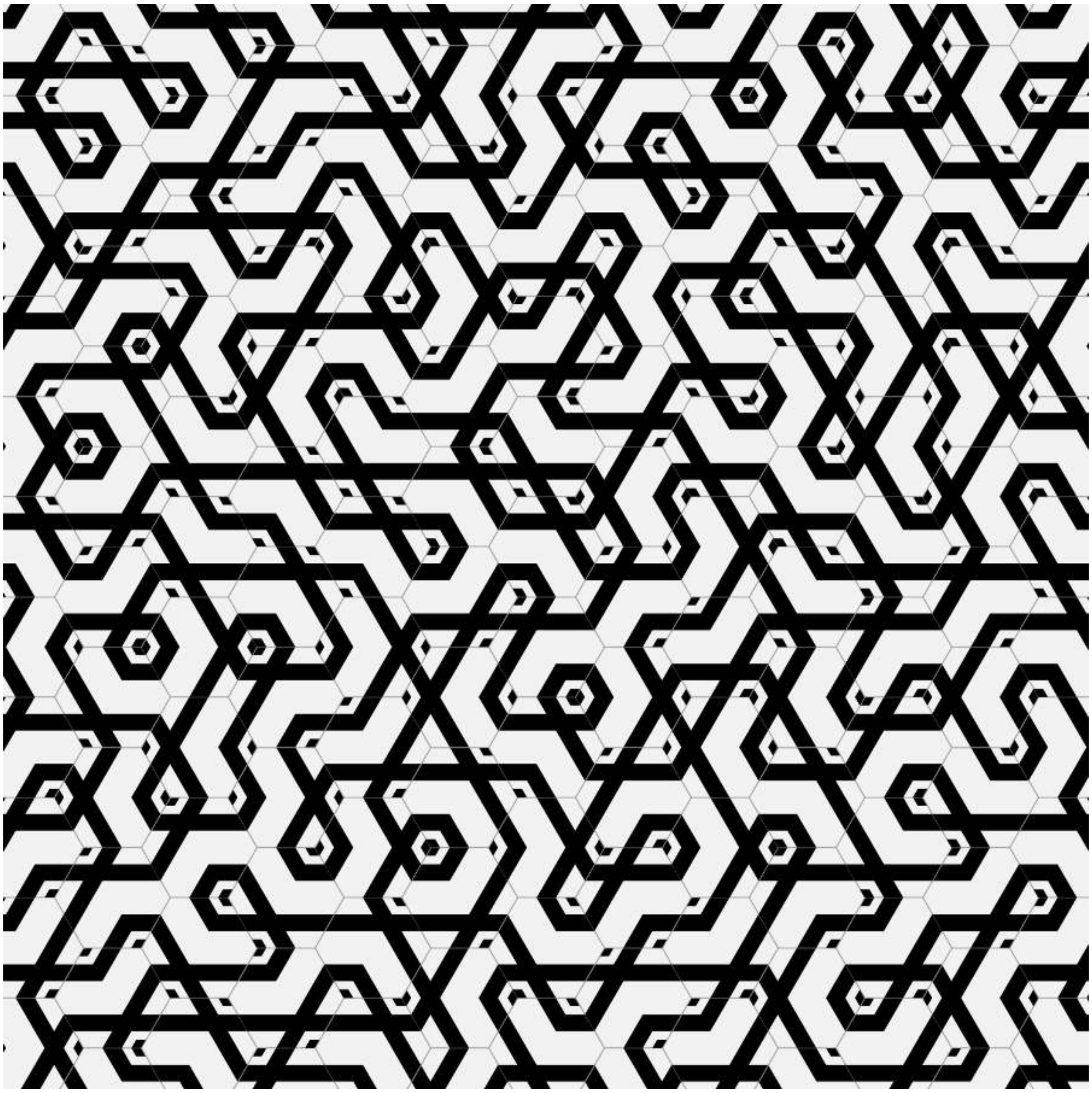


Figure 2 : *An example random C_1 tessellation pattern using the motif in Figure 1A.*

References

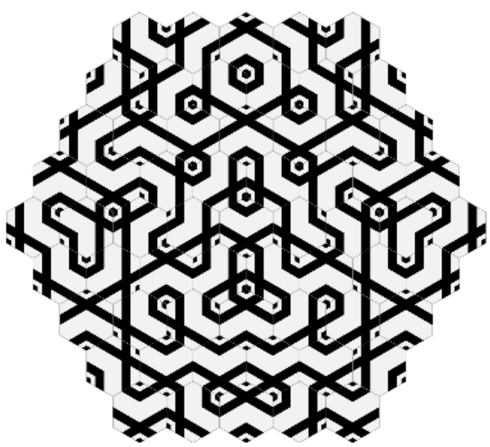
[1] B. Ernst. *The magic mirror of M.C. Escher*. Random house New York, 1976.

[2] G.A. Escher. M.C. Escher at work. In H. M. S. Coxeter, M. Emmer, R. Penrose, and M. L. Teuber, editors, *M.C. Escher: Art and Science*, pages 1–11, 1986.

[3] M.C. Escher. Tegels. unpublished, Gemeentemuseum archives, viewed April 2011.

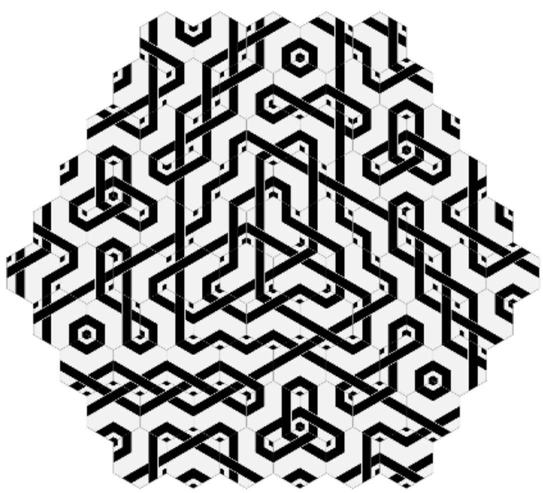
[4] David A. Reimann. Point symmetry patterns on a regular hexagonal tessellation. In Robert Bosch, Douglas McKenna, and Reza Sarhangi, editors, *Bridges Towson 2012: Mathematics, Music, Art, Architecture, Culture*, pages 365–368, Towson, Maryland, 25–29 July 2012.

[5] D. Schattschneider. *M.C. Escher: Visions of Symmetry*. Harry N. Abrams, 2004.

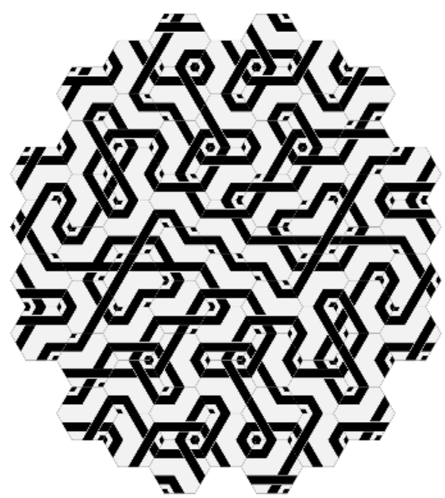


A

B



C



D

Figure 3: *Example point symmetric patterns using the motifs in Figure 1. Subfigure A shows a* $D_6 : D_1$ *symmetric pattern with a mirror line running vertically through the figure. Subfigure B shows a* $D_3 : D_3$ *symmetric pattern with three mirror lines at* 120° *apart meeting at the center vertex. Subfigure C shows a* $D_3 : C_3$ *symmetric pattern with the fixed point located at the center vertex. Subfigure D shows a* $D_2 : C_2$ *symmetric pattern with the fixed point located at an edge midpoint. These figures also show some of the types of local patterns possible using these motifs.*