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Polarization and anisotropy of the relict radiation in an anisotropic universe

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Summary. A rigorous first order solution with respect to anisotropy is obtained for the equation of polarized radiation transfer in a homogeneous anisotropic universe with a flat co-moving space. The degree of polarization of the background radiation is shown to be very sensitive to the recombination dynamics and to the secondary reheating epoch. Provided that a quadrupole anisotropy of the background radiation is established, the measurements of its polarization degree enable one to set severe limitations on the conditions of secondary ionization.

1 Introduction

The high degree of isotropy of the relict (cosmic background) radiation (large-scale temperature fluctuations in the Rayleigh–Jeans region $\Delta T/T < 10^{-3}$ (Alpher & Herman 1975; Smoot, Gorenstein & Muller 1977)) provides one of the basic arguments for the Universe to be adequately described by a homogeneous and isotropic Friedmann model. But one cannot exclude a strongly anisotropic expansion in the remote past, which smoothed out to the recombination epoch $t = t_r$ and gave rise to only a small anisotropy of the relict radiation at present. After being scattered by a free electron, a weakly anisotropic radiation acquires a small degree of linear polarization. Earlier, the polarization of the relict radiation in an axisymmetric cosmology had been discussed by Rees (1968). But since the precision of observations is steadily growing, we found it appropriate to revisit the problem and give it a somewhat more thorough and general treatment.

In this present paper an exact solution of the first order radiative transfer equation with respect to anisotropy is obtained for a homogeneous cosmological model with a flat co-moving space (Bianchi type I (Zeldovich & Novikov 1975)), in which the rates of expansion along all three axes are different. Calculations based on this solution confirm most of the qualitative conclusions by Rees (1968), though some numerical coefficients are quite different. Astrophysical applications are discussed.

2 Solution of the equation of radiative transfer

Consider a homogeneous cosmological model with a Hubble law

$$v_x = H_x x, \quad v_y = H_y y, \quad v_z = H_z z, \tag{1}$$

where

$$H_i(t) = (d/dt)[\ln a_i(t)]. \tag{2}$$

Below we shall use the notations

$$H(t) = \tfrac{1}{3}[H_x(t) + H_y(t) + H_z(t)]; \tag{3a}$$

$$\Delta H(t) = H_z(t) - \tfrac{1}{2}[H_x(t) + H_y(t)]; \quad \overline{\Delta H}(t) = H_x(t) - H_y(t). \tag{3b}$$

It is known that beginning from the redshifts $z \sim 3 \times 10^6$ and up to the present moment the dominant opacity mechanism in the Universe is Thomson scattering. To describe the polarization of radiation in the course of scattering, we introduce (following Chandrasekhar 1960) a symbolic vector **n** consisting of components n_l, n_r and n_u of the polarization tensor, which in the axes – one complanar with the z-axis (index l) and the other perpendicular to it (index r) – reads

$$\rho_{\alpha\beta} = \frac{1}{n}\begin{pmatrix} n_l & \tfrac{1}{2}n_u \\ \tfrac{1}{2}n_u & n_r \end{pmatrix}. \tag{4}$$

Here $n = n_l + n_r$ is the photon occupation number. Vector $\mathbf{n} = \mathbf{n}(t, \nu, \mu, \phi)$ is a function of time t, of the photon frequency ν, and of the direction of photon propagation characterized by the polar angle $\theta = \arccos \mu$ between the wave vector and z-axis, and by azimuth ϕ measured from the x-axis of the co-moving cartesian coordinate system. The kinetic equation for the photons is

$$\frac{\partial \mathbf{n}}{\partial t} = -\frac{\partial \mathbf{n}}{\partial \nu}\frac{d\nu}{dt} - \frac{\partial \mathbf{n}}{\partial \mu}\frac{d\mu}{dt} - \frac{\partial \mathbf{n}}{\partial \phi}\frac{d\phi}{dt} - \mathbf{R}(t, \mu, \phi)\,\mathbf{n} - c\sigma_T N_e(t) \times \left[\mathbf{n} - \frac{1}{4\pi}\int_{-1}^{+1}\int_0^{2\pi} \mathbf{P}(\mu, \phi, \mu', \phi')\,\mathbf{n}(t, \nu, \mu', \phi')\,d\mu'\,d\phi'\right], \tag{5}$$

where σ_T is the Thomson cross-section, $N_e(t)$ is the number of electrons per unit volume of the co-moving space, and the scattering matrix **P** is given in the Appendix.

The frequency change along the ray, due to the Doppler effect for the local expansion law (1) is

$$\frac{d\nu}{dt} = -\nu[H + \Delta H(\mu^2 - \tfrac{1}{3}) + \overline{\Delta H}\,\tfrac{1}{2}(1 - \mu^2)\cos 2\phi], \tag{6}$$

where H, ΔH and $\overline{\Delta H}$ are defined in equations (3).

For the purposes of this paper we do not need explicit forms of the angular operators $(d\mu/dt)(\partial/\partial\mu)$ and $(d\phi/dt)(\partial/\partial\phi)$, and of the matrix **R**, describing respectively the aberration and rotation of the polarization axes (Brans 1975; Caderni *et al.* 1978a) by parallel transport along null geodesics. The reason for this is that we solve equation (5) in the framework of the perturbation theory. In a zero approximation with respect to anisotropy (which is a small parameter in our problem) the radiation field is isotropic and unpolarized. The effect of the above operators on such a radiation field is identically zero. And since the operators themselves are proportional to small quantities ΔH and $\overline{\Delta H}$, they have to be accounted for in the second and higher approximations only, which we do not consider here.

From equation (4) it follows that the angular dependence of $\mathbf{n}(t, \nu, \pm 1, \phi)$ for photons travelling along the z-axis should be of the form (Landau & Lifshitz 1976):

$$\mathbf{n}(t, \nu, \pm 1, \phi) = \frac{n}{2}\begin{pmatrix} 1 + p \cos 2(\phi - \phi_0) \\ 1 - p \cos 2(\phi - \phi_0) \\ \pm 2p \sin 2(\phi - \phi_0) \end{pmatrix}, \tag{7}$$

where n and p are independent of ϕ, and ϕ_0 is the azimuth of one of the principal axes of the polarization tensor. Solutions of equation (4) which do not satisfy equation (7) have no physical meaning.

Let us assume now that

$$\mathbf{n}(t, \nu, \mu, \phi) = n_0(t, \nu_0)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mathbf{n}_1(t, \nu_0, \mu, \phi), \tag{8}$$

where $|n_{1l}| \ll n_0$, $|n_{1r}| \ll n_0$, $|n_{1u}| \ll n_0$, and the frequency

$$\nu_0 = \nu\left[\frac{a_x(t)\, a_y(t)\, a_z(t)}{a_x(t_0)\, a_y(t_0)\, a_z(t_0)}\right]^{1/3} \tag{9}$$

is an 'integral of motion' in zero approximation. Substituting equations (6), (8) and (9) into equation (5) and retaining only zero and first order terms, we get

$$\frac{\partial n_0}{\partial t} = 0; \tag{10}$$

$$\frac{\partial \mathbf{n}_1}{\partial t} = \nu_0 \frac{dn_0}{d\nu_0}\left[\Delta H(\mu^2 - 1/3) + 1/2\, \overline{\Delta H}(1 - \mu^2)\cos 2\phi\right]\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - c\sigma_{\mathrm{T}} N_{\mathrm{e}} \times \left[\mathbf{n}_1 - \frac{1}{4\pi}\int_{-1}^{+1}\int_0^{2\pi} \mathbf{P}\mathbf{n}_1(t, \nu_0, \mu', \phi')\, d\mu'\, d\phi'\right]. \tag{11}$$

From equation (11) one sees that an anisotropic expansion generates a polar and azimuthal anisotropies in the background radiation of the form

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}(\mu^2 - 1/3), \quad \bar{\mathbf{a}} = 1/2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}(1 - \mu^2)\cos 2\phi. \tag{12}$$

The collisional integral in the right-hand part of equation (11) (acting on 'spin' and angular variables only) transforms vector $\mathbf{a}$ into a linear combination of itself and vector $\mathbf{b}$, while vector $\bar{\mathbf{a}}$ is being transformed into a linear combination of itself and vector $\bar{\mathbf{b}}$, where

$$\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}(1 - \mu^2); \quad \bar{\mathbf{b}} = 1/2 \begin{pmatrix} (1 + \mu^2)\cos 2\phi \\ -(1 + \mu^2)\cos 2\phi \\ 4\mu \sin 2\phi \end{pmatrix}. \tag{13}$$

One can easily verify that vector **b** in its turn transforms into a linear combination of **a** and **b**, while $\bar{\mathbf{b}}$ – into a linear combination of $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$. Thus, a solution of equation (11) which is of interest here must be of the form

$$\mathbf{n}_1 = n_0(\nu_0)[\alpha(t,\nu_0)\,\mathbf{a} + \beta(t,\nu_0)\,\mathbf{b} + \bar{\alpha}(t,\nu_0)\,\bar{\mathbf{a}} + \bar{\beta}(t,\nu_0)\,\bar{\mathbf{b}}]. \tag{14}$$

The quantities α and $\bar{\alpha}$ represent the degree of anisotropy of the relict radiation, while the quantities β and $\bar{\beta}$ – the degree of its polarization. The general form (equation 14) of a solution is such that the condition (7) is satisfied automatically.

Note that according to equations (12)–(14) the net contribution to the total photon number density (summed over polarizations and integrated over angles) vanishes in the first order of the perturbation theory. The latter quantity is determined entirely by $n_0(\nu_0)$. This implies, in particular, that though anisotropic and polarized radiation components may contribute to spectral distortions in fixed directions, the total spectral shape averaged over all directions and polarization states is determined by the function $n_0(\nu_0)$ only. To calculate n_0 one has to make a proper account of Compton scattering and true absorption mechanisms in the epochs $10^3 \lesssim z \lesssim 10^7$ (Illarionov & Sunyaev 1974; Zeldovich, Illarionov & Sunyaev 1972).

Substituting equation (14) into equation (11), we obtain a system of ordinary differential equations for the quantities α and β:

$$\frac{d\alpha}{dt} = \frac{d\ln n_0}{d\ln \nu_0}\Delta H - c\sigma_{\mathrm{T}} N_{\mathrm{e}}(\tfrac{9}{10}\alpha + \tfrac{3}{5}\beta), \tag{15a}$$

$$\frac{d\beta}{dt} = -c\sigma_{\mathrm{T}} N_{\mathrm{e}}(\tfrac{1}{10}\alpha + \tfrac{2}{5}\beta). \tag{15b}$$

The functions $\bar{\alpha}$ and $\bar{\beta}$ obey the same system with ΔH replaced by $\overline{\Delta H}$. This latter circumstance enables us to make a straightforward generalization of all the results obtained for an axisymmetric expansion (when $\overline{\Delta H} = \bar{\alpha} = \bar{\beta} = 0$) to a three-axial expansion since *all the formulae and numerical relationships obtained for α and β are also valid for $\bar{\alpha}$ and $\bar{\beta}$ when ΔH is replaced by $\overline{\Delta H}$.*

The system (15) is not equivalent to a corresponding system of Rees (1968). We suspect that Rees has made an error when calculating the matrix of the system, but since he had not published the details of the derivation, we were unable to pinpoint it. This difference infers systematic discrepancy in numerical coefficients and some qualitative conclusions.

The solution of equations (15) is

$$\alpha(t,\nu_0) = \frac{d\ln n_0}{d\ln \nu_0}\frac{1}{7}\int_0^t \Delta H(t')\{6\exp[-\tau(t,t')] + \exp[-\tfrac{3}{10}\tau(t,t')]\}\,dt', \tag{16a}$$

$$\beta(t,\nu_0) = \frac{d\ln n_0}{d\ln \nu_0}\frac{1}{7}\int_0^t \Delta H(t')\{\exp[-\tau(t,t')] - \exp[-\tfrac{3}{10}\tau(t,t')]\}\,dt', \tag{16b}$$

where

$$\tau(t,t') = c\sigma_{\mathrm{T}}\int_{t'}^{t} N_{\mathrm{e}}(t'')\,dt'' \tag{17}$$

is the optical depth along the ray between the moments t' and t.

3 General properties of the solution

From expressions (16) we see that the influence of cosmological parameters on α and β falls off exponentially with the optical depth $\tau(t, t')$. It means that, in order to evaluate α and β at the present epoch, one has to know the plasma parameters and the expansion law beginning from the recombination epoch only, namely from $z = z_r \sim 1300$. An interesting property of the solution (16) is the fact that the weight functions

$$f_\alpha(\tau) = \tfrac{6}{7} \exp(-\tau) + \tfrac{1}{7} \exp(-\tfrac{3}{10}\tau), \tag{18a}$$

$$f_\beta(\tau) = \tfrac{1}{7} [\exp(-\tfrac{3}{10}\tau) - \exp(-\tau)] \tag{18b}$$

drop as $\exp(-\tfrac{3}{10}\tau)$, and not as $\exp(-\tau)$. The reason is that in the course of scattering an anisotropic non-polarized component of the radiation field does not simply become isotropic (as would be the case for the scattering of a scalar field), but partly transforms into a polarized component. The scattering of the latter results in its partial depolarization, with a certain fraction being transformed into the anisotropic non-polarized component. Thus, the anisotropic and the polarized components of the radiation field make a single complex (the reason for that stems from the transverse nature of electromagnetic waves) which decays on the whole slower than $\exp(-\tau)$.

Having considered a ratio of functions (18)

$$\frac{f_\beta}{f_\alpha} = \frac{1 - \exp(-\tfrac{7}{10}\tau)}{1 + 6\exp(-\tfrac{7}{10}\tau)} < 1, \tag{19}$$

we immediately arrive at a conclusion that *the degree of polarization* $|\beta|$ *is always less than the degree of anisotropy* $|\alpha|$. This result contradicts the contention of Rees (1968) that in some cases $|\beta| \sim 3|\alpha|$. The function f_β reaches its maximum $f_\beta(\tau_m) = 0.060$ at $\tau_m = \tfrac{10}{7} \ln \tfrac{10}{3} = 1.72$.

The solution (16) has been obtained under the assumption $|\alpha| \ll 1$ (which infers $|\beta| \ll 1$). Now we can write down the conditions that guarantee its validity:

$$|\Delta H|/c\sigma_T N_e \ll 1, \quad \text{if} \quad z > z_r, \tag{20a}$$

$$|\Delta H/H| \ll 1, \quad \text{if} \quad z < z_r. \tag{20b}$$

Indeed, from equations (16) one finds that before the recombination when the mean time between scatterings $(c\sigma_T N_e)^{-1}$ is much less than the cosmological time-scale H^{-1} – the degree of anisotropy $|\alpha| \sim |\Delta H|/c\sigma_T N_e$. After the recombination, when the universe is transparent, the degree of anisotropy $|\alpha| \sim |\Delta H/H|$, where the last ratio should be evaluated at the epoch of recombination because it decreases in the course of expansion. In all calculations below we assume that the stronger condition (20b) is fulfilled.

4 The Universe transparent after recombination

If the average matter density in the universe at the present epoch $t = t_0$ is close to a critical value,

$$\Omega \equiv \frac{\rho_0}{\rho_c} \equiv \rho_0 \frac{8\pi G}{3H_0^2} \approx 1, \tag{21}$$

then we may treat a co-moving space as a flat one at all stages of expansion. In this case we can evaluate the polarization and anisotropy of the relict radiation directly from equations

(16). The expansion at $z < 4.2 \times 10^4 h^2 \Omega$ ($h = H/100\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$), when non-relativistic nucleons dominate in matter density, is governed by the Schücking–Heckmann (1958) solution for $P = 0$:

$$\begin{aligned} a_x(t) &= a_{x0} t^{p_1} (t + t_\mathrm{i})^{2/3 - p_1}, \\ a_y(t) &= a_{y0} t^{p_2} (t + t_\mathrm{i})^{2/3 - p_2}, \\ a_z(t) &= a_{z0} t^{p_3} (t + t_\mathrm{i})^{2/3 - p_3}, \end{aligned} \tag{22}$$

where

$$p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1, \tag{23}$$

and the isotropization moment $t_\mathrm{i} \ll t_\mathrm{r}$. In this case

$$\Delta H(z) = \Delta H_0 (1+z)^3; \quad \overline{\Delta H}(z) = \overline{\Delta H}_0 (1+z)^3 \tag{24}$$

(z is the redshift). The relationship (24) implies that prior to recombination, the ratio $\Delta H / c\sigma_\mathrm{T} N_\mathrm{e}$ had been independent of time, and the integrals in equations (16) can be evaluated analytically:

$$\alpha = \frac{4}{3} \frac{\Delta H}{c\sigma_\mathrm{T} N_\mathrm{e}} \frac{d \ln n_0}{d \ln \nu_0}; \quad \beta = -\tfrac{1}{4}\alpha. \tag{25}$$

Substituting equation (24) into equations (16) and changing the integration variable, we get for $t = t_0$:

$$\alpha_0 = \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0} \int_0^\infty [\![\tfrac{6}{7} \exp[-\tau(z')] + \tfrac{1}{7} \exp[-\tfrac{3}{10}\tau(z')]]\!] (1+z')^{1/2}\, dz', \tag{26a}$$

$$\beta_0 = \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0} \frac{1}{7} \int_0^\infty [\![\exp[-\tau(z')] - \exp[-\tfrac{3}{10}\tau(z')]]\!] (1+z')^{1/2}\, dz', \tag{26b}$$

where

$$\tau(z) = \tau_0 \int_0^z x_\mathrm{e}(z')(1+z')^{1/2}\, dz', \tag{27}$$

$\tau_0 = c\sigma_\mathrm{T} \rho_\mathrm{c} / m_\mathrm{p} H_0 = 0.069\,\mathrm{h}$, $x_\mathrm{e}(z)$ is the degree of ionization.

First, we shall estimate α_0 and β_0 under the assumption that the recombination occurs instantaneously at $z_\mathrm{r} = 1500$ (when $x_\mathrm{e} = 0.5$). In this case the integrals in equations (26) can be evaluated analytically, the results being

$$\begin{aligned} \alpha_0 &\approx \tfrac{2}{3} z_\mathrm{r}^{3/2} \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0} = 3.87 \times 10^4 \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0}, \\ \beta_0 &\approx -\alpha_0 / 2\tau_0 z_\mathrm{r}^{3/2} = -1.25 \times 10^{-4} \alpha_0 \end{aligned} \tag{28}$$

(here and below all numbers are given for $h = 1$). As the next step, we shall account for the non-instantaneous character of the recombination, making use of an approximate formula for the optical depth from Sunyaev & Zeldovich (1970):

$$\tau(z) \approx 27.3 z^{3/2} \exp(-14\,600/z). \tag{29}$$

This formula is valid in the interval $900 \lesssim z \lesssim 1300$ (we ignore for the moment a residual optical depth $\tau(900)$ which according to Sunyaev & Zeldovich (1970) is ~ 0.4). Substituting equation (29) into equations (26), we obtain

$$\alpha_0 = 2.24 \times 10^4 \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0}; \quad \beta_0 = -0.018 \alpha_0. \tag{30}$$

Comparing equations (30) with equations (28), we conclude that, as contrasted with α, *the degree of polarization β is very sensitive to the dynamics of the recombination*. Somewhat more accurate integration with the aid of the table for x_e from Peebles (1968) gives

$$\alpha_0 = 2.21 \times 10^4 \frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0}; \quad \beta_0 = -0.022 \alpha_0, \tag{31}$$

which exceeds the analogous estimate of Rees (1968) for β_0 by a factor of 2.

5 The effect of secondary ionization

Since the 'source' of polarization is anisotropy, the major part of which builds up after recombination when β ceases to grow, a secondary ionization of the intergalactic gas at $z = z_{rh} \ll z_r$ may result in a considerable increase of β and decrease of α. We shall illustrate this effect for $\Omega = 1$. As was shown by Ozernoi & Chernomordik (1976), the radiation of young galaxies is capable of completely ionizing the residual intergalactic gas as early as $z \sim 30$. We shall assume that 70 per cent of the total mass has clumped, while the other 30 per cent remains uniformly distributed and has been ionized at $z = z_{rh}$. The values of α and β calculated for different values of z_{rh} are given in Table 1. In this table τ_{rh} is the optical depth along the ray from $z = z_{rh}$ to $z = 0$. For large τ_{rh} the values of α and β must approach the solution (25), but this occurs at $\tau_{rh} \gtrsim 15$, and not at $\tau_{rh} \gtrsim 5$ as estimated by Rees (1968).

Table 1.

z_{rh}	0	10	20	30	40
τ_{rh}	0	0.49	1.31	2.36	3.6
$\alpha_0 \times \left(\frac{\Delta H_0}{H_0} \frac{d \ln n_0}{d \ln \nu_0}\right)^{-1}$	2.21×10^4	1.44×10^4	7.4×10^3	3.5×10^3	1.75×10^3
β_0/α_0	−0.022	−0.082	−0.22	−0.42	−0.64

6 Discussion and main conclusions

In this paper we have restricted ourselves to a cosmological model of a Bianchi type I ($\Omega = 1$). If the universe is open, one has to consider more complicated models with a spacial curvature. In this latter case the co-moving space is asymptotically flat for $z \gg \Omega^{-1}$, and one can proceed as follows: first, to calculate $\mathbf{n}_1$ from equations (12)–(14), (16) for some epoch $t = t_1$ with a redshift z_1, $\Omega^{-1} \ll z_1 \ll z_r$, and then to transform the result to the present epoch with the aid of null geodesic equations in a spatially curved model. Such a transformation significantly modifies only the angular dependence of the anisotropic and polarized components of the relict radiation. If for $\Omega = 1$ this dependence had a quadrupole form, then for $\Omega \ll 1$ it will appear as a spot on the sky with an angular size $\sim \Omega^{-1}$ (Zeldovich & Novikov 1975; Doroshkevich, Lukash & Novikov 1974). The situation becomes more complicated when $\Omega < 1$ and a large optical depth builds up due to the secondary ionization. In this case one must solve a more complicated problem of the polarized radiation transfer in a curved co-moving space.

Concerning possible depolarization mechanisms, we shall mention the following two factors. If a global magnetic field is present in the universe, and its strength now is $\mathscr{H}_0$, then the Faraday rotation of the polarization plane will substantially depolarize the relict radiation at a wavelength λ_0 when $\mathscr{H}_0 \gtrsim 10^{-8}\,(1\,\mathrm{cm}/\lambda_0)^2\,\mathrm{G}$. Another depolarizing effect is the rotation of the polarization plane with respect to a co-moving reference frame by parallel transport along geodesics (Brans 1975; Caderni *et al.* 1978a). But according to calculations by Caderni *et al.* (1978a) it may become important only in the case of early secondary ionization in an open universe.

Only upper bounds on the large-scale anisotropy and polarization of the relict radiation have been obtained up to now (Smoot, Gorenstein & Muller 1977; Caderni *et al.* 1978b; Lubin & Smoot 1979). Taking into account that $|\beta| < |\alpha|$, as well as possible depolarization factors and high sensitivity of β on the recombination dynamics and secondary ionization epoch, we arrive at a conclusion that from the point of view of detecting possible expansion anisotropies the most promising approach would still be to search for a large-scale anisotropy of the background radiation. But, provided that the value of α is known, the measurements of β will become of special interest because they will enable one to place severe limitations on the parameters of secondary ionization, or, if $\tau_{\mathrm{rh}} \ll 1$, to check experimentally the calculations of the recombination dynamics (Peebles 1968; Zeldovich, Kurt & Sunyaev 1968). We wish to emphasize once more that all said about α and β, is valid also for $\bar{\alpha}$ and $\bar{\beta}$ when expansion is anisotropic along all three axes.

Finally, it should be noted that both the degree of anisotropy and the degree of polarization of the relict radiation do not depend on the frequency in Rayleigh–Jeans region (see equations (16)), and vary as $h\nu_0/kT$ in the Wien part of the blackbody spectrum.

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Appendix

The scattering matrix $\mathbf{P}(\mu, \phi, \mu', \phi')$ (Chandrasekhar 1960) has the form

$$\mathbf{P} = \mathbf{Q}[\mathbf{P}_0(\mu, \mu') + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\,\mathbf{P}_1(\mu, \phi, \mu', \phi') + \mathbf{P}_2(\mu, \phi, \mu', \phi')],$$

where

$$\mathbf{Q} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix};$$

$$\mathbf{P}_0 = \frac{3}{4}\begin{pmatrix} 2(1-\mu^2)(1-\mu'^2) + \mu^2\mu'^2 & \mu^2 & 0 \\ \mu'^2 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{P}_1 = \frac{3}{4}\begin{pmatrix} 4\mu\mu' \cos(\phi' - \phi) & 0 & 2\mu \sin(\phi' - \phi) \\ 0 & 0 & 0 \\ -2\mu' \sin(\phi' - \phi) & 0 & \cos(\phi' - \phi) \end{pmatrix};$$

$$\mathbf{P}_2 = \frac{3}{4}\begin{pmatrix} \mu^2\mu'^2 \cos 2(\phi' - \phi) & -\mu^2 \cos 2(\phi' - \phi) & \mu^2\mu' \sin 2(\phi' - \phi) \\ -\mu'^2 \cos 2(\phi' - \phi) & \cos 2(\phi' - \phi) & -\mu' \sin 2(\phi' - \phi) \\ -\mu\mu'^2 \sin 2(\phi' - \phi) & \mu \sin 2(\phi' - \phi) & \mu\mu' \cos 2(\phi' - \phi) \end{pmatrix}.$$

Note added in proof

After this paper had been submitted to the journal, the measurements of G. P. Nanos were published (1979, *Astrophys. J.*, **232**, 341) who obtained an upper limit for the quadrupole component of the polarization degree $|\beta| < 6 \times 10^{-4}$. He also derived a correct system of differential equations for the polarization and anisotropy degrees in the axisymmetric universe, which is equivalent to our equations (15).