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POLARIZATION SCATTERING MATRICES FOR POLARIMETRIC RADAR

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A study of radar cross section in terms of transmitted and received polarizations is presented to aid in assessing the potential for polarimetric radar. The polarization scattering matrices for various options are related. (Author)

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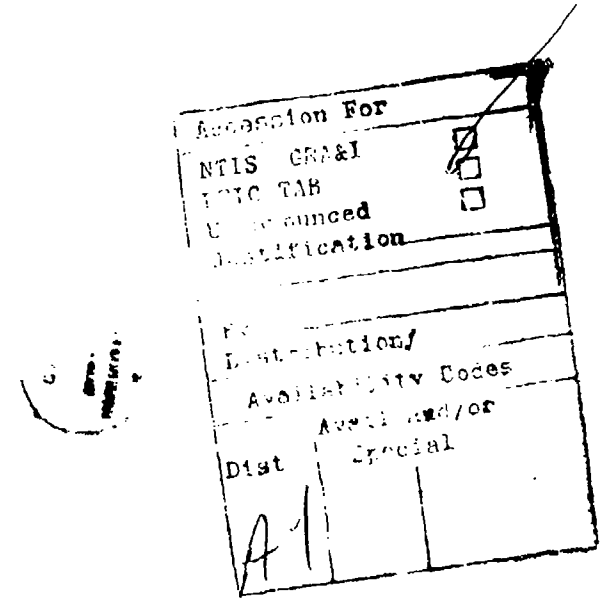

I. INTRODUCTION

There has been considerable interest recently, in using polarization-diverse radar to improve target detection, tracking, and identification for tactical land-combat weapon systems. Both linear and circular polarizations are of interest, and also, a combination of the two, wherein circular polarization is transmitted, while a two-channel receiver measures orthogonal linear polarization components. This latter technique has been called "polarimetric radar," but we would like to extend this definition to include any radar which does more (with polarization) than transmit and receive the same, single polarization state.

This report represents an in-depth study of radar cross section in terms of transmitted and received polarizations in an attempt to better understand the potential of polarimetric radar. The basis for this analysis is the polarization scattering matrix.

The polarization scattering matrix, as shown in section II, is a generalization of the concept of radar cross section, and includes amplitude, phase, and polarization. Through the use of unit vectors, equations for transforming matrices from linear to circular polarization (and vice versa) are explicitly derived.

The measurement of the elements of the polarization scattering matrix is discussed in Section III. Matrix elements are shown to be related to measureable radar parameters.

The polarization scattering matrices (both for linear and circular polarizations), as discussed in section IV, are derived for three types of simple targets: the dipole, the odd bounce reflector, and the dihedral corner reflector.

In section V, the derivations of section II are applied to a "mixed" polarization scattering matrix for which the transmitted polarization states are different than the received polarization states. Transformation equations for the mixed matrix are determined, and mixed matrices are given for the simple targets discussed in section IV.

In section VI, the concept of the polarization scattering matrix is extended to targets consisting of more than one reflector.

II. THE POLARIZATION SCATTERING MATRIX

The radar cross section of a target is a (fictional) area such that if this area scattered the incident power isotropically, the power received by the radar would be the same as that from the target; radar cross section can be defined mathematically as[1/]

$$\sigma \equiv \lim_{R\to\infty} \; 4\pi R^2 \left| \frac{E^R}{E^T} \right|^2 , \tag{1}$$

[1/] See Reference 1.

where:

σ = radar cross section,
R = distance between radar and target,
E^R = reflected field strength at radar,
E^T = transmitted field strength at target.

Polarization is implicit in this definition of radar cross section, and usually, it is assumed that a single polarization is employed for both the transmitted and received fields. This assumption is not required, however, and radar cross sections can be defined for arbitrary polarization of transmitted and received fields. An arbitrarily polarized plane wave[2/] can be expressed as the sum of two plane waves having orthogonal, but otherwise general polarizations. Following Long[3/], in phasor notation, the transmitted field is expressible as

$$\bar{E}^T = \bar{E}_1^T + \bar{E}_2^T , \tag{2}$$

where the subscripts 1 and 2 refer to any pair of orthogonal polarizations and the overbar indicates a vector quantity. The received fields can be considered to be related to the transmitted fields by a set of reflection coefficients, a_{ij} , as follows:

$$E_{11}^R = a_{11} E_1^T \tag{3a}$$

$$E_{12}^R = a_{12} E_2^T \tag{3b}$$

$$E_{21}^R = a_{21} E_1^T \tag{3c}$$

$$E_{22}^R = a_{22} E_2^T , \tag{3d}$$

where the a_{ij} are, in general, complex quantities.[4/] By superposition, the received fields can be combined as

$$E_1^R = a_{11} E_1^T + a_{12} E_2^T \tag{4a}$$

and

$$E_2^R = a_{21} E_1^T + a_{22} E_2^T . \tag{4b}$$

[2/] The limit in equation (1) assures that the wave is planar.

[3/] See Reference 2.

[4/] When two subscripts are used, the first subscript refers to the polarization of the received wave, and the second subscript refers to the polarization of the transmitted wave.

In matrix notation equations (4a) and (4b) can be expressed as

$$\begin{bmatrix} E_1^R \\ E_2^R \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} E_1^T \\ E_2^T \end{bmatrix}. \tag{5}$$

The matrix [S], given by

$$[S] \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{6}$$

is defined as the polarization scattering matrix. Since the a_{ij} are, in general, complex, the scattering matrix can be written as

$$[S] = \begin{bmatrix} |a_{11}| e^{j\phi_{11}} & |a_{12}| e^{j\phi_{12}} \\ |a_{21}| e^{j\phi_{21}} & |a_{22}| e^{j\phi_{22}} \end{bmatrix}, \tag{7}$$

where, of course, the $|a_{ij}|$ and ϕ_{ij} represent the amplitudes and phase, respectively, of the a_{ij}. By using equations (1) through (7), the concept of radar cross section can be generalized to the case of arbitrary polarization. In terms of scattering matrix notation, the result would be

$$[S] = \frac{1}{\sqrt{4\pi}\, R} \begin{bmatrix} \sqrt{\sigma_{11}}\, e^{j\phi_{11}} & \sqrt{\sigma_{12}}\, e^{j\phi_{12}} \\ \sqrt{\sigma_{21}}\, e^{j\phi_{21}} & \sqrt{\sigma_{22}}\, e^{j\phi_{22}} \end{bmatrix}. \tag{8}$$

The scattering matrix, for a given frequency and given orientations of radar and target, contains all the information concerning the scattering properties of the target. The plausibility of this can be visualized by noting that a plane wave can be represented by any orthogonal but otherwise general polarization(s), and that a polarization basis of one kind can be transformed into any other polarization basis that satisfies the orthogonality requirement.[5/] Hence, a scattering matrix established by one polarization basis can be transformed to any other basis.

[5/] The orthogonality requirement could be reduced to the requirement that $u_1 \times u_2 \neq 0$, where u_1 and u_2 are the unit vector for the two polarization components; this is not done herein, however, as the orthogonality requirement greatly simplifies the mathematics.

Following a procedure similiar to that outlined by Ruck, et al,[6] we will now consider specific forms of polarization and transformations among polarizations for both transmitted and received waves. With these transformations we will have the tools needed for transforming scattering matrices from one polarization basis to another. We begin by defining a Cartesian coordinate system specified by unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, where

$$\hat{x} \times \hat{y} = \hat{z}. \tag{9}$$

We are dealing with plane waves and these waves will be expressed in forms so that transmitted waves propagate in the $+\hat{z}$ direction and received waves propagate in the $-\hat{z}$ direction. Further, for linearly polarized waves, $\hat{x}$ and $\hat{y}$ will represent the direction of the horizontal and vertical components, respectively.

Circular polarization can be formed by splitting a linear-polarized wave of amplitude E into two orthogonal components, each of amplitude $E/\sqrt{2}$, then phase shifting one of these components by 90°. Note that if we express a wave travelling along the positive z axis by E exp [j (wt-kz)], a phase shift θ results in a wave expressed by E exp [j (wt-kz+θ)]. Thus, a phase shift of 90° is given by $\exp(j\pi/2) = \cos \pi/2 + j \sin \pi/2 = j$. Circular polarization can also be mathematically formed by advancing one component by 90°. In this case, the phase shift is given by -j. Right circular polarization can be formed by advancing the y component by 90°; this results in an electric field vector of magnitude $E/\sqrt{2}$ which rotates about the z-axis as it travels down the axis. Looking from the origin along the positive z axis, the tip of this vector is seen to trace out a counterclockwise corkscrew as it travels in the $\hat{z}$ direction. Consider, however, looking at a given plane parallel to the x-y plane and observing the rotation of the electric field vector in this plane as a function of time; looking from the origin in the positive $\hat{z}$ direction, the rotation in time is clockwise. Hence, one must specify a spatial rotation or a temporal rotation in order to define right and left-hand circular polarization clearly.[7]

An arbitrarily polarized plane wave, $\bar{E}^T$, propagating in the $+\hat{z}$ direction, can be expressed in terms of horizontal and vertical components, i.e.

$$\bar{E}^T = H^T \hat{x} + V^T \hat{y}, \tag{10}$$

where, in general, H^T and V^T are complex. This same field can also be represented by the sum of right and left circularly polarized fields as

$$\begin{aligned} \bar{E}^T &= R^T \hat{r} + L^T \hat{l}, \\ &= \bar{E}_r^T + \bar{E}_l^T \end{aligned} \tag{11}$$

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[6] See Reference 3.

[7] The IEEE definition for circular polarization is used herein; i.e., for right-hand polarization the electric field vector rotates CCW (in time) for an approaching wave and CW for a receding wave, and for left-hand polarization the electric field vector rotates CW for an approaching wave and CCW for a receding wave.

where $\hat{r}$ and $\hat{l}$ are unit vectors corresponding to right-hand and left-hand circularity.

In order to understand the meaning of the unit vectors $\hat{r}$ and $\hat{l}$, consider a right circular transmitted field, $\bar{E}_r^T$. In terms of circular polarization

$$\bar{E}_r^T = R^T \hat{r} = E^T \hat{r}. \tag{12}$$

In terms of horizontal and vertical polarizations, a transmitted wave is right circular when $(V/H) = -j$, i.e., and the y component (vertical component) is advanced by 90°. Therefore, $\bar{E}_r^T$ can also be expressed as

$$\bar{E}_r^T = \frac{E^T}{\sqrt{2}} \hat{x} - j \frac{E^T}{\sqrt{2}} \hat{y}, \tag{13}$$

where $H = E^T/\sqrt{2}$ and $V = -jE^T/\sqrt{2}$. Equating (12) and (13) and solving for the right circular unit vector we have

$$\hat{r} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \quad \text{(transmitted wave)}. \tag{14}$$

Note that $|\hat{r}| = 1$. In a similar manner, a left circular transmitted field, $\bar{E}_l^T$, can be expressed in terms of circular polarization as

$$\bar{E}_l^T = L^T \hat{l} = E^T \hat{l}. \tag{15}$$

In terms of horizontal and vertical polarizations, a transmitted wave is left circular when $(V/H) = j$. $\bar{E}_l^T$ can thus be expressed as

$$\bar{E}_l^T = \frac{E^T}{\sqrt{2}} \hat{x} + j \frac{E^T}{\sqrt{2}} \hat{y}, \tag{16}$$

where $H = E^T$ and $V = jE^T$. Equating (15) and (16) and solving for $\hat{l}$ we have

$$\hat{l} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}} \quad \text{(transmitted wave)}. \tag{17}$$

Now, using equations (14) and (17) in equation (11) and equating the result to equation (10) one obtains

$$R^T\left[\frac{\hat{x} - j\hat{y}}{\sqrt{2}}\right] + L^T\left[\frac{\hat{x} + j\hat{y}}{\sqrt{2}}\right] = H^T \hat{x} + V^T \hat{y}. \tag{18}$$

By equating all $\hat{x}$ terms and by equating all $\hat{y}$ terms in equation (18), two equations are obtained. Solving these equations gives

$$R^T = \frac{H^T + jV^T}{\sqrt{2}} \quad , \quad L^T = \frac{H^T - jV^T}{\sqrt{2}} \tag{19}$$

and

$$H^T = \frac{R^T + L^T}{\sqrt{2}} \quad , \quad V^T = \frac{-jR^T + jL^T}{\sqrt{2}} \quad . \tag{20}$$

It should be apparent that R^T, for example, is defined by the magnitude and phase of the initial linear electric field vector before it is broken into x and y components and converted to right circular polarization. Equations (19) and (20) can be expressed in matrix notation as

$$\begin{bmatrix} R^T \\ L^T \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} H^T \\ V^T \end{bmatrix} = [T_{LC}] \begin{bmatrix} H^T \\ V^T \end{bmatrix} \tag{21}$$

and

$$\begin{bmatrix} H^T \\ V^T \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} R^T \\ L^T \end{bmatrix} = [T_{LC}]^{-1} \begin{bmatrix} R^T \\ L^T \end{bmatrix} \quad . \tag{22}$$

The matrix $[T_{LC}]$ and its inverse $[T_{LC}]^{-1}$ can be used to transform the representation of the polarization from linear polarization to circular polarization and vice versa.

The transformation matrices for received fields are somewhat different, due to the changes in propagation direction. However, the method used to determine the transform matrices for transmitted fields can be used to determine the transform matrices for received fields. Consider an arbitrarily polarized plane wave, $\bar{E}^R$, propagating in the $-\hat{z}$ direction. In terms of horizontal and vertical components this wave can be expressed as

$$\bar{E}^R = H^R \, \hat{x} + V^R \, \hat{y} \, , \tag{23}$$

and in terms of circularly polarized fields this field can be expressed as

$$\bar{E}^R = R^R \, \hat{r} + L^R \, \hat{l} \, . \tag{24}$$

A right circular received field, $\overline{E}_r^R$, can be expressed as

$$\overline{E}_r^R = R^R \hat{r} = E^R \hat{r}. \tag{25}$$

In terms of horizontal and vertical polarization, a received wave is right circularly polarized when (V/H) = j; therefore, $\overline{E}_r^R$ can be expressed as

$$\overline{E}_r^R = \frac{E^R}{\sqrt{2}} \hat{x} + j \frac{E^R}{\sqrt{2}} \hat{y}, \tag{26}$$

where $H = E^R/\sqrt{2}$ and $V = jE^R/\sqrt{2}$. From equations (25) and (26) we obtain

$$\hat{r} = \frac{\hat{x} + j\hat{y}}{\sqrt{2}} \qquad \text{(received wave)}. \tag{27}$$

For a left circular received field, $\overline{E}_l^R$, can be written in terms of circular polarization as

$$\overline{E}_l^R = L^R \hat{l} = E^R \hat{l}. \tag{28}$$

In terms of horizontal and vertical polarization, a received wave is left circularly polarized when (V/H) = -j; therefore, we can write

$$\overline{E}_l^R = \frac{E^R}{\sqrt{2}} \hat{x} - j \frac{E^R}{\sqrt{2}} \hat{y} \tag{29}$$

where $H = E^R/\sqrt{2}$ and $V = -j\ E^R/\sqrt{2}$ from equations (28) and (29) we obtain

$$\hat{l} = \frac{\hat{x} - j\hat{y}}{\sqrt{2}} \qquad \text{(received wave)}. \tag{30}$$

Note that the circular polarization unit vectors for the received wave are complex conjuga-es of those for the transmitted wave.

From equations (23), (24), (27), and (29) it can easily be shown that

$$R^R = \frac{H^R - jV^R}{\sqrt{2}}, \quad L^R = \frac{H^R + jV^R}{\sqrt{2}} \tag{31}$$

and

$$H^R = \frac{R^R + jL^R}{\sqrt{2}}, \quad V^R = \frac{jR^R - jL^R}{\sqrt{2}}. \tag{32}$$

Again, R^R is the electric field (amplitude and phase) <u>after</u> the circularly polarized wave has been converted back to a linearly polarized wave. This is achieved by delaying the same component as for the transmitted wave, and then vectorially summing $\hat{x}$ and $\hat{y}$ components. In matrix notation

$$\begin{bmatrix} R^R \\ L^R \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ 1 & j \end{bmatrix} \begin{bmatrix} H^R \\ V^R \end{bmatrix} = [R_{LC}] \begin{bmatrix} H^R \\ V^R \end{bmatrix} , \tag{33}$$

with the reverse transformation given by

$$\begin{bmatrix} H^R \\ V^R \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix} \begin{bmatrix} R^R \\ L^R \end{bmatrix} = [R_{LC}]^{-1} \begin{bmatrix} R^R \\ L^R \end{bmatrix} . \tag{34}$$

Scattering matrices are generally expressed in terms of linear or circular polarizations. From equation (5), we have for linearly polarized fields

$$\begin{bmatrix} H^R \\ V^R \end{bmatrix} = \begin{bmatrix} a_{HH} & a_{HV} \\ a_{VH} & a_{VV} \end{bmatrix} \begin{bmatrix} H^T \\ V^T \end{bmatrix}$$

$$= [S_L] \begin{bmatrix} H^T \\ V^T \end{bmatrix} , \tag{35}$$

and for circularly polarized fields

$$\begin{bmatrix} R^R \\ L^R \end{bmatrix} = \begin{bmatrix} a_{RR} & a_{RL} \\ a_{LR} & a_{LL} \end{bmatrix} \begin{bmatrix} R^T \\ L^T \end{bmatrix}$$

$$= [S_C] \begin{bmatrix} R^T \\ L^T \end{bmatrix} . \tag{36}$$

It should be noted that we have abandoned the often used order of subscripts (transmitted polarization first, received polarization second) for the mathematically conventional order.

Using the transforms given in equations (21), (22), (33), and (34), the elements of $[S_L]$ and $[S_C]$ can be related, giving

$$[S_C] = [R_{LC}] \; [S_L] \; [T_{LC}]^{-1} , \tag{37}$$

and

$$[S_L] = [R_{LC}]^{-1} \; [S_C] \; [T_{LC}] . \tag{38}$$

The relationship between the linear and circular scattering matrices, as expressed in equations (37) and (38), have been expanded; the results are presented in Table 1 below.

TABLE 1. FUNCTIONAL RELATIONSHIPS AMONG THE COMPONENTS OF THE LINEAR AND CIRCULAR SCATTERING MATRICES

Linear to Circular	Circular to Linear
$a_{RR} = \left(\frac{a_{HH} - a_{VV}}{2}\right) - j\left(\frac{a_{HV} + a_{VH}}{2}\right)$	$a_{HH} = \left(\frac{a_{RL} + a_{LR}}{2}\right) + \left(\frac{a_{RR} + a_{LL}}{2}\right)$
$a_{RL} = \left(\frac{a_{HH} + a_{VV}}{2}\right) + j\left(\frac{a_{HV} - a_{VH}}{2}\right)$	$a_{HV} = j\left[\left(\frac{a_{RR} - a_{LL}}{2}\right) - \left(\frac{a_{RL} - a_{LR}}{2}\right)\right]$
$a_{LR} = \left(\frac{a_{HH} + a_{VV}}{2}\right) - j\left(\frac{a_{HV} - a_{VH}}{2}\right)$	$a_{VH} = j\left[\left(\frac{a_{RR} - a_{LL}}{2}\right) + \left(\frac{a_{RL} - a_{LR}}{2}\right)\right]$
$a_{LL} = \left(\frac{a_{HH} - a_{VV}}{2}\right) + j\left(\frac{a_{HV} + a_{VH}}{2}\right)$	$a_{VV} = \left(\frac{a_{RL} + a_{LR}}{2}\right) - \left(\frac{a_{RR} + a_{LL}}{2}\right)$

By invoking the reciprocity theorem, Berkowitz [8/] has shown that for a monostatic radar, the scattering matrix is symmetrical about its main diagonal. In terms of equation (6), the scattering matrix for a monostatic radar, $[S]_M$, can be expressed

$$[S]_M = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} . \qquad (39)$$

The functional relationships among the components of the linear and circular scattering matrices for the monostatic radar can be determined from equations (37) and (38), or they can be determined from Table 1 by setting $a_{LR} = a_{RL}$ and $a_{HV} = a_{VH}$. Results are presented in Table 2 below.

8/ See Reference 4.

TABLE 2. FUNCTIONAL RELATIONSHIPS AMOUNG THE COMPONENTS OF THE LINEAR AND CIRCULAR SCATTERING MATRICES FOR THE MONONSTATIC RADAR CASE.

Linear to Circular	Cicrular to Linear
$a_{RR} = \frac{(a_{HH} - a_{VV})}{2} - j\, a_{HV}$	$a_{HH} = a_{RL} + \frac{(a_{RR} + a_{LL})}{2}$
$a_{RL} = a_{LR} = \frac{(a_{HH} + a_{VV})}{2}$	$a_{HV} = a_{VH} = j\,\frac{(a_{RR} - a_{LL})}{2}$
$a_{LL} = \frac{(a_{HH} - a_{VV}) + j\, a_{HV}}{2}$	$a_{VV} = a_{RL} - \frac{(a_{RR} + a_{LL})}{2}$

III. POLARIZATION SCATTERING MATRIX MEASUREMENT

In general, four amplitudes and four phases are required to completely specify the scattering matrix. The phases are measured relative to the transmitted wave, and include the change in phase of the transmitted wave over the two way path, from radar to target and return, plus the phase change experienced upon reflection from the target. Using equations (3a) through (3d), the elements of the scattering matrix can be expressed in terms of the transmitted and received fields, i.e.,

$$a_{11} = \left[\frac{E^R_{11}}{E^T_1}\right], \tag{40a}$$

$$a_{12} = \left[\frac{E^R_{12}}{E^T_2}\right], \tag{40b}$$

$$a_{21} = \left[\frac{E^R_{21}}{E^T_1}\right], \tag{40c}$$

and

$$a_{22} = \left[\frac{E^R_{22}}{E^T_2}\right], \tag{40d}$$

where it should be remembered that the double subscript, ij, refers to the polarization state of the received and transmitted fields, respectively. In matrix notation

$$[S] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \left(\frac{E^R_{11}}{E^T_1}\right) & \left(\frac{E^R_{12}}{E^T_2}\right) \\ \left(\frac{E^R_{21}}{E^T_1}\right) & \left(\frac{E^R_{22}}{E^T_2}\right) \end{bmatrix} . \tag{41}$$

Clearly, measuring the a_{ij}'s requires first transmitting one polarization state while receiving that state and the orthogonal state simultaneously, followed by transmitting the orthogonal polarization and again receiving both polarizations. A "two-pulse" measurement is needed.

The amplitude of the signals in each of the two receive channels can be achieved by square-law detection. To determine the phases, the received signal must be processed coherently; that is, it must be mixed with the transmitted signal to produce a dc voltage which is a function of the relative phase. Such a measurement is not easy, particularly for a pulsed system, since the transmitted signal must be stored.

It is much easier, for the monostatic case, to measure the phases of the received signals relative to each other.

Using the notation, as given in equation (7) and choosing ϕ_{12} as the phase term to be factored, the scattering matrix can be written as

$$[S] = e^{j\phi_{12}} \begin{bmatrix} |a_{11}| e^{j(\phi_{11}-\phi_{12})} & |a_{12}| \\ |a_{21}| e^{j(\phi_{21}-\phi_{12})} & |a_{22}| e^{j(\phi_{22}-\phi_{12})} \end{bmatrix} . \tag{42}$$

Note that, in general, one must retain the first pulse received signals to compare phase with the second pulse received signals.

For the monostatic radar case, which is of primary interest herein, $\phi_{12} = \phi_{21}$ and $|a_{12}| = |a_{21}|$; therefore, for this case, equation (42) can be rewritten as

$$[S]_M = e^{j\phi_{12}} \begin{bmatrix} |a_{11}| e^{j(\phi_{11} - \phi_{12})} & |a_{12}| \\ |a_{12}| & |a_{22}| e^{j(\phi_{22}-\phi_{12})} \end{bmatrix} . \tag{43}$$

All that is required is a relative phase measurement between the orthogonal receive channels for each pulse separately.

Suppose now we have a monostatic radar that is designed to sequentially transmit $\begin{bmatrix} E_1^T \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ E_2^T \end{bmatrix}$ and to simultaneously receive E_1^R and E_2^R for both transmissions, where subscripts 1 and 2 indicate orthogonal polarization states. For this radar, the relative phase measurements can be made directly from the received fields; the importance of this is that it is not required to retain the phases of the transmitted fields. Using equations (40a) through (40d) we readily obtain

$$|a_{11}| = \frac{|E_{11}^R|}{|E_1^T|}, \tag{44a}$$

$$|a_{21}| = |a_{12}| = \frac{|E_{12}^R|}{|E_2^T|} = \frac{|E_{21}^R|}{|E_1^T|}, \tag{44b}$$

$$|a_{22}| = \frac{|E_{22}^R|}{|E_2^T|}, \tag{44c}$$

$$e^{j(\phi_{11} - \phi_{12})} = \frac{E_{11}^R}{|E_{11}^R|} \cdot \frac{|E_{21}^R|}{E_{21}^R} \tag{44d}$$

$$= \exp\left[j\left(\text{phase of } E_{11}^R - \text{phase of } E_{21}^R\right)\right],$$

and

$$e^{j(\phi_{22} - \phi_{12})} = \frac{E_{22}^R}{|E_{22}^R|} \cdot \frac{|E_{12}^R|}{E_{12}^R} \tag{44e}$$

$$= \exp\left[j\left(\text{phase of } E_{22}^R - \text{phase of } E_{12}^R\right)\right].$$

As can be seen by inspection of the latter two equations, the relative phases are not functions of the phases of the transmitted fields. In fact, with the assumed dual channel receiver (i.e., a receiver that simultaneously measures E_1^R and E_2^R), the relative phase measurements are determined solely from simultaneously received signals. This greatly eases the phase measurement burden.

For completeness, we substitute equations (44a) through (44e) into equation (43); the result is

$$[S]_M = e^{j\phi 12} \begin{bmatrix} \left(\frac{E^R_{11}}{|E^T_1|} \cdot \frac{|E^R_{21}|}{E^R_{21}}\right)\left(\frac{|E^R_{12}|}{|E^T_2|}\right) = \left(\frac{|E^R_{21}|}{|E^T_1|}\right) \\ \left(\frac{|E^R_{12}|}{|E^T_1|}\right) = \left(\frac{|E^R_{21}|}{|E^T_1|}\right)\left(\frac{E^R_{22}}{|E^T_2|} \cdot \frac{|E^R_{12}|}{E^R_{12}}\right) \end{bmatrix} . \tag{45}$$

The term $e^{j\phi 12}$ is a function of radar-to-target distance, and is therefore not easily measurable. Thus, except for this phase term, $[S]_M$ can be specified in terms of readily measurable radar quantities. We now specifically address circular and linear polarization states.

In terms of circular polarization, equation (43) can be written as

$$[S_C]_M = e^{j\phi_{RL}} \begin{bmatrix} |a_{RR}|\, e^{j(\phi_{RR} - \phi_{LR})} & |a_{LR}| \\ |a_{RL}| & |a_{LL}|\, e^{j(\phi_{LL} - \phi_{RL})} \end{bmatrix}, \tag{46}$$

and, for linear polarization, we have

$$[S_L]_M = e^{j\phi_{HV}} \begin{bmatrix} |a_{HH}|\, e^{j(\phi_{HH} - \phi_{VH})} & |a_{VH}| \\ |a_{HV}| & |a_{VV}|\, e^{j(\phi_{VV} - \phi_{HV})} \end{bmatrix} . \tag{47}$$

Using these two equations we define $[S_C]_m$ and $[S_L]_m$, via

$$[S_c]_M = e^{j\phi_{RL}}\, [S_c]_m \tag{48}$$

and

$$[S_L]_M = e^{j\phi_{HV}}\, [S_L]_m \tag{49}$$

It is of importance to note that $[S_C]_m$ and $[S_L]_m$ are those portions of $[S_C]_M$ and $[S_L]_M$, respectively, that can be quantified in terms of the radar measures specified by equations (44a) through (44e) or by equation (45). By substituting equations (48) and (49) into equations (37) and (38), and then rearranging slightly, we obtain

$$e^{j(\phi_{RL} - \phi_{HV})} [S_C]_m = [R_{LC}] [S_L]_m [T_{LC}]^{-1} \tag{50}$$

and

$$e^{j \phi_{RL} - \phi_{HV}} [[S_L]_m = [R_{LC}]^{-1} [S_C]_m [T_{LC}]. \tag{51}$$

Except for the constant phase difference of $\pm (\phi_{RL}-\phi_{HV})$, these equations are the same as equations (37) and (38), respectively; the functional relation listed in Table 2 are essentially valid for $[S_C]_m$ and $[S_L]_m$.

In this section we have developed relationships specifying scattering matrix elements in terms of received and transmitted fields. However, we have been somewhat lax in our use of these field quantities. In particular, we have equated the received field to the field reflected by the target, and we have equated the transmitted field to the field incident on the target. Although the received and transmitted fields can be scaled to the reflected and incident fields, respectively, we should be more specific about these field quantities.

Consider a monostatic radar operating in a lossless homogeneous medium. From the radar range equation, the received power, P_{ri}, is related to the transmitted power, P_{tj}, by

$$P_{ri} = \left(\frac{P_{tj} G}{4\pi R^2} \right) \left(\frac{\sigma_{ij} A}{4\pi R^2} \right), \tag{52}$$

where the subscript i and j refer to the polarization states of the received and transmitted fields, respectively; G is the effective antenna gain, σ_{ij} is the cross section of the target for the case of received polarization of state "i" and transmitted polarization of state "j", A is the effective area of the antenna, and R is the distance between radar and target. Let us further suppose that there are two permissible polarization states denoted by (1,2), with the two polarization states being orthogonal. In general then, i and j can take on any combination of values denoted by

$$i,j = 1,2 .$$

Rearranging equation (52), we can express σ_{ij} as

$$\sigma_{ij} = 4\pi R^2 \left[\frac{\left(P_{ri}/A \right)}{\left(P_{tj} G/4\pi R^2 \right)} \right] . \tag{53}$$

The quanity (P_{ri}/A) is the power density (power per unit area), or the intensity, of reflected signal at the radar, and the quanity $\left[\frac{P_{tj}G}{4\pi R^2}\right]$ is the intensity of the radar signal incident on the target. Referring to equation (1), it should be obvious that the magnitudes of the reflected fields at the radar and the transmitted fields at the target are given by

$$\left|E^R_{ij}\right| = a\left(P_{ri}/A\right)^{1/2}, \tag{54a}$$

and

$$\left|E^T_j\right| = a\left(\frac{P_{tj}G}{4\pi R^2}\right)^{1/2}, \tag{54b}$$

where the subscripts are included so as to maintain the polarization references, and a is a constant.

Although not quite as straight forward, the above procedure can be used to determine $\left|E^R_{ij}\right|$ and $\left|E^T_j\right|$ for the case of a monostatic radar operating in a homogenous, lossy medium. The results are expressible as

$$\left|E^R_{ij}\right| = a\left(\frac{P_{ri}}{A}\, e^{-2\alpha R}\right)^{1/2} \tag{55a}$$

and

$$\left|E^T_j\right| = a\left(\frac{P_{tj}G}{4\pi R^2}\, e^{-2\alpha R}\right)^{1/2}, \tag{55b}$$

where α represents the attenuation coefficient of the medium.

We wish to emphasize that $\left|E^R_{ij}\right|$ and $\left|E^T_j\right|$ are not direct radar measures, but they can be related to measures of received and transmitted power. In addition, by determining the phases of the radar signals, the phases of E^R_{ij} and E^T_j can also be determined. Thus, when we refer to received and transmitted fields we are, strictly speaking, referring to E^R_{ij} and E^T_j.

IV. POLARIZATION SCATTERING MATRICES FOR SIMPLE TARGETS

In this section, we heuristically derive monostatic scattering matrices for a dipole, an odd-bounce reflector (e.g., plate, sphere, curved surface, or trihedral corner reflector), and a dihedral corner reflector. Results are presented for both linear and circular polarizations. It will be noted that these scattering matrices provide for partitioning polarization and for the phase change of each polarization component, but they do not account for amplitude. To account for amplitude, the scattering matrix must be multipled by a real non-negative constant. This constant can be estimated in the following manner: first determine the cross section of the target under con-

sideration, via the procedures given in many radar texts[9/] (this cross section is usually given for same sense linearly polarized transmitted and received signals), and then scaled via the radar range equation; the constant is the square root of this scaled cross section.

A. DIPOLE TARGET

Assume that a thin wire (diameter << wavelength) or a sharp edge of a conducting body is constrained to lie parallel to the $\hat{x}$, $\hat{y}$ plane and at an angle α with respect to the $\hat{x}$, $\hat{z}$ plane (see Figure 1). Assume further that a transmitted field $\bar{E}^T = H^T \hat{x} + V^T \hat{y}$ is normally incident on the wire. For such a field, the induced current along the dipole is proportional to the projection of incident electric field along the axis of the dipole. The reradiated field is proportional to the induced current, and it is polarized along the axis of the dipole. Thus, the induced current, i_d, is expressible as

$$i_d \propto H^T \cos \gamma + V^T \sin \gamma, \tag{56}$$

and the reradiated field, $\bar{E}_d$, is, therefore, given by

$$\bar{E}_d \propto (H^T \cos \gamma + V^T \sin \gamma)\, \hat{\gamma}. \tag{57}$$

where $\hat{\gamma}$ is a unit vector parallel to the wire. This unit vector, expressed in terms of $\hat{x}$ and $\hat{y}$, can be written

$$\hat{\alpha} = \cos \gamma\, \hat{x} + \sin \gamma\, \hat{y}\ . \tag{58}$$

The reradiated field and the received field, $\bar{E}^R = H^R \hat{x} + V^R \hat{y}$, are also proportional; therefore,

$$H^R \propto H^T \cos^2 \gamma + V^T \sin \gamma \cos \gamma, \tag{59a}$$

and

$$V^R \propto H^T \sin \gamma \cos \gamma + V^T \sin^2 \gamma. \tag{59b}$$

Referring to equations (3a) through (3d), one can easily show that the dipole scattering matrix, $[S_L]_{\text{dipole}}$, for linear representation of polarization is given by

$$[S_L]_{\text{dipole}} = \begin{bmatrix} \cos^2 & \sin \gamma \cos \gamma \\ \sin \gamma \cos \gamma & \sin^2 \gamma \end{bmatrix} \tag{60}$$

9/ See Reference 5.

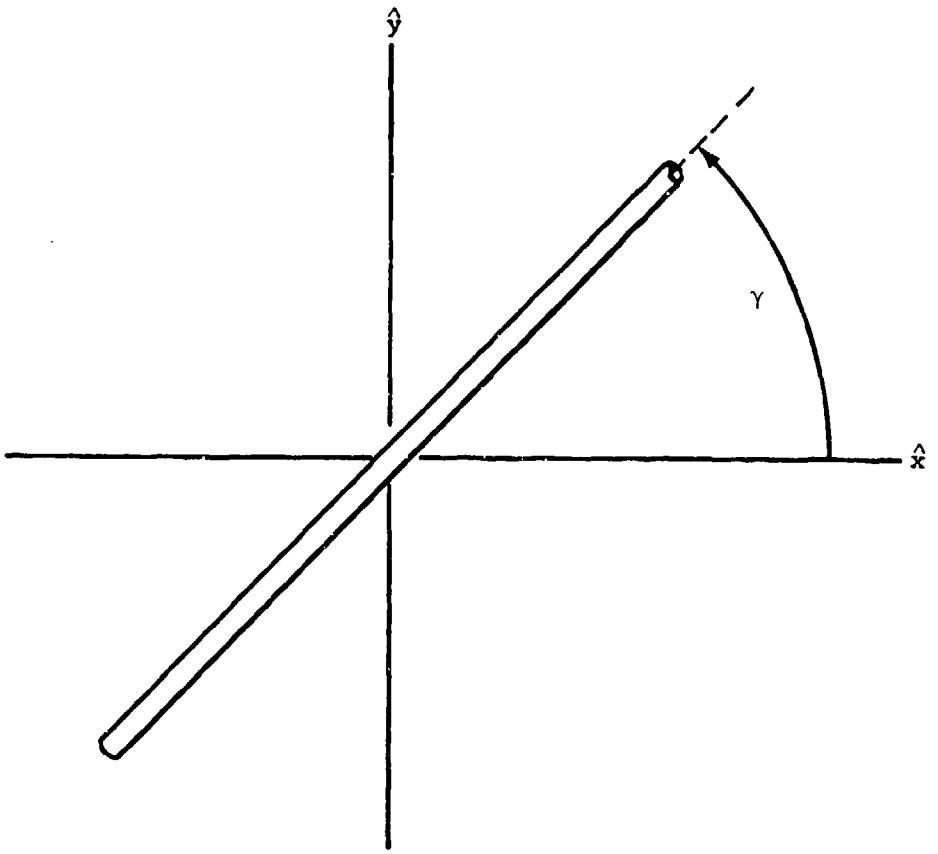

Figure 1. Dipole parallel to the $\hat{x}$, $\hat{y}$ plane.

The dipole scattering matrix for circular polarization, $[S_C]_{dipole}$, can be obtained directly from equation (47) and Table 1. The result is

$$[S_C]_{dipole} = \begin{bmatrix} e^{-j2\gamma} & 1 \\ 1 & e^{j2\gamma} \end{bmatrix} . \tag{61}$$

B. ODD-BOUNCE TARGETS

Odd-bounce targets refer to area targets having dimensions >> wavelength; this includes flat plates, trihedrals, and curved surfaces, where the curved surfaces are also conditioned by the requirement that their radii of curvature >> wavelength. Consider a flat, perfectly conducting plate aligned parallel to the $\hat{x}$, $\hat{y}$ plane, with a transmitted field $\overline{E}^T = H^T\hat{x} + V^T\hat{y}$ incident on the plate. To maintain the boundary requirement of zero field on the surface of the plate, the reflected field, $\overline{E}^R$ must be equal to $-\overline{E}^T$ at the surface of the plate; therefore, we have

$$\bar{E}^R = -H^T \hat{x} - V^T \hat{y} . \tag{62}$$

The linear scattering matrix, $[S_L]_{\text{odd-bounce}}$, is thus obviously given by

$$[S_L]_{\text{odd-bounce}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{63}$$

where we have suppressed the (-1) coefficient.[10/]

Again using Table 1, it can be readily shown that in circular representation the scattering matrix can be expressed

$$[S_C]_{\text{odd-bounce}} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}. \tag{64}$$

It is of some interest to compare the linear and circular scattering matrices given by equations (63) and (64). In linear notation, the return is co-polarized (same polarization received as transmitted), and in circular notation, the return is cross-polarized (opposite polarization received as transmitted).

C. DIHEDRAL TARGET

Assume that a dihedral is located so that its seam is parallel to the x, y plane, that its seam makes an angle α with respect to the x, z plane, that its retro-reflecting face is composed of perfectly conducting planes, and that all dihedral dimensions >> wavelength. The incident radiation is normal to the seam.

Now consider a transmitted field, $\bar{E}^T = H^T\hat{x} + V^T\hat{y}$, incident on the dihedral. This incident field can be written as

$$\bar{E}^T = (H^T \cos\alpha + V^T \sin\alpha)\,\hat{\alpha} + (H^T \sin\alpha - V^T \cos\alpha)\,\hat{\beta} , \tag{65}$$

where $\hat{\alpha}$ is a unit vector paralled to the dihedral seam, $\hat{\beta}$ is a unit vector perpendicular to $\hat{\alpha}$, and $\hat{\beta} \times \hat{\alpha} = \hat{z}$. In terms of the unit vectors x and y we have,

$$\hat{\alpha} = \cos\alpha\,\hat{x} + \sin\alpha\,\hat{y} , \tag{66a}$$

10/ This turn would not affect relative phase, and therefore, since we are interested only in relative phase, suppressing it does not affect our results.

and

$$\hat{\beta} = \sin\alpha\,\hat{x} - \cos\alpha\,\hat{y}\,. \tag{66b}$$

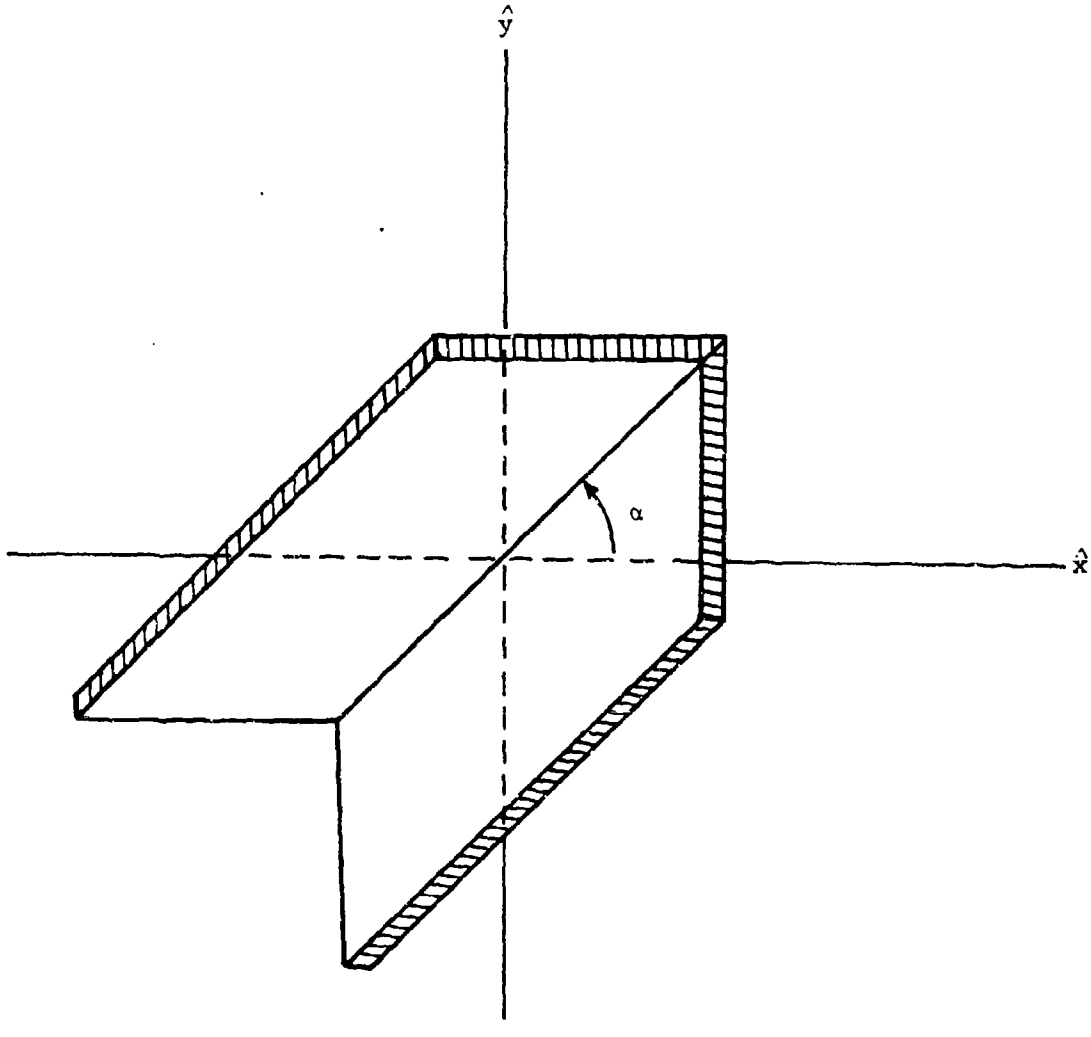

Figure 2. Dihedral parallel to the $\hat{x}$, $\hat{y}$ plane.

The term $(H^T \cos\alpha + V^T \sin\alpha)\,\hat{\alpha}$ is the portion of the incident field that is polarized parallel to the dihedral seam; the remaining portion of the incident field $(H^T \sin\alpha - V^T \cos\alpha)\,\hat{\beta}$ is polarized perpendicular to the dihedral seam. First, consider the parallel component of the incident field; boundary conditions of zero field on the dihedral surfaces require that, upon each reflection, the phase of the field be shifted by 180°. Thus, for two reflections, the reflected field would be the same as the incident field. In a similiar manner, it can be reasoned that the perpendicular portion of the incident field experiences a total of 180° phase shift upon reflection from the dihedral; thus, the refected portion is $(-H \sin\alpha + V \cos\alpha)\,\hat{\beta}$. The total reflected field, $\bar{E}^R$, can therefore be expressed as

$$\bar{E}^R = (H^T \cos\alpha + V^T \sin\alpha)\,\hat{\alpha} + (-H^T \sin\alpha + V^T \cos\alpha)\,\hat{\beta} \,. \qquad (67)$$

Using equations (66a) and (66b) we can write

$$\bar{E}^R = (H^T \cos 2\alpha + V^T \sin 2\alpha)\,\hat{x} + (H^T \sin 2\alpha - V^T \cos 2\alpha)\,\hat{y} \,. \qquad (68)$$

Setting $\bar{E}^R = H^R\hat{x} + V^R\hat{y}$, and using matrix notation we have

$$\begin{bmatrix} H^R \\ V^R \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \begin{bmatrix} H^T \\ V^T \end{bmatrix} \,. \qquad (69)$$

Therefore, the linear scattering matrix for a dihedral can be written

$$[S_L]_{\text{dihedral}} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix} \,. \qquad (70)$$

The corresponding circular scattering matrix can be obtained directly from this equation and Table 1;

$$[S_C]_{\text{diherdral}} = \begin{bmatrix} e^{-j2\alpha} & 0 \\ 0 & e^{j2\alpha} \end{bmatrix} \,. \qquad (71)$$

V. MIXED POLARIZATION SCATTERING MATRIX

The polarization scattering matrix, as previously stated, is defined for a pair of orthogonal polarizations and for transmitting and receiving these same polarizations. We can, however, define a matrix that relates different received and transmitted polarizations, that is, a "mixed matrix."

A mixed matrix, of some interest, is that resulting from relating circularly polarized transmitted fields to linearly polarized received fields. This mixed matrix can be defined, via equation (5), as

$$\begin{bmatrix} H^R \\ V^R \end{bmatrix} = \begin{bmatrix} a_{HR} & a_{HL} \\ a_{VR} & a_{VL} \end{bmatrix} \begin{bmatrix} R^T \\ L^T \end{bmatrix}$$

$$= [M_{CL}] \begin{bmatrix} R^T \\ L^T \end{bmatrix} \,. \qquad (72)$$

Using equation (34), the mixed matrix can be expressed in terms of the circular scattering matrix as

$$[M_{CL}] = [R_{LC}]^{-1} [S_C] \cdot \tag{73}$$

The reverse transformation is therefore given by

$$[S_C] = [R_{LC}] [M_{CL}] \cdot \tag{74}$$

Using equation (22), the mixed matrix can be related to the linear scattering matrix, i.e.,

$$[M_{CL}] = [S_L] [T_{LC}]^{-1} ; \tag{75}$$

the reverse relationship is expressed as

$$[S_L] = [M_{CL}] [T_{LC}] \cdot \tag{76}$$

The relationship among the matrix components, as specified in equations (73) through (76) are tabulated in Tables 3 and 4 below. For the monostatic radar case, $a_{12} = a_{21}$ for both linear and circular scattering matrices. However, in general, the mixed matrix is not symmeterical about the main diagonal. This results from the fact that the mixed matrix is defined for different states of transmitted and received polarizations; whereas, scattering matrices are defined only for orthogonal transmitted and received polarizations pairs.

TABLE 3. FUNCTIONAL RELATIONSHIPS AMONG THE COMPONENTS OF THE CIRCULAR SCATTERING MATRIX AND THE MIXED MATRIX

Circular to Mixed	Mixed to Circular
$a_{HR} = \frac{a_{RR} + a_{LR}}{\sqrt{2}}$	$a_{RR} = \frac{a_{HR} - ja_{VR}}{\sqrt{2}}$
$a_{HL} = \frac{a_{LL} + a_{RL}}{\sqrt{2}}$	$a_{RL} = \frac{a_{HL} - ja_{VL}}{\sqrt{2}}$
$a_{VR} = \frac{j\left(a_{RR} - a_{LR}\right)}{\sqrt{2}}$	$a_{LR} = \frac{a_{HR} + ja_{VR}}{\sqrt{2}}$
$a_{VL} = \frac{-j\left(a_{LL} - a_{RL}\right)}{\sqrt{2}}$	$a_{LL} = \frac{a_{HL} + ja_{VL}}{\sqrt{2}}$

TABLE 4. FUNCTIONAL RELATIONSHIPS AMONG THE COMPONENTS OF THE LINEAR SCATTERING MATRIX AND THE MIX MATRIX

Linear to Mixed	Mixed to Linear
$a_{HR} = \frac{a_{HH} - ja_{HV}}{\sqrt{2}}$	$a_{HH} = \frac{a_{HR} + a_{HL}}{\sqrt{2}}$
$a_{HL} = \frac{a_{HH} + ja_{HV}}{\sqrt{2}}$	$a_{HV} = \frac{j\,a_{HR} - a_{HL}}{\sqrt{2}}$
$a_{VR} = \frac{-ja_{VV} + a_{HH}}{2\sqrt{2}}$	$a_{VH} = \frac{a_{VR} + a_{VL}}{\sqrt{2}}$
$a_{VL} = \frac{ja_{VV} + a_{VH}}{\sqrt{2}}$	$a_{VV} = \frac{j\,a_{VR} - a_{VL}}{\sqrt{2}}$

As a further result of this nonsymmetry, the ability to determine all essential parameters of the polarization scattering matrix, using two-channel relative phase measurements, is lost.

Using the relations in Table 3 or Table 4, one can derive the mixed matrix for the simple targets discussed in Section IV. These matrices are given in Table 5.

TABLE 5. MIXED MATRICES FOR SIMPLE TARGETS

Target	Mixed Matrix
Dipole	$\frac{1}{2\sqrt{2}} \begin{bmatrix} 1+e^{j2\alpha} & 1+e^{j2\alpha} \\ -j\ 1-e^{j2\alpha} & j\ 1-e^{j2\alpha} \end{bmatrix}$
Odd Bounce	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix}$
Dihedral Corner	$\frac{1}{\sqrt{2}} \begin{bmatrix} e^{-j2\alpha} & e^{j2\alpha} \\ je^{-j2\alpha} & -je^{j2\alpha} \end{bmatrix}$

VI. POLARIZATION SCATTERING MATRIX FOR COMPLEX TARGETS

The concept of the polarization scattering matrix, in the following development, is extended to complex (i.e., "complicated ") targets under the assumption that the target RCS is dominated by specular reflections. The target consists of n reflectors each described by a matrix

$$[S^i] = \begin{bmatrix} a^i_{11} & a^i_{12} \\ a^i_{21} & a^i_{22} \end{bmatrix}, \tag{77}$$

where the superscript denotes the i^{th} reflector.

The total scattering matrix elements will be a phasor sum of each of the a^i_{jk} taking into account the phase shifts resulting from the range distribution of the reflectors. If the matrix for the total target is

$$[S_T] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tag{78}$$

and if R_i is the range to the i^{th} reflector, then, for the monostatic case,

$$A_{11} = \sum_{i=1}^{n} a^i_{11} \exp\left[-j2\pi\left(\frac{2R_i}{c}\right) f\right], \tag{79}$$

$$A_{22} = \sum_{i=1}^{n} a^i_{22} \exp\left[j2\pi\left(\frac{2R_i}{c}\right) f\right], \tag{80}$$

and

$$A_{12} = A_{21} = \sum_{i=1}^{n} a^i_{12} \exp\left[-j2\pi\left(\frac{2R_i}{c}\right)\right] f \quad , \tag{81}$$

Again, the various phase shifts resulting from surface reflections need to be included in the a's. It is argued, however, that since the target extent is much larger than the wavelength, and since any given phase shift is equivalent to a range displacement of less than a wavelength, the general polarization properties of the target can be grasped without including these phase shifts.

Equations (79) through (81) demonstrate very clearly the dependence of the matrix elements of transmitted frequency and target orientation, even if the target is a collection of spheres. Note that it has been assumed, that all reflectors are equally illuminated by the radar. Also, the equations indicate that the returns from a pair of reflectors may constructively or destructively interfere, and the nature of the interference changes with changing target orientation or with changing transmitted frequency. The first phenomenon is well known as amplitude scintillation. The latter phenomenon has been investigated as a target classification tool referred to as "frequency-diverse target scintillation."[11/]

Applying equation (79) to the case of same-sense linear polarization and recalling that

$$a_{ik} = \sqrt{\sigma_{ik}}\ e^{j\phi_{ik}}, \tag{82}$$

and

$$\sigma_{ik} = \left|a_{ik}\right|^2, \tag{83}$$

the total radar cross section is found to be the well-known result,

$$\sigma_T = |A|^2 = \left|\sum_{i=1}^{n} \sqrt{\sigma_i}\ e^{j\phi_i}\right|^2, \tag{84}$$

where the phase factors ϕ_i include phase change on reflection and phase delay because of the distance from the radar.

An important, although perhaps intuitive, result of equations (79) through (81) is that the target retains the polarization character of its individual reflectors. For example, if the individual reflectors are largely odd-bouce (a_{RL}^i), then A_{RL} will, on the average, be large, and the target can be considered an odd-bounce target. The "average" referred to above is an average over a number of frequencies, a number of aspect angles, or a number of different targets (clutter patches, for example) with random spatial distributions.

11/ See Reference 6.

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