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Polarizations and Asymmetries in Stripping Reactions

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Spin dependences of interactions appearing in the DWBA stripping amplitude are theoretically investigated by the invariant-amplitude method for polarizations of emitted protons from unpolarized targets, $P(\theta)$, and cross-section asymmetries from polarized targets, $A(\theta)$. In the case treated, the nuclear spins are 1/2 and zero for the target and residual nuclei, the neutron being captured into an S-state. The theory gives a definite value of $P(\theta)/A(\theta)$ for each spin dependence without any numerical calculation. In particular, the conventional proton-spin-orbit assumption leads to $P(\theta)/A(\theta)=-3$, which is incompatible with the experimental data for the $He^3(d,p)He^4$ reactions. Possibilities of explaining the data are discussed.

§ 1. Introduction

The distorted-wave Born approximation (DWBA) has been successful in analyses of (d, p) reactions. In the theory, the transition amplitude in the post form is transformed by the Gell-Mann-Goldberger theorem; an optical potential is assumed for the outgoing proton.[1] The total wave function of the initial state is approximately replaced by the distorted-wave function provided by an optical potential of the deuteron. The resultant transition amplitude is given by[2]

$$T(d+A\rightarrow p+B)=\langle\chi_{pB}^{(-)}\varphi_B,\ (V_{pn}+V_{pA}-U_{pB})\chi_{dA}^{(+)}\varphi_d\varphi_A\rangle\,, \tag{1·1}$$

where V_{pn} and V_{pA} are the interaction of the proton with the neutron and that with the target nucleus A, respectively, and U_{pB} is the optical potential of the proton from the residual nucleus B. The wave functions φ_A, φ_B and φ_d describe the internal motions of the target nucleus, the residual nucleus and the deuteron. The distorted wave functions $\chi_{pB}^{(-)}$ and $\chi_{dA}^{(+)}$ are calculated by U_{pB} and U_{dA}, respectively, U_{dA} being the optical potential of the deuteron. Most of the conventional analyses of the reaction assume a central potential for U_{dA} and a central-plus-spin (proton spin)-orbit potential for U_{pB} and entirely neglect $V_{pA}-U_{pB}$.

In many cases, the theoretical predictions are in worse agreement with the experimental data in the polarizations of the emitted protons than in the cross sections.[3] This matter will be interpreted as follows: the cross section is mainly determined by the fundamental assumption of the reaction mechanism and scarcely depends on details of the interactions, while the polarization depends seriously on the details, for example the spin dependence of the interactions, and the conventional treatment is not refined sufficiently for analyses of polarization phenomena. That is to say, some of the theoretical approximations are not always valid for polarization phenomena; for example, the neglect of $V_{pA}-U_{pB}$ in Eq. (1·1) is not always justified because of differences between the spin dependences of V_{pA} and U_{pB}. In fact, as is discussed later, nucleon-nucleus interactions sometimes depend on the spin of the nucleus, and the spin of the nucleus A is always different from that of B in the stripping reaction. Such inadequate treatments of the interactions will presumably introduce significant errors in theoretical results for polarization phenomena. The purpose of the present paper is to postulate a method of investigation of the spin dependence of the interactions and to criticize the theoretical approximation as its application.

Since the spin dependences of the interactions are specifically reflected in the polarization phenomena, our study will be limited to this subject. The general features of the phenomena in the stripping reaction have been discussed earlier,[4),5)] i.e. the polarization of the emitted proton, the polarization of the residual nucleus, and the effects of a polarized target and of a polarized beam are studied. Of these phenomena, we discuss the ratio of the proton polarization from unpolarized targets to a left-right asymmetry of the cross section from polarized targets. In particular, we treat the special case in which the neutron is captured into an S-state by the target nucleus of spin 1/2 forming the final nucleus of spin 0. In this case, the asymmetry has been measured only for the $He^3(d,p)He^4$ reactions.[6)] Tanifuji[7)] has discussed the effect of the proton-spin-orbit interactions and of a proton-target tensor force on the polarization-asymmetry ratio in this reaction in the so-called prior formalism. For the same reaction, Duck[8)] has calculated numerically the proton polarization and the cross-section asymmetry by the invariant-amplitude method with a limited number of partial waves, and Csonka et al.[9)] have also investigated the properties of the invariant amplitudes, but the spin dependence of the interaction has never been studied by this method. In this paper, the invariant-amplitude method is extended to a more general case where particles with arbitrary spins are interacting with each other. The method is applied to the DWBA stripping amplitude for which the spin dependences of the interactions are investigated, $V_{pA}-U_{pB}$ being taken into account. As an example, the $He^3(d,p)He^4$ reaction is discussed in detail. Since the partial-wave expansion is not used at the present, the result is free from the limitation of the number of partial waves.

In this paper, the following force assumptions are particularly examined; 1) proton-spin-orbit interactions, 2) target-spin-orbit interactions, 3) a tensor-type interaction between the proton and the target nucleus, 4) deuteron-spin-orbit interactions, 5) deuteron tensor forces, 6) a spin-spin interaction between the deuteron and the target nucleus, 7) a spin-spin interaction between the proton and the target nucleus, and 8) a neutron-proton tensor force. The mathematical expressions of these interactions are listed in Table I of § 4. Of these interactions, a proton-spin-orbit potential has been found in analyses of the elastic scattering of protons by nuclei and is conventionally used in the DWBA analyses, while for the others, 2) ~7), their effects have not been clarified. A spin-orbit potential of a Be^9 nucleus has been postulated for α-Be^9 elastic scatterings.[10] Spin-orbit interactions of nuclei are a reasonable extension of this idea. The interaction 3) is introduced for V_{pA} by analogy with the nucleon-nucleon tensor force, the effect of which is well known for low-energy nucleon-nucleon scattering and the properties of the deuteron bound state. The range and depth of this interaction are almost the same as those of the nucleon-nucleon central force.[11] Although the derivation of nucleon-nucleus potentials from the two-nucleon interactions is not clear at present, the existence of the nucleon-nucleus tensor force is probable. Recently, spin-orbit potentials and tensor forces of deuterons have been studied for elastic scattering by nuclei and have been found to be important in obtaining good fits to data.[12] A spin-spin interaction between a neutron and a nucleus has been studied by several authors.[13],[14] The results suggest the investigation of the similar interaction 7) in the present case to be valuable. In the DWBA theory, V_{pn} is eliminated from the transition amplitude by the use of the Schrödinger equation for the deuteron internal motion, the property of the interaction being fully reflected in the deuteron wave function φ_d. Thus, the effect of the neutron-proton tensor force can be taken into account by including the D-state admixture in φ_d. This note will discuss the effect of the D-state admixture instead of the tensor force itself.

Effects of exchange processes have been studied by both the cutoff Born approximation[15],[16] and the distorted-wave theory.[17] This study discusses the exchange effect, for which the spin-dependent interactions are assumed analogous to those for the direct process. In later sections, we first treat almost all the possible spin dependences of the interactions in the first order. Then we show that for most cases of physical significance, the obtained results are valid to any order. As shown later, for each case the theory can predict a definite polarization-asymmetry ratio without any numerical calculation. Since the ratio obtained depends strongly on the assumptions of the spin dependence of the interactions and the results are completely independent of the parameters of the potentials, the results are useful in providing a criticism of the theory and the potential assumptions, though the cases treated are rather special.

§ 2. Cross-section asymmetries and polarizations of emitted protons

A general formula of the cross section from a polarized target was discussed by Goldfarb and Bromley[4] and is given by

$$\left.\frac{d\sigma}{d\Omega}\right)_{\mathrm{pol}} \propto \mathrm{Tr}\,(T\rho^{i}T^{\dagger}),$$

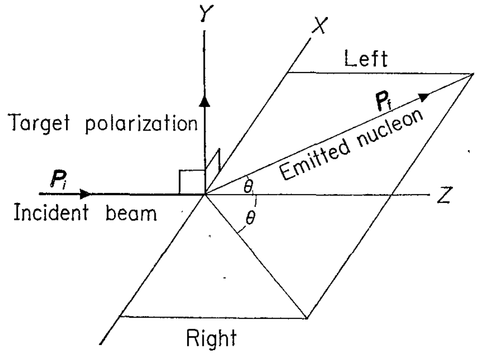

Fig. 1. Orientation of coordinate axes. Cross sections are defined in the x-z plane.

where ρ^i is the density matrix of the target spin and T denotes the reaction matrix, the elements of which give the transition amplitudes. This formula is calculated for the case of target spin 1/2 in the special frame of coordinate axis illustrated in Fig. 1, where the direction of momentum of the incident deuteron is chosen as the z-axis and where the direction of the target polarization, which is perpendicular to this axis, is taken as the y-axis; the results are

$$\left.\frac{d\sigma}{d\Omega}\right)^{(R)}_{\text{pol}\,(L)} = \left.\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}$$

$$\pm C(q_{1/2}-q_{-1/2}) \sum_{\nu_B,\nu_d,\nu_p} \mathrm{Im}\{\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A=\tfrac{1}{2},\nu_d;\boldsymbol{k}_i\rangle^*\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A=-\tfrac{1}{2},\nu_d;\boldsymbol{k}_i\rangle\}, \tag{2·1}$$

where $d\sigma/d\Omega)^{(L)}_{\text{pol}}$ and $d\sigma/d\Omega)^{(R)}_{\text{pol}}$ are the left-side and right-side cross sections for the polarized target and $d\sigma/d\Omega)_{\text{unpol}}$ is the cross section for an unpolarized target. Also, C is a constant for a particular choice of the momenta and the masses of the incident deuteron and the emitted nucleon. The magnitude of the polarization of the target nucleus is given by the populations of its spin-substate $q_{\pm 1/2}$. The transition amplitude is specified by the initial and final momenta, $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ and the z-components of the spins; ν_A (the target nucleus), ν_d (the deuteron), ν_B (the residual nucleus) and ν_p (the emitted nucleon).

The general formula also gives the cross section for an unpolarized target as a special case,

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{C}{2} \sum_{\nu_A,\nu_B,\nu_d,\nu_p} |\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle|^2. \tag{2·2}$$

The left-right asymmetry of the cross section is defined by

$$A(\theta) = \frac{1}{(q_{1/2}-q_{-1/2})} \cdot \frac{\left.\frac{d\sigma}{d\Omega}\right)^{(L)}_{\text{pol}} - \left.\frac{d\sigma}{d\Omega}\right)^{(R)}_{\text{pol}}}{\left.\frac{d\sigma}{d\Omega}\right)^{(L)}_{\text{pol}} + \left.\frac{d\sigma}{d\Omega}\right)^{(R)}_{\text{pol}}} \tag{2·3}$$

and we obtain, by Eqs. (2·1) ~ (2·3),

$$A(\theta) = -2\frac{\sum_{\nu_B,\nu_d,\nu_p} \mathrm{Im}\,\{\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A=\tfrac{1}{2},\nu_d;\boldsymbol{k}_i\rangle^*\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A=-\tfrac{1}{2},\nu_d;\boldsymbol{k}_i\rangle\}}{\sum_{\nu_A,\nu_B,\nu_d,\nu_p} |\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle|^2}. \tag{2·4}$$

The polarization, $P(\theta)$, of the emitted proton along the y-axis is calculated for an unpolarized target[18] and is given by

$$P(\theta)=2\frac{\sum_{\nu_B,\nu_d,\nu_A}\mathrm{Im}\{\langle\nu_B,\nu_p=\frac{1}{2};\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle^*\langle\nu_B,\nu_p=-\frac{1}{2};\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle\}}{\sum_{\nu_B,\nu_A,\nu_p,\nu_d}|\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle|^2}. \tag{2·5}$$

The physical significance of $A(\theta)$ can be clarified by the observation that $A(\theta)$ for the reaction $A(d,p)B$ is just the polarization of the particle A for the time-reversed process $p(B,A)d$. Now, in the usual (d,p) reaction, it is well known that the polarization of the emitted proton is strongly affected by the proton-spin-dependent interaction. Therefore, it seems probable that the interaction which includes the target spin plays an important role in the calculation of $A(\theta)$. This point of view is developed in later sections.

For the convenience of further development, the spin density matrices ρ_p and ρ_A are introduced as follows:

$$\langle\nu_p|\rho_p|\nu_p'\rangle=\frac{\sum_{\nu_B,\nu_d,\nu_A}\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle^*\langle\nu_B,\nu_p';\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle}{\sum_{\nu_B,\nu_A,\nu_p,\nu_d}|\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle|^2} \tag{2·6}$$

and

$$\langle\nu_A|\rho_A|\nu_A'\rangle=\frac{\sum_{\nu_B,\nu_p,\nu_d}\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A',\nu_d;\boldsymbol{k}_i\rangle^*}{\sum_{\nu_B,\nu_A,\nu_p,\nu_d}|\langle\nu_B,\nu_p;\boldsymbol{k}_f|T|\nu_A,\nu_d;\boldsymbol{k}_i\rangle|^2}, \tag{2·7}$$

where ρ_p describes the spin density of the proton after the reaction $A(d,p)B$ and ρ_A is the spin density of the nucleus A after the reaction, $p(B,A)d$. With these quantities, the asymmetry and the polarization given by Eqs. (2·4) and (2·5) are expressed as

$$P(\theta)=\frac{1}{i}\left\{\left\langle\frac{1}{2}|\rho_p|-\frac{1}{2}\right\rangle-\left\langle-\frac{1}{2}|\rho_p|\frac{1}{2}\right\rangle\right\} \tag{2·8}$$

and

$$A(\theta)=\frac{1}{i}\left\{\left\langle\frac{1}{2}|\rho_A|-\frac{1}{2}\right\rangle-\left\langle-\frac{1}{2}|\rho_A|\frac{1}{2}\right\rangle\right\}. \tag{2·9}$$

§ 3. Invariant-amplitude method

For the study of polarization phenomena, particularly of the spin dependence of the interaction, it is convenient to describe the transition amplitude by tensors in the spin space, because the spin dependence of the interaction can also be classified as tensors in the spin space. In reference 8), a method similar to ours has been presented but the terms, by which the transition amplitude is expanded, are not classified as such tensors. To develop the present method we will treat a reaction,

$$a+A\to b+B\,,$$

and the intrinsic spins of the particles a, A, b and B are denoted by $\boldsymbol{s}_a$, $\boldsymbol{s}_A$, $\boldsymbol{s}_b$ and $\boldsymbol{s}_B$, respectively. When we take the coupling scheme

$$\boldsymbol{s}_a+\boldsymbol{s}_A=\boldsymbol{S}_{aA},\qquad \boldsymbol{s}_b+\boldsymbol{s}_B=\boldsymbol{S}_{bB}$$

and

$$\boldsymbol{S}_{aA}+\boldsymbol{S}_{bB}=\boldsymbol{S}\,,$$

the spin tensor can be completely specified by the numbers, S_{aA}, S_{bB} and S, the rank of the tensor being given by S. Since the transition amplitude is a scalar in the spin-coordinate space, this tensor must form a scalar product with a tensor of rank S in the coordinate space which is to be constructed from the momenta, $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$. After the factorization of this scalar product, the residual part, say F, becomes invariant under rotation in the coordinate space and can be considered as a function of E and $\cos\theta$, where E is the total energy of the system and θ is the angle between $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$. In the following, the function F is referred to as the invariant amplitude analogous to the definition in high energy physics. This section is devoted to the derivation of the expression for the transition amplitude, the differential cross section, and the density matrix in terms of the invariant amplitudes.

The tensors of rank S constructed from $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are as follows:

$$[C_r(\Omega_i)\times C_{\bar{S}-r}(\Omega_f)]^S \qquad \text{with} \quad r=\bar{S}-S,\ \cdots,\ S\,,$$

where

$$\bar{S}=S \qquad \text{for} \quad S=\text{even}$$

and

$$\bar{S}=S+1 \quad \text{for} \quad S=\text{odd}\,,$$

when the total parity of the particles is not changed by the reaction, and

$$\bar{S}=S+1 \quad \text{for} \quad S=\text{even}$$

and

$$\bar{S}=S \qquad \text{for} \quad S=\text{odd}\,,$$

when the parity is changed. Here, Ω_i and Ω_f are the angular variables of $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$, respectively. The quantity $C_{lm}(\Omega)$ is related to the spherical harmonics, $Y_l^m(\Omega)$, by

$$C_{lm}(\Omega)=\sqrt{\frac{4\pi}{2l+1}}\,Y_l^m(\Omega)\,.$$

Therefore, the invariant amplitude F is specified by the numbers, S_{aA}, S_{bB}, S and r.

The transition amplitude $\langle\nu_B, \nu_b, \boldsymbol{k}_f|T|\nu_A, \nu_a; \boldsymbol{k}_i\rangle$ is given in terms of the invariant amplitude, $F(S_{aA}, S_{bB}, S, r; E, \cos\theta)$, by

$$\langle\nu_B, \nu_b; \boldsymbol{k}_f|T|\nu_A, \nu_a; \boldsymbol{k}_i\rangle=\frac{2\pi}{\sqrt{M_fM_ik_fk_i}}\sum_{S_{aA}, S_{bB}, S, r}(s_a\nu_a s_A\nu_A|S_{aA}\nu_{aA})$$

$$\times(s_b\nu_b s_B\nu_B|S_{bB}\nu_{bB})(S_{aA}\nu_{aA}S_{bB}-\nu_{bB}|S\nu_{aA}-\nu_{bB})(-)^{S_{bB}-\nu_{bB}}$$

$$\times[C_r(\Omega_i)\times C_{\bar{S}-r}(\Omega_f)]^S_{\nu_{aA}-\nu_{bB}}F(S_{aA}, S_{bB}, S, r; E, \cos\theta), \qquad (3\cdot1)$$

where the ν's denote the respective z-components of the spins and the M's are the reduced masses in the initial state and in the final state. Matrix elements of the spin tensor can be calculated by the Wigner-Eckart theorem. In Eq. (3·1) the product of three Clebsch-Gordan coefficients is the geometrical factor of the element of the spin tensor, the physical part being included in F.

One can define the invariant amplitude in different ways which depend on the choice of the coupling scheme, i.e. the invariant amplitude $G(S_{ab}, S_{AB}, S, r; E, \cos\theta)$ for the coupling scheme

$$\boldsymbol{s}_a+\boldsymbol{s}_b=\boldsymbol{S}_{ab}, \quad \boldsymbol{s}_A+\boldsymbol{s}_B=\boldsymbol{S}_{AB}$$

and

$$\boldsymbol{S}_{ab}+\boldsymbol{S}_{AB}=\boldsymbol{S},$$

and the amplitude $H(S_{aB}, S_{Ab}, S, r; E, \cos\theta)$ for the coupling scheme

$$\boldsymbol{s}_a+\boldsymbol{s}_B=\boldsymbol{S}_{aB}, \quad \boldsymbol{s}_b+\boldsymbol{s}_A=\boldsymbol{S}_{Ab}$$

and

$$\boldsymbol{S}_{aB}+\boldsymbol{S}_{Ab}=\boldsymbol{S}.$$

The amplitudes G and H are related to F by the transformation with the 9-j symbol,

$$G(S_{ab}, S_{AB}, S, r; E, \cos\theta)$$

$$=\sum_{S_{aA}S_{bB}}\sqrt{(2S_{ab}+1)(2S_{AB}+1)(2S_{aA}+1)(2S_{bB}+1)}\begin{Bmatrix} s_a & s_A & S_{aA} \\ s_b & s_B & S_{bB} \\ S_{ab} & S_{AB} & S \end{Bmatrix}$$

$$\times F(S_{aA}, S_{bB}, S, r; E, \cos\theta) \qquad (3\cdot2)$$

and

$$H(S_{aB}, S_{Ab}, S, r; E, \cos\theta)$$

$$=\sum_{S_{aA}, S_{bB}}\sqrt{(2S_{aB}+1)(2S_{Ab}+1)(2S_{aA}+1)(2S_{bB}+1)}\begin{Bmatrix} s_a & s_A & S_{aA} \\ s_B & s_b & S_{bB} \\ S_{aB} & S_{Ab} & S \end{Bmatrix}(-)^{s_b+s_B-S_{bB}}$$

$$\times F(S_{aA}, S_{bB}, S, r; E, \cos\theta). \qquad (3\cdot3)$$

The physical quantities, for example, the differential cross section and the density matrix can be expressed by any one of F, G and H. In the following, these quantities are given in terms of H for the convenience of application in § 4. Similar expressions can be obtained by use of F or G. The transition amplitude is given in terms of H:

$$\langle \nu_B, \nu_b; \boldsymbol{k}_f | T | \nu_A, \nu_a; \boldsymbol{k}_i \rangle = \frac{2\pi}{\sqrt{M_f M_i k_f k_i}} \sum_{S_{aB}, S_{Ab}, S, r} (s_a \nu_a s_B - \nu_B | S_{aB}\, \nu_a - \nu_B)\, (-)^{s_B - \nu_B}$$

$$\times (s_A \nu_A s_b \nu_b | S_{Ab} \nu_A - \nu_b)\, (-)^{s_b - \nu_b} (S_{aB}\, \nu_a - \nu_B\, S_{Ab}\, \nu_A - \nu_b | S\, \nu_a + \nu_A - \nu_b - \nu_B)$$

$$\times [C_r(\Omega_i) \times C_{\bar{S}-r}(\Omega_f)]^S_{\nu_a+\nu_A-\nu_b-\nu_B} H(S_{aB}, S_{Ab}, S, r; E, \cos\theta). \qquad (3\cdot1')$$

By substituting Eq. $(3\cdot1')$ into Eq. $(2\cdot2)$ and noting that

$$C = \frac{1}{2\pi^2} \cdot \frac{M_f M_i k_f}{k_i} \cdot \frac{1}{(2s_a+1)(2s_A+1)}, \qquad (3\cdot4)$$

one gets the unpolarized cross section,

$$\left.\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} (a + A \to b + B) = \frac{1}{k_i^2 (2s_a+1)(2s_A+1)} N(E, \cos\theta),$$

with

$$N(E, \cos\theta) = \sum_{S_{aB}, S_{Ab}, S, r, r'} (2S+1) H^*(S_{aB}, S_{Ab}, S, r; E, \cos\theta)$$

$$\times H(S_{aB}, S_{Ab}, S, r'; E, \cos\theta) \sum_p W(r\, \bar{S}-r\, r'\, \bar{S}-r'; Sp)\, (r0r'0|p0)\, (-)^{S+p}$$

$$\times (\bar{S}-r\ 0\ \bar{S}-r'\ 0 | p0)\, P_p(\cos\theta). \qquad (3\cdot5)$$

The elements of the density matrices $\langle \nu_b | \rho_b | \nu_b' \rangle$ and $\langle \nu_A | \rho_A | \nu_A' \rangle$ can also be calculated by the use of Eq. $(3\cdot1')$. To clarify the transformation properties, the matrix element is expanded in terms of irreducible tensors, for instance,

$$\langle \nu_b | \rho_b | \nu_b' \rangle = \sum_k (s_b \nu_b'\ k\ \nu_b - \nu_b' | s_b \nu_b)\, \rho_b^{(k)}(\nu_b - \nu_b'), \qquad (3\cdot6)$$

where $\rho_b^{(k)}(\nu)$ is the so-called statistical tensor[4] of rank k, and is further expanded as

$$\rho_b^{(k)}(\nu) = \sum_{q=\bar{k}-k}^{k} [C_q(\Omega_i) \times C_{\bar{k}-q}(\Omega_f)]_\nu^k \rho_b^{(k)}(q; E, \cos\theta) \qquad (3\cdot7)$$

with $\bar{k} = k$ for k = even and $\bar{k} = k+1$ for k = odd. Here, $\rho_b^{(k)}(q; E, \cos\theta)$ is given in terms of H by

$$\rho_b^{(k)}(q; E, \cos\theta) = \frac{1}{N(E, \cos\theta)} \sqrt{\frac{2k+1}{2s_b+1}} \sum_{S_{aB}, S_{Ab}, S'_{Ab}, S, S', r, r', p, p'} (-)^{S+S'-\bar{S}}$$

$$\times (2S+1)(2S'+1)(2S_{Ab}+1)(2S'_{Ab}+1)(2p+1)(2p'+1)\, W(S'_{Ab} s_A k s_b; s_b S_{Ab})$$

$$\times W(S_{Ab}S_{aB}kS'; SS'_{Ab})\begin{Bmatrix} r & \bar{S}-r & S \\ r' & \bar{S}'-r' & S' \\ p & p' & k \end{Bmatrix}(r0r'0|p0)\,(\bar{S}-r0\bar{S}'-r'0|p'0)$$

$$\times A(pp'qk; \cos\theta)H^*(S_{aB}, S_{Ab}, S, r; E, \cos\theta)\; H(S_{aB}, S'_{Ab}, S', r'; E, \cos\theta), \tag{3·8}$$

where $A(pp'qk; \cos\theta)$ is the coefficient function for the space tensor defined by

$$[C_p(\Omega_i)\times C_{p'}(\Omega_f)]^k=\sum_{q=\bar{k}-k}^{k} A(pp'qk; \cos\theta)\,[C_q(\Omega_i)\times C_{\bar{k}-q}(\Omega_f)]^k \tag{3·9}$$

with $\bar{k}=k$ for $p+p'+k=$even and $\bar{k}=k+1$ for $p+p'+k=$odd. The reduction formula for evaluating the function $A(pp'qk: \cos\theta)$ is given in Appendix B where some properties of this function are also discussed. From Eq. (3·8), using the symmetry properties of the 6-j and 9-j symbols, one can easily show that $\rho_b^{(k)}(q; E, \cos\theta)$ is real for even k and imaginary for odd k. The invariant amplitude can be expanded in partial waves. Such a treatment will be useful for numerical computations of physical quantities and is discussed in Appendix C.

§ 4. Application to special stripping reactions

The invariant-amplitude method developed in the previous section is applied to the special stripping reaction, where the orbital angular momentum of the captured neutron is zero and the nuclear spins are 1/2 and zero for the target nucleus and the residual nucleus, respectively, i.e. the proton polarization and the cross-section asymmetry are calculated with the spin values specified and the ratio of the polarization to the asymmetry is investigated for the assumptions of the spin dependence for the interactions given in Table I. The theoretical result is discussed in comparison with the experimental data for the $He^3(d, p)He^4$ reactions.

In the present case, a, A, b and B are assumed to be the deuteron, the target nucleus, the proton, and the residual nucleus, respectively, the spin assignment being

$$s_a=1, \quad s_A=1/2, \quad s_b=1/2 \text{ and } s_B=0\,.$$

To discuss the relation between the polarization of the particle b and the asymmetry from the polarized target A, it is convenient to take the coupling scheme which leads us to the invariant amplitude H. For the above values of the spins, the following four sets of the spin-coupling parameters are available:

i) $S_{Ab}=1$ and $S=0$,

ii) $S_{Ab}=1$ and $S=1$,

iii) $S_{Ab}=0$ and $S=1$

and iv) $S_{Ab}=1$ and $S=2$,

with $S_{aB}=1$. Correspondingly, we get the following space tensors under the condition of no parity change,

$$
\begin{array}{llll}
1 & \text{for } S=0 & \text{and} & r=0, \\
[C_1(\Omega_i)\times C_1(\Omega_f)]^1 & \text{for } S=1 & \text{and} & r=1, \\
C_2(\Omega_f) & \text{for } S=2 & \text{and} & r=0, \\
[C_1(\Omega_i)\times C_1(\Omega_f)]^2 & \text{for } S=2 & \text{and} & r=1
\end{array}
$$

and $C_2(\Omega_i)$ for $S=2$ and $r=2$.

The transition amplitude is given in terms of H,

$$
\begin{aligned}
\langle\nu_B, \nu_p;\, \boldsymbol{k}_f|T|\nu_A, \nu_d;\, \boldsymbol{k}_i\rangle &= 4\pi(-)^{1/2-\nu_p}(M_fM_ik_fk_i)^{-1/2} \\
&\times[(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|1\,\nu_A-\nu_p)(1\nu_d\,1\,\nu_A-\nu_p|00)H_1 \\
&+(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|1\,\nu_A-\nu_p)(1\nu_d\,1\,\nu_A-\nu_p|1\,\nu_A-\nu_p+\nu_d)[C_1(\Omega_i)\times C_1(\Omega_f)]^1_{\nu_A-\nu_p+\nu_d}H_2 \\
&+(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|00)(1\nu_d00|1\nu_d)[C_1(\Omega_i)\times C_1(\Omega_f)]^1_{\nu_d}H_3 \\
&+(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|1\,\nu_A-\nu_p)(1\nu_d1\,\nu_A-\nu_p|2\,\nu_A-\nu_p+\nu_d)\{C_{2\,\nu_A-\nu_p+\nu_d}(\Omega_f)H_4 \\
&+[C_1(\Omega_i)\times C_1(\Omega_f)]^2_{\nu_A-\nu_p+\nu_d}H_5+C_{2\nu_A-\nu_p+\nu_d}(\Omega_i)H_6\}],
\end{aligned}
\tag{4·1}
$$

where

$$
\begin{aligned}
H_1 &\equiv H(S_{dB}=1,\, S_{Ap}=1,\, S=0,\, r=0), \\
H_2 &\equiv H(S_{dB}=1,\, S_{Ap}=1,\, S=1,\, r=1), \\
H_3 &\equiv H(S_{dB}=1,\, S_{Ap}=0,\, S=1,\, r=1), \\
H_4 &\equiv H(S_{dB}=1,\, S_{Ap}=1,\, S=2,\, r=0), \\
H_5 &\equiv H(S_{dB}=1,\, S_{Ap}=1,\, S=2,\, r=1)
\end{aligned}
$$

and

$$H_6\equiv H(S_{dB}=1,\, S_{Ap}=1,\, S=2,\, r=2). \tag{4·2}$$

The proton polarization and the cross-section asymmetry given by Eqs. (2·8) and (2·9) are calculated as

$$P(\theta)=\frac{2}{i}\sqrt{\frac{1}{6}}\sin\theta\rho_p{}^{(1)}(1;\cos\theta) \tag{4·3}$$

and

$$A(\theta)=-\frac{2}{i}\sqrt{\frac{1}{6}}\sin\theta\rho_A{}^{(1)}(1;\cos\theta), \tag{4·4}$$

where

$$\rho_p{}^{(1)}(1;\cos\theta)$$
$$=i\frac{\sqrt{2}}{N(E,\cos\theta)}\,\mathrm{Im}\left\{H_1{}^*\left(H_2-\frac{1}{\sqrt{2}}H_3\right)+\frac{\sqrt{2}}{4}(H_2{}^*+\sqrt{2}H_3{}^*)(H_4+H_6)\right.$$
$$\left.+\frac{\sqrt{3}}{6}(H_2{}^*+\sqrt{2}H_3{}^*)H_5\cos\theta-\frac{3\sqrt{2}}{4}(H_4{}^*H_5-H_6{}^*H_5)+\frac{3\sqrt{2}}{2}H_4{}^*H_6\cos\theta\right\} \tag{4·5}$$

and

$$\rho_A{}^{(1)}(1;\cos\theta)=\rho_p{}^{(1)}(1;\cos\theta)\,(H_3\to -H_3). \tag{4·6}$$

The denominator $N(E,\cos\theta)$ is given by Eq. (3·5) and is expressed in terms of H as

$$N(E,\cos\theta)=\frac{1}{2}\left\{|H_1|^2+\frac{\sin^2\theta}{2}(|H_2|^2+|H_3|^2)+|H_4|^2+\frac{3+\cos^2\theta}{6}|H_5|^2\right.$$
$$\left.+|H_6|^2+\frac{2\sqrt{6}}{3}\mathrm{Re}(H_4{}^*H_5+H_5{}^*H_6)+(3\cos^2\theta-1)\mathrm{Re}(H_4{}^*H_6)\right\}. \tag{4·7}$$

It should be noticed that the difference between $\rho_p{}^{(1)}(1;\cos\theta)$ and $\rho_A(1;\cos\theta)$ in Eqs. (4·5) and (4·6) is only in the sign of H_3, which allows us to find easily the polarization-asymmetry ratio. These equations are similar to Eqs. (8′) and (9′) in reference 8). However, the present expressions are more convenient than the latter for investigation of the spin dependence of the interactions. The relationship between the present formulae and those in the reference is given in Appendix A.

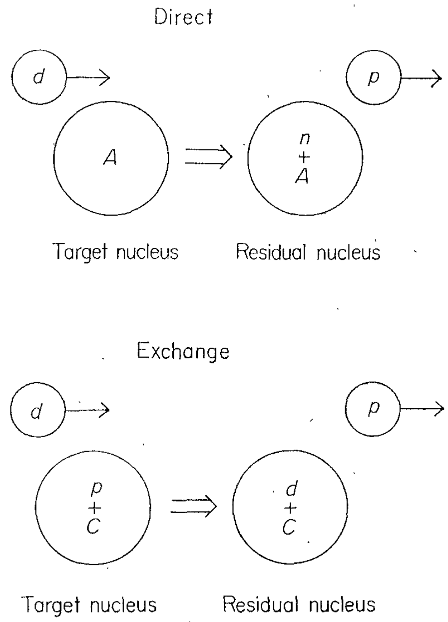

Fig. 2. Schematic representation of reaction modes. Symbols A, d, n and p represent the target nucleus, the incident deuteron, the captured neutron and the proton emitted into the final state, respectively. The symbol C represents the core part of the target nucleus, the target minus one proton.

As is seen in Eq. (1·1), the direct amplitude in the DWBA theory is determined by four interactions: V_{pn}, V_{pA}, U_{pB} and U_{dA}. Similarly, V_{pC}, V_{pd}, U_{pB} and U_{dA} take part in the exchange stripping reaction illustrated in Fig. 2, the transition amplitude, $T^{(E)}$, being

$$T^{(E)}(d+A\to p+B)$$
$$=\langle\chi_{pB}^{(-)}\varphi_B,(V_{pC}+V_{pd}-U_{pB})\chi_{dA}^{(+)}\varphi_d\varphi_A\rangle\,.$$

At this time, the suffix p denotes the proton of the target nucleus which is emitted into the final state. The wave function of the residual nucleus, φ_B, consists of the wave functions of the bound deuteron and the core part of the target nucleus, C. In the following, we study all of the interactions with their spin dependence listed in Table I. In the table, the possible spin dependences of the interactions are given as tensors in the spin space, their ranks being denoted by S. There, $\boldsymbol{s}_i$ is the spin operator of the particle i, $s_d^{(2)}$ is the tensor of rank 2 constructed by the deuteron spin, and $\boldsymbol{E}_d$ denotes an operator which gives rise to the transition between the deuteron intrinsic states, the singlet and the triplet states. The vector $\boldsymbol{l}$ represents an axial vector constructed by the space coordinates, for example, the orbital-angular-momentum operator of the related particle. For the interaction between composite particles, $\boldsymbol{l}$ can also include the internal variables. Some discussions of the spin dependent interactions have been given in the introduction. The table also contains other interactions that are kinematically acceptable. Physically, they are expected to be derived from two-nucleon forces. In a derivation of the proton deuteron interaction from the two-nucleon forces, the $(\boldsymbol{s}_p\cdot\boldsymbol{E}_d)$ and $(\boldsymbol{l}_{pd}\cdot\boldsymbol{E}_d)$ terms will arise from the $(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})$ and spin-orbit terms of the nucleon-nucleon interaction, respectively. Similarly, in the derivation, the $\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{E}_d]^1$ term will appear as a second order effect of

Table I. Spin dependence of interactions. All of the interactions listed in the table are classified by their rank as tensors (S). Vacant columns of the spin dependence in the exchange amplitude are read to be the same as the corresponding columns in the direct amplitude.

S	Direct amplitude		Exchange amplitude	
	Interaction	Spin dependence	Interaction	Spin dependence
0	U_{dA}	$\boldsymbol{s}_d\cdot\boldsymbol{s}_A$	U_{dA}	
	V_{pA}	$\boldsymbol{s}_p\cdot\boldsymbol{s}_A$	V_{pd}	$\boldsymbol{s}_p\cdot\boldsymbol{s}_d$ $\boldsymbol{s}_p\cdot\boldsymbol{E}_d$
1	U_{pB}	$\boldsymbol{l}_{pB}\cdot\boldsymbol{s}_p$	U_{pB}	
	U_{dA}	$\boldsymbol{l}_{dA}\cdot\boldsymbol{s}_d$ $\boldsymbol{l}_{dA}\cdot\boldsymbol{s}_A$ $\boldsymbol{l}_{dA}\cdot[\boldsymbol{s}_d\times\boldsymbol{s}_A]^1$ $\boldsymbol{l}_{dA}\cdot[s_d^{(2)}\times\boldsymbol{s}_A]^1$	U_{dA}	
	V_{pA}	$\boldsymbol{l}_{pA}\cdot\boldsymbol{s}_p$ $\boldsymbol{l}_{pA}\cdot\boldsymbol{s}_A$ $\boldsymbol{l}_{pA}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_A]^1$	V_{pd}	$\boldsymbol{l}_{pd}\cdot\boldsymbol{s}_p$ $\boldsymbol{l}_{pd}\cdot\boldsymbol{s}_d$ $\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_d]^1$ $\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times s_d^{(2)}]^1$ $\boldsymbol{l}_{pd}\cdot\boldsymbol{E}_d$ $\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{E}_d]^1$
2	U_{dA} V_{pA} φ_D	tensor tensor D-state admixture	U_{dA} V_{pd} φ_D	tensor

the spin-orbit term of the nuclear force. This second order effect also gives rise to the $\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_d]^1$ term in the V_{pd}, $\boldsymbol{l}_{dA}\cdot[\boldsymbol{s}_d\times\boldsymbol{s}_A]^1$ term in U_{dA} and the $\boldsymbol{l}_{pA}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_A]^1$ term in V_{pA}. The interactions, $\boldsymbol{l}_{dA}\cdot[s_d^{(2)}\times\boldsymbol{s}_A]^1$ in U_{dA} and $\boldsymbol{l}_{pd}\cdot[s_d^{(2)}\times\boldsymbol{s}_p]^1$ in V_{pd}, can arise as a combined effect of the spin-orbit and the tensor terms. In the table, the tensor terms are not specified because the explicit forms are not necessary for the later discussion. The effect of the interaction V_{pn} is treated as that of the D-state admixture in the deuteron internal motion.

In general, some of the interactions listed above will take part together in actual reactions, their effects in polarization phenomena being mixed up with each other. Thus, the quantitative determination of their contribution to P and A requires laborious numerical calculations. In this section, we discuss the ratio P/A in two simplified ways: First, for each spin dependence, P/A is calculated up to the linear term both in P and A. The result may be useful for study of a special combination of the spin dependence in the sense of the first order approximation. Secondly, in the light of the above results, P/A is investigated without the linear approximation for several physically important interactions. The results obtained are exact to any order within the limitation of the DWBA theory.

In the linear approximation, Eqs. (4·5) and (4·6) are reduced to

$$\rho_p^{(1)}(1;\cos\theta)\approx i\frac{\sqrt{2}}{N(E,\cos\theta)}\,\mathrm{Im}\left\{H_1^*\left(H_2-\frac{1}{\sqrt{2}}H_3\right)\right\} \tag{4·5′}$$

and

$$\rho_A^{(1)}(1;\cos\theta)\approx i\frac{\sqrt{2}}{N(E,\cos\theta)}\,\mathrm{Im}\left\{H_1^*\left(H_2+\frac{1}{\sqrt{2}}H_3\right)\right\}. \tag{4·6′}$$

Since H_2 and H_3 are characterized by $S=1$, in the table only the interactions referred by $S=1$ can contribute to P and A in this approximation. The detail of the method of calculating P/A will be given for the special spin dependence $\boldsymbol{l}_{pB}\cdot\boldsymbol{s}_p$ for U_{pB}. Other spin dependences are similarly treated and the results are listed in Table II. From the diagram in Fig. 3, the contribution of the $\boldsymbol{l}_{pB}\cdot\boldsymbol{s}_p$ term to the transition amplitude can be shown to be proportional to the matrix element,

$$\langle\nu_p|\boldsymbol{s}_p|\nu_A,\ \nu_d\rangle(-)^{1-\nu_d-\nu_A+\nu_p},$$

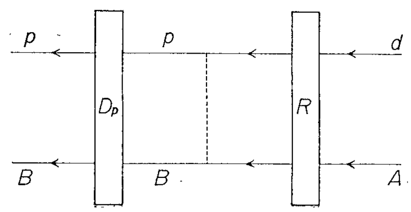

Fig. 3. The first-order diagram with respect to the spin-orbit term of U_{pB}. Symbols A, B, d and p represent the target nucleus, the final nucleus, the incident deuteron and the emitted proton, respectively. The box denoted by D_p represents spin-independent distortions of the proton and the dotted line represents the proton-spin-orbit interaction. The stripping process takes place at the box denoted by R, where any spin-independent distortions are allowed.

Table II. Calculated P/A in the linear approximation.

Interaction	Spin depedence	H_3/H_2	P/A
U_{pB}	$\boldsymbol{l}_{pB}\cdot\boldsymbol{s}_p$	$-1/\sqrt{2}$	-3
U_{dA}	$\boldsymbol{l}_{dA}\cdot\boldsymbol{s}_d$	0	-1
	$\boldsymbol{l}_{dA}\cdot\boldsymbol{s}_A$	$1/\sqrt{2}$	$-1/3$
	$\boldsymbol{l}_{dA}\cdot[\boldsymbol{s}_d\times\boldsymbol{s}_A]^1$	$-\sqrt{2}$	$A=0$
	$\boldsymbol{l}_{dA}\cdot[s_d^{(2)}\times\boldsymbol{s}_A]^1$	$\sqrt{2}$	$P=0$
V_{pA}	$\boldsymbol{l}_{pA}\cdot\boldsymbol{s}_p$	$-1/\sqrt{2}$	-3
	$\boldsymbol{l}_{pA}\cdot\boldsymbol{s}_A$	$1/\sqrt{2}$	$-1/3$
	$\boldsymbol{l}_{pA}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_A]^1$	$H_2=0$	1
V_{pd}	$\boldsymbol{l}_{pd}\cdot\boldsymbol{s}_p$	$-1/\sqrt{2}$	-3
	$\boldsymbol{l}_{pd}\cdot\boldsymbol{s}_d$	0	-1
	$\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{s}_d]^1$	$\sqrt{2}$	$P=0$
	$\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times s_d^{(2)}]^1$	$-\sqrt{2}$	$A=0$
	$\boldsymbol{l}_{pd}\cdot\boldsymbol{E}_d$	$H_2=0$	1
	$\boldsymbol{l}_{pd}\cdot[\boldsymbol{s}_p\times\boldsymbol{E}_d]^1$	0	-1

the geometrical part of which is given by the use of the Wigner-Eckart theorem as

$$(-)^{1/2-\nu_p}(1\nu_d\tfrac{1}{2}\nu_A|\tfrac{1}{2}\nu_d+\nu_A)\,(\tfrac{1}{2}\nu_d+\nu_A\tfrac{1}{2}-\nu_p|1\nu_d+\nu_A-\nu_p)\,, \tag{4·8}$$

where the first Clebsh-Gordan coefficient represents the reaction part (R) in the diagram. The product of the Clebsh-Gordan coefficients (4·8) is rewritten as

$$\begin{aligned}&(1\nu_d\tfrac{1}{2}\nu_A|\tfrac{1}{2}\,\nu_d+\nu_A)\,(\tfrac{1}{2}\,\nu_d+\nu_A\,\tfrac{1}{2}-\nu_p|1\,\nu_d+\nu_A-\nu_p)\\
&=\sum_{S=0}^{1}\sqrt{2(2S+1)}\,W(1\tfrac{1}{2}1\tfrac{1}{2};\,1S)\,(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|S\,\nu_A-\nu_p)\,(1\nu_d\,S\,\nu_A-\nu_p|1\,\nu_d+\nu_A-\nu_p)\\
&=\sqrt{\frac{2}{3}}\Big\{(\tfrac{1}{2}\nu_A\tfrac{1}{2}-\nu_p|1\,\nu_A-\nu_p)\,(1\nu_d\,1\,\nu_A-\nu_p|1\,\nu_d+\nu_A-\nu_p)\\
&\qquad-\frac{1}{\sqrt{2}}(\frac{1}{2}\nu_A\frac{1}{2}-\nu_p|00)\,(1\nu_d00|1\nu_d)\Big\}.\end{aligned} \tag{4·9}$$

By comparing the content of the ket { } with Eq. (4·2), one gets

$$H_3/H_2=-1/\sqrt{2}\,,$$

which gives P/A by the aid of Eqs. (4·5′) and (4·6′),

$$P/A=-3\,.$$

Now, some special spin dependences are investigated without the linear approximation, ρ_p and ρ_A being given by their full expressions (4·5) and (4·6). The method of the investigation is quite similar to that in the linear case. As an example, we will consider a higher order term of the proton-spin-orbit inter-

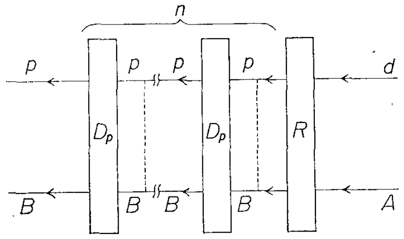

Fig. 4. The n-th order diagram with respect to the spin-orbit term of U_{pB}. The symbols are read similarly to those in Fig. 3.

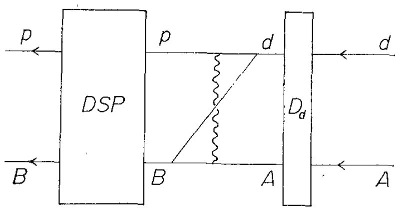

Fig. 5. Combined diagram of the proton-spin-orbit interaction and the spin-spin interaction of V_{pA}. The box DSP describes distortions of the proton wave, both the spin-independent interaction and the proton-spin-orbit interaction being allowed to any order. The box D_d represents the spin-independent distortion of the deuteron wave. The part which consists of the wavy line and the slanting line describes the neutron capture by the spin-spin interaction of V_{pA}. Other symbols are read similarly to those in Fig. 3.

action. The n-th order contribution of the interaction (see Fig. 4) will be proportional to the matrix element

$$\langle \nu_p | \boldsymbol{s}_p{}^n | \nu_A, \nu_d \rangle .$$

Since the proton has spin 1/2, it is always possible to reduce the product $\boldsymbol{s}_p{}^n$ to

$$a + b\boldsymbol{s}_p \, .$$

Therefore, the higher order effects of the $(\boldsymbol{l}_{pB} \cdot \boldsymbol{s}_p)$ term contribute to the $S=0$ and $S=1$ amplitudes, the geometrical factor of the latter being the same as the first order contribution. Since the full expressions (4·5) and (4·6) for ρ_p and ρ_A are reduced to the right-hand side of (4·5′) and (4·6′) in the absence of the $S=2$ amplitudes, one gets $P/A=-3$ for the proton-spin-orbit interaction in the n-th order. Further, it is shown that the same result is obtained when an additional spin-spin interaction of V_{pA} is taken into account for the process. From the Feynman diagram shown in Fig. 5, one can see that the contribution of the combined effect of the $(\boldsymbol{l}_{pB} \cdot \boldsymbol{s}_p)$ term in the U_{pB} and $(\boldsymbol{s}_p \cdot \boldsymbol{s}_A)$ term in V_{pA} is proportional to

$$\langle \nu_p | \boldsymbol{s}_p (\boldsymbol{s}_p \cdot \boldsymbol{s}_A) | \nu_A, \nu_d \rangle = \sum_{\nu_p'} \langle \nu_p | \boldsymbol{s}_p | \nu_p' \rangle \langle \nu_p' | (\boldsymbol{s}_p \cdot \boldsymbol{s}_A) | \nu_A, \nu_d \rangle \, .$$

Since $(\boldsymbol{s}_p \cdot \boldsymbol{s}_A)$ is a scalar in spin space, only $\nu_p' = \nu_d + \nu_A$ can contribute to the matrix element and the geometrical factor of the above expression is shown to be the same as that of the matrix element of $\boldsymbol{s}_p$. The similar arguments are applied to the higher order effects of the target-spin-orbit interactions.

Next, we will discuss the assumption of a linear combination of the proton-spin-orbit interaction and the target-spin-orbit interaction, $\boldsymbol{l}_{pA} \cdot (a\boldsymbol{s}_p + b\boldsymbol{s}_A)$, for V_{pA}, neglecting other spin-dependent interactions completely. As mentioned in § 1, the DWBA transition amplitude is derived from the original post-form amplitude by the Gell-Mann Goldberger transformation,[1] where U_{pB} is mathematically an arbitrary function of the variables of the proton. In this viewpoint, one can choose U_{pB} to be a central potential, the spin dependence being considered in V_{pA}. In the above linear combination, the choice, $b=0$ for example, is expected to give a result similar to that from the proton-spin-orbit interaction for U_{pB} with the neglect of $V_{pA}-U_{pB}$. From such considerations, one can expect that the linear combination provides the main feature of the combined effect of both kinds of spin-orbit interactions. The calculation of P/A for this assumption is quite similar to those for the proton-spin-orbit interaction and the target-spin orbit interaction. The result will be given later together with those for higher order effects of other spin dependences.

As is noted previously, the relation $P/A=-1$ holds exactly in the absence of H_3, the amplitude for $S_{pA}=0$. When the spin dependence is concerned only with the deuteron spin, the proton and the target spins must be coupled to one ($S_{pA}=1$) and then $P/A=-1$. Similarly, $P/A=-1$ is obtained for the tensor term in V_{pA}. These considerations can be summarized as follows:

1) For any combination of the proton-spin-orbit interactions with or without the spin-spin interaction in V_{pA} (or V_{pd} for the exchange term), one gets $P/A=-3$.

2) For any combination of the target spin-orbit interactions with or without the spin-spin interaction in V_{pA} (or V_{pd} for the exchange term), one gets $P/A=-1/3$.

3) The spin-dependent interaction relating to only the deuteron spin, the tensor term in V_{pA} (or V_{pd} for the exchange term), the D-state admixture and their combined effect give $P/A=-1$, with or without the spin-spin interaction in V_{pA}, and, similarly,

4) for a combination of the proton-spin-orbit interaction and the target-spin-orbit interaction for V_{pA}, i.e. $\boldsymbol{l}_{pA}\cdot(a\boldsymbol{s}_p+b\boldsymbol{s}_A)$, one also finds

$$\frac{P}{A}=\frac{-3a+b}{a-3b},$$

where the form factor of the proton-spin-orbit interaction is assumed to be the same as that of the target-spin-orbit interaction.

The cross-section asymmetry has been measured only for the $He^3(d,p)He^4$ reactions. Since the general agreement of the DWBA result with experimental data seems to be better for the heavy nuclei than for the light, the reaction on He^3 target may not provide the best test of the validity of the theory. On the other hand, the configuration of the final nucleus, He^4, has little ambiguity in the ground state since the D-state amplitude is almost negligibly small. In fact, forward angular distributions of the cross section show typical S-state captures.[19] This matter rather simplifies the analysis of the reaction. Under these circumstances, it is worthwhile to compare the theoretical results with the experimental data of this reaction. The measured polarization[20] and asymmetry[6] are shown in Fig. 6. The main features are (1) in the forward angular region, the asymmetry and the polarization show a quite similar shape and magnitude but have opposite signs, i.e. $P(\theta)\approx -A(\theta)$, (2) the maximum magnitude of the asymmetry is certainly larger than 50 % at 6, 8 and 10 MeV, (3) at larger angles, the magnitude of the asymmetry appears to be smaller than that of the polarization especially at 6 MeV, and (4) the shape of the angular distribution of the asymmetry appears to be shifted to larger angles at 6 and 8 MeV when compared with the polarization.

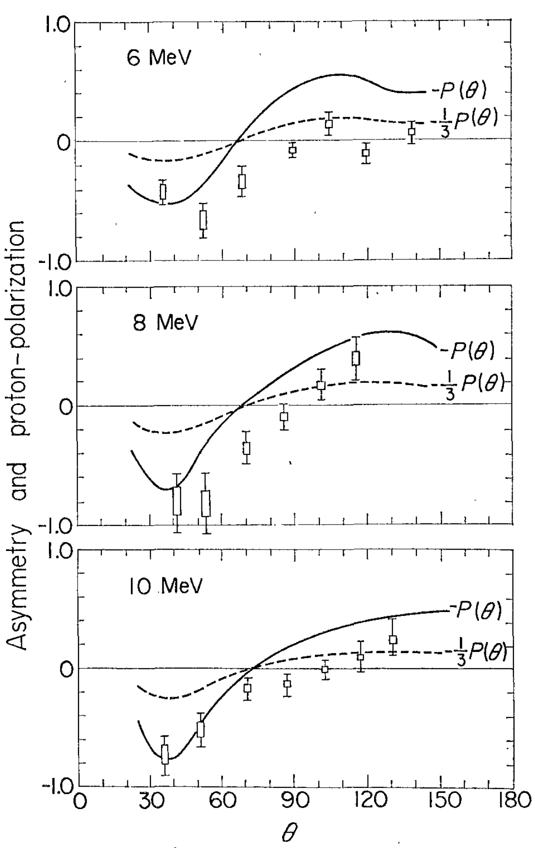

Fig. 6. The measured values of $A(\theta)$ and $P(\theta)$ at 6, 8 and 10 MeV for the $He^3(d, p)He^4$ (*g.s.*) reaction. The experimental points are the measured $A(\theta)$ of reference 6) and the solid curve represents the measured $P(\theta)$ of reference 20) but the sign has been changed for comparison with $A(\theta)$. The dashed curve represents $-\frac{1}{3}P(\theta)$.

These features, in particular (1) and (2) do not favour the conventional proton-spin-orbit assumption because of the too large absolute values of the theoretical P/A at forward angles. Agreement between the theoretical and experimental results cannot be obtained by including the spin-spin interactions. These circumstances are not changed even if the exchange effect is taken into account. Obviously, these conclusions are independent of the range of V_{pn} and the finite-range calculation[21] does not improve the theoretical results. The target-spin-orbit interactions do not explain the data because of the too small values of theoretical P/A. The linear combination of these two kinds of spin-orbit interactions for V_{pA} can give $P/A=-1$ when $a=-b$ is assumed. The validity of this assumption can be investigated by comparison of measured polarizations of protons with those of He^3 nuclei in proton-He^3 elastic scatterings. The He^3 polarization is equal to cross-section asymmetries in elastic scatterings for polarized He^3 targets[18] because of the time-reversal theorem. The latter has been measured at $E_p=4\sim11$ MeV and in most energies has been found to have the same sign as that of the proton polarization.[22] This suggests that the assumption $a=-b$ is invalid, though in the (d, p) reaction the scattering energy is not definite.

Recently, De Facio et al.[23] studied scatterings and reactions in the He^4 plus one-nucleon system using an elementary-particle model and taking account of the effect of the coupling between the elastic and reaction channels, for example, the He^4 plus proton channel and the He^3 plus deuteron channel. For the $He^3(d, p)He^4$ reaction, they found that the coupled-channel effect can reproduce the observed shift of the asymmetry and that tensor-force couplings are necessary in explaining the experimental data. Although the physical interpretation of the tensor coupling is not so clear because the model is much different from the usual nuclear model, the tensor interaction of V_{pA} will be included in the tensor coupling in the reference. To explain the variation of the experimental P/A both with the incident energy and with the angle, it is necessary to simultaneously consider several kinds of spin-dependent interactions. Since detailed numerical calculations are not attempted in this work, definite conclusions cannot be drawn on the relative importance of such spin dependences. However, from the above studies, it is speculated that some of the proton-He^3 tensor force, the deuteron-spin-dependent interaction, and the D-state admixture should be taken into account in the numerical calculations to explain the data. In the backward angular region, an appreciable increase of the observed cross section[19] together with the smallness of the magnitude of the asymmetry suggests the importance of the heavy-particle stripping reaction[15] with the proton-spin-orbit interaction.[19] Through the above analysis, $\boldsymbol{E}_d$-term and the terms due to the second-order effect of the two-nucleon forces are assumed to be small because of no evidence of such interactions in the actual nuclear phenomena.

§ 5. Remarks

The method presented here can be applied to the study of the spin dependence of interactions in other stripping theories,[24] since, as seen in the preceding sections, the method needs only the interactions and their time order in the given theoretical frame, from which one can describe the transition by the Feynman diagram and then get the geometrical factor of the transition amplitude which determines the properties of the invariant amplitudes. Also, it can be shown that the present analysis can be applied to another special case where the spin of the residual nucleus is one, instead of zero. In this case, the method gives definite values of P/A for several kinds of the spin-dependent assumptions. The measurement of the proton polarization and the cross-section asymmetry for this case will further test the validity of the DWBA theory. Further, the method can be used for analyses of other reactions, for example, inelastic scatterings of nucleons. In fact, some of the present results can be applied to inelastic scatterings of protons accompanied by $0^+ \to 1^+$ nuclear excitations, for which the theory gives the ratio of the proton polarization for unpolarized beam to the cross-section asymmetry for polarized beam.

Finally, it should be emphasized that because the disagreement between the prediction by the conventional proton-spin-orbit assumption and the experimental results for P/A is remarkable, measurements of both the polarization and the asymmetry for other target nuclei are valuable. Such measurements will determine whether or not the disagreement is the general tendency of the stripping reaction,

Acknowledgements

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Appendix A

The relation between Duck's invariant amplitudes[8] and the present ones

In reference 8), the six invariant amplitudes F are defined by

$$M=F_1(\boldsymbol{\sigma}\cdot\boldsymbol{E})+F_2\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k}_i)(\boldsymbol{E}\cdot\boldsymbol{k}_f)}{k_ik_f}+F_3\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k}_f)(\boldsymbol{E}\cdot\boldsymbol{k}_i)}{k_ik_f}$$

$$+F_4\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k}_i)(\boldsymbol{E}\cdot\boldsymbol{k}_i)}{k_i^2}+F_5\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k}_f)(\boldsymbol{E}\cdot\boldsymbol{k}_f)}{k_f^2}+F_6\frac{i(\boldsymbol{E}\cdot\boldsymbol{k}_i\times\boldsymbol{k}_f)}{k_ik_f}, \qquad (\mathrm{A}\cdot 1)$$

where $\boldsymbol{\sigma}$ is the spin matrix, the element of which is defined between the target spin state and the proton spin state and $\boldsymbol{E}$ represents the spin of the deuteron. Taking the matrix elements of these operators, one can easily show that the invariant amplitudes F defined above are related to our invariant amplitudes H as follows:

$$\sqrt{6k_fk_i}F_1=\frac{\sqrt{2}}{2}H_1+\frac{\sqrt{6}}{6}H_5\cos\theta+\frac{1}{2}(H_4+H_6),$$

$$\sqrt{6k_fk_i}F_2=\frac{\sqrt{6}}{4}(H_2-H_5),$$

$$\sqrt{6k_fk_i}F_3=-\frac{\sqrt{6}}{4}(H_2+H_5),$$

$$\sqrt{6k_fk_i}F_4=-\frac{3}{2}H_6,$$

$$\sqrt{6k_fk_i}F_5=-\frac{3}{2}H_4$$

and

$$\sqrt{6k_fk_i}F_6=\frac{\sqrt{3}}{2}H_3. \qquad (\mathrm{A}\cdot 2)$$

Appendix B

The coefficient function $A(pp'qk:\cos\theta)$

In the text, the coefficient function $A(pp'qk:\cos\theta)$ is defined by

$$[C_p(1)\times C_{p'}(2)]^k\equiv\sum_{q=\bar{k}-k}^{k}A(pp'qk:\cos\theta)[C_q(1)\times C_{\bar{k}-q}(2)]^k, \qquad (\mathrm{B}\cdot 1)$$

where $\bar{k}=k$ for $p+p'+k=$ even and $\bar{k}=k+1$ for $p+p'+k=$ odd. In order to obtain a recurrence formula for $A(pp'qk\colon \cos\theta)$, we rewrite $[C_p(1)\times C_{p'}(2)]^k$ by using the identity, $(p-1010|p0)C_p=[C_{p-1}\times C_1]^p$. Then

$$[C_p(1)\times C_{p'}(2)]^k=\frac{1}{(p-1010|p0)\,(p'-1010|p'0)}[[C_{p-1}(1)\times C_1(1)]^p \times[C_{p'-1}(2)\times C_1(2)]^{p'}]^k$$

$$=\frac{1}{(p-1010|p0)\,(p'-1010|p'0)}\sum_{s,r}\sqrt{(2p+1)\,(2p'+1)\,(2s+1)\,(2r+1)} \times\begin{Bmatrix} p-1 & 1 & p \\ p'-1 & 1 & p' \\ s & r & k \end{Bmatrix}[[C_{p-1}(1)\times C_{p'-1}(2)]^s\times[C_1(1)\times C_1(2)]^r]^k. \quad (\mathrm{B}\cdot 2)$$

In the sum over r and s, the $r=0$ $(s=k)$ term can be evaluated as

$$\frac{1}{(p-1010|p0)\,(p'-1010|p'0)}\sqrt{(2p+1)\,(2p'+1)\,(2k+1)} \times\begin{Bmatrix} p-1 & 1 & p \\ p'-1 & 1 & p' \\ k & 0 & k \end{Bmatrix}\left(-\frac{1}{\sqrt{3}}\cos\theta\right)[C_{p-1}(1)\times C_{p'-1}(2)]^k. \quad (\mathrm{B}\cdot 3)$$

The other terms are again rewritten to give

$$\sum_{\substack{s\\ r\neq 0}}\sqrt{(2p+1)\,(2p'+1)\,(2s+1)\,(2r+1)}\begin{Bmatrix} p-1 & 1 & p \\ p'-1 & 1 & p' \\ s & r & k \end{Bmatrix} \times[[C_{p-1}(1)\times C_{p'-1}(2)]^s\times[C_1(1)\times C_1(2)]^r]^k$$

$$=\sum_{\substack{s,r\neq 0\\ p_1,p_1'}}\sqrt{(2p+1)\,(2p'+1)\,(2p_1+1)\,(2p_1'+1)}\,(2s+1)\,(2r+1) \times\begin{Bmatrix} p-1 & 1 & p \\ p'-1 & 1 & p' \\ s & r & k \end{Bmatrix}\begin{Bmatrix} p-1 & 1 & p_1 \\ p'-1 & 1 & p_1' \\ s & r & k \end{Bmatrix} \times(p-1010|p_10)\,(p'-1010|p_1'0)\,[C_{p_1}(1)\times C_{p_1'}(2)]^k, \quad (\mathrm{B}\cdot 4)$$

where $p_1=p$ or $p-2$ and $p_1'=p'$ or $p'-2$. Equations (B·3) and (B·4) can now be used to express $[C_p(1)\times C_{p'}(2)]^k$ in terms of $[C_{p-1}(1)\times C_{p'-1}(2)]^k$, $[C_p(1)\times C_{p'-2}(2)]^k$, $[C_{p-2}(1)\times C_{p'}(2)]^k$ and $[C_{p-2}(1)\times C_{p'-2}(2)]^k$. Using the orthogonality relations for $9j$-symbols, to carry out the sum over s and r, we obtain

$$\sqrt{(2p+1)(2p'+1)(2k+1)}\begin{Bmatrix} p-1 & 1 & p \\ p'-1 & 1 & p' \\ k & 0 & k \end{Bmatrix}$$

$$\times (p-1010|p0)(p'-1010|p'0)[C_p(1)\times C_{p'}(2)]^k$$

$$=-\frac{1}{\sqrt{3}}\cos\theta[C_{p-1}(1)\times C_{p'-1}(2)]^k$$

$$-\sum_{\substack{p_1,p_1' \\ (p_1,p_1')\neq(p,p')}}\sqrt{(2p_1+1)(2p_1'+1)(2k+1)}\begin{Bmatrix} p-1 & 1 & p_1 \\ p'-1 & 1 & p_1' \\ k & 0 & k \end{Bmatrix}$$

$$\times (p-1010|p_10)(p'-1010|p_1'0)[C_{p_1}(1)\times C_{p_1'}(2)]^k. \qquad (\mathrm{B}\cdot 5)$$

Since Eq. (B·5) guarantees the validity of Eq. (B·1), we can derive the recurrence formula for $A(pp'qk;\cos\theta)$ by the use of Eq. (B·1) on both sides of Eq. (B·5) as

$$A(pp'qk:\cos\theta)=-\frac{2(2p-1)(2p'-1)\cos\theta}{\sqrt{(p+p'+k+1)(p+p'+k)(p+p'-k)(p+p'-k-1)}}$$

$$\times A(p-1\ p'-1\ q\ k:\cos\theta)$$

$$+\sqrt{\frac{(k+p-p'+2)(k+p-p'+1)(k-p+p')(k-p+p'-1)}{(p+p'+k+1)(p+p'+k)(p+p'-k)(p+p'-k-1)}}$$

$$\times A(p\ p'-2\ q\ k:\cos\theta)$$

$$+\sqrt{\frac{(k+p'-p+2)(k+p'-p+1)(k-p'+p)(k-p'+p-1)}{(p+p'+k+1)(p+p'+k)(p+p'-k)(p+p'-k-1)}}$$

$$\times A(p-2\ p'\ q\ k:\cos\theta)$$

$$-\sqrt{\frac{(p+p'+k-1)(p'+p+k-2)(p+p'-k-2)(p+p'-k-3)}{(p+p'+k+1)(p+p'+k)(p+p'-k)(p+p'-k-1)}}$$

$$\times A(p-2\ p'-2\ q\ k:\cos\theta). \qquad (\mathrm{B}\cdot 6)$$

Equation (B·6) together with the trivial relation,

$$A(q'\ \bar{k}-q'\ qk:\cos\theta)=\delta_{qq'}, \qquad (\mathrm{B}\cdot 7)$$

determines all the coefficient functions, $A(pp'qk:\cos\theta)$.

In some cases, the following equation which is easily derived from Eq. (B·1) is useful for the evaluation of $A(pp'qk:\cos\theta)$:

$$C_{pm}(\theta,0)(pmp'0|km)=\sum_q A(pp'qk:\cos\theta)C_{qm}(\theta,0)(qm\ \bar{k}-q\ 0|km). \qquad (\mathrm{B}\cdot 8)$$

For example, $A(2211:\cos\theta)$, which is necessary to derive Eq. (4·5) in the text, can be calculated by

$$C_{21}(\theta,0)\,(2120|11)=A(2211:\cos\theta)\,C_{11}(\theta,0)\,(1110|11) \tag{B·9}$$

with the result

$$A(2211:\cos\theta)=-\frac{3\sqrt{5}}{5}\cos\theta\,. \tag{B·10}$$

Appendix C

The partial wave expansion

Here we discuss the relation between the invariant amplitude F defined by Eq. (3·1) in the text and the usual partial wave amplitudes. The invariant amplitude F can be expanded in terms of Legendre polynomials as follows:

$$F(S_{aA}S_{bB}Sr:E,\cos\theta)=\sum_l(2l+1)P_l(\cos\theta)F(S_{aA}S_{bB}Sr:E,l)\,. \tag{C·1}$$

On the other hand, the usual partial wave amplitudes are defined by

$$\begin{aligned}&\langle\nu_B,\nu_b;\boldsymbol{k}_f|T|\nu_A,\nu_a;\boldsymbol{k}_i\rangle\\&=\frac{2\pi}{\sqrt{M_fM_ik_fk_i}}\sum_{\substack{l_i\mu_i\\l_f\mu_f\\S_{aA}S_{bB}\\J}}\sqrt{(2l_i+1)(2l_f+1)}\,C_{l_f\mu_f}(\Omega_f)C_{l_i\mu_i}(\Omega_i)\\&\quad\times(s_a\nu_as_A\nu_A|S_{aA}\,\nu_a+\nu_A)\,(s_b\nu_bs_B\nu_B|S_{bB}\,\nu_b+\nu_B)\,(l_i\mu_i\,S_{aA}\,\nu_a+\nu_A|JM)\\&\quad\times(l_f\mu_fS_{bB}\nu_b+\nu_B|JM)\,T^J(l_il_fS_{aA}S_{bB},E)\,.\end{aligned} \tag{C·2}$$

By substituting Eq. (C·1) into Eq. (3·1) and using the relation,

$$\begin{aligned}&P_l(\cos\theta)\,[C_r(\Omega_i)\times C_{\bar{S}-r}(\Omega_f)]^S\\&=(-)^{r-S}\sum_{l_1l_2}(-)^{l_2}W(r\,\bar{S}-r\,l_1l_2:Sl)\,[C_{l_1}(\Omega_i)\times C_{l_2}(\Omega_f)]^S,\end{aligned} \tag{C·3}$$

we obtain

$$\begin{aligned}&T^J(l_il_fS_{aA}S_{bB};E)\\&=\frac{(-)^{l_i+l_f+S_{bB}-J}}{\sqrt{2J+1}}\sum_{\substack{Sr\\l}}(-)^r(2l+1)\,W(l_iS_{aA}l_fS_{bB};JS)\,W(r\,\bar{S}-r\,l_il_f;Sl)\\&\quad\times(l0r0|l_i0)\,(l0\,\bar{S}-r\,0|l_f0)F(S_{aA}S_{bB}Sr;E,l)\,.\end{aligned} \tag{C·4}$$

Analogous expressions in terms of the invariant amplitudes G and H can be derived in a similar way.

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