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Policy Variability and Economic Growth*

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Abstract

This paper explores the effect of policy variability (or frequency of regime switching) on the level and characteristics of growth. We find that, contrary to prior views, the lack of persistence in policies per se need not be welfare reducing and that it can enhance growth. We also find that it is important to distinguish the type of policies that alternate in order to asses its impact. Two types of alternating policies are considered: i) policies that differ in the degree to which all investment is subsidized; ii) policies where the specific sector being subsidized changes over time. For the first case, we find that variability is likely to decrease growth, while in the second case it is likely to increase growth. These differences are linked to the role of income and substitution effects. For the case of sectoral policies, we also find that, in average, growth rates increase temporarily following a change in regime.

1. Introduction.

The connection between government policies and economic growth has recently received considerable attention. For the case of developing countries, the work of Chenery et al. (1986), Easterly and Wetzel (1989) and Krueger (1978) suggests that policies that distort relative prices and resource allocation are an important source of differences in performance among countries. Examples of such policies are differential import tariffs, export and investment subsidies, allocation of foreign exchange, taxes (see Easterly (1990)).

Policies not only differ across countries, but there is also a significant variation over time within countries. Often as a consequence of political instability, regimes with different economic incentives alternate over time, with different degrees of persistence. According to Easterly, King et al. (1990) the government current expenditure variables – like consumption and education spending – are quite persistent while government investment, trade orientation, and black market premium are not very persistent[1]. Harrison (1991) also finds that over the last thirty years most developing countries have experienced large variations in commercial and exchange rate policies[2].

The purpose of this paper is to analyze the implications of such policy variability or regime switching on the level and characteristics of economic growth. Though there is a fairly widespread view that the lack of persistence could lead to slower growth and reduced welfare, to our knowledge no serious theoretical attempt has been made to consider the question in a more rigorous way. This paper explores this issue in a neoclassical growth model.

The type of policies we consider are given by the level and distribution of investment subsidies. Many policies, such as investment tax credits and regional or sectoral subsidies to investment fall into this category. The scope is even broader if one considers other policies which, indirectly, have a similar effect, such as tariffs or import quotas, quantitative allocation of credit or foreign exchange and, investment licenses or other regulatory requirements. Throughout the paper we take these policies as given and do not model the political process by which they are instituted or changed.

The growth effects of government policies that distort the allocation of resources across sectors was first analyzed by Easterly (1990), Jones and Manuelli (1990) and King and Rebelo (1990). By altering the rate of return to capital, these policies can either stimulate or discourage investments in different sectors of the economy, thus affecting their capital accumulation rates. In particular, if there are two generic types of capital their work implies that any distortion that increases the price of one capital good will have a negative impact on growth. If instead the distortion decreases the price of one capital good, the result will be the reverse (i.e there will be too much growth). Easterly (1990) proceeds to test empirically this hypothesis for developing countries and his results suggest that distortions have a significant effect on growth.

In contrast, little attention has been given to the effects of persistence of policy regimes –or policy variability – on growth. Aizenman and Marion (1991) study a model with two

[1] As an example they find that the cross section correlation between 1960's and 1970's for government education spending is 0.75 while for trade orientation is 0.56.

[2] See also FIEL (1988) and (1990) for the case of Argentina.

possible tax regimes, characterized by a high or low profit tax. They find that increases in policy uncertainty (defined as the gap between the two policy regimes) can have positive or negative effects on growth depending on the degree of regime persistence. In addition, they show that in the absence of persistence, policy uncertainty does not affect growth. Our paper focuses, precisely, on the implications of the *degree* of policy persistence on the level and characteristics of growth and on welfare.

The main objective of the paper is to understand some of the basic mechanisms by which policy variability affects growth and welfare. For this purpose we analyze a fairly specific class of linear neoclassical growth models. In the first part of the paper we consider a one sector growth model. The policy we analyze is one where investment is subsidized at a uniform rate but this rate can be either positive or zero. Thus there are two possible regimes: the *subsidy* and *nonsubsidy* regimes. Subsidies are financed with lump sum taxation. The degree of policy variability of an economy is given by the arrival rate of a regime switch. The setup is similar to Aizenman and Marion (1991), but with two differences. Firstly, we allow for income effects, which by assumption are absent in their setup. Secondly, as indicated above, we focus on policies that affect the cost of investment rather than its return.[3]

The results obtained for this one sector model show that, surprisingly, higher variability leads to higher welfare and can result in higher growth. The mechanism by which increased welfare results is an interesting one, where income effects play a crucial role. Since there are no externalities in capital accumulation, investment subsidies lead to an intertemporal distortion and excessive investment in periods of subsidy. This raises the value of consumption in periods of subsidy and, in anticipation, the expected return on investment in periods of no subsidy. Consequently, in periods of no subsidy investment is also above its corresponding value for a zero subsidy economy: subsidies to investment *spill over* periods of no subsidy.

Higher variability implies more frequent changes in the level of investment and consumption. But by creating a stronger intertemporal link across the two regimes, investment in periods of no subsidy increases with variability, while it decreases in periods of subsidy. This reduces the amplitude of the fluctuations in consumption, thereby increasing welfare. We prove that average long run growth rates increase with variability when the latter is close to zero. However, our numerical simulations suggest that this is a local result and that for more reasonable levels of variability the opposite direction prevails.

Policies are often targeted to specific sectors of the economy or have a differential impact on them. In either case, a sectoral impact results. In this context, regime changes are likely to result in an important sectoral reallocation of resources. To capture this effect, the second part of the paper considers a two sector model, which in absence of investment subsidies exhibits exactly the same aggregate behavior as the first model. The investment subsidy rate is positive and constant, but the sector being subsidized varies over time. There are also two regimes in this case, depending on which investment good is subsidized.

Our results for the two sector model indicate that more frequent regime switching leads to higher average growth rates, larger intersectoral distortions and to some extent

[3]For the one sector model, similar results are obtained with taxes on returns.

more variable consumption. As a consequence, welfare decreases with variability. Adjustment dynamics, which are inexistent in the first model, play an important role here. As the type of capital being subsidized changes, so does the desired relative capital intensity. Since investment is irreversible, the adjustment takes place through positive investment in the subsidized capital (and depreciation of the nonsubsidized). The reallocation incentives lead to high levels of investment and a larger growth rate, which monotonically decrease over time while the desired capital intensity is approached. This has two interesting consequences. Firstly, growth rates are in average higher when regimes change. Policy makers can indeed claim their new policy has stimulated growth! Secondly, higher variability tends to increase the targeted capital intensity ratios and also the time spent by the economy in periods of reallocation. This explains the higher growth rates.

The paper contains analytical results and numerical computations. The methods developed for the computation of the two sector model are of independent interest. (It is a continuous time model with irreversible investment and a jump process, which involves corner solutions.) We use an iterative scheme with successive ODE problems. To compute statistics of the long run behavior, we derive explicitly a differential equation for the density function and provide a simple method for its computation.

The paper is organized as follows. Section 2 considers the one sector model and Section 3 the two sector case. Each section includes first theoretical results and then a subsection with numerical results. Section 4 concludes. The computation method is described in an appendix.

2. One Sector Model.

We consider a simple endogenous growth model where all factors of production are reproducible and their quantity is summarized by the composite capital good k (see King and Rebelo (1990) and Rebelo (1991)).

The production technology is given by:

$$y_t = Ak_t \tag{1}$$

where $A > 0$ is a time invariant productivity parameter. Capital depreciates at rate δ and investment is irreversible:

$$\dot{k}_t = x_t - \delta k_t \tag{2}$$

where $\dot{k}_t = \partial k_t/\partial t \geq \delta$. There is a large number (constant over time) of identical agents that are endowed with an initial amount of capital. Agents maximize expected utility defined as $U = E\int_0^\infty e^{-\rho t}\, u(c_t)\, dt$, where $\rho > 0$ is the constant rate of time preference, c_t is consumption at time t and, $u(c_t)$ is the instantaneous utility. We assume that $u(c_t)$ is of the constant elasticity type so the expected utility is:

$$U = E\int_0^\infty e^{-\rho t}\, \frac{c_t^{1-\sigma}}{1-\sigma}\, dt \tag{3}$$

where $\sigma > 0$ is the reciprocal of the intertemporal elasticity of substitution.

To study the role of policy variability we analyze a very simple subsidy policy. We assume that the government grants investment subsidies that vary over time. Specifically, there are two possible regimes: one where investment is subsidized at a constant rate $0 < \tau < 1$ and another, where there is no subsidy. Let z_t be a random variable that takes values one and zero in the subsidy and no subsidy regimes respectively. Let λ represent the arrival rate of a regime change so the expected duration of a regime is then $1/\lambda$. Agents observe the realization of z_t before making their period t decisions. The investment subsidy per unit of capital at time t is then

$$\tau(z_t) = \begin{cases} \tau & \text{if } z_t = 1 \\ 0 & \text{if } z_t = 0 \end{cases} \tag{4}$$

Since investment is irreversible, aggregate investment can not be negative. Because agents are identical, we may assume without loss of generality that the representative agent is subject to this constraint. Throughout our analysis, investment is strictly positive and the growth rate constant within each regime. The price of capital is then identical to that of the consumption good, which is used as a numeraire.

Subsidies are financed by lump sum taxation. Since all agents are identical and the economy is closed, feasibility requires that the government budget be balanced in each period. Let $T_t(z_t)$ represent the lump sum taxes at t measured in units of consumption as a function of the regime, then

$$T_t(z_t) = \begin{cases} \tau x_t & \text{if } z_t = 1 \\ 0 & \text{if } z_t = 0. \end{cases} \tag{5}$$

We study the competitive equilibrium for this stochastic economy. Firms maximize profits each period while agents choose their consumption and capital accumulation plans so as to maximize their expected lifetime utility given by (3). In this representative agent framework, capital investment is the only way to shift consumption over time. The agent's budget constraint at time t is given by:

$$c_t + (1 - \tau(z_t))x_t \leq Ak_t - T_t(z_t) \tag{6}$$

where by constant returns to scale the rental price of capital is equal to A.

For the particular preferences used and since technology is linear, it is a well established fact that the equilibrium decision rules are linear in k_t and that along a balanced stochastic growth path the interest rate is independent of k_t (though both, decision rules and interest rate will depend on the regime, z_t). Furthermore (3) implies that the optimal growth rate of consumption $g(z_t)$ satisfies the following:

$$g(z_t) = \frac{r(z_t) - \rho}{\sigma} \tag{7}$$

where $r(z_t)$ is the real interest rate (see Rebelo (1991)). Notice that the agent's lifetime utility will be finite if the expected growth rate of the momentary utility $(1-\sigma)E(g(z_t))$ is lower than the discount rate ρ. This is also equivalent to the transversality condition of the agent's problem.

Let $c(z_t)$ represent an agent's consumption per unit of capital at time t as a function of the regime z_t. Then $c(0)$ and $c(1)$ represent time t consumption when $z_t = 0$ and $z_t = 1$, respectively, and $k = 1$. Define $\tilde{R}$ to be the relative net price of capital between subsidy and non subsidy regimes measured in consumption units of the nonsubsidized regime, i.e.:

$$\tilde{R} = (1-\tau)\,\frac{u'(c(1))}{u'(c(0))} = (1-\tau)\left(\frac{c(0)}{c(1)}\right)^{\sigma}. \tag{8}$$

The variable $\tilde{R}$ captures the change in the value of a stock of capital due to regime changes. Suppose that at time t the no subsidy regime is in place and let $c(0)$ be the numeraire. The instantaneous capital gain from a change in regime[4] is given by $(1-\tau)\frac{u'(c(1))}{u'(c(0))} - 1 = \tilde{R} - 1$. If instead the prevailing regime is that of subsidy and $c(1)$ the numeraire, the corresponding gain is $\frac{u'(c(0))}{u'(c(1))} - (1-\tau) = (1-\tau)(\tilde{R}^{-1} - 1)$.

Equilibrium in the capital market requires that the interest rate equals the expected return divided by the price of capital. The expected return is equal to the marginal product net of depreciation plus the expected capital gains. Since investment is always positive, capital gains are zero unless there is a regime change. Thus, the expected capital gains are $\lambda(\tilde{R}-1)$ when z_t is equal to zero and $\lambda(1-\tau)(\tilde{R}^{-1}-1)$ when z_t equals one.

Letting $r(0)$ and $r(1)$ represent the interest rates when $z_t = 0$ and $z_t = 1$ respectively,

$$r(0) = A - \delta + \lambda(\tilde{R} - 1) \tag{9}$$

$$r(1) = \frac{A - \delta(1-\tau) + \lambda(1-\tau)(\tilde{R}^{-1} - 1)}{(1-\tau)}. \tag{10}$$

Per capita consumption in both regimes is constrained by

$$c_t(0) = (A - g_k(0) - \delta)k_t \tag{11}$$

$$c_t(1) = (A - g_k(1) - \delta)k_t, \tag{12}$$

where $g_k(0)$ and $g_k(1)$ are the growth rates of capital in the two regimes. Notice that (11) and (12) imply that if growth rates are independent of k_t and only depend on the regime in place at time t, consumption and capital will grow at the same rate (i.e., $g(0) = g_k(0)$ and $g(1) = g_k(1)$).

The equilibrium for this economy is then characterized by:

$$\sigma g(0) + \rho = A - \delta + \lambda(\tilde{R} - 1) \tag{13}$$

$$\sigma g(1) + \rho = \frac{A}{(1-\tau)} - \delta + \lambda(\tilde{R}^{-1} - 1) \tag{14}$$

$$\tilde{R} = (1-\tau)\left(\frac{A - g(0) - \delta}{A - g(1) - \delta}\right)^{\sigma}. \tag{15}$$

We now turn to the properties of the competitive equilibrium. We compare the two growth rates and analyze the effect of changes in the subsidy rate and the degree of policy variability. The first proposition shows that the growth rate (and therefore the interest rate) is higher in the subsidy regime than in the no subsidy regime.

[4]Notice that this capital gain can be positive or negative. In fact, as we will show later, it will be positive when $z_t = 0$ and negative when $z_t = 1$.

Proposition 1. *Suppose* $0 < \tau < 1$ *and* $\lambda > 0$, *then* $g(1) > g(0)$.

Proof. Suppose, by way of contradiction, that $g(1) \leq g(0)$. Then $r(1) \leq r(0)$ and by (9) and (10):

$$\lambda(\tilde{R} - \tilde{R}^{-1}) \geq A(\frac{\tau}{1-\tau}).$$

If $\tau > 0$ the previous equation implies that $\tilde{R} > 1$. On the other hand, (15) implies that $\tilde{R} < 1$, a contradiction.

The following Proposition considers the effect of changes in the level of subsidy.

Proposition 2. *For* $0 \leq \tau < 1$ *and* $\lambda > 0$, $\partial g(0)/\partial\tau$ *and* $\partial g(1)/\partial\tau$ *are strictly positive.*

Proof. See Appendix.

Proposition 2 shows that a higher subsidy results in higher growth rates in both regimes. The growth rate in the subsidy regime ($g(1)$) moves in the expected direction: increases in the subsidy rate decrease both income and the price of capital. A higher subsidy makes investment more attractive and although higher lump sum taxes discourage investment, the first effect is stronger and $g(1)$ increases. The positive effect on $g(0)$ on the other hand is more indirect. As $g(1)$ increases, consumption in periods of subsidy decreases, raising marginal utility of consumption. Since $\lambda > 0$ this has a positive effect on the interest rate in periods of no subsidy, which leads to a higher $g(0)$. One may conjecture that this investment spillover effect will be more important the higher λ is. (In particular, when $\lambda = 0$ there is no such spillover).

Proposition 2 and equation (13) imply that $\tilde{R}_\tau = \partial\tilde{R}/\partial\tau > 0$. Since $\tilde{R} = 1$ when $\tau = 0$, $\tilde{R}$ is greater than one for $\tau > 0$. Thus taking as a numeraire the consumption good in a period of no subsidy, the price of capital is higher in the subsidy regime than in the non subsidy regime.

We now turn to the effect of changes in the degree of policy variability on growth rates. Notice that since λ represents the arrival rate of a regime change, economies with higher λ are economies with more variability.

To study the effect of λ, a useful benchmark is the case where no regime switching occurs ($\lambda = 0$). This corresponds to the standard nonstochastic case. Let $g^\lambda(z_t)$ and $r^\lambda(z_t)$ represent the growth and interest rates for an economy with variability λ as a function of the regime z_t. Using (9) and (10) for λ equal to zero,

$$r^0(0) + \delta = A \quad (16)$$

$$(r^0(1) + \delta)(1 - \tau) = A. \quad (17)$$

For $\lambda > 0$, using (9), (10), (16), (17) and rearranging,

$$r^\lambda(0) - r^0(0) = \lambda(\tilde{R} - 1) \quad (18)$$

$$r^\lambda(1) - r^0(1) = \lambda(\tilde{R}^{-1} - 1). \quad (19)$$

We have previously shown that $\tilde{R} > 1$ and therefore $r^{\lambda}(0) > r^0(0)$ and $r^0(1) > r^{\lambda}(1)$. Using (7) implies that $g^{\lambda}(0) > g^0(0)$ and $g^0(1) > g^{\lambda}(1)$. By Proposition 1, $g^{\lambda}(1) > g^{\lambda}(0)$. We have thus established that:

$$g^0(1) > g^{\lambda}(1) > g^{\lambda}(0) > g^0(0).$$

The following Proposition provides a slightly stronger result.

Proposition 3. *For $0 < \tau < 1$, (1) $\frac{\partial g(1)}{\partial \lambda} < 0$ for any $\lambda > 0$, (2) $\frac{\partial g(0)}{\partial \lambda} > 0$ for all λ close to zero.*

Proof. See Appendix.

Proposition 3 indicates that policy variability can moderate the response of investment to a subsidy. This result arises because higher variability implies a stronger link between the two different states of the economy: subsidized and non subsidized periods. This is also reflected in the fact that $\frac{\partial \tilde{R}}{\partial \lambda} < 0$ (which can be easily established), indicating that capital gains (losses) are lower when there is more variability. Notice also that in proving the above proposition we have also used the fact that $\tilde{R} > 1$, which is a consequence of the income effects that are due to lump sum taxation. If agents could insure themselves from these policy shocks, marginal utility of consumption would be equated across states and $\tilde{R}$ would equal $1 - \tau$ which is smaller than one, leading to the opposite conclusion. This also suggests the important differences in behavior that arise when interest rates are not taken as given and general equilibrium effects considered.

We have shown that higher variability will decrease the growth rate in subsidy periods for any λ and increase it in no subsidy periods when λ is close to zero. It is also of interest to evaluate what happens to the average growth rates. Since the value for λ is independent of the regime in place, the limiting probability of each regime is equal to one half. The average long run growth rate for the economy is approximately $\frac{g(0)+g(1)}{2}$, which is monotonically increasing in $r(0) + r(1)$. Using (18) and (19) :

$$\frac{r^{\lambda}(0) + r^{\lambda}(1)}{2} = \frac{r^0(0) + r^0(1)}{2} + \lambda a > \frac{r^0(0) + r^0(1)}{2},$$

since $a \doteq \frac{(\tilde{R}-1)+(\tilde{R}^{\Gamma 1}-1)}{2}$ (the average capital gain due to regime changes) is strictly positive. We have thus proved:

Proposition 4. *For $0 < \tau < 1$ and λ close to zero, $\frac{(g^{\smile}(0)+g^{\smile}(1))}{2}$ is increasing in λ.*

So asymptotic growth can be stimulated by policy variability. However, this is a local result. Since $\tilde{R}$ is decreasing in λ, $\frac{\partial a}{\partial \lambda} < 0$. Given that for larger values of λ this effect is likely to dominate, higher variability could lead to lower growth. The simulations presented in the following section show that this is indeed the case and that λ need not be very large for it to happen.

We now turn to welfare considerations. Our numerical computations below indicate that higher variability results in higher average discounted utility. Though this may seem counterintuitive, it has a straightforward explanation. It is important to observe that here higher variability does not mean larger amplitude of oscillation but higher frequency. Moreover, the higher frequency of regime switching leads indeed to smoother consumption paths and hence lower amplitude (see Proposition 3). Because of the curvature of the utility function, average welfare is higher for these consumption paths than for those with larger but less frequent oscillations.

Simulations of the one sector model.

Assuming that investment is positive in both regimes and given values for the parameters $(A, \delta, \rho, \sigma)$, equations (13)-(15) provide a nonlinear set of equations to solve for ($g(0)$ and $g(1)$). Parameter values where assigned as follows: Given values for δ and σ, A and ρ can be determined to match the real interest rate (6%) and per capita growth rate (2%) of the economy, taking $\tau = 0$ as a benchmark. Using parameter values that are standard in the real business cycle literature, the assignments were: $\delta = 0.10$, $\sigma = 2$, $\rho = 0.02$ and A=0.16. As the invariant distribution puts equal weight in either regime the expected value of relevant variables were calculated as the simple average of their values in the two regimes. The results of the simulations for subsidies of 10% and 20% are presented in Tables 1 and 2, respectively.

By comparing the values in Tables 1 and 2 we can analyze the effect of an increase in the subsidy rate for a given degree of policy variability. Growth rates in both regimes increase while expected welfare decreases. As expected, increases in the gap between the two policy regimes are detrimental for welfare but have a positive effect on growth.

Increases in variability decrease the growth rate in the subsidy period and increase it in the non subsidy period. Except for extremely low levels of variability, the expected growth rate decreases with variability.

Tables 1 and 2 also provide three welfare indices, W_0, W_1 and $\overline{W}$. The first (second) one represent the expected discounted utility when the initial regime is one without (with) subsidy and $k_0 = 1$. $\overline{W}$ is the simple average of W_0 and W_1, providing a measure of long run average welfare.[5] Discounted utility for the undistorted economy ($\tau = 0$) is 625, which corresponds to W_0 at $\lambda = 0\,(ED = \infty)$. The ratio of any two welfare values gives roughly the percentage by which consumption would have to increase (decrease) in all periods to compensate for the lower (higher) welfare. For example, for $\tau = 0.20$ and $\lambda = 0.01$ ($ED = 100$) a 21.1% increase in consumption per period would be needed to compensate for the lower welfare that results from the distortion due to investment subsidies when starting in a period of subsidy ($\frac{757}{625} - 1$), while only 5.1% increase would be required if starting in a period of no subsidy.

As variability increases, expected discounted utility when the initial period is one with subsidy and it decreases if the initial period is one with no subsidy, and the two figures approach each other. This results from the fact that as λ increases, growth rates decrease towards the optimal levels in periods of subsidy and increase away from it in periods of no subsidy. The figure for average utility increases with variability, reflecting the decreased amplitude of consumption variations. For instance, in order for ex-ante average utility in an economy with no variability to equate the level corresponding to $\lambda = 2\,(ED = 0.5)$, consumption per period would have to increase by almost 9%.

[5]It also corresponds to the ex ante utility of an economy with probabilities $(\frac{1}{2}, \frac{1}{2})$ of starting in either regime.

3. Two sector model.

It is often the case that policies that alternate imply different sectoral or locational incentives. Exchange rate policies affect the returns to investments in traded and non traded sectors and the cost of those investments based on imported capital goods. Industrial policies often provide investment tax credits or low interest credit to promote specific industries or geographical areas. Furthermore, when resource reallocation is costly, the adjustment to policy changes is not likely to be immediate, as opposed to the one sector model case where all adjustments to a regime change were instantaneous. To study the characteristics and costs of such resource reallocations, this section considers a simple two sector model.

The simplest extension of the one sector model presented in the previous section consists of disaggregating the composite capital into two types of capital ($k_1$and k_2). We assume that these two types of capital are perfectly symmetric both in their usage and accumulation laws. The production technology for the consumption good is given by:

$$y_t = f(k_{1t}, k_{2t}) = AF(k_{1t}, k_{2t}) \tag{20}$$

where $F(.)$is twice differentiable, strictly concave, homogeneous of degree one, $F_i(.) > 0$, $i = 1,2$ for any $0 < \frac{k_{1t}}{k_{2t}} < \infty$, $F_{ij} > 0$, $i,j = 1,2$ and symmetric (i.e. $F(k_{1t}, k_{2t}) = F(k_{2t}, k_{1t})$). As before investment is irreversible and the law of motion for each capital is:

$$\dot{k}_{it} = x_{it} - \delta k_{it}, \qquad i = 1,2. \tag{21}$$

Agents are endowed with an initial amount of both types of capital and their preferences are given by (3). Notice that in the absence of distortions, this economy exhibits the same aggregate behavior as the one sector model.

Policies that affect relative prices and that fluctuate over time are quite common in many developing countries (see Chenery et al. (1986), Easterly and Wetzel (1989), Krueger (1978), and FIEL (1988) (1990)). Examples are differential import tariffs, export and investment subsidies, allocation of foreign exchange, taxes (see Easterly (1990)).

We analyze a very simple subsidy policy that nevertheless captures what we think are the fundamental aspects of these policies. In any given period, only one type of capital is subsidized at a constant rate $0 < \tau < 1$. There are then two possible regimes: one where investment in k_1 is subsidized and another, where investment in k_2 is subsidized. We assume that the regime variable (z_t) takes a value of one in the first case and a value of two in the latter. Let τ_1 and τ_2 represent the subsidies to the two types of investments, then:

$$\tau_1(z_t) = \begin{cases} \tau & \text{if } z_t = 1 \\ 0 & \text{if } z_t = 2; \end{cases}$$

$$\tau_2(z_t) = \begin{cases} 0 & \text{if } z_t = 1 \\ \tau & \text{if } z_t = 2; \end{cases}$$

Since all agents are identical and the economy is closed, feasibility requires a balanced government budget in each period. Consequently, lump sum taxes are:

$$T_t(z_t) = \begin{cases} \tau x_{1t} & \text{if } z_t = 1 \\ \tau x_{2t} & \text{if } z_t = 2. \end{cases} \tag{22}$$

The agent's budget constraint at time t is given by:

$$c_t + (1 - \tau_1(z_t))x_{1t} + (1 - \tau_2(z_t))x_{2t} \leq q_{1t}k_{1t} + q_{2t}k_{2t} - T_t(z_t), \tag{23}$$

where q_{1t} and q_{2t} are the rental prices of the two types of capital.

Since both types of capital enter symmetrically in the production function and the subsidy rate is constant, the relevant state variables are the stocks of subsidized and nonsubsidized capital (k_{st}, k_{nt}). To simplify the exposition we use the convention that the subsidized capital is the first argument in the production function. Notice that because of the symmetry of the production function in the two capitals, the ratio of output over total capital is maximized when $k_{st} = k_{nt}$. Therefore, subsidizing investment in only one type of capital introduces both a static and a dynamic distortion.

The homogeneity of preferences given by (3) and the constant returns to scale imply that the equilibrium decision rules are homogeneous of degree one in (k_{st}, k_{nt}). In addition, along a balanced stochastic growth path the interest rate, the growth rate and all relative prices will only depend on a single state variable: the ratio of subsidized to nonsubsidized capital $(k_t = k_{st}/k_{nt})$. The optimal growth rate of consumption $g(k_t)$ is then:

$$g(k_t) = \frac{r(k_t) - \rho}{\sigma}. \tag{24}$$

Notice that $(1 - \sigma)E(g(k_t)) < \rho$ is necessary and sufficient for the transversality condition of the agent's problem to hold. Let $c(k_1, k_2)$ represent an agent's consumption when the stocks of subsidized and nonsubsidized capital are $k_1$and k_2 respectively. If the regime changes, the agent's consumption will instead be $c(k_2, k_1)$. By homogeneity, $c(k_1, k_2) = k_2 c(k, 1)$, and $c(k_2, k_1) = k_1 c(\frac{1}{k}, 1)$, where $k = \frac{k_1}{k_2}$ is the ratio of subsidized to nonsubsidized capital. Let $R(.)$ be the ratio of marginal utilities in the two regimes, which only depends on the ratio k and is given by:

$$R(k) = \frac{u'(c(k_2, k_1))}{u'(c(k_1, k_2))} = \left[\frac{c(k, 1)}{kc(\frac{1}{k}, 1)}\right]^{\sigma} = \left[\frac{c(k, 1)}{c(1, k)}\right]^{\sigma}. \tag{25}$$

We now derive the equations relating the interest rate and the values of the subsidized and nonsubsidized capitals. Notice first that if investment is positive for both, these values are $(1 - \tau)$ and one, respectively. Let $w(k_t)$ be the value of the nonsubsidized capital at k_t and assume that investment in the subsidized capital is always positive. As investment in the nonsubsidized capital can be zero, $w(k_t)$ can take values smaller than one.

Notice that for both types of capital the expected return is the sum of the expected capital gains[6] and the marginal product net of depreciation. Let $k_t = k$ and consider first the subsidized capital. Since investment is always positive for the subsidized good, capital gains are zero unless there is a regime change. The instantaneous capital gain from a change in regime -measured in units of consumption prior to the change- is $w(\frac{1}{k})R(k) - (1 - \tau)$, where $w(\frac{1}{k})R(k)$ is the value of the nonsubsidized capital when $k_t = \frac{1}{k}$ (relative to the same numeraire).

[6]These gains can be negative or positive

Consider now the nonsubsidized capital. Since $w(k)$ is its current value, the instantaneous capital gain is $(1-\tau)R(k) - w(k)$ if the regime changes and $w'(k)\ \dot{k}$ if it does not. The next two capital market equilibrium conditions follow from the above remarks:

$$r(k) + \delta = \frac{f_1(k,1) + \lambda\ \left[w(\frac{1}{k})R(k) - (1-\tau)\right]}{(1-\tau)} \tag{26}$$

$$r(k) + \delta = \frac{f_2(k,1) + w'(k)\ \dot{k} + \lambda\ [(1-\tau)R(k) - w(k)]}{w(k)}. \tag{27}$$

The resource constraint for the economy is given by

$$c(k_s, k_n) = f(k_s, k_n) - \delta(k_s + k_n) - \dot{k}_s - \dot{k}_n. \tag{28}$$

Assuming $k_n = 1$ and $k = \frac{k_s}{k_n} = k_s$ it follows that $\dot{k} = \dot{k}_s - k\dot{k}_n$, so after rearranging (28) and using the fact that consumption is homogeneous of degree one in (k_s, k_n), the resource constraint can be written as follows:

$$c(k,1) = f(k,1) - (\delta + \dot{k}_n)(1+k) - \dot{k} \tag{29}$$

Note that if investment in the nonsubsidized capital is zero, $\dot{k}_n = -\delta$ so this equation simplifies to

$$c(k,1) = f(k,1) - \dot{k}. \tag{30}$$

On the other hand, if $\dot{k} = 0$ it simplifies to

$$c(k,1) = f(k,1) - (\delta + \dot{k}_n)(1+k). \tag{31}$$

Using equation (24), the growth rate of consumption satisfies

$$r(k) = \rho + \sigma g(k) \tag{32}$$

Equations (25), (26), (27), (29) and (32) plus the transversality condition characterize the equilibrium for this economy.

To develop some intuition about the equilibrium behavior, it is useful to consider first the extreme case where $\lambda = 0$, so no future change of regime occurs. For that case the above system reduces to:

$$[r(k) + \delta](1-\tau) = f_1(k,1) \tag{33}$$

$$[r(k) + \delta]w(k) = f_2(k,1) + w'(k)\dot{k}. \tag{34}$$

Once the economy reaches a balanced growth path, the interest rate remains constant and both capitals grow at the same rate, with their ratio constant at some value $k^* > 1$ and $w(k^*) = 1$. For values of $k_t < k^*$,$w(k_t)$ is less than one, so all investment goes to the subsidized capital. As k_t approaches k^*, $w(k_t)$ increases to its limiting value. From equation (33) it follows that $r(k)$ is decreasing and consequently consumption grows at a decreasing rate (see King and Rebelo [1989] and Mulligan and Sala i Martin [1991] for a discussion of the dynamics of the nonstochastic two sector model).

Consider now the case where $\lambda > 0$. From the above discussion, one may conjecture that the equilibrium path has the following characteristics:

1. There exists some ratio of subsidized to nonsubsidized capital $k^* > 1$ such that if $k_t = k^*$ the economy grows at a constant rate with positive investment of both capital goods and k_t unchanged. If $k_t < k^*$, all investment will go to the subsidized capital and k_t will increase over time until it reaches k^* or a change of regime occurs.

2. $w'(k_t) > 0$, and along this adjustment path $w(k_t)$ will increase to one.

3. The interest rate $r(k_t)$ is decreasing in k_t. As a result, along the adjustment path consumption grows at a decreasing rate. Investment also decreases and consequently output grows at a decreasing rate.

Suppose a change of regime occurs after a long period of stability. The economy is at k^* and with the regime change the state switches to $\frac{1}{k\Lambda}$. This leads to a jump in the growth rate of the economy, to later decrease over time. Policy makers can indeed claim their new policy has stimulated growth! The described adjustment suggests a more general result: policy shocks that result in persistent changes in the relative price of different types of capital will lead over time to a change in relative capital intensity. With irreversible investment, this adjustment will translate into a sizable jump in total investment, followed by higher growth rates.

We now return to the analysis of policy variability. It is useful to decompose the effect of changes in λ in two parts: 1) the effect on the boundary k^* and the growth rate of the economy at this boundary; 2) the effect on the adjustment path. A general analysis including the two effects is quite complex and is not carried out here. Equilibrium paths are numerically computed and described in Section 3.2. The effect of changes of λ on the boundary can be isolated and analytical results obtained by considering a discrete time version of the above model in which all adjustment takes place in one period. This is a reasonable approximation to the case where depreciation rate is high or the subsidy rate low and thus the period of adjustment a short one. The following section provides a complete characterization of this case.

3.1. **The one period adjustment case.** The discrete time version of the model is as follows. Agents maximize

$$U = E\left(\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}\right), \tag{35}$$

where $0 < \beta < 1$ is the discount factor. The production function is the same as before and the law of motion for capital is

$$k_{it+1} = (1-\delta)k_{it} + x_{it}, \quad i = 1,2. \tag{36}$$

Since the subsidy policy is the same as before, subsidies, lump sum taxes and the budget constraint are unchanged. Let λ represent now the probability of a regime change, so with probability $1-\lambda$ the economy will remain in the same regime. As before, decision rules are homogeneous of degree one and relative prices homogeneous of degree zero in (k_{st}, k_{nt}).

Consider parameter values such that the equilibrium is always interior, with positive investment of both, subsidized and nonsubsidized capital. Letting k^* denote the boundary very short time. Consequently, as $\lambda \to \infty$, capital gains or losses due to regime changes disappear. Equations (26) and (27) simplify to

$$r(k^*) + \delta = \frac{f_1(k^*, 1)}{1 - \tau}$$

and

$$r(k^*) + \delta = f_2(k^*, 1).$$

These two equations, which implicitly define k^*, are exactly the same two equations that correspond to the case $\lambda = 0$. Hence the value of k^* when $\lambda = 0$ is the same as the limiting value as $\lambda \to \infty$. This just reflects the fact that in both extremes there are no capital gains or losses. The two cases, however, differ significantly in behavior. While at $\lambda = 0$ the economy remains at k^*, this is not true for high values of λ.[7]To consider the behavior of the economy for intermediate values of λ, we now analyze some numerical results.

3.2. **Simulations of the two sector model** To simulate the model we restrict the production function to be of the CES type and use a version of the model that is consistent with the US long term experience. Output is then given by: $y_t = A(k_{1t}^{\gamma} + k_{2t}^{\gamma})^{\frac{1}{\gamma}}$, where $1/(1-\gamma)$ is the elasticity of substitution. We set γ equal to 0.5, σ equal to 2, the depreciation rate equal to 10%, and the instantaneous discount factor (ρ) equal to 2% . Finally, the value of A is chosen so that in the absence of subsidies the steady state growth rate of the economy is 2% ($A = 0.08$). For $k_1 = k_2 = k$ the production function reduces to the one used in the one sector model. For $\tau = 0$ the competitive equilibrium has the same aggregate allocations as in the one sector model.

The quantitative experiments are performed for different degrees of policy variability and subsidy rates. A summary of the results is presented in Figures 1, 2 and 3 and Tables 3 and 4.

Figure 1 plots the growth rates of consumption, capital and output along the adjustment path for values of $\tau = 0.20$ and $\lambda = 0.10$. Recall that investment in the subsidized capital is always positive; investment in the nonsubsidized capital is zero unless the ratio $k_t = \frac{k_{st}}{k_{nt}}$ is bigger or equal to the corresponding boundary value (k^*); if $k_t = k^*$, investment in both types of capital is positive and k_t is unchanged. Though in average for this economy, consumption, capital and output grow at the same rate, the paths are quite different. Along the adjustment path the growth rate of capital is fairly constant, while output and consumption grow at decreasing rates. This reflects the fact that throughout the adjustment path, investment is positive for the subsidized capital only, so growth occurs at the *intensive margin.* The output-capital elasticity -along this margin- is decreasing in the ratio k. At $\frac{1}{k\lambda}$ this elasticity is 1.24 while at k^* it is only 0.88.

At k^* the growth rates of output, capital and consumption are equal. Along the adjustment path, the growth rates of capital and consumption converge to their corresponding values at k^*. In contrast, the growth rate of output jumps at this point. This reflects the fact that at k^* investment of both capital goods is positive and k_t constant, so growth takes place at the *extensive margin.* The output-capital elasticity at this point is one.

[7]We conjecture that as $\lambda \to \infty$ the assymptotic distribution for k converges to a point mass at $k = 1$.

value, there are only two possible values for k_t, namely k^* and $\frac{1}{k^*}$. If the regime does not change between t and $t+1$, k_{t+1} will be equal k^* while if it changes, k_{t+1} will equal $\frac{1}{k^*}$. Note that regardless of whether a change in regime occurred or not between periods t and $t+1$, at time $t+1$ the ratio of the capital that was *subsidized in period t* to the capital that was *not subsidized in period t* will always be k^*. Also, since investment in both types of capital is positive, the values of the subsidized and nonsubsidized capitals are $(1-\tau)$ and one, respectively.

The equilibrium conditions for capital markets are given by:

$$(1-\tau)r(k_t) = (1-\lambda)\ [f_1(k^*,1)+(1-\delta)(1-\tau)] + \lambda\,\theta\,[f_1(k^*,1)+(1-\delta)] \quad (37)$$

$$r(k_t) = (1-\lambda)\ [f_2(k^*,1)+(1-\delta)] + \lambda\,\theta\,[f_2(k^*,1)+(1-\delta)(1-\tau)], \quad (38)$$

where $r(k_t)$ is the interest rate and $\theta = u'(c(1,k^*))/u'(c(k^*,1)) = [c(k^*,1)/c(1,k^*)]^\sigma$.

Since the right hand side of either of these equations does not depend on k_t, it follows immediately that the interest rate is independent of k_t and thus constant over time. In the Appendix (Lemma 1) we also establish that $c(k^*,1) = c(1,k^*)$, so $\theta = 1$. By adding these two equations we obtain:

$$(2-\tau)\ r = f_1(k^*,1) + f_2(k^*,1) + (1-\delta)(2-\tau),$$

where r is the constant interest rate. The left hand side is the capital cost of a portfolio of one unit of capital of each type and the right hand side the return of the portfolio.

Since the growth rate is monotonically related to the interest rate, the above equation can be used to determine the growth effects of higher variability. As shown in Lemma 2 in the Appendix, an increase in λ results in higher k^*. A straightforward calculation shows that since f is homogeneous of degree one and symmetric, $f_1(k,1)+f_2(k,1)$ is increasing in k. It follows that r increases with λ and so does the growth rate. We have thus established

Proposition 5. *For $0<\tau<1$ and $0<\lambda<1$, both the boundary k^* and the growth rate are increasing in the degree of policy variability λ.*

In contrast to the one sector case, higher variability here results in larger welfare losses. This can be seen as follows. Firstly, as λ increases so does k^* and thus output is produced less efficiently. Secondly, note that aside form the lower productive efficiency, along the balanced growth path described this economy looks just like the linear one sector model with production function $\tilde{A}k$, with $\tilde{A} = AF(k^*,1)/(1+k^*)$ but with a growth rate that exceeds the optimal level. Since welfare in the one sector model is a quasiconcave function of the growth rate, it decreases as growth rates increase with variability.

Proposition 5 establishes a positive association between the boundary value k^* and λ for the discrete time case. It is interesting to note that for the continuous time case, though this also holds true for small $\lambda = 0$ (see Lemma 3), it does not hold globally. This can be analyzed by considering the limiting case as $\lambda \to \infty$. For high λ the capital losses associated to a regime change must be small, since the original regime is likely to recur in

When the regime changes and the economy is initially at k^*, a positive jump in investment and consequent decrease in consumption (approximately 5.6%) occurs. Along with the higher marginal product of investment described above, this explains the higher growth rates of consumption following this jump. (The growth rate of consumption at $\frac{1}{k\lambda}$ is more than two times higher than at k^*.) Note also that along the adjustment path the rate of growth of consumption is greater than the rate of growth of output. The gap is closed by the positive jump of output at k^* and the negative jump of consumption at $\frac{1}{k\lambda}$. Figure 2 gives the price of nonsubsidized capital, $w(k)$ for $\tau = 0.20$ and $\lambda = 0.10$. Since investment in this type of capital is positive at k^*, $w(k^*) = 1$. But for values of $k < k^*$ there is no investment in this capital and thus the price per unit is lower than one. This price increases with k_t (at a decreasing rate), approaching one as k_t reaches k^*.

The process for regime switching and equilibrium adjustment rules imply a stationary stochastic process for k_t with support in $\left[\frac{1}{k\lambda}, k^*\right]$ and a unique limiting distribution. In the appendix we provide a method for its computation. This distribution and the corresponding density functions, are plotted for several values of λ in Figures 3a and 3b . Since k_t is increasing except for periods in which a regime switch occurs, the distribution puts more weight on higher values of k_t and has a point mass on k^*. The invariant distribution can be used to compute some statistics that describe the long run behavior of the economy. These are presented in tables 3 and 4, the first one corresponding to a subsidy rate of 10% and the second one to a rate of 20%.

Let $k_t = k$ and suppose a change of regime occurs. Since the state switches to $\frac{1}{k}$, the change in the growth rate depends on whether $k \gtrless 1$. If k is equal to one, the growth rate remains constant but if k is bigger than one, the growth rate increases. Since the economy spends relatively more time in higher values of k_t, one may expect the model would predict that in average growth rates increase after regime changes. To verify this conjecture, average growth rates conditional on a regime change taking place were computed as well as the unconditional ones. Tables 3 and 4 show that the growth rates after regime changes are more than 50% higher than the average values.

We now turn to the effect of variability. Consider first the effect on the boundary value k^*. As discussed above, a monotonic relationship is not globally predicted by the theory. Yet for the range of values considered -with the exception of just one case- higher variability results in a higher boundary k^*.

The effects of changes in variability on the invariant distribution of k_t are presented in Figure 3. Economies with higher variability spend more time on the adjustment path and less at the extremes. Consequently, there is more mass in values of k_t closer to one. This can also be seen in the last column of Tables 3 and 4, which indicates the proportion of the time spent by an economy at k^*. Notice also that in the cases where k^* increases with variability, the support of the distribution increases in both directions. Therefore, variability has two opposite effects: it assigns positive mass to more extreme values but also increases the total mass for values closer to one. Since the long run indicators are calculated using the invariant distribution, the effects of variability depend on the strengths of these two effects. The third column of tables 3 an 4 gives the mean values for k_t. For low values of λ the first effect predominates and the mean increases. But for higher values, the second effect dominates and the mean decreases.

The effects of variability on efficiency (measured as the average output/capital ratio) depend on the size of the distortion and the initial degree of variability. We found almost no variation in output per unit of capital. This is due to the high elasticity of substitution between the two types of capital, (implying that very large distortions are necessary for the output/capital ratios to exhibit variation) and the relatively small differences in the mean values of k_t for different levels of variability.

We now turn to the effect of variability on growth. With the exception of only one case ($\tau = 0.20$ and $\lambda = 2$), average growth rates increase with variability. This occurs mostly because as variability increases, the economies spend an increasing proportion of the time in periods of adjustment where investment is positive only for the subsidized capital. For instance, the last column of Table 4 shows that when $\lambda = 2$ ($ED = 0.5$) the economy spends only 8% of the time at k^* and the remaining time in periods of adjustment. Since we argued that growth rates are higher in periods of adjustment, the higher growth in economies with more variability can be thus explained. This also implies that, as variability increases, an increasing share of investment is subsidized and thus the average subsidy per unit of capital increases. This is shown in column 6 of Tables 3 and 4. For instance, for $\tau = 0.20$ an economy with $\lambda = 2$ has a subsidy/capital ratio almost twice as high as an economy with no variability.[8] Thus, the effective intertemporal distortion increases with variability.

The welfare effects follow a corresponding pattern, mirroring the changes in the growth rates. The values obtained indicate that in this two sector model higher variability leads to lower welfare.

4. Summary and Conclusions.

This paper has explored the effect of policy variability on the level and characteristics of growth. We find that, contrary to prior views, the lack of persistence in policies per se need not be welfare reducing and that it can enhance growth. We also find that it is important to distinguish the type of policies that alternate in order to assess its impact.

In a neoclassical model, the main determinant of the growth rate of an economy is the rate of return to investment. For the representative agent economy, in absence of externalities or market failure, a competitive equilibrium results in an optimal allocation. Consequently, policies that raise the rate of return to investment lead to higher growth rates and lower welfare. This suggests that the consequences of policy variability are closely linked to the way it affects the rate of return to investment.

Policy variability, or the degree of regime switching, affects the extent to which policies that will prevail in a future regime may, in anticipation, affect investment decisions in a prior one. In the extreme, when there is no variability, regimes are completely isolated and have no impact on each other. But when a change of regime is likely to occur, future policies can play an important role in current rate of return calculations. Our analysis distinguishes two sources of cross-regime effects, one operating through income and the other through substitution effects. The first dominate when the policies that alternate differ in the degree to which investment (vis a vis consumption) is subsidized, while the type of capital subsidized does not change significantly over time. This was the case

[8]Notice the contrast with the one sector model where, by reducing investment in periods of subsidy, higher variability resulted in a lower subsidy/capital ratio.

studied in the one sector model. The substitution effect predominates when it is not the level but the mix of subsidies that vary over time. This was the case studied in the two sector model.

The anticipation of a period of high (low) consumption -and thus lower (higher) marginal utility- has a negative (positive) effect on the rate of return to investment. In the one sector model, periods of subsidy are periods of low consumption and periods of no subsidy are periods of high consumption. Thus, lower persistence tends to reduce the rate of return to investment and total investment in periods of subsidy and increase them in periods of no subsidy. This decreases investment rates in periods of subsidy and increases them in periods of no subsidy, thus reducing the amplitude of fluctuations in consumption. This is the income effect.

In the one sector model we find that variability is welfare improving and that it is likely to decrease growth. The first result may perhaps seem counterintuitive given that the standard intuition suggests that uncertain consumption profiles are typically associated with lower welfare. Our analysis indicates that it is important to distinguish between the *frequency* and *amplitude* of consumption oscillations. Larger amplitude of oscillations are clearly welfare reducing. But as indicated above, the higher frequency of oscillations that results from higher variability leads to lower amplitude. This explains the welfare effect.

If regimes affect the type of capital that is subsidized and not the rate of subsidy -as occurs in the two sector model- the substitution effect prevails. The anticipation of a change in the type of capital to be subsidized, increases the rate of return to investment in the currently subsidized one. Higher variability has the effect of further delaying the investment in capital goods that are not currently subsidized and concentrating most investment in those that are. As a consequence, the rate of subsidy per unit of investment increases, as the share of subsidized investment does so. This increases the average growth rate but decreases welfare.

Another interesting result obtained from the two sector model is a sort of "spring" effect that occurs after regimes change. More precisely, we find that regime changes are likely to be followed by a jump in investment and growth rates. This occurs because the delay in the investment of nonsubsidized capital goods in anticipation of future subsidies, has the effect of raising the desired investment rate at the time the change in regime occurs and the new subsidies appear. Following the initial jump, growth rates monotonically decrease over time and the spring is over.

Transitional dynamics play an important role in the two sector model, particularly when variability is high and thus the economy spends a large fraction of the time in periods of adjustment. In the presence of *reallocation* shocks as the ones considered here, aggregating capital and abstracting from the sectoral component can thus be misleading.

Appendix

1. Proofs

Proof of Proposition 2: Let $\tilde{R}_0$ and $\tilde{R}_1$ represent the partial derivatives of $\tilde{R}$ with respect to $g(0)$ and $g(1)$ respectively. Substituting (15) into (13) and (14), differentiating with respect to τ and rearranging:

$$\begin{pmatrix} \sigma - \lambda \tilde{R}_0 & -\lambda \tilde{R}_1 \\ \frac{\lambda \tilde{R}_0}{\tilde{R}^2} & \sigma + \frac{\lambda \tilde{R}_1}{\tilde{R}^2} \end{pmatrix} \begin{pmatrix} \frac{\partial g(0)}{\partial \tau} \\ \frac{\partial g(1)}{\partial \tau} \end{pmatrix} = \begin{pmatrix} -\lambda \frac{\tilde{R}}{(1-\tau)} \\ \frac{A}{(1-\tau)^2} + \frac{\lambda}{(1-\tau)\tilde{R}} \end{pmatrix}$$

Solving this system and simplifying we obtain

$$\frac{\partial g(0)}{\partial \tau} = \frac{\sigma \lambda \tilde{R}}{D(1-\tau)} \left[\frac{A}{(1-\tau)(A - g(1) - \delta)} - 1 \right] > 0, \tag{39}$$

$$\frac{\partial g(1)}{\partial \tau} = \frac{\sigma A}{D(1-\tau)^2} \left[1 + \lambda \tilde{R}(A - g(0) - \delta) + \frac{\lambda(1-\tau)}{\tilde{R} A} \right] > 0, \tag{40}$$

where D, the determinant of the system, satisfies

$$D = \sigma^2 - \lambda \tilde{R}_0 \sigma + \lambda \sigma \frac{\tilde{R}_1}{\tilde{R}^2} > 0,$$

because $\tilde{R}_0 < 0$ and $\tilde{R}_1 > 0$.

Proof of Proposition 3: Substituting (15) into (13) and (14), differentiating with respect to λ and rearranging:

$$\begin{pmatrix} \sigma - \lambda \tilde{R}_0 & -\lambda \tilde{R}_1 \\ \frac{\lambda \tilde{R}_0}{\tilde{R}^2} & \sigma + \frac{\lambda \tilde{R}_1}{\tilde{R}^2} \end{pmatrix} \begin{pmatrix} \frac{\partial g(0)}{\partial \lambda} \\ \frac{\partial g(1)}{\partial \lambda} \end{pmatrix} = \begin{pmatrix} \tilde{R} - 1 \\ \tilde{R}^{-1} - 1 \end{pmatrix}$$

Solving the system and simplifying we obtain

$$\frac{\partial g(0)}{\partial \lambda} = \frac{\sigma}{D} \left[(\tilde{R} - 1) - \frac{\lambda (\tilde{R} - 1)^2}{\tilde{R}(A - g(1) - \delta)} \right] \tag{41}$$

$$\frac{\partial g(1)}{\partial \lambda} = \frac{\sigma}{D \tilde{R}} \left[-(\tilde{R} - 1) - \frac{\lambda (\tilde{R} - 1)^2}{(A - g(0) - \delta)} \right]. \tag{42}$$

Since $\tilde{R} > 1$, $\frac{\partial g(1)}{\partial \lambda} < 0$ for all $\lambda > 0$. The sign of $\frac{\partial g(0)}{\partial \lambda}$ depends on λ. If λ is sufficiently close to zero, $\frac{\partial g(0)}{\partial \lambda} > 0$, but for larger λ the opposite will be true.

Lemma 1. *$c(k^*, 1) = c(1, k^*)$.*

Proof. Let $g(k_t) = E\left(\frac{c(k_{st+1}, k_{nt+1})}{c(k_{st}, k_{nt})}\right)$ represent the growth rate of consumption when $\frac{k_{st}}{k_{nt}} = k_t$. Let $g_s(k_t)$ and $g_n(k_t)$ represent the growth rates of k_{st} and k_{nt}. Notice that if $k_t = k^*$, both types of capital grow at the same rate (i.e., $g_s(k^*) = g_n(k^*)$). If instead $k_t = \frac{1}{k_t^\lambda}$, k_{t+1} has to equal k^* if the regime does not change and therefore $g_s(\frac{1}{k_t^\lambda}) = k^{*2} g_n(\frac{1}{k_t^\lambda})$. Using these properties of the growth rates of capital we get:

$$g(k^*) = \frac{g_s(k^*)}{c(k^*,1)}\left[(1-\lambda)c(k^*,1) + \lambda c(1,k^*)\right] \tag{43}$$

$$g(\frac{1}{k^*}) = \frac{k^* g_n(\frac{1}{k_t^\lambda})}{c(1,k^*)}\left[(1-\lambda)c(k^*,1) + \lambda c(1,k^*)\right]. \tag{44}$$

Since the interest rate is independent of k_t and monotonically related to $g(k_t)$ it follows that $g(k^*) = g(\frac{1}{k^\lambda})$. Using (43) and (44):

$$\frac{c(k^*,1)}{c(1,k^*)} = \frac{g_s(k^*)}{k^* g_n(\frac{1}{k_t^\lambda})} \quad . \tag{45}$$

Notice that :

$$c(k^*,1) = f(k^*,1) - [g_s(k^*) - (1-\delta)](1+k^*) \tag{46}$$

$$c(1,k^*) = f(1,k^*) - [g_n(\frac{1}{k_t^*})k^* - (1-\delta)](1+k^*) \; . \tag{47}$$

Using (45), (46), (47) and the symmetry of the production function we can conclude that $c(k^*,1) = c(1,k^*)$.

Lemma 2. *Consider the discrete time two sector model with $0 < \lambda < 1$, $0 < \tau < 1$ and assume that the adjustment takes place in one period, then k^* is increasing in λ.*

Proof. We divide (37) by (38) to get:

$$f_2(k^*,1)(1-\tau) - f_1(k^*,1) = \lambda(1-\delta)\tau(2-\tau). \tag{48}$$

Notice that as $f_{21} > 0$ and $f_{11} < 0$ the left hand side of (48) is increasing in k^*. Because the right hand side is always positive and the left hand side is negative at $k^* = 1$, k^* is bigger than one. Assume λ increases, this increases the right hand side and therefore k^* has to increase.

Lemma 3. *Consider the continuous time two sector model with $\lambda > 0$ and $0 < \tau < 1$, then the boundary value k^* is increasing in λ at $\lambda = 0$.*

Proof. Using equations (26) and (27) at $k = k^*$, we get:

$$f_1(k^*,1) - f_2(k*,1)(1-\tau) - \lambda R(k^*)\left[(1-\tau)^2 - w(\frac{1}{k^*})\right] = 0 \tag{49}$$

Differentiating (49) with respect to λ and evaluating the derivative at $\lambda = 0$ we have:

$$\frac{\partial k^*}{\partial \lambda}\Big|_{\lambda=0} = \frac{R(k^*)\left[\,(1-\tau)^2 - w(\frac{1}{k_t^\lambda})\right]}{f_{11}(k^*,1) - f_{21}(k^*,1)(1-\tau)} \tag{50}$$

Notice that if $w(\frac{1}{k_*}) = (1-\tau)^2$, the relative prices of the two types of capital at k^* and $\frac{1}{k^\lambda}$ coincide. Therefore, $w(\frac{1}{k_t^\lambda}) > (1-\tau)^2$. Since $f_{11} < 0$ and $f_{21} > 0$, (50) is strictly positive.

2. METHOD OF COMPUTATION.

This section describes the method used to compute the equilibrium in the two sector model. Using (25)-(27) the following equations are derived:

$$(1-\tau)\left[r(k)+\lambda+\delta\right] = f_1(k,1) + \lambda w(\frac{1}{k})\left[\frac{c(k,1)}{kc(\frac{1}{k},1)}\right]^{\sigma} \tag{51}$$

and

$$w(k)\left[r(k)+\lambda+\delta\right] = f_2(k,1) + w'(k)\dot{k} + \lambda(1-\tau)\left[\frac{c(k,1)}{kc(\frac{1}{k},1)}\right]^{\sigma}, \tag{52}$$

where

$$r(k) = \rho + \sigma\left[\dot{k}_n + \frac{c'(k,1)}{c(k,1)}\dot{k}\right] \tag{53}$$

and $c(k,1)$ satisfies

$$c(k,1) = f(k,1) - \delta(1+k) - \dot{k} - \dot{k}_n(1+k) \tag{54}$$

where $\dot{k}_n$ is the time derivative of the non-subsidized capital.

The above equations define a differential system in the k-space which can be used to solve for the unknown functions of k. From our previous analysis, the state space can be partitioned in the two sets: a) $k = k^*$, where $\dot{k} = 0$, $w(k^*) = 1$ and $\dot{k}_n$ is equal to the rate of growth of the economy; b) $k < k^*$, where $\dot{k}_n = -\delta$. In the first case, equation (54) can be used to eliminate $\dot{k}_n$ and equation (53) to eliminate $r(k)$. Similar procedure can be followed in case (b). The resulting differential system involves the unknowns: $(k^*, w(k), c(k,1))$ and the derivatives of the last two functions.

The solution is obtained by iteration on these functions. Given two differentiable functions $w_n(k)$ and $c_n(k)$ defined on a domain $[0, k_n^*]$, c_{n+1} and w_{n+1} are obtained as follows.

Step 1. Determination of k_{n+1}^*. Solving for $r(k_{n+1}^*)$ as indicated above we get

$$r(k_{n+1}^*) = \rho + \sigma\left[\frac{f(k_{n+1}^*,1) - c_{n+1}(k_{n+1}^*,1)}{1+k_{n+1}^*} - \delta\right]$$

Using this equation, letting $\dot{k} = 0$, and using the functions w_n and c_n to evaluate $w_{n+1}(\frac{1}{k_{n+1}^{\lambda}})$ and $c_{n+1}(\frac{1}{k_{n+1}^{\lambda}},1)$, equations (51) and (52) solve for k_{n+1}^* and $c_{n+1}(k_{n+1}^*,1)$.

Step 2. Determination of w_{n+1} and c_{n+1}. Assuming the growth rate of consumption (and thus r) is continuous at k^* it follows that

$$\frac{f(k_{n+1}^*,1) - c_{n+1}(k_{n+1}^*,1)}{1+k_{n+1}^*} - \delta = \frac{c_{n+1}'(k_{n+1}^*,1)}{c_{n+1}(k_{n+1}^*,1)}\left(f(k_{n+1}^*,1) - c_{n+1}(k_{n+1}^*,1)\right) - \delta$$

which implies that

$$c_{n+1}'(k_{n+1}^*,1) = \frac{c_{n+1}(k_{n+1}^*,1)}{1+k_{n+1}^*}.$$

Assuming w_{n+1} is differentiable, it follows that $w_{n+1}'(k_{n+1}^*) = 0$.

Choose a grid $\{k_j\}_{j=0}^{m}$ on $\left[\frac{1}{k^{\Lambda}_{n+1}}, k^*_{n+1}\right]$ such that $k_j = \frac{1}{k_{m\Gamma j}}$. Abusing notation we will let $c_j = c_{n+1}(k_j, 1)$ and the same for w_j and the derivatives of both functions. Using equations (54) and (53) to eliminate $\dot{k}$ and $r(k)$, equations (51) and (52) yield:

$$(1-\tau)\left[\rho + \sigma\left[\frac{c'_j}{c_j}(f(k_j,1) - c_j)\right] + \lambda + \delta(1-\sigma)\right] =$$
$$f_1(k_j,1) + \lambda w_n(k_{m-j})\left[\frac{c_j}{k_j c_n(k_{m-j},1)}\right]^{\sigma}$$

and

$$w_j\left[\rho + \frac{\sigma}{1+k_j}\left[\frac{c'_j}{c_j}(f(k_j,1) - c_j)\right] + \lambda + \delta(1-\sigma)\right] =$$
$$f_2(k_j,1) + w'_j(f(k_j,1) - c_j) + \lambda(1-\tau)\left[\frac{c_j}{k_j c_n(k_{m-j},1)}\right]^{\sigma}$$

The values for $c_n(k_{m-j}, 1)$ and $w_n(k_{m-j})$ are obtained by using the functions previously interpolated . Given those values and (w_j, c_j), these two equations can be used to solve linearly for c'_j and w'_j. Setting $w_{j+1} = w_j + w'_j(k_{j+1} - k_j)$ and similarly for c_{j+1}, the procedure continues until all values are computed. By polynomial interpolation the new functions c_{n+1} and w_{n+1} are thus obtained. This process is continued until the change in the functions and in k^*_n is below some given critical level.

Note that by using the functions c_n and w_n in the way just described, the above set of equations define an ordinary differential equation system parameterized by these functions. This defines implicitly a mapping from the space of c and w functions into itself. The iterations that are followed seek to find the fixed point of this map.

Invariant distribution.

The stochastic process for $k(t)$ is given by the following:

- A differential equation $\dot{k} = g(k)$, where $g(\bar{k}) = 0$ and $g(k) > 0$ for $k < \bar{k}$.
- Poisson jumps with arrival rate λ, that change the state from k to $\frac{1}{k}$.

The invariant probability measure for this process has support on $[\frac{1}{\bar{k}}, \bar{k}]$ and is the sum of an absolutely continuous measure with density u and a point mass on $\{\bar{k}\}$.

We now proceed to characterize this invariant probability measure.

It is convenient to treat the deterministic and stochastic part separately. Let $H(k,t) = \{x : k(t) = k \text{ given } k(0) = x\}$, assuming $k(t)$ satisfies the differential equation $\dot{k} = g(k)$. Fix $\frac{1}{\bar{k}} < k_0 < k_1 < \bar{k}$. Pick t small enough so that $\frac{1}{\bar{k}} < H(k_0,t) < k_1$ and $k_0 < H(k_1,t) < \bar{k}$. If there are not Poisson jumps, the change in the mass on the interval $[k_0, k_1]$ in a length of time t is given by

$$\phi(t, k_0, k_1) = \int_{H(k_0,t)}^{k_0} u(x)dx - \int_{H(k_1,t)}^{k_1} u(x)dx \tag{55}$$

Taking the derivative of this function with respect to t we obtain

$$\phi_1(t,k_0,k_1) = -H_2(k_0,t)u(H(k_0,t)) - H_2(k_1,t)u(H(k_1,t))$$

For small t, $H(k,t) \doteq k - g(k)t$. Hence

$$\phi_1(0,k_0,k_1) = g(k_0)u(k_0) - g(k_1)u(k_1)$$

This expression gives the time derivative of the mass contained in $[k_0,k_1]$ conditional on no Poisson jump in k. To obtain a similar expression for the density note that

$$D_t u(k_0)\mid_{no\,poisson\,jump} = \lim_{k_1 \searrow k_0} \frac{\phi_1(0,k_0,k_1)}{k_1-k_0} = -g'(k_0)u(k_0) - g(k_0)u'(k_0) \tag{56}$$

The net change per unit time in $u(k_0)$ due to the Poisson jumps only is given by

$$D_t u(k_0)|_{poisson\,jump\,only} = \lambda\left[u(\frac{1}{k_0}) - u(k_0)\right] \tag{57}$$

Combining equations (56) and (57) we obtain

$$D_t u(k) = -g'(k)u(k) - g(k)u'(k) + \lambda\left[u(\frac{1}{k}) - u(k)\right] \tag{58}$$

For all $k \in (\frac{1}{\bar{k}}, \bar{k})$ and since u is part of an invariant density, equating equation (58) to zero we obtain the differential equation for u

$$u'(k) = \frac{\lambda\left[u(\frac{1}{k}) - u(k)\right] - u(k)g'(k)}{g(k)} \tag{59}$$

We now need to take care of the endpoints. Let δ be the mass at $\bar{k}$. The inflow to point $\frac{1}{\bar{k}}$ is this mass δ at an arrival rate λ.

The outflow can be calculated as

$$\lim_{t\searrow 0}\frac{1}{t}\int_{\frac{1}{\bar{k}}}^{\frac{1}{\bar{k}}+g(\frac{1}{\bar{k}})t} u(x)dx = -u(\frac{1}{\bar{k}})g(\frac{1}{\bar{k}}).$$

This gives

$$D_t u(\frac{1}{\bar{k}}) = \lambda\delta - u(\frac{1}{\bar{k}})g(\frac{1}{\bar{k}}).$$

which equating to zero gives

$$u(\frac{1}{\bar{k}}) = \frac{\lambda\delta}{g(\frac{1}{\bar{k}})}. \tag{60}$$

Finally, the density u and the point mass at $\bar{k}$ must integrate to one. This provides the following equation

$$\int_{\frac{1}{\bar{k}}}^{\bar{k}} u(x)dx = 1-\delta \tag{61}$$

Equations (59), (60) and (61) can be used to solve uniquely for the density u and mass δ.

Computation.

Take a grid $\{k_n, k_{-n+1}, ..., k_{-1}, k_0, k_1, ..., k_{n-1}, k_n\}$, where $1 = k_0 < k_1 < ... < k_{n-1} < k_n = \overline{k}$ and $k_{-j} = \frac{1}{k_j}$ for all j.

1. Set $u(1) = 1$ and using equation (59) compute $u'(1)$.
2. Use linear approximation of u at $k_0 = 1$ to compute $u(k_{-1})$ and $u(k_1)$.
3. Recursive step: Having estimates for $u(k_{-j})$ and $u(k_j)$, use (59) to compute $u'(k_{-j})$ and $u'(k_j)$. Use these estimates to approximately linearly u at k_{-j} and k_j respectively to compute $u(k_{-j-1})$ and $u(k_{j+1})$.
4. Continue step (3) until $\overline{k}$ is reached. Denote by $\tilde{u}$ the density thus constructed.
5. Note that if we have started by setting $u(1) = \beta > 0$, the procedure would have given the density $\beta\tilde{u}$. Using this and equations (60) and (61) we obtain the following two linear equations to solve for β and δ

$$\begin{aligned} \beta\left(\int_{\frac{1}{\overline{k}}}^{\overline{k}} \tilde{u}(x)dx\right) &= 1 - \delta \\ \beta\tilde{u}\left(\frac{1}{k}\right) &= \frac{\lambda\delta}{g(\frac{1}{k})} \end{aligned}$$

Welfare Analysis.

One Sector Model

Let $V(k)$ denote the expected discounted utility when capital is k and the current regime is one of subsidy. Let $W(k)$ be the corresponding value when there is no subsidy. These functions satisfy the following equations

$$\rho V(k) = \frac{c_t(1)^{1-\sigma}}{1-\sigma} + V'(k)\dot{k} + \lambda(W(k) - V(k))$$

and

$$\rho W(k) = \frac{c_t(0)^{1-\sigma}}{1-\sigma} + W'(k)\dot{k} + \lambda(V(k) - W(k)).$$

It is immediate to show that $V(k) = vk^{1-\sigma}$ and $W(k) = wk^{1-\sigma}$ for some v and w that satisfy

$$\rho v = \frac{c(1)^{1-\sigma}}{1-\sigma} + g(1)(1-\sigma)v + \lambda(w - v) \tag{62}$$

and

$$\rho w = \frac{c(0)^{1-\sigma}}{1-\sigma} + g(0)(1-\sigma)w + \lambda(v - w). \tag{63}$$

Equations (62) and (63) can be easily solved for v and w.

Two sector model.

Let $V(k, x)$ denote the expected discounted utility of the representative agent if the initial state is $k_n = x$ and $k_s = kx$. For $k < k^*$, V satisfies the following functional equation

$$\rho V(k,x) = x^{1-\sigma}\frac{c(k,1)^{1-\sigma}}{1-\sigma} + V_1(k,x)\dot{k} - \delta V_2(k,x)x + \lambda\left[V(\frac{1}{k},kx) - V(k,x)\right].$$

It is easy to check that the solution to this equation must satisfy

$$V(k,x) = x^{1-\sigma}v(k)$$

and

$$\rho v(k) = \frac{c(k,1)^{1-\sigma}}{1-\sigma} + v'(k)\dot{k} - \delta(1-\sigma)v(k) + \lambda\left[k^{1-\sigma}v(\frac{1}{k}) - v(k)\right]. \quad (64)$$

For $k = k^*$ and letting g^* denote the growth rate at k^*, the following equation holds

$$\rho v(k^*) = \frac{c(k^*)^{1-\sigma}}{1-\sigma} + g^*(1-\sigma)v(k^*) + \lambda\left[k^{*1-\sigma}v(\frac{1}{k^*}) - v(k^*)\right] \quad (65)$$

Replacing $c(k,1)$ and $\dot{k}$ by the equilibrium decision rules, equations (64) and (65) provide an ODE and a boundary condition to solve for v.

This equation can be easily solved by numerical method. The following procedure was used. For any arbitrary value of $v(1)$, start at $k = 1$ and move in both directions (*e.g.* first use equation (64) to calculate $v'(1)$. Then set $v(1+\Delta) \doteq v(1) + v'(1)\Delta$ and $v(\frac{1}{1+\Delta}) = v(1) - v'(1)\frac{\Delta}{1+\Delta}$. Then use equation (64) again to obtain $v'(1+\Delta)$ and $v'(\frac{1}{1+\Delta})$ and continue in the same fashion). Once the endpoints are reached, subtract the right hand side of(65) from left hand side. This defines a mapping $F(v(1))$, the zero of which gives the right initial condition.

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Table 1: One Sector Model ($\tau = 0.10$)

ED[a]	g_0[b]	g_1	$\frac{g_0+g_1}{2}$	$E\left(\frac{T_t}{k_t}\right) \times 100$	W_0	W_1	$\overline{W}$
∞	2.00	2.89	2.44	0.644	-625	-658	-641
100	2.14	2.78	2.46	0.639	-630	-646	-638
50	2.20	2.72	2.46	0.636	-631	-642	-637
10	2.31	2.60	2.45	0.630	-633	-635	-634
5	2.33	2.57	2.45	0.629	-633	-634	-634
2	2.34	2.55	2.45	0.628	-633	-634	-633
1	2.35	2.55	2.45	0.627	-633	-633	-633
0.5	2.35	2.54	2.45	0.627	-633	-633	-633

[a] ED=$\frac{1}{\lambda}$

[b] In percentage terms. g_0 =growth rate in the no subsidy regime; g_1 =growth rate in the subsidy regime

Table 2: One Sector Model ($\tau = 0.20$)

ED[a]	g_0[b]	g_1	$\frac{g_0+g_1}{2}$	$E\left(\frac{T_t}{k_t}\right) \times 100$	W_0	W_1	$\overline{W}$
∞	2.00	4.00	3.00	1.400	-625	-833	-729
100	2.48	3.75	3.12	1.375	-657	-757	-707
50	2.62	3.62	3.12	1.362	-666	-726	-696
10	2.78	3.32	3.05	1.332	-672	-682	-677
5	2.81	3.25	3.03	1.325	-671	-675	-673
2	2.82	3.20	3.01	1.320	-670	-671	-671
1	2.83	3.19	3.01	1.319	-669	-670	-670
0.5	2.83	3.18	3.01	1.318	-669	-669	-669

[a] ED=$\frac{1}{\lambda}$

[b] In percentage terms. g_0 =growth rate in the no subsidy regime; g_1 =growth rate in the subsidy regime

Table 3: Two Sector Model ($\tau = 0.10$)

ED[a]	k^*	$\bar{k}$	$E\left(\frac{\dot{y}_t}{y_t}\right)^b$	$E\left(\frac{\dot{y}_t}{y_t}\mid switch\right)$	$E\left(\frac{T_t}{k_{1t}+k_{2t}}\right) \times 100$	EV [c]	% time at k^*
∞	1.23	1.23	2.44	3.20	0.686	-641	100.0
100	1.26	1.26	2.45	3.28	0.704	-643	98.2
50	1.29	1.28	2.46	3.35	0.722	-645	96.1
10	1.44	1.35	2.51	3.66	0.854	-657	77.6
5	1.55	1.36	2.56	3.73	0.964	-668	59.6
2	1.65	1.29	2.65	3.62	1.100	-673	35.3
1	1.65	1.23	2.71	3.50	1.170	-674	22.3
0.5	1.61	1.18	2.79	3.45	1.220	-677	13.9

[a] ED=$\frac{1}{\lambda}$
[b] In percentage terms.
[c] EV =expected discounted utility when $\frac{k_1}{k_2} = k$ and $k_1 + k_2 = 1$.

Table 4: Two Sector Model ($\tau = 0.20$)

ED[a]	k^*	$\bar{k}$	$E\left(\frac{\dot{y}_t}{y_t}\right)^b$	$E\left(\frac{\dot{y}_t}{y_t}\mid switch\right)$	$E\left(\frac{T_t}{k_{1t}+k_{2t}}\right) \times 100$	EV [c]	% time at k^*
∞	1.56	1.56	3.00	4.89	1.59	-713	100.0
100	1.62	1.60	3.01	5.03	1.65	-722	96.5
50	1.67	1.63	3.04	5.15	1.70	-732	92.8
10	1.98	1.71	3.19	5.52	2.06	-787	66.5
5	2.16	1.67	3.31	5.51	2.28	-822	47.0
2	2.31	1.57	3.42	5.38	2.51	-855	25.0
1	2.32	1.52	3.43	5.33	2.61	-866	14.6
0.5	2.28	1.48	3.39	5.30	2.67	-865	8.3

[a] ED=$\frac{1}{\lambda}$
[b] In percentage terms.
[c] EV =expected discounted utility when $\frac{k_1}{k_2} = k$ and $k_1 + k_2 = 1$.

Figure 1. Growth Rate Dynamics

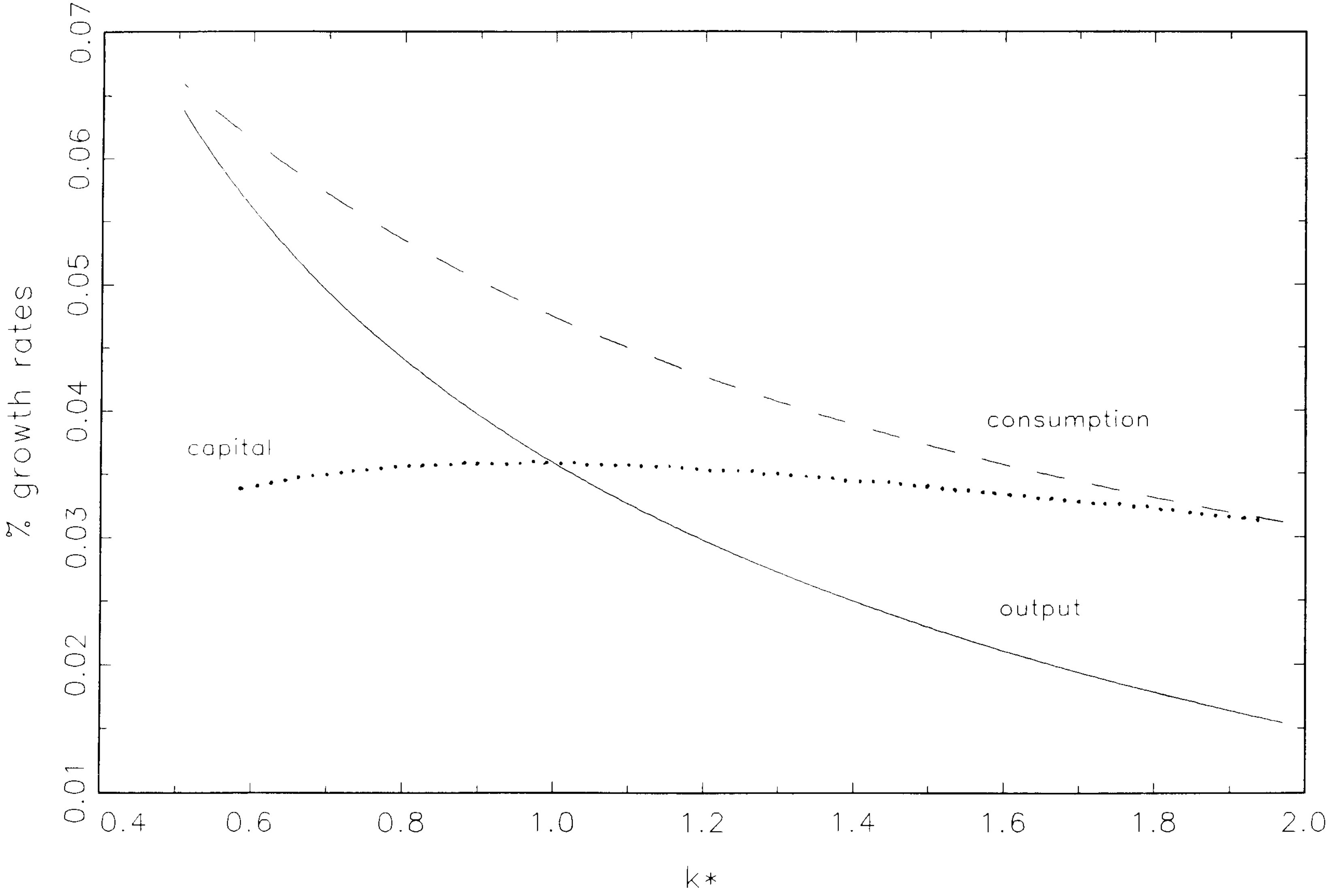


Figure 2. Price of Nonsubsidized Capital

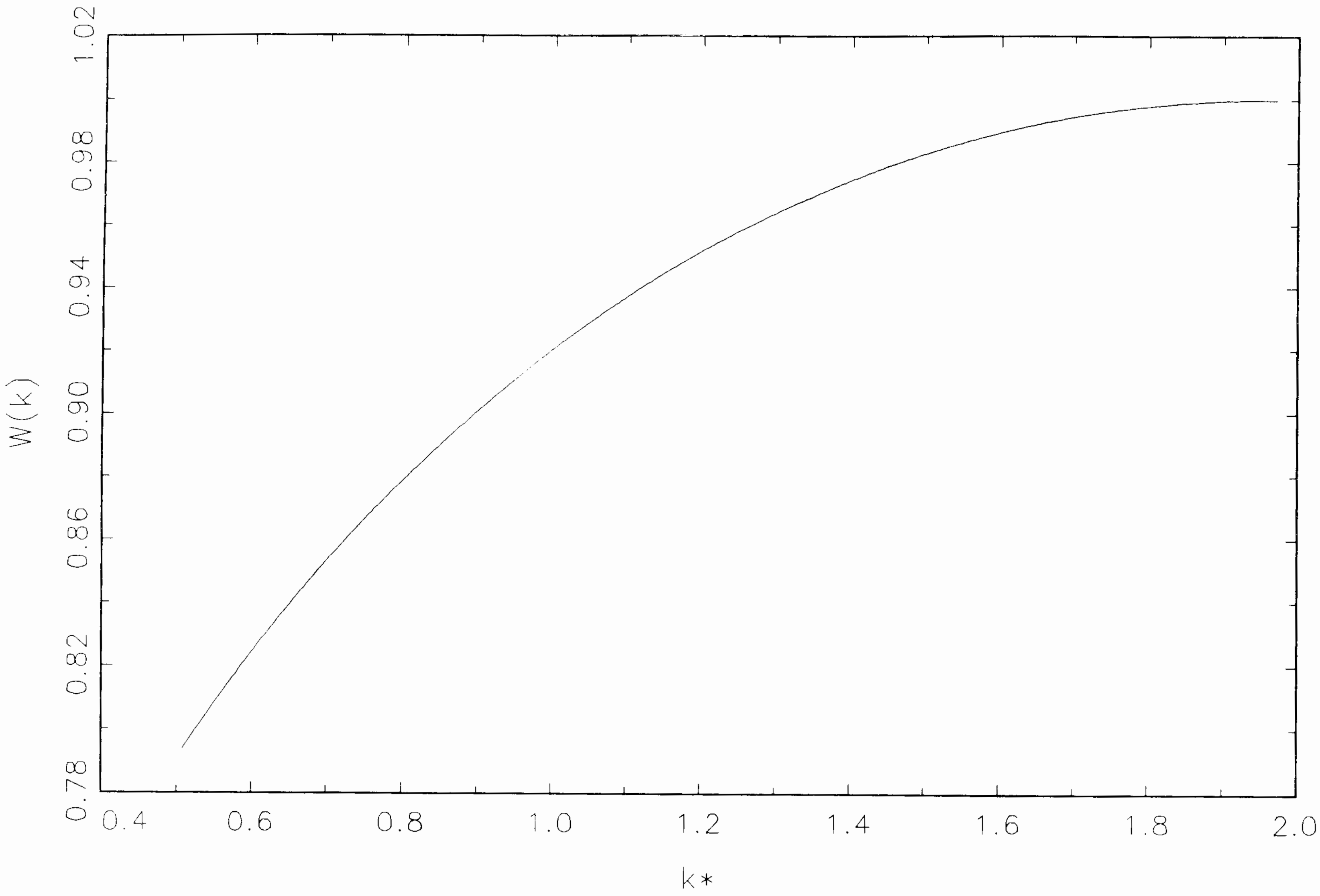


Figure 3.a. Distribution Functions

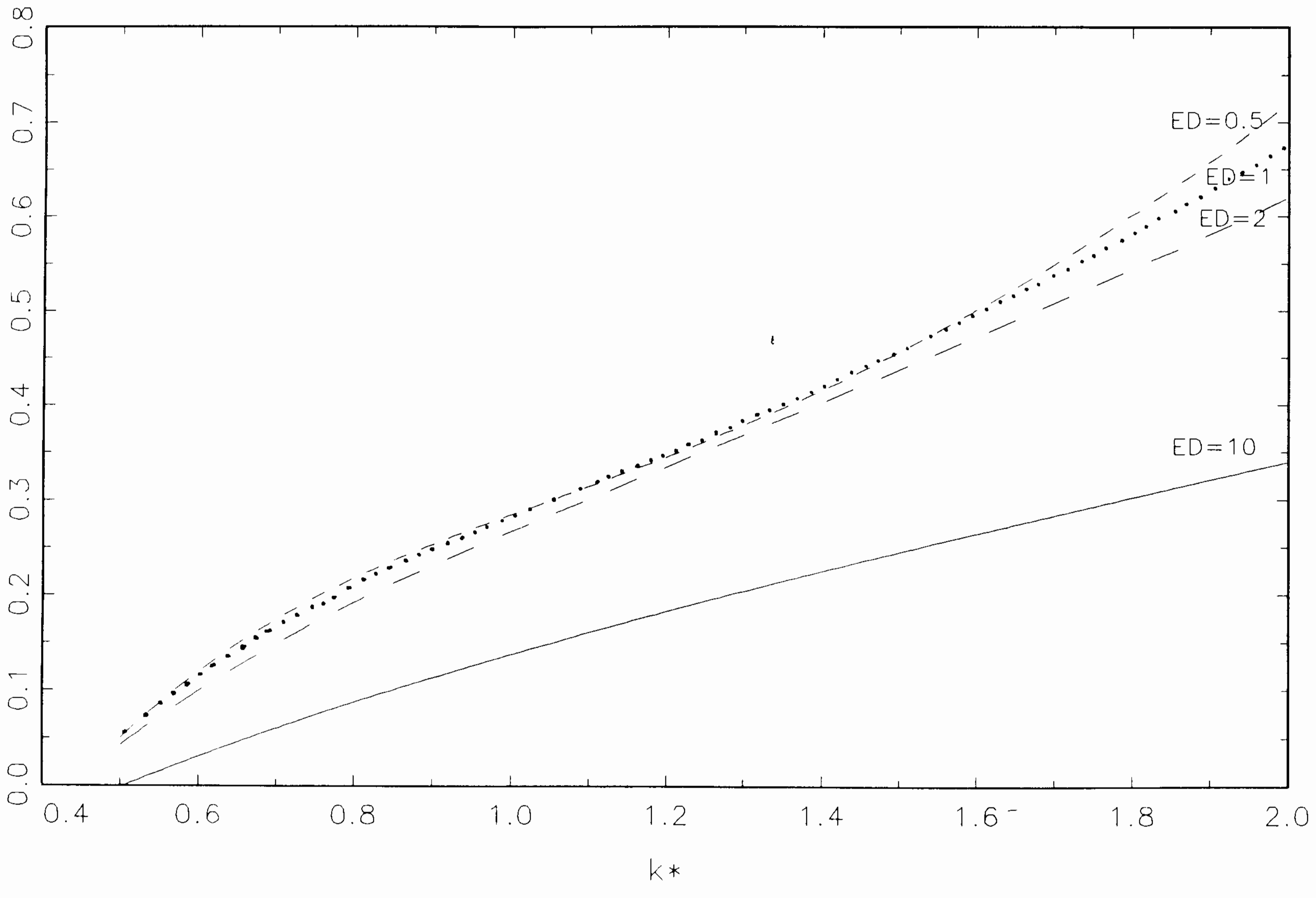


Figure 3.b. Density Functions

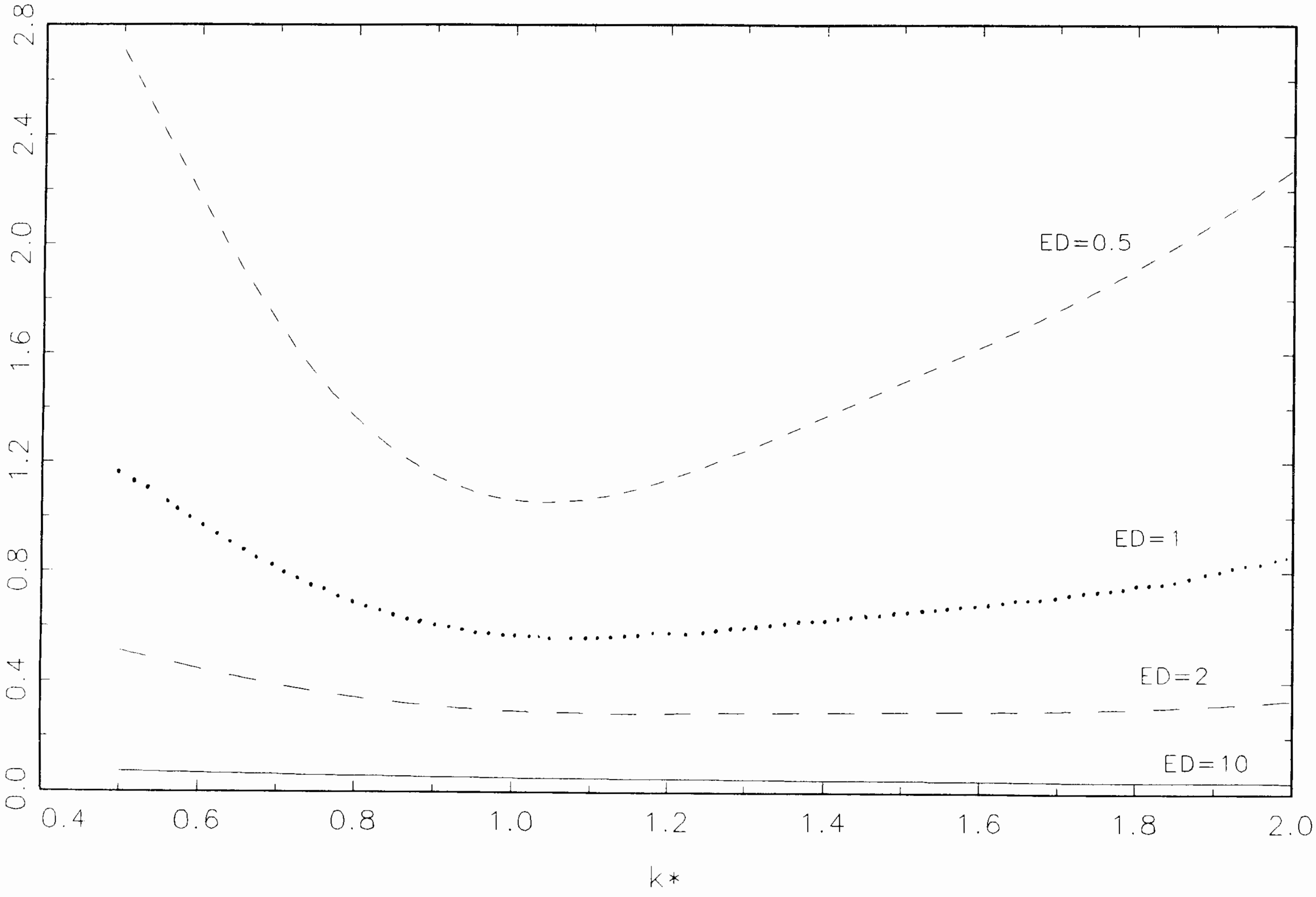


Figure 3.a. Distribution Functions

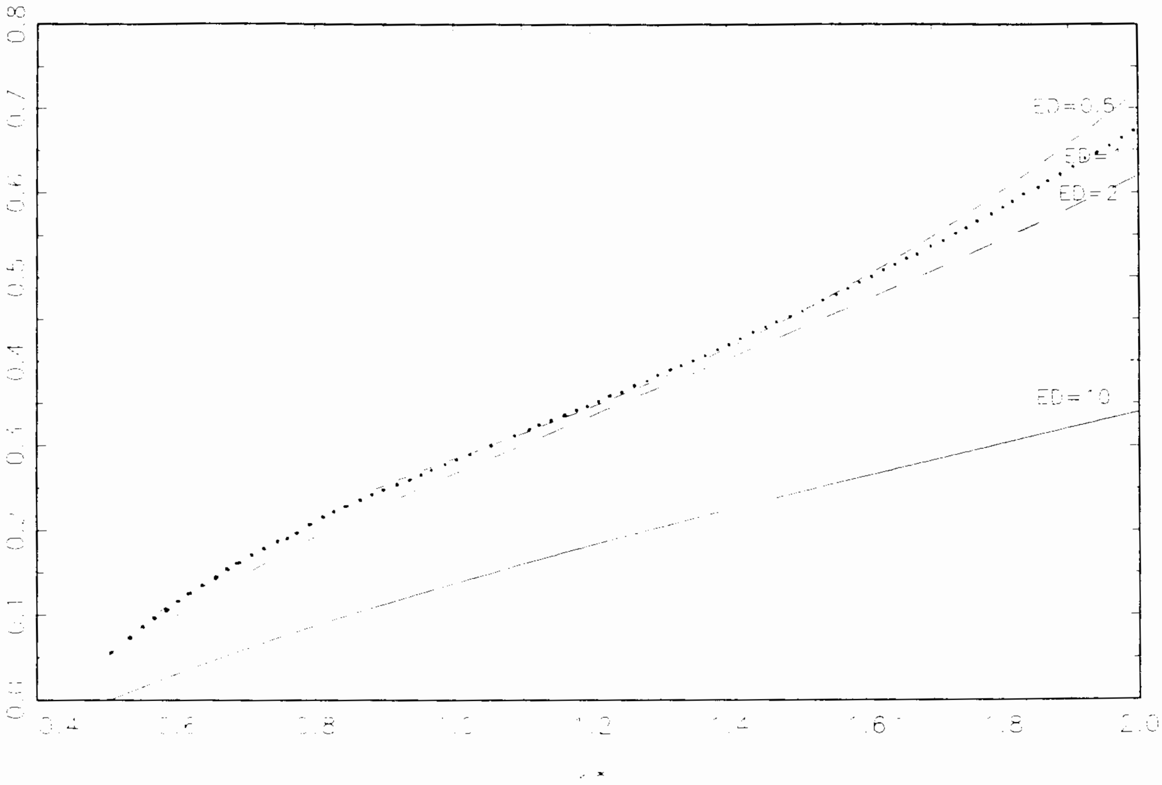
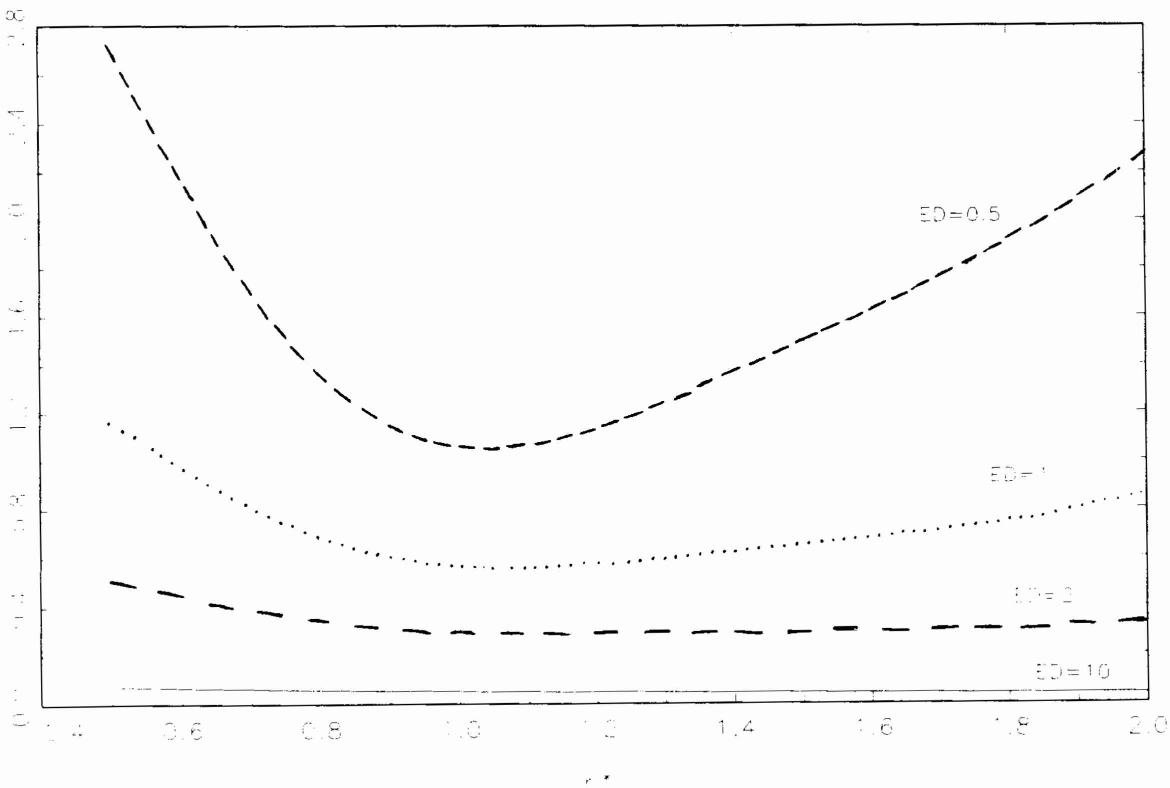


Figure 3.a. Density Functions



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