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## Political Election on Legal Retirement Age

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### Abstract

We use a lifecycle model in which individuals differ by age and by wage in order to analyze a pairwise majority voting process on the legal retirement age. We consider two different retirement regimes. In the first one the retirees do not return to the labor market, regardless the new retirement age. In the second one, they have to return if this

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age is higher than her own age. We show that the final outcome of the voting process will crucially depend on the retirement regime as well as on the parameters of the Social Security, that is, the redistributive character of the system and the present legal retirement age.

## 1 Introduction

Reforms of Social Security systems is now one of the main issues of most of industrialized countries' economic policy agenda. It is widely considered that, unless there are serious changes, the rise in the number of retirees relative to workers will threaten the viability of pay-as-you-go public pension systems in the long-run. With the aim of eliminating these future financing problems, the central reforms that are being proposed are raising taxes, cutting pension benefits and/or raising the age of retirement, see Blondal and Scarpetta (1998) or Gruber and Wise (1997).

In order to achieve this latter reform, the main economic policy measures are either to allow a greater flexibility in Social Security's retirement rules (e.g. Germany, Italy or Sweden) or to postpone the pensionable age.<sup>1</sup> In point of fact, this second measure is one of the policy conclusions of *Maintaining Prosperity in an Ageing Society*, OECD (1998): "...a direct way to encourage people to work longer would be to raise the pensionable age".

However, according to recent surveys, most of workers declare that they are happy with the current retirement age (see Cremer and Pestieau, 2003), which suggests that reforms on the legal retirement age are becoming a delicate matter for governments. In this paper we analyze this option, the reform of the pensionable age. We concentrate on this specific issue by modeling a pairwise majority voting process on the legal retirement age in a steady-state setting with given discount rates and where the rest of Social Security parameters are also given.

Earlier literature dealing with retirement in a political economy environment has mainly focussed only on the effects of Social Security systems on

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<sup>1</sup>In Switzerland, for instance, in 1998 there was a referendum on a single issue, in which the voters approved of a delay of two years in the female retirement age within the public pension from 62 to 64 (Bütler, 2002).

the *individual* retirement decision.<sup>2</sup> Our paper examines the *legal* retirement age, which allows us to emphasize the relevance of the indirect 'macro' effects of changing the pensionable age, that is, the effects on pension benefits via the dependency ratio.

The term 'legal retirement age' usually refers to the age at which benefits are available. However, since there are strong incentives to stop working after this standard entitlement age, in this model we consider the legal retirement age as the age at which workers have to leave the labor force, that is, as a mandatory retirement.<sup>3</sup> Indeed, the average retirement age in some OECD countries is very close to this standard retirement age (e.g. the United Kingdom, Portugal or Ireland); see Blondal and Scarpetta (1998).<sup>4</sup>

Given that public pension systems often redistribute not only from younger to older generations but also from high- to low-wage workers, we consider that individuals differ by age and by wage. In such a context we study the optimal legal retirement age of voters and obtain the elected one. We also show that this majority equilibrium retirement age depends crucially on the composition of the population as well as on the existing parameters of the pension system.

Since individuals differ not only according to wage but also according to age, population is divided into two completely separated groups: workers and retirees. In order to see the relevance of the effects of pension reforms on present retirees, we show how final results change depending on whether retired people have to go back to labor market after the election or not. So, we first assume that they will not return to the labor market, even although the elected retirement age were higher than her own age. This implies that they will behave as a homogeneous group and that, given that their pensions are positively related to the retirement age via dependency ratio, they will always prefer the highest one. We will also calculate the majority equilibrium

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<sup>2</sup>See for instance, Sheshinski (1978), Crawford and Lilien (1981), Kahn (1988), or Fabel (1994).

<sup>3</sup>In some countries there are direct restrictions on work above the standard age (Portugal or Spain make entitlements to pension benefits beyond the standard age conditional on complete withdrawal from work) or frequently, individuals have to leave their current jobs to receive their pensions; see Blondal and Scarpetta (1998) or Gruber and Wise (1999).

<sup>4</sup>If there is a possibility to have an early access to pension benefits with some adjustment in the value of retirement benefits, the average retirement age is usually found between this age at which pensions can be accessed and the standard retirement age; see Blondal and Scarpetta (1998) or Samwick (1998).

when retirees have to return to work if the elected retirement age is higher than her own age. Under this assumption retired people will no longer behave as a homogeneous group which will affect to the final outcome of the voting process.

The main findings of this study are the following. First, the older the worker, the more difficult for her to change her retirement age. The reason is the bigger weight of the wealth on the optimal retirement decisions as workers get older and the link between the wealth and the status quo retirement age. Second, workers with wages below average will delay their optimal legal retirement ages as the Social Security system is more and more redistributive. This result suggests that it may be appropriate to accompany the deferment of the legal retirement age with an increase in the redistribution level of pension system in order to ensure a bigger political support. Third, the great importance not only of retirees but also of retirement conditions in the final outcome of the voting process. This is highlighted by showing that the relation between the redistribution level of the system and the elected retirement age may be completely different depending on whether retirees do or do not return to labor market. At last, it is worth noting a change in the legal retirement age affect not only the worker's decision, but also the composition of the electorate, since it divides the population into workers and retirees. So, although we prove that the workers' preferred legal retirement ages are positively related to the current one, it cannot be ruled out that, under some circumstances, the macro effect of a lower current legal retirement age be large enough to overcompensate the micro one and, therefore, imply a higher elected one.

## 2 The model

Consider a constant population where individuals are continuously and uniformly distributed by age  $a$  with  $a \in [0, T]$ . They are also continuously distributed by wage, from a minimum to a maximum wage level,  $[w_m, w_M]$ . Individuals know with certainty that they live for exactly  $T$  years, believing that  $R^{sq}$  of which will be spent working and  $T - R^{sq}$  of which will be spent in retirement, being  $R^{sq}$  the status quo legal retirement age.

Individuals have a stationary and temporally independent utility func-

tion, separable and strictly increasing in consumption and leisure<sup>5</sup>. This instantaneous utility function is written as  $u(c_t) + v(l_t)$ , where  $u(c_t)$  is the utility from consumption of goods  $c_t$  at time  $t$  and  $v(l_t)$  is the utility from leisure  $l_t$ , at time  $t$ . We assume that  $u$  is strictly concave for all  $c_t$ . It is also assumed that the coefficient of relative risk aversion  $\rho_r(x) = -xu''(x)/u'(x)$  is non-increasing and smaller than one. The amount of hours labored while working cannot be varied, it is institutionally set. We can, therefore, assume that individuals have a fixed utility from leisure,  $v$ , independent of age. That is, the utility of leisure is  $v(l_t^w) = 0$  and  $v(l_t^R) = v$ , where  $l_t^w$  and  $l_t^R$  indicate hours of leisure while working and retired.

While working individuals earn a fixed gross wage per unit of time  $w \in [w_m, w_M]$  independent of age. While retired they receive a constant stream of pension benefits per unit of time  $p(R^{sq}; w)$ .

For the sake of simplicity, we assume that savings earn no interest and that individuals do not discount the future. Then, the lifetime utility of the individual of wage  $w$  can therefore be written as

$$\int_0^T U(c_t, l_t) dt = \int_0^{R^{sq}} u(c_t) dt + \int_{R^{sq}}^T [u(c_t) + v] dt, \quad (1)$$

and her lifetime budget constraint as

$$\int_0^T c_t dt = \int_0^{R^{sq}} w(1 - \tau) dt + \int_{R^{sq}}^T p(R^{sq}; w) dt. \quad (2)$$

Separability and concavity of the instantaneous utility function, certain lifetimes, and perfect capital markets imply that each individual entering the labor force, at age 0, will set a constant level of consumption in order to maximize (1) subject to (2)

$$c = \frac{1}{T} (R^{sq}w(1 - \tau) + (T - R^{sq})p(R^{sq}; w)). \quad (3)$$

The Social Security system is defined by a constant contribution rate  $\tau \in [0, 1]$  and by a constant intra-generational redistribution degree  $\alpha \in [0, 1]$ . It is based on the Pay-As-You-Go principle. That is, pensions of retirees are

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<sup>5</sup>Similar to Crawford and Lilien, (1981) or Sheshinski (1978).

financed by contributions of workers. Therefore, from the Social Security budget constraint, we obtain the pension benefits per unit of time of the individual with wage  $w$

$$p(R^{sq}; w) = \frac{R^{sq}}{T - R^{sq}} \tau [(1 - \alpha) \varpi + \alpha w], \quad (4)$$

where  $R^{sq}/(T - R^{sq})$  is the well-known dependency ratio and  $[(1 - \alpha) \varpi + \alpha w]$  a linear combination of the mean wage,  $\varpi$ , and the individual's wage,  $w$ . Thus, depending on the level of  $\alpha$ , the type of Social Security may range from a totally uniform pension benefits scheme ( $\alpha = 0$ ), usually referred as Beveridgean, to a type in which pension benefits are actuarially fair ( $\alpha = 1$ ), usually referred as Bismarckian.<sup>6</sup>

It is assumed that people face an unexpected voting process on the legal retirement age at an arbitrary moment of time  $t > 0$ . In order to avoid strategic behaviour, we assume that they cannot anticipate it and that the elected retirement age is believed by everybody to remain indefinitely valid.

At the moment of the voting process each individual can be characterized both by her age and by her wage. They will also have a different amount of accumulated wealth,  $\pi(a, w)$  which is given by the total income earned minus total consumption up to the instant at which the voting process takes place

$$\pi(a, w) = \begin{cases} a \left(1 - \frac{R^{sq}}{T}\right) (w(1 - \tau) - p(R^{sq}; w)) & a \leq R^{sq}; \\ R^{sq} \left(1 - \frac{a}{T}\right) (w(1 - \tau) - p(R^{sq}; w)) & a \geq R^{sq}. \end{cases} \quad (5)$$

This expression describes the pattern of wealth accumulation.

There exists a threshold wage  $\tilde{w}$  such that  $\tilde{w}(1 - \tau) = p(\tilde{w})$ . In particular

$$\tilde{w} = \frac{R^{sq} \tau (1 - \alpha) \varpi}{(1 - \tau) (T - R^{sq}) - R^{sq} \tau \alpha}. \quad (6)$$

For any  $w > \tilde{w}$ , wealth increases linearly with age up to the point in which  $a = R^{sq}$ . In other words, they save money for retirement. Beyond that point the agents start to spend their accumulated wealth until  $a = T$ , when wealth is zero. For any  $w < \tilde{w}$ , wealth decreases linearly from  $a = 0$  till  $a = R^{sq}$ .

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<sup>6</sup>See Casamatta et al. (2000) for a classification of several OECD countries depending on the redistribution character of the Social Security system.

That is, the agents accumulate debt in a linear way. In other words, they are borrowing against their pension wealth. From  $a = R^{sq}$  till  $a = T$ , wealth increases (or debt decreases). Again, when  $a = T$ , wealth is zero.<sup>7</sup>

### 3 Preferred Legal Retirement Ages

Let us now analyze the preferred legal retirement age of the different agents, the retirees, with  $a > R^{sq}$ , and the workers, with  $a \leq R^{sq}$ . Let  $R^*(a, w)$  be the optimal legal retirement age of an individual of age  $a$  and wage  $w$ .

#### 3.1 The retirees

The indirect utility function of a retiree is

$$U(c, a) = (T - a)(u(c) + v), \quad (7)$$

where the constant consumption is

$$c = \frac{1}{T - a} ((T - a)p(R; w) + \pi(a, w)), \quad (8)$$

being  $R$  the legal retirement age,  $p(R; w)$  her constant pension benefits per unit of time, and  $\pi(a, w)$  her accumulated wealth.<sup>8</sup>

Given that the dependency ratio is increasing in  $R$ , pension benefits will be positively related to the retirement age, and since retirees do not return to the labor market, their utility functions will also be increasing in  $R$ .

Nevertheless, they will be indifferent between retirement ages within this interval,  $R \in [R^{sq} + (T - a), T]$ . The reason is the following. Since retirees will not come back to the labor force, an increase in the legal retirement age will improve their pension benefits year by year as workers keep working up to the new legal retirement age. So, retirees will be indifferent between the retirement ages that have the same effect on their pension benefits, a continuous increase until  $t = T$ .<sup>9</sup>

<sup>7</sup>So, if  $w < \tilde{w}$  for any  $w \in [w_m, w_M]$ , then the accumulated wealth is negative for all values of  $a \in (0, T)$ .

<sup>8</sup>The formula of pension benefits of retirees is very extensive and not necessary for the analysis. The only important thing is that it depends positively on the retirement age.

<sup>9</sup>For example, if  $R^{sq} = 60$  and  $T = 80$ , a retiree with age  $a = 70$  will be indifferent

### 3.2 The workers

The indirect utility function of a worker is given by

$$U(R; a, w) = \begin{cases} (T - a)(u(c) + v) & R \leq a; \\ (T - a)u(c) + (T - R)v & R \geq a. \end{cases} \quad (9)$$

It is easy to check that (9) is continuous for all  $R$ . The constant consumption per unit of time of a worker is

$$c = \begin{cases} \frac{1}{T-a} ((T - a)p(R; w) + \pi(a, w)) & R \leq a; \\ \frac{1}{T-a} ((R - a)w(1 - \tau) + (T - R)p(R; w) + \pi(a, w)) & R \geq a; \end{cases} \quad (10)$$

and the constant pension benefits per unit of time is given by

$$p(R; w) = \frac{R}{T - R} \tau [(1 - \alpha) \varpi + \alpha w], \quad (11)$$

being  $\frac{R}{T-R}$  the new dependency ratio derived from the retirement age.

From (9), (10) and (11) the following proposition can be stated.

**Proposition 1** *Let  $a \leq R^{sq}$ . The utility function  $U(R; a, w)$  is single-peaked in  $R$ . Moreover,  $R^*(a, w) \geq a$ .*

**Proof.** See Appendix.

The proposition tells us first, that any worker has an optimal legal retirement age, and secondly, that a worker never prefers a legal retirement age lower than her own age. In other words, voting for  $R^*(a, w) < a$  is always worse than voting for  $R^*(a, w) = a$ , since both have the same effect on leisure (the worker would be retired immediately after the election day in both cases), but the first case implies lower pension benefits, due to the dependency ratio effect on pensions.

The effect of the age and of the wage on the preferred legal retirement age of a worker is characterized in the next proposition.

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with reference to retirement ages comprised between (70, 80). If the new retirement were  $R = 70$ , pension benefits would reach the stability after 10 years. This implies that pension benefits would be increasing until this retiree were 80 years old,  $a = 80$ . But pension benefits would also be increasing until  $a = 80$  if  $R > 70$ .



**Proposition 2** Let  $R^*(a, w) > a$ . The preferred legal retirement ages have the following properties:

- i)  $\frac{\partial R^*(a, w)}{\partial a} > (<)0$  if  $R^{sq} > (<)R^*(a, w)$ ;
- ii)  $\frac{\partial R^*(a, w)}{\partial w} > 0$ .

**Proof.** From the F.O.C. of the maximization problem of the utility function (9), using the implicit function theorem and after some simplifications we obtain respectively:

i)

$$\frac{\partial R^*(a, w)}{\partial a} = \frac{R^{sq} - R^*(a, w)}{T - a}. \quad (12)$$

So, if  $R^{sq} > (<)R^*(a, w)$ , then  $\partial R^*(a, w) / \partial a > (<)0$ .

ii)

$$\frac{\partial R^*(a, w)}{\partial w} = - \frac{[1 - \tau(1 - \alpha)] [u'(c)(1 - \rho_r)]}{(w(1 - \tau) + \tau W)^2 u''(c) \frac{1}{T - a}}. \quad (13)$$

This equation is strictly positive since the elasticity of marginal utility  $\rho_r$  is less than one. Q.E.D.

The first point of proposition states that the older the worker, the closer her optimal retirement age to that of the status quo. In other words, workers with the same wage level have their optimal legal retirement ages monotonically ordered with respect to age toward the status quo retirement age,  $R^{sq}$ .<sup>10</sup> The underlying economic intuition is as follows. Workers had planned their consumptions regarding the previous retirement age,  $R^{sq}$ , and therefore, their accumulated wealths will be closely related to the status quo situation. Besides, since the absolute value of the accumulated wealth is strictly increasing up to  $R^{sq}$ , the weight of  $\pi(a, w)$  in the workers' retirement decision raises with the age. Consequently, the effect of the accumulated wealth, acting like a magnet towards  $R^{sq}$ , implies that increases in the age takes  $R^*(a, w)$  closer to  $R^{sq}$ .

The second point says that optimal retirement ages are increasing with wage. This result arises because the negative substitution effect on leisure of a higher wage outweighs the positive income effect.<sup>11</sup>

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<sup>10</sup>It has to be noticed that if a worker of wage  $w$  has her  $R^*(a, w)$  lower than  $R^{sq}$ , then, any other worker with the same wage but different age,  $\hat{a}$ , cannot have her  $R^*(\hat{a}, w)$  higher than  $R^{sq}$ .

<sup>11</sup>This result is similar to that obtained in the analysis of optimal *individual* retirement decision of previous literature.

It should be stressed that this proposition refers only to the cases in which  $R^*(a, w) > a$ . If  $R^*(a, w) = a$  it would be possible that changes in wage does not alter  $R^*(a, w)$ . The reason is the following. The utility function (9) is continuous in all  $R$  but it is not derivable in  $R = a$ , so, there may be individuals of the same age with the same optimal legal retirement age, her own age, in spite of their different wages. This case,  $R^*(a, w) = a$ , will be more likely when workers are older. In other words, it is more probably that old workers have the same optimal legal retirement age.<sup>12</sup>

## 4 Majority voting process

Let us now explain the pairwise majority voting process on the legal retirement age.<sup>13</sup> Since retirees do not return to the labor market, they vote for the highest retirement proposal in order to improve their pension benefits via dependency ratio.

In spite of some retirees are indifferent between the two retirement proposals, we assume that they will always choose the highest one. Our reason is the following. Retired people usually behave as an homogeneous group, and since there will always be some retirees who will prefer the highest proposal, those ones with ages slightly higher than  $R^{sq}$ , the rest of retirees will support them. It is also considered that retired people are less than fifty per cent of the population, which is the usual case in most of industrialized countries, thus, decisive voters will belong to the working group.

With respect to the workers' choice, let us begin with individuals who have just entered to the labor market, that is, workers of age  $a = 0$ . Since  $\pi(0, w) = 0$ , their optimal legal retirement ages are independent of the previous status quo one. Besides, given that  $R^*(0, w) > 0$  for any wage, these optimal ages will be continuous and monotonically increasing distributed, from  $R^*(0, w_m)$  to  $R^*(0, w_M)$ .<sup>14</sup>

There exists a wage  $\hat{w}$  such that  $R^*(0, \hat{w}) = R^{sq}$ .<sup>15</sup> From now on, we

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<sup>12</sup>Needless to say, for a same wage level, if  $R^*(a, w) = a$ , increases in  $a$  leads to increases in the optimal retirement age,  $R^*(a, w)$ , since  $R^*(a, w) \geq a$  for any age.

<sup>13</sup>In such a process,  $R^{sq}$  does not have to be one of the two alternatives.

<sup>14</sup>We assume a constant value of the parameter  $v$  such that  $R^*(0, w_m) > 0$ .

<sup>15</sup>Given that  $\frac{\partial R^*(a, w)}{\partial a} = \frac{R^{sq} - R^*(a, w)}{T - a}$ , it can be derived that  $R^*(a, \hat{w}) = R^{sq}$  for any age  $a$ .

consider that  $\hat{w} \in [w_m, w_M]$ . This implies that  $R^*(a, w) < (>)R^{sq}$  for any wage  $w < (>)\hat{w}$ , regardless the age.<sup>16</sup> Optimal legal retirement ages of working population are represented in figure 1.

As we can observe, differences between optimal retirement ages due to the wage are smaller and smaller as workers gets older.

Summing up, since the utility function (9) is single-peaked with respect to  $R$ , the majority voting process leads to a Condorcet winner legal retirement age,  $R^e$ , that divides the society into two groups of equal size. An illustrative example can be observed in figure 2.

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<sup>16</sup>If  $\hat{w}$  were higher (lower) than any  $w \in [w_m, w_M]$ , then all working population will prefer a legal retirement age lower (higher) than the status quo one.

Decisive voters are those with their  $R^*(a, w) = R^e$ . They form a continuum of individuals and differ in terms of age. If  $R^e \geq R^{sq}$ , they also differ in terms of wage.

If  $R^e < R^{sq}$ , all workers from a given age, poor and rich old ones, would prefer a retirement age higher than  $R^e$ . On the contrary, in this case, poor young workers will always prefer a greater advance in  $R^e$ . Needless to say,  $R^e < R^{sq}$  implies that  $\hat{w} > w_{med}$ , being  $w_{med}$  the median wage.

If  $\hat{w} \leq w_{med}$ , the legal retirement age will be delayed,  $R^e > R^{sq}$ . We should note that although  $\hat{w} = w_{med}$ , that is, although 50% of working population prefers a retirement age lower than  $R^{sq}$ , the elected one would be higher due to the presence of retirees.<sup>17</sup>

Only if  $R^e = R^{sq}$ , decisive voters would have the same wage,  $\hat{w}$ , and they would only differ in age. In this case, those who prefer a legal retirement age lower than  $R^{sq}$ , all workers with wages below  $\hat{w}$ , constitute half of the population.<sup>18</sup>

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<sup>17</sup>In a situation with a great number of retirees, it is possible that the delay in the retirement age were so high, that the coalition in favor of a retirement age even higher than  $R^e$  were composed by young rich workers and retirees. In such a case, old rich workers, together with all poor workers, would prefer a smaller postponement.

<sup>18</sup>The other half of population will prefer a retirement age higher than  $R^{sq}$ , and it will be formed by those workers with wages above  $\hat{w}$  and the retirees.

## 5 Comparative Statics

To see how Social Security parameters affect to the majority equilibrium retirement age, let us now analyze how the decision of the working population changes by altering the redistribution level of the system, determined by  $\alpha$  and  $\tau$ , and the status quo retirement age  $R^{sq}$ .

### 5.1 The Redistributive Character of the Pension System

The next proposition states the effect of  $\alpha$  and  $\tau$  on the preferred legal retirement age of workers.

**Proposition 3** *Let  $R^*(a, w) > a$ . Consider a worker with a wage level  $w < \varpi$  ( $w > \varpi$ ). The more redistributive the Social Security system, the higher (lower) her preferred retirement age.*

**Proof.** From F.O.C. of maximization problem (12), the implicit function theorem and after some simplifications, we obtain

$$\frac{\partial R^*(a, w)}{\partial \alpha} = - \frac{[u'(c)(1 - \rho_r(c))]\tau(w - \varpi)}{(w(1 - \tau) + \tau W)^2 u''(c) \frac{1}{T-a}} \quad (14)$$

and

$$\frac{\partial R^*(a, w)}{\partial \tau} = - \frac{[u'(c)(1 - \rho_r(c))](1 - \alpha)(\varpi - w)}{(w(1 - \tau) + \tau W)^2 u''(c) \frac{1}{T-a}}. \quad (15)$$

Since  $\rho_r(c) < 1$ , if  $w < \varpi$  ( $w > \varpi$ ) then  $\partial R^*(a, w)/\partial \alpha < 0$  ( $> 0$ ) and  $\partial R^*(a, w)/\partial \tau > 0$  ( $< 0$ ). Q.E.D.

An increase in the redistribution level of the pension system causes a positive effect on the optimal decision of workers with  $w < \varpi$ , regardless the redistributive parameter.<sup>19</sup>

A more redistributive system implies that public pension is more attractive for low-wage workers. So, they prefer to delay the legal retirement age

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<sup>19</sup>Since the two discount factors are equal to zero and there is no borrowing constraints, both redistributive parameters,  $\alpha$  and  $\tau$ , cause the same effect on the preferred legal retirement age of workers.

in order to increase the size of the Social Security system and to reduce their private savings.

From this result we can deduce the following. If in a pension system with a determined redistribution level, the half of population with optimal retirement ages lower than  $R^e$  in the voting process had all wages lower than the mean one, then, for the same wage distribution and with the same status quo retirement age  $R^{sq}$ , a more redistributive pension system would lead to a higher  $R^e$ .

This result contrasts with that obtained in models in which the pension system allows for flexible retirement. In those cases, when the retirement decision is analyzed, it is found that a more redistributive system reduces optimal *individual* retirement ages. It is considered, first, that the pension system imposes a implicit tax on postponing retirement and secondly, that this implicit tax is higher, the more redistributive is the system. (See Casamatta et al, 2002)

Therefore, reforms of public pension systems aiming to raise the age of retirement by increasing the flexibility in the retirement decision should be implemented together with increases in the actuarial fairness of the system. In this way, disincentives to work would be lower and therefore retirement decisions of workers would be delayed.<sup>20</sup>

But, when the pension system reform is a postponement of the standard age of entitlement (as in New Zealand, Japan or Italy; see Blondal and Scarpetta, 1998), our model suggests that, in order to increase the political support, the reform should be accompanied by increases in the redistributive character of the system, since it would reduce the rejection of the majority of workers, those with wages lower than the mean one, by improving their pension benefits.

## 5.2 The Status Quo Retirement Age

Let us now analyze the effect of  $R^{sq}$  on the retirement decision of workers.

**Proposition 4** *Let  $R^*(a, w) > a$ . The higher the previous retirement age, the higher the preferred one.*

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<sup>20</sup>A more actuarially fair pension system implies a closer relation between lifetime contributions and pension benefits.

**Proof.** We need to find out the sign of  $\partial R^*(a, w) / \partial R^{sq}$ . From F.O.C. of maximization problem (12), the implicit function theorem and after some simplifications, we obtain

$$\frac{\partial R^*(a, w)}{\partial R^{sq}} = \frac{a(T - a)}{T} \quad (16)$$

Needless to say  $\partial R^*(a, w) / \partial R^{sq} > 0$ . Q.E.D.

A higher status quo retirement age implies a greater constant consumption per unit of time previous to the voting process, and therefore a smaller accumulated wealth for all workers with  $a > 0$ .<sup>21</sup> Consequently, the negative income effect on the leisure leads to a delay in the preferred legal retirement age.

In spite of this result, it is not obvious that, given two identical pension systems differentiated only by  $R^{sq}$ , the higher elected retirement age be associated with the higher status quo one. The reason is the different number of retirees between the two economies. The influence of  $R^{sq}$  on the elected one  $R^e$  is stated in the following proposition.

**Proposition 5** *Let  $\tau_1 = \tau_2$ ,  $\alpha_1 = \alpha_2$  and  $R_1^{sq} < R_2^{sq}$ .*

- i) If  $R_1^e \leq R_1^{sq}$  then  $R_1^e < R_2^e$ .*
- ii) If  $R_1^e > R_1^{sq}$ , and  $R_2^*(a, w) > R_1^e$  for any age  $a \in [R_1^{sq}, R_2^{sq}]$  and for any wage  $w \in [w_m, w_M]$ , then  $R_1^e < R_2^e$ .*

Proof: See Appendix.

The first point of the proposition says that when the voting process does not postpone the retirement age in the pension system with the lower  $R^{sq}$ , this pension system will be related to a lower  $R^e$ . The reason is that in both systems individuals with ages between  $R_1^{sq}$  and  $R_2^{sq}$  prefer a legal retirement age higher than  $R_1^e$  disregarding they are workers or retirees. So, the difference between the two voting outcomes arises due to the higher optimal retirement ages of most of workers with ages from 0 to  $R_1^{sq}$  in the economy with the higher status quo retirement age.<sup>22</sup>

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<sup>21</sup>Either a smaller positive or a bigger negative accumulated wealth.

<sup>22</sup>We cannot guarantee that all workers from 0 to  $R_1^{sq}$  will have higher optimal retirement ages in the economy 2 since the utility function is not derivable at  $R = a$ . As before, we cannot rule out the possibility of workers with  $R^*(a, w) = a$  in both economies.

The importance of retirees in the voting process is highlighted in the second point of the proposition. If  $R_1^e > R_1^{sq}$ , we can only guarantee that  $R_2^e > R_1^e$  under the conditions named in the proposition. In spite of most of working population have a higher preferred legal retirement age in the economy 2, it would be possible that the elected one were higher in the economy 1. The reason is the different behaviour of those who change her labor situation depending of the pension system they belong.

If  $R_1^e$  were even higher than  $R_2^{sq}$ , then individuals with ages between  $R_1^{sq}$  and  $R_2^{sq}$  and wages between  $w_m$  and  $\hat{w}_2$  would change her mind relative to  $R_1^e$  depending on they were workers or retirees. If they were in the economy 2, they would be workers and consequently they would be in favor of a legal retirement lower than  $R_1^e$ , but if they were in economy 1, they would be retired and therefore they would prefer a legal retirement age higher than  $R_1^e$ .

The bigger percentage of retirees in the voting process in economy 1 prevents us to exclude the possibility that the economy with the lower status quo retirement age be related to the higher elected one. This highlights the crucial role of  $R^{sq}$  in the society. It does not only determine, at micro level, the working period of an individual, but, at macro level, it also divides population into workers and retirees, which may be decisive in most of public choice issues. Given that retirees usually behave as a homogeneous group, they will have a bigger or a smaller political power depending on her number.

## 6 Extension: Retirees Can Return to the Labor Market

So far, we had assumed that retirees did not return to the labor market. Let us now analyze the implications that arise when they have to return to work if the elected retirement age is higher than her own age. With this regime the problem of the retirees is drastically changed.

The indirect utility function of a retiree is now given by (9), and the constant consumption per unit of time by (10). The accumulated wealth is now

$$\pi(a, w) = R^{sq} \left(1 - \frac{a}{T}\right) (w(1 - \tau) - p(R^{sq}; w)). \quad (17)$$

Substituting (17) in (10) the following proposition is stated.



**Proposition 6** *Let  $a > R^{sq}$ . i) The utility function  $U(R; a, w)$  is single-peaked in  $R$ . Moreover,  $R^*(a, w) \geq a$ .*

*ii) If  $R^*(a, w) > a$  then  $\partial R^*(a, w) / \partial w > 0$ .*

*iii) Let  $R^*(a, w) > a$ . If  $w(1 - \tau) > (<)p(R; w)$ , then  $\partial R^*(a, w) / \partial a > (<)0$ .*

**Proof.** See Appendix.

If retirees may go back to labor market, they will no longer prefer a retirement age as high as possible. Now, her preferred age will depend on her age and on her wage. In order to obtain higher pension benefits, her preferred retirement age will be, at least, as high as her own age.<sup>23</sup>

The second and the third point of the proposition characterize the behaviour of retirees in the cases in which they want to return to work. As in the working population case, optimal legal retirement ages are increasing in wage. Again the positive substitution effect of a higher wage on the retirement decision outweighs the negative income effect.

The proposition also tells us that, if the net wage is bigger (smaller) than pension benefits, then, the older the retiree, the higher (lower) her optimal retirement age, with the aim of reduce the period with the lower income per unit of time.

On the other hand, it is easy to check that a recent retiree has the same optimal retirement age as a worker of age  $R^{sq}$ .<sup>24</sup> In other words, there is a continuous distribution of optimal retirement ages from  $a = 0$  to  $a = T$ . This can be observed in figure 3, where an example with  $w_M(1 - \tau) > p(R; w_M)$  is showed.

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<sup>23</sup>Since retirees are older than workers, it will be more likely to find retired people with optimal retirement ages equal to her own age.

<sup>24</sup>Since their accumulated wealths are equal.

In order to compare the majority equilibrium under each retirement regime, let  $R_h^e$  be the elected retirement age when retirees behave as an homogeneous group, the former case, and  $R_b^e$  when retirees have to go back to work.

**Proposition 7** *i) If  $R_h^e \leq R^{sq}$ , then  $R_h^e = R_b^e$ . If  $R_h^e > R^{sq}$  then  $R_h^e > R_b^e$ .*

*ii) If  $R_h^e > R^*(a, \varpi)$  for any  $a \in [0, R^{sq}]$ ,  $R^*(a, \varpi) > R_b^e$  for any  $a \in [0, R^{sq}]$ ,  $R^*(R^{sq}, \varpi) = R_b^e$  and  $\varpi(1 - \tau) = p(R; \varpi)$ , then, a higher intra-generational redistribution level, lower  $\alpha$ , will imply a lower  $R_h^e$  but a higher  $R_b^e$ .*

**Proof.** See Appendix.

Since retirees are older than  $R^{sq}$ , they will never prefer a retirement age lower than  $R^{sq}$ . So, if the elected retirement age is lower than the status quo one, the situation of retirees will not affect the voting outcome since in both regimes they will prefer a legal retirement age higher than the elected one.

When the elected retirement age is higher than the previous one in the two situations, since there will always be retirees in the back-to-work scheme with optimal ages below the elected one,  $R_h^e$  will always be higher than  $R_b^e$ .

The second part of the proposition shows the importance of retirement conditions on the final outcome of the voting process. It tells us that, under some circumstances and if retirees may go back to work, a more redistributive

pension system will imply a higher elected retirement age. On the other hand, under the same circumstances, if retirees did not return to the labor market, the more redistributive system would be related to a lower retirement age. In other words, depending how pension reforms affect to present retirees, the political support behind a delay in the retirement age could be completely different.

At last, and with reference to the influence of  $R^{sq}$  on the elected one  $R_b^e$ , we should note that it cannot be ruled out either the possibility that, due to the different composition of retirees and workers in the population, an economy with a lower  $R^{sq}$  may lead to a higher elected retirement age. Although retirees may go back to work, there will again be voters that will change her mind relative to the elected retirement age depending on they are workers or retirees.

## 7 Concluding Remarks

In this paper we have employed a lifecycle model to study a majority voting process on the legal retirement age. One result which emerges from our analysis is that as workers get older, their preferred legal retirement ages get closer to the present one, which suggests that a reform changing the retirement age might come up against the opposition of the older workers.

It has also been suggested that in order to obtain a bigger political support, it would be better to combine different reforms. If the proposed reform is to delay the pensionable age, it would be appropriate to associate it with an increase in the redistributive character of the pension system, since it would delay preferred legal retirement ages of the majority of workers by improving their pension benefits. This result suggests that it will not always be useful to strength the link between life-time contributions and pension benefits, one of the main measures that is being proposed to encourage people to work longer. It should only be applied together with reforms aimed to increase the flexibility of the pension system's retirement rules.

Finally, we want to emphasize another issue that runs through our results. The crucial importance in the final outcome of any voting process of retirees and, consequently, of the legal retirement age. Since this age divides population into workers and retirees, and given that these latter usually behave as an homogeneous group, political parties should take very into account

the indirect consequences on any public choice issue of changing the legal retirement age.

## 8 Appendix

### 8.0.1 Proposition 1

We can rewrite the indirect utility function (9) as

$$U(R; a, w) = \begin{cases} (T - a) \left( u \left( \frac{R}{T-R} \tau W + \frac{1}{T-a} \pi(a, w) \right) + v \right) & R \leq a \\ (T - a) u \left( \frac{1}{T-a} ((R - a) w (1 - \tau) + R \tau W + \pi(a, w)) \right) + (T - R) v & R \geq a \end{cases} \quad (18)$$

where  $W = (1 - \alpha) \varpi + \alpha w$ . The first and second derivatives of the utility function of an individual of age  $a$  and wage  $w$  are:

$$\frac{\partial U(\cdot)}{\partial R} = \begin{cases} (T - a) u'(c) \tau W \frac{T}{(T-R)^2} & R \leq a \\ u'(c) (w(1 - \tau) + \tau((1 - \alpha) \varpi + \alpha w)) - v & R \geq a \end{cases} \quad (19)$$

$$\frac{\partial^2 U(\cdot)}{\partial R^2} = \begin{cases} (T - a) \tau W \left( u'(c) \frac{2T}{(T-R)^3} + u''(c) \tau W \frac{T^2}{(T-R)^4} \right) & R \leq a \\ \frac{1}{T-a} u''(c) (w(1 - \tau) + \tau((1 - \alpha) \varpi + \alpha w))^2 & R \geq a \end{cases} \quad (20)$$

From (19) and (20) we obtain that the function is strictly increasing with respect to the retirement age for  $R \leq a$ , and it is strictly concave for  $R \geq a$ . Therefore, preferences are single-peaked on retirement age and  $R^*(a, w) \geq a$  or to the right of his age,  $R^*(a, w) > a$ . Q.E.D.

### 8.0.2 Proposition 5

The following relations are used for proving the proposition:  $\partial R^*(a, w) / \partial w > 0$ ,  $\partial R^*(a, w) / \partial R^{sq} > 0$  and  $\partial R^*(a, w) / \partial a > (<)0$  if  $R^* < (>)R^{sq}$  for any  $R^*(a, w) > a$ .

i) We have to distinguish three situations:

- If  $R_1^e = R_1^{sq}$ . Then,  $R_1^*(a, \hat{w}) = R_1^e$  for any age  $a \in [0, R_1^e]$  (recall that  $\hat{w}$  is the wage such that  $R^*(0, \hat{w}) = R^{sq}$ ) and there exists an age  $\hat{a}$ ,  $0 < \hat{a} < R_1^e$  such that  $R_2^*(\hat{a}, \hat{w}) > R_1^e$ ; there also exist two wages,  $\check{w}$  and  $\tilde{w}$ , with  $\check{w} < \tilde{w} < \hat{w}$ , such that  $R_2^*(\hat{a}, \check{w}) = R_1^e$  and  $R_2^*(\hat{a}, \tilde{w}) > R_1^e$ . Since  $R_2^*(\hat{a}, \tilde{w}) > R_1^e$ , we can find an age  $\hat{b}$ ,  $0 < \hat{b} < \hat{a}$ , such that  $R_2^*(\hat{b}, \tilde{w}) = R_1^e$  and  $R_2^*(\hat{b}, \hat{w}) > R_1^e$ . The population who prefer a legal retirement age lower than  $R_1^e$  with  $R_1^{sq}$  are half of the total population. Since for any age  $a \in [\hat{b}, \hat{a}]$  and for any wage  $w \in [\tilde{w}, \hat{w})$ , we get  $R_1^*(a, w) < R_1^e$  and  $R_2^*(a, w) > R_1^e$ , then  $R_1^e < R_2^e$ .

- If  $R_1^e < R_1^{sq}$  and  $R_1^*(R_1^e, w_m) > R_1^e$ . There exists an age  $\hat{a}$ ,  $0 < \hat{a} < R_1^e$ , such that  $R_1^*(\hat{a}, w_m) = R_1^e$  and  $R_2^*(\hat{a}, w_m) > R_1^e$ ; there exists an age  $\hat{b}$ ,  $0 < \hat{b} < \hat{a}$ , such that  $R_1^*(\hat{b}, w_m) < R_1^e$  and  $R_2^*(\hat{b}, w_m) = R_1^e$ ; and therefore there exists an age  $\hat{c}$ ,  $\hat{b} < \hat{c} < \hat{a}$ , such that  $R_1^*(\hat{c}, w_m) < R_1^e$  and  $R_2^*(\hat{c}, w_m) > R_1^e$ . There also exists a wage  $\check{w} > w_m$  such that  $R_1^*(\hat{b}, \check{w}) = R_1^e$  and  $R_2^*(\hat{b}, \check{w}) > R_1^e$ ; and a wage  $\tilde{w}$ ,  $w_m < \tilde{w} < \check{w}$ , such that  $R_1^*(\hat{c}, \tilde{w}) = R_1^e$ ,  $R_2^*(\hat{c}, \tilde{w}) > R_1^e$ , and  $R_1^*(\hat{b}, \tilde{w}) < R_1^e$ . The population who prefer a legal retirement age lower than  $R_1^e$  with  $R_1^{sq}$  are half of the total population. Since for any age  $a \in [\hat{b}, \hat{c}]$  and for any wage  $w \in [w_m, \tilde{w})$ , we get  $R_1^*(a, w) < R_1^e$  and  $R_2^*(a, w) > R_1^e$ , then  $R_1^e < R_2^e$ .

- If  $R_1^e < R_1^{sq}$  and  $R_1^*(R_1^e, w_m) = R_1^e$ . There exists a wage  $\check{w}$ ,  $w_m < \check{w} < \hat{w}$ , and an age  $\hat{a}$ ,  $0 < \hat{a} < R_1^e$ , such that  $R_1^*(\hat{a}, \check{w}) = R_1^e$  and  $R_2^*(\hat{a}, \check{w}) > R_1^e$ ; there exists an age  $\hat{b}$ ,  $0 < \hat{b} < \hat{a}$ , such that  $R_1^*(\hat{b}, \check{w}) < R_1^e$  and  $R_2^*(\hat{b}, \check{w}) = R_1^e$ ; and therefore there exists an age  $\hat{c}$ ,  $\hat{b} < \hat{c} < \hat{a}$ , such that  $R_1^*(\hat{c}, \check{w}) < R_1^e$  and  $R_2^*(\hat{c}, \check{w}) > R_1^e$ . There also exists a wage  $\tilde{w}$ ,  $\check{w} < \tilde{w} < \hat{w}$ , such that  $R_1^*(\hat{b}, \tilde{w}) = R_1^e$  and  $R_2^*(\hat{b}, \tilde{w}) > R_1^e$ ; and a wage  $\ddot{w}$ ,  $\tilde{w} < \ddot{w} < \check{w}$ , such that  $R_1^*(\hat{c}, \ddot{w}) = R_1^e$ ,  $R_2^*(\hat{c}, \ddot{w}) > R_1^e$ , and  $R_1^*(\hat{b}, \ddot{w}) < R_1^e$ . The population who prefer a legal retirement age lower than  $R_1^e$  with  $R_1^{sq}$  are half of the total population. Since for any age  $a \in [\hat{b}, \hat{c}]$  and for any wage  $w \in [\tilde{w}, \ddot{w}]$ , we get  $R_1^*(a, w) < R_1^e$  and  $R_2^*(a, w) > R_1^e$ , then  $R_1^e < R_2^e$ .

ii) Since  $R_2^*(a, w) > R_1^e$  for any age  $a \in [R_1^{sq}, R_2^{sq}]$  and for any wage  $w \in [w_m, w_M]$ , we only have to prove that the population who prefer a legal retirement age lower than  $R_1^e$  is reduced from a system with  $R_1^{sq}$  to a system with  $R_2^{sq}$ . Since  $R_2^*(R_1^{sq}, w_m) > R_1^e$ , there exists an age  $\hat{a}$ ,  $0 < \hat{a} < R_1^{sq}$ , such that  $R_2^*(\hat{a}, w_m) = R_1^e$ . The population who prefer a legal retirement age lower than  $R_1^e$  with  $R_1^{sq}$  are half of the total population. Since for any age  $a \in (\hat{a}, R_1^{sq}]$  and for any wage  $w \in [w_m, \hat{w}]$ , we get  $R_1^*(a, w) < R_1^e$  and  $R_2^*(a, w) > R_1^e$ , then  $R_1^e < R_2^e$ . Q.E.D.

### 8.0.3 Proposition 6

i) Equal to proof of Proposition 1.

ii) From the F.O.C. of the maximization problem of the utility function (9), the implicit function theorem and after some simplifications we obtain

$$\frac{\partial R^*(a, w)}{\partial w} = - \frac{[1 - \tau(1 - \alpha)] u'(c) (1 - \rho_r) + \varphi(a)}{(w(1 - \tau) + \tau W)^2 u''(c) \frac{1}{T-a}} \quad (21)$$

with  $\varphi(a) = u''(c) \frac{1}{T-a} (R^{sq} - a) (1 - \alpha) (1 - \tau) \varpi \left(1 + \frac{\tau R^{sq}}{T - R^{sq}}\right) > 0$  since  $a > R^{sq}$ . (21) is strictly positive given that  $\rho_r < 1$  and  $\varphi(a) > 0$ .

iii) From the F.O.C. of the maximization problem of the utility function (9), using the implicit function theorem and after some simplifications we obtain

$$\frac{\partial R^*(a, w)}{\partial a} = \frac{(T - R^*(a, w)) (w(1 - \tau) - p(w))}{(T - a) (w(1 - \tau) + \tau W)}. \quad (22)$$

If  $w(1 - \tau) > (<) p(w)$ , then  $\pi(a, w) > (<) 0$ , and  $\partial R^*(a, w) / \partial a > (<) 0$ . Q.E.D.

### 8.0.4 Proposition 7

Working population have the same optimal retirement ages under the two retirement regimes. Therefore, the difference between the two elected retirement ages will arise due to the different behaviour of retirees. So, in order to distinguish this different behaviour, let  $R_h^*(a, w)$  be the optimal legal retirement age of retirees under the former regime, when they do not return to the labor market and vote for the highest retirement age, and  $R_b^*(a, w)$  be the optimal age of retirees when they may have to go back to work.

i) If  $R_h^e \leq R^{sq}$ . Retirees would have their optimal retirement ages above  $R_h^e$  in the two retirement regimes and therefore  $R_h^e = R_b^e$ .

If  $R_h^e > R^{sq}$ . Let  $\check{w} > \hat{w}$  be the wage such that  $R^*(R^{sq}, \check{w}) = R_h^e$ . If  $\check{w}(1 - \tau) \leq p(R; \check{w})$ , then  $\partial R_b^*(a, \check{w}) / \partial a \leq 0$ . Consequently, for any age  $a \in (R^{sq}, R_h^e]$  and for any wage  $w \in [w_m, \check{w})$ , we get  $R_b^*(a, w) < R_h^e$  and  $R_h^*(a, w) > R_h^e$ , which implies that in order to  $R_b^e$  divides population in two groups of equal size,  $R_b^e < R_h^e$ . If  $\check{w}(1 - \tau) > p(R; \check{w})$ , we can find a wage  $\dot{w} \in [w_m, \check{w}]$  and an age  $\hat{a}$ , with  $R^{sq} < \hat{a} \leq R_h^e$ , such that  $R^*(\hat{a}, \dot{w}) = R_h^e$ . Thus, for any age  $a \in (R^{sq}, \hat{a})$  and for any wage  $w \in [w_m, \dot{w})$ , we get  $R_b^*(a, w) < R_h^e$  and  $R_h^*(a, w) > R_h^e$ , which implies that in order to  $R_b^e$  divides population in two groups of equal size,  $R_b^e < R_h^e$ .

ii) The following relations are used for proving the second part of the proposition:  $\partial R^*(a, w) / \partial w > 0$ ,  $\partial R^*(a, w) / \partial \alpha > 0$  if  $w < (>) \varpi$  and  $\partial R^*(a, w) / \partial a > (<) 0$  if  $R^* < (>) R^{sq}$  for any  $R^*(a, w) > a$ . Let  $\tau_1 = \tau_2$ ,  $R_1^{sq} = R_2^{sq}$  and  $\alpha_1 > \alpha_2$ .

First, we prove the positive relation between  $\alpha$  and  $R_h^e$ .  $R_{h_1}^e > R_1^*(a, \varpi)$  for any  $a \in [0, R^{sq}]$  implies the following. There exists a wage  $\check{w} > \varpi$  and an age  $\hat{a} > 0$ , such that  $R_1^*(0, \check{w}) > R_{h_1}^e$ ,  $R_2^*(0, \check{w}) = R_{h_1}^e$  and  $R_1^*(\hat{a}, \check{w}) \geq R_{h_1}^e$ . There also exist two wages  $\check{w}$  and  $\ddot{w}$ , with  $\check{w} < \ddot{w} < \check{w}$ , and an age  $\hat{b}$ , with  $0 < \hat{b} < \hat{a}$ , such that  $R_1^*(0, \check{w}) = R_{h_1}^e$ ,  $R_1^*(0, \ddot{w}) > R_{h_1}^e$  and  $R_1^*(\hat{b}, \ddot{w}) \geq R_{h_1}^e$ . The population who prefer a legal retirement age higher than  $R_{h_1}^e$  with  $\alpha_1$  are half of the total population. Since for any age  $a \in [0, \hat{b})$  and for any wage  $w \in [\ddot{w}, \check{w}]$ , we get  $R_1^*(a, w) > R_{h_1}^e$  and  $R_2^*(a, w) < R_{h_1}^e$ , then  $R_{h_1}^e > R_{h_2}^e$ .

Now, we prove the negative relation between  $\alpha$  and  $R_b^e$ . In this case, we have to distinguish between workers and retirees. With reference to the workers:  $R_1^*(a, \varpi) > R_{b_1}^e$  for any  $a \in [0, R^{sq}]$  implies the following. There exists a wage  $\check{w} < \varpi$  and an age  $\hat{a} > 0$ , such that  $R_1^*(0, \check{w}) = R_{b_1}^e$ ,  $R_2^*(0, \check{w}) > R_{b_1}^e$  and  $R_2^*(\hat{a}, \check{w}) \geq R_{b_1}^e$ . There also exist two wages  $\check{w}$  and  $\ddot{w}$ , with  $\check{w} < \ddot{w} < \check{w}$ , and an age  $\hat{b}$ , with  $0 < \hat{b} < \hat{a}$ , such that  $R_2^*(0, \check{w}) = R_{b_1}^e$ ,  $R_2^*(0, \ddot{w}) > R_{b_1}^e$  and  $R_2^*(\hat{b}, \ddot{w}) \geq R_{b_1}^e$ .

With respect to the retirees, we have to calculate  $\partial R_b^*(a, w) / \partial \alpha$ .

$$\frac{\partial R_b^*(a, w)}{\partial \alpha} = - \frac{(u'(c)(1 - \rho_r) + \phi(a)) \tau (w - \varpi)}{(w(1 - \tau) + \tau W)^2 u''(c) \frac{1}{T-a}} \quad (23)$$

with  $\phi(a) = \frac{1}{T-a} u''(c) (a - R^{sq} w (1 - \tau) \frac{T-a}{T-R^{sq}})$ . So, since  $\varpi(1-\tau) = p(R; \varpi)$  implies  $\partial R_b^*(a, \varpi) / \partial a = 0$ , we obtain the following. Given that  $R_1^*(R^{sq}, \varpi) = R_{b_1}^e$  and  $\partial R_b^*(a, \varpi) / \partial \alpha = 0$ , the percentage of retirees with optimal ages below or equal to  $R_{b_1}^e$  will be those with age  $a \in (R^{sq}, R_{b_1}^e]$  and wage  $w \in [w_m, \varpi]$ , regardless the intragenerational redistribution degree.

Summing up, the population who prefer a legal retirement age lower than  $R_{b_1}^e$  with  $\alpha_1$  are half of the total population. Since for any age  $a \in [0, \hat{b})$  and for any wage  $w \in [\check{w}, \tilde{w})$ , we get  $R_1^*(a, w) < R_{b_1}^e$  and  $R_2^*(a, w) > R_{b_1}^e$ , then  $R_{b_1}^e < R_{b_2}^e$ . Q.E.D.

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