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**Abstract**

This paper explores the theoretical relationship between the population growth rate and asset prices implied by an overlapping-generations model. The model shows that changes in a population's age distribution affect asset prices but such changes generate low frequency movements in asset prices. The model also shows that the treatment of expectations matter; a small response of individuals to changes in asset prices has large implications for the path of asset prices. Finally, the model shows that incorporating a supply of assets by interpreting an asset as a claim on physical capital diminishes the magnitude of the relationship but does not change the sign or timing of the relationship between a population's age distribution and asset prices.

**Keywords:** asset pricing, demographics

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## I. Introduction

Currently, 12.8 percent of the population is over the age of 65; by 2030, the U.S. Bureau of Census estimates that the figure will be over 20 percent. Such large demographic changes have significant implications for government programs like Social Security and Medicare, but do they affect the price of assets?

Three papers examine the relationship between a population's age distribution and asset prices. Mankiw and Weil [1989] argue that the maturing of the baby boomers during the 1970's accelerated the rate of household formation, which in turn, increased the demand for housing and its price. Bakshi and Chen [1994] incorporate Mankiw and Weil's findings into a "life-cycle investment hypothesis" which argues that individuals change their allocations of wealth as they age so an aging population alters aggregate demand for assets and thus their prices. Yoo [1994], motivated by a similar intuition, finds that the real return to U.S. T-bills is negatively correlated with the size of the age group that has the highest increment to its wealth.

All three papers cited above show that an individual's demand for an asset varies with age. They then argue that because aggregate demand is merely the sum of individual demands, changes in a population's age distribution affects the aggregate demand for that asset and thus affect the price of the asset.

Although all three papers share a common intuition, they are mostly empirical. This paper examines the theoretical underpinnings of the relationship between a population's age distribution and asset prices. It presents a general equilibrium model that aggregates individual's optimizing behavior to derive equilibrium asset prices. The theoretical relationship between the two variables are shown by simulations based on the model.

The model suggests four conclusions about the relationship between asset prices and a population's age distribution. First, changes in a population's age distribution affect asset prices, as noted by the empirical literature. Second, although fluctuations in the population growth rate like the US post-war baby boom affect asset prices, such changes generate low frequency movements in asset prices. Third, the treatment of expectations matter; a small response of individuals to changes in asset prices has large implications for the path of asset prices. Finally, incorporating a supply of assets by interpreting an asset as a claim on physical capital diminishes the magnitude of the relationship but does not change the sign or timing of the relationship between a population's age distribution and asset prices.

## II. A Model of Age Distribution and Asset Prices

The intuition that explains the potential relationship between a population's age distribution and asset prices is rather simple. First, it assumes that an individual's demand for an asset varies with age. This premise underlies all three empirical papers cited and it is consistent with life-cycle behavior if an individual saves in assets, rather than saves storable consumption goods. Given the assumption, variations in a populations age distribution will alter the aggregate demand for that asset by changing the distribution of asset holders. So, a young population has many savers which generates a high total demand for assets, but an old population has many dissavers so total demand for assets is low. It is this variation in aggregate demand for an asset that produces a relationship between a population's age distribution and asset prices.

### A. A Simple Model

Let $D_t$ be the aggregate quantity of consumption goods invested in an asset in period $t$. Also, let $S_t$ be the number of shares of that asset outstanding in the economy. Then in equilibrium, the price of the asset, $P_t$ adjusts so that,

$$D_t = P_t S_t. \tag{1}$$

If an agent lives for $T_d$ periods and saves $a_{t,s}$ when $s$ years old in period $t$, and $N_{t,s}$ is the number of individuals $s$ years old, then the aggregate demand for the asset is

$$D_t = \sum_{s=1}^{T_d} N_{t,s} a_{t,s}. \tag{2}$$

Combining the two equations shows how changes in a population's age distribution affects asset prices in a manner suggested by the empirical literature,

$$P_t = \frac{\sum_{s=1}^{T_d} N_{t,s} a_{t,s}}{S_t}. \tag{3}$$

An additional individual aged $s$ affects asset prices by an amount proportional to $a_{t,s}$. So variations in a population's age distribution affect asset prices as long as age is a related to an individual's demand for the asset.

A pricing equation very similar to equation (3) can also be derived by combining an individual's budget constraint and an aggregate resource constraint for an endowment economy. The individual's budget constraint is,

$$a_{t,s} = \frac{P_t}{P_{t-1}} a_{t-1,s-1} + e_s - c_{t,s}, \tag{4}$$

where $e_s$ is the endowment of a consumption good received by an individual aged $s$ and $c_{t,s}$ is the consumption of that agent in period $t$. If the endowment goods are non-storable, then in equilibrium, total consumption equals total endowment each period,

$$\sum_{s=1}^{T_d} N_{t,s} c_{t,s} = \sum_{s=1}^{T_d} N_{t,s} e_s. \tag{5}$$

Substituting (4) into (5) yields

$$P_t = \frac{\sum_{s=1}^{T_d} N_{t,s} a_{t,s}}{\sum_{s=1}^{T_d} N_{t-1,s} a_{t-1,s}} P_{t-1}. \tag{6}$$

The two pricing equations (3) and (6) are equal if

$$S_t = \frac{\sum_{s=1}^{T_d} N_{t-1,s} a_{t-1,s}}{P_{t-1}}.$$

Although the derivations of the asset pricing equations are fairly simple, both equations capture the intuition and the methodology of the empirical literature. The empirical findings start with a cross- sectional estimate of an individual's demand for an asset and aggregate using the population's age distribution to determine how demographic factors affect asset prices.

## B. Simulating the Baby Boom

One useful benchmark for simulating the relationship between age distribution and asset prices is the post-war baby boom. The simulation uses a stylized baby boom; the steady-state annual population growth rate is 1 percent, and the baby boom temporarily doubles the growth rate to 2 percent for 15 years. This simplification roughly matches the actual population growth rate of the US since 1930. Figure 1 shows the Census Bureau's estimates of the annual growth rate of the US population between 1930 and 1980, as well as the baby boom used for the simulation.

Like the empirical studies, the simulation proceeds in two stages. First, I derive an age dependent demand for an asset by solving a 55 period overlapping-generations model. An individual receives an age dependent endowment $e_s$ of a non-storable consumption good during the first 45 periods of her life. She can save for her retirement by purchasing an unbacked asset for $P_t$. So, an individual born in period $t$ maximizes her lifetime utility, $U(t)$,

$$U(t) = \sum_{s=1}^{55} \beta^{s-1} \frac{c_{t+s-1,s}^{1-\rho}}{1-\rho}, \tag{7}$$

subject to the budget constraint in (4). $\beta$ is the subjective discount rate, and $\rho$ is reciprocal of the intertemporal elasticity of substitution. Table 1 shows the parameter values used in the simulations.

Next, I combine the age-wealth profile from the steady-state and the age distributions generated by the stylized baby boom to simulate the response of asset prices to a baby boom. The second panel of figure 1 shows the simulation results. Note that the simulation starts in 1966 because the model does not incorporate childhood.

Asset prices clearly respond to a baby boom. Initially, the baby boom has little effect on the growth of asset prices. The simulation shows that asset prices grow 19 percent during the first 15 years (the years of high population growth), a rate near the steady state growth rate of 1 percent per year. This small initial effect reflects the relatively small size of the baby boomers and the small savings of young individuals. The growth of asset prices then accelerates to 73 percent during the next 30 years as the baby boomers mature and increase their savings. The year 2010 marks the retirement of the first baby boomer which reduces the demand for assets as the baby boomers begin to consume some of their wealth. The net effect slows the grow of asset prices and eventually the large numbers of baby boomers consuming their wealth causes asset prices to fall. The decline starts in 2018 and continues for 9 years beyond the death of the first baby boomer in 2020. This fall in asset prices supports Schieber and Shoven's [1994] speculation that the retirement of baby boomers will be a net drain on pension plans, thereby place a downward pressure on asset prices.

The third panel shows the simulated asset prices relative to those generated without a baby boom and it suggests that the potential effect of the baby boom on asset prices is large. The baby boomer's demand for savings increases asset prices as much as 33 percent above the steady state and the baby boom induces a 10 percent permanent increase in asset prices.

Although asset prices respond to changes in a population's age distribution, such fluctuations induce relatively low frequency movements in asset prices. A baby boom first increases asset prices then decreases it but this cycle requires a length of time that is of similar order of magnitude as an individual's life expectancy.

The simulation also shows that the average age does not sufficiently capture the age distribution as suggested by Bakshi and Chen. The model clearly suggests that asset prices grow at population growth rate in steady state while the average age remains constant. The simulation also suggest that the relationship does not hold during a demographic transition like a baby boom. The bottom panel of figure 1 shows the average age of the simulation. Although the average age and asset prices move together for long periods of time, there are several periods when they move in opposite directions: 1965-1980, 2019-2020, and 2029-2035.

## III. Modifying the Model's Assumptions

The basic model presented in the previous section and implied by the empirical papers make two strong assumptions when using equation (3) to show the relationship between the age distribution and asset prices. This section examines the impact of relaxing the assumptions about expectations and supply of assets.

### A. Expectations

Estimating and simulating (6) in two steps assumes that an individual's demand for an asset does not respond to its price, a strong assumption given that saving is implicitly a forward-looking behavior. This section presents a simulation that replaces the static expectations assumption with a perfect foresight one. Equation (6) still captures the relationship between asset prices and age distributions but future asset prices now affect $a_{t,s}$.

The simulation strategy is similar to that of the previous section. The population growth rates generate the $N_{t,s}$'s and (7) supplies the $a_{t,s}$'s for the asset pricing equation but unlike the previous simulation, individuals now consider the future path of asset prices when making their saving decisions. Incorporating forward-looking behavior generates a perfect foresight path for the price of an asset.

Computationally, I use an iterative technique to simulate a perfect foresight path for asset prices. Agents in each iteration use the path of prices calculated by the previous iteration as the future path of asset prices. The iterations continue until each period's price changes by a negligible amount. The system is closed by assuming that asset prices return to their steady state growth rate within a finite horizon. Auerbach and Kotlikoff use this technique to simulate their model.

Changing the assumption about individual's expectations has a noticeable effect on the response of asset prices to a baby boom. The top two panels of figure 2 shows a comparison of asset prices assuming static expectations and perfect foresight. The model under either assumption about expectations shows that demographic shocks like a baby boom affects asset prices albeit with different timing and magnitudes. Unlike the simulation with static expectations, the perfect foresight simulation produces asset prices that rise quickly, 32 percent within the first 15 years versus 19 percent in the previous section's simulation. Asset prices with perfect foresight also reach their peak 8 years earlier in 2010, the year that the baby boomers begin to retire. The retirement of the baby boomers depresses asset prices sooner so that prices reach their trough 8 years earlier as well. The second panel indicates that asset prices deviate more from their no baby boom path under the perfect foresight assumption than static expectations.

Assumptions about the responsiveness of $a_{t,s}$ to changes in asset prices are responsible for the differences between the responses of asset prices to a baby boom. The third panel of figure 2 shows the difference between the age-wealth profiles of an individual living in steady state and an individual born during the first period of the baby boom. Not surprisingly, the profiles are very similar, but the first generation baby boomer's profile is noticeably higher, nearly 14 percent at peak savings. This difference between the two generations is similar to the finding in Mankiw and Weil that shows a higher cross-sectional demand for housing in 1980 than in 1970, although the profiles are similar in shape.

## B. Supply of Assets

The model specified above shows how the demand for an asset changes with demographic changes but it assumes that the supply of the asset is fixed. This section addresses that problem by incorporating a production function into the economy so that assets now represent claims to physical capital.

An individual still maximizes her utility function (7) but wages replace the age-dependent endowments in her budget constraint. So,

$$a_{t,s} = \frac{P_t}{P_{t-1}} a_{t-1,s-1} + e_s w_t - c_{t,s}, \tag{8}$$

where $e_s$ now represents an age-dependent labor productivity parameter and $w_t$ is the wage paid per effective unit of labor.

A Cobb-Douglas production function represents the productive capacity of the economy.

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{9}$$

where $Y_t$ is the net output of the economy,

$$K_t = \sum_{s=1}^{55} N_{t-1,s} a_{t-1,s},$$

and

$$L_t = \sum_{s=1}^{45} N_{t,s} e_s.$$

The factors of production receive their marginal product, so that

$$\frac{P_t}{P_{t-1}} = \alpha k_t^{\alpha-1} \tag{10}$$

and

$$w_t = (1-\alpha) k_t^{\alpha}, \tag{11}$$

where $k_t$ is the capital-labor ratio in $t$.

The above equations replace (5) and (6) in determining the equilibrium asset prices of the economy. Otherwise the simulation strategy is similar to that of the endowment economy. As before, the simulation uses an iterative technique to determine the perfect foresight path of asset prices.

The modified model still shows a relationship between asset prices and a baby boom, as seen in figure 3. Asset prices relative to those without a baby boom peak in 1999 in the endowment and production economies. In both cases, prices rise with the aging of the baby boomers and fall as the baby boomers approach retirement. Prices begin to rise again as the baby boomers pass from the economy. However, asset prices in a productive economy deviate much less from their steady state growth rate than in an endowment economy as the number of claims increase with the capital stock. Asset prices in the endowment economy rise to a peak of 37 percent above the no baby boom prices but they reach less than half of that rate, 16 percent, in the productive economy.

## IV. Conclusion

The results presented in this paper suggest that demographic variables play a role in the determination of the low frequency movements in the prices of assets through individuals' saving decisions. The model also indicates that assumptions about expectations have large effects on the response of asset prices to a demographic shock. But the model also shows that asset prices respond to demographic changes even with perfect foresight. Finally, the model shows that linking assets to productive capital damps the response of asset prices to a baby boom but they still respond in a qualitatively similar manner as a model with unbacked assets.

Although the simulations suggest that a baby boom affects asset prices, capturing its effects empirically may be difficult. The simulations with increasing supply of assets indicate that a baby boom adds 16 percent to asset prices over 33 years, an increase in the average annual return of less than one-half percent. Moreover, the existing empirical work does not adequately incorporate the effects of expectations into their estimates so forecasts of turning points based on their procedures are likely to be biased.

Table 1: Model Parameters

| parameters | | value |
|---|---|---|
| $T_d$ | lifespan | 55 |
| $T_r$ | retirement age | 45 |
| $\beta$ | subjective discount rate | $\frac{1}{1.015}$ |
| $\frac{1}{\rho}$ | intertemporal elasticity of substitution | 0.25 |
| $\alpha$ | capital's share | 0.25 |

## References


Auerbach, A., and Kotlikoff, L. J. *Dynamic Fiscal Policy.* Cambridge: Cambridge University Press, 1987.

Bakshi, G. and Chen, Z. "Baby Boom, Population Aging and Capital Markets." *Journal of Business* 67(1994): 165-202.

Mankiw, N. G., and Weil, D. N. "The Baby Boom, the Baby Bust and the Housing Market." *Regional Science and Urban Economics* 19(1989): 235-258.

Schieber, S. J., and Shoven, J. B. "The Consequences of Population Aging on Private Pension Fund Saving and Asset Markets." *NBER Working Paper* 4665, (1994).

Yoo, P. S. "Age Distributions and Returns to Financial Assets." Federal Reserve Bank of St. Louis Working Paper 94-002B, (1994).

Figure 1

Annual Population Growth Rate (percent)

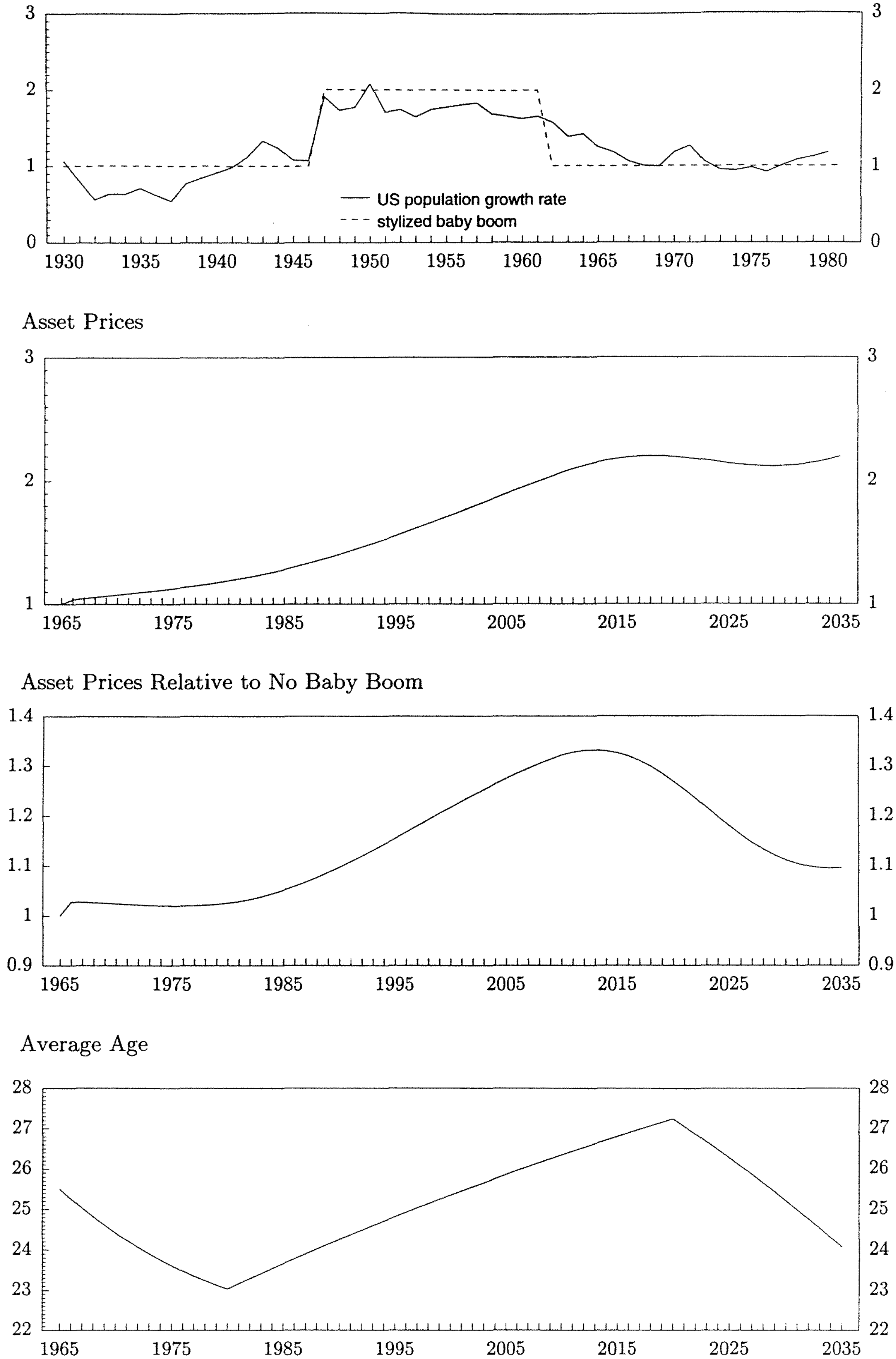
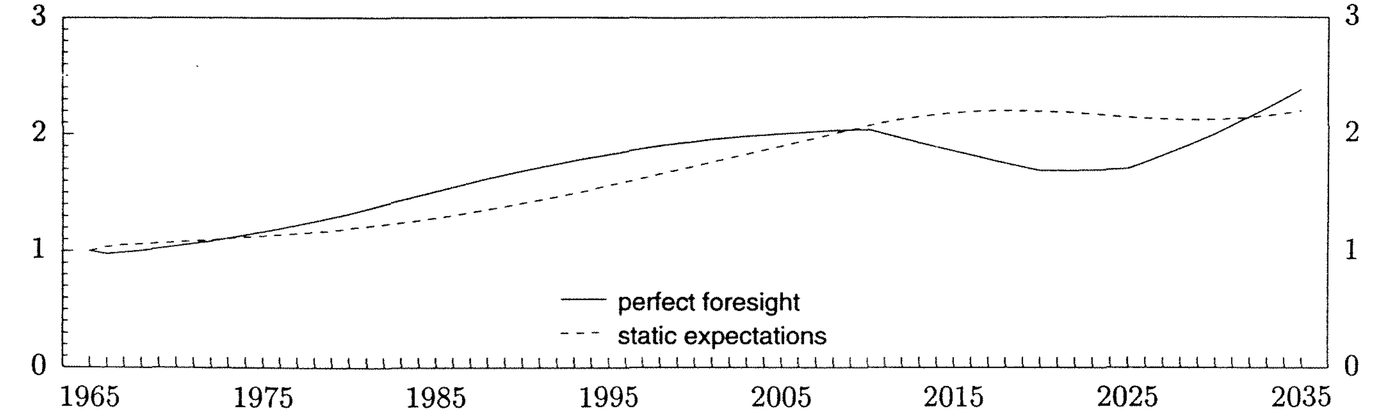
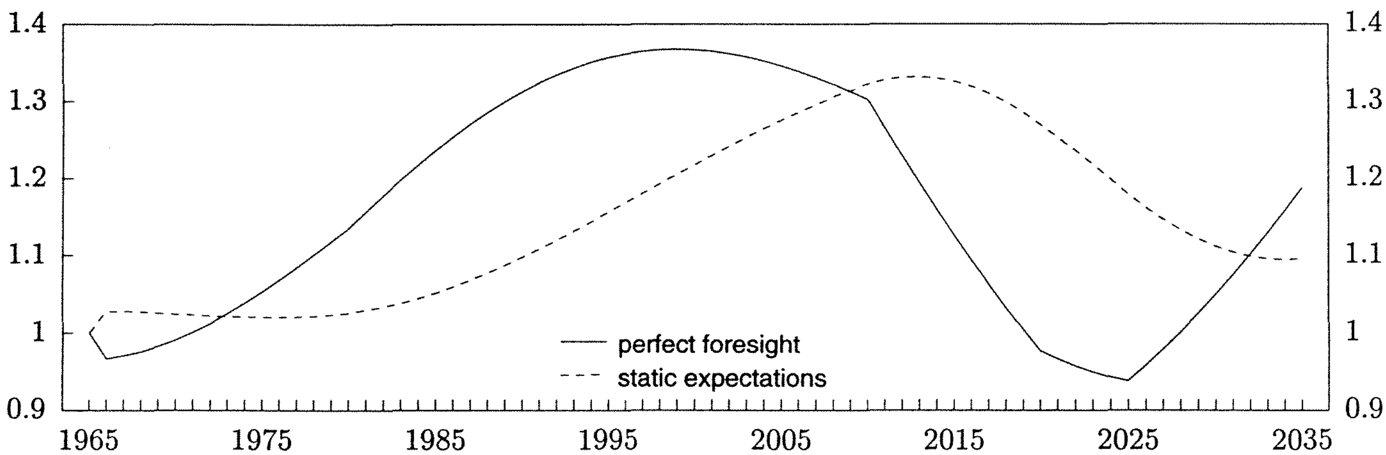


Figure 2
Comparison of Asset Prices




Asset Prices Relative to No Baby Boom




Comparison of Age-Wealth Profiles

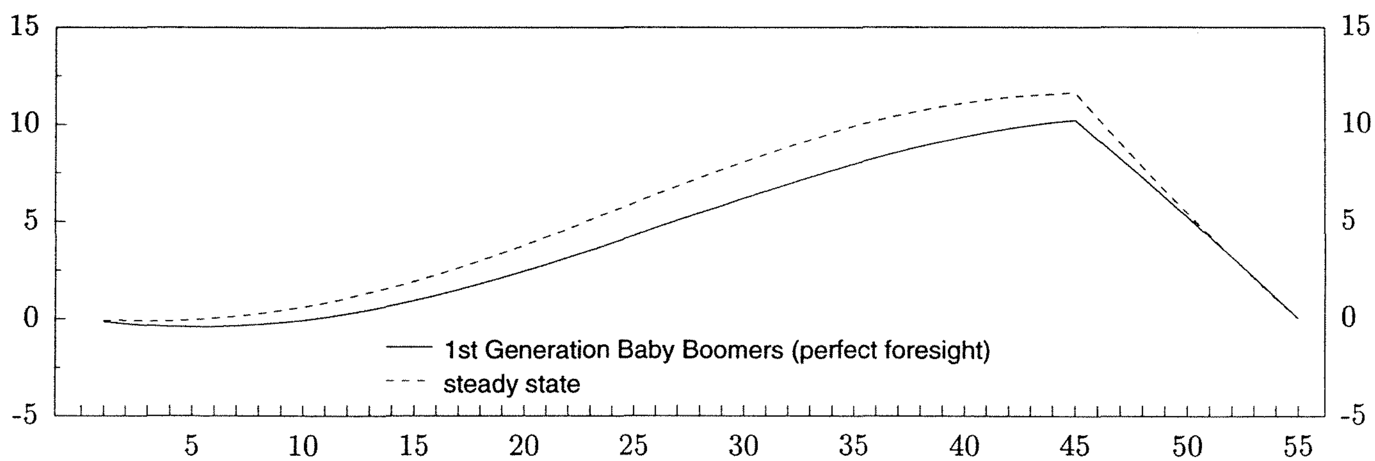


Figure 3
Asset Prices Relative to No Baby Boom