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Population synthesis for double white dwarfs

II. Semi-detached systems: AM CVn stars

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Abstract. We study two models for AM CVn stars: white dwarfs accreting (i) from a helium white dwarf companion and (ii) from a helium-star donor. We show that in the first model possibly no accretion disk forms at the onset of the mass transfer. The stability and the rate of mass transfer then depend on the tidal coupling between the accretor and the orbital motion. In the second model the formation of AM CVn stars may be prevented by detonation of the CO white dwarf accretor and the disruption of the system. With the most favourable conditions for the formation of AM CVn stars we find a current Galactic birth rate of 6.8 10^{-3} yr^{-1}. Unfavourable conditions give 1.1 10^{-3} yr^{-1}. The expected total number of the systems in the Galaxy is 9.4 10^7 and 1.6 10^7, respectively. We model very simple selection effects to get some idea about the currently expected observable population and discuss the (quite good) agreement with the observed systems.

Key words. stars: white dwarfs – stars: statistics – binaries: close – binaries: evolution

1. Introduction

AM CVn stars are helium-rich faint blue objects that exhibit variability on time-scales of ∼1000 s. Smak (1967) discovered that the prototype of the class, AM CVn (=HZ 29), shows photometric variability with a period ∼18 min and suggested that it is a binary. Immediately Paczyński (1967) realised that it could be a semi-detached pair of degenerate dwarfs in which mass transfer is driven by loss of angular momentum due to gravitational wave radiation. After flickering, typical for cataclysmic binary systems, was found in AM CVn by Warner & Robinson (1972), this model was used to explain this system by Faulkner et al. (1972). There are currently 8 AM CVn candidates (Table 2) which have been studied photometrically in great detail (for reviews see Ulla 1994; Warner 1995; Solheim 1995). AM CVn stars also attracted attention as possible sources of gravitational waves (Hils & Bender 2000, and references therein).

After the introduction of the concept of common envelope evolution for the formation of cataclysmic variables and X-ray binaries (Paczyński 1976), the formation of close double white dwarfs through two of such phases was anticipated by Tutukov & Yungelson (1979, 1981). The emission of gravitational waves would subsequently bring the two white dwarfs into a semi-detached phase. Nather et al. (1981) independently suggested this scenario for the formation of AM CVn itself. In an alternative scenario the white dwarf donor is replaced by a helium star that becomes semi-degenerate during the mass transfer (Iben & Tutukov 1991).

Throughout this paper we use the term AM CVn for binaries in which a white dwarf accretes from an another white dwarf or from a semi-degenerate helium star, irrespective how they would be classified observationally.

This paper continues our study on the formation and evolution of the Galactic population of close double white dwarfs (Nelemans et al. 2000, 2001b). Here we study the population that becomes semi-detached and transfers mass in a stable way. In addition we examine the alternative case where the donor is a semi-degenerate helium star.

In Sect. 2 we outline the evolution of binaries driven by gravitational wave radiation. We review the models for the formation of AM CVn stars and discuss the stability of the mass transfer in Sect. 3. The results of our population synthesis and a comparison with observations are presented in Sect. 4. The differences with previous studies are discussed in Sect. 5 after which the conclusions follow.

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2. Mass transfer in close binaries driven by gravitational wave radiation

The rate of angular momentum loss ($\dot{J}$) of a binary system with a circular orbit due to gravitational wave radiation (GWR) is (Landau & Lifshitz 1971):

$$\left(\frac{\dot{J}}{J}\right)_{\rm GWR} = -\frac{32}{5}\,\frac{G^3}{c^5}\frac{M\,m\,(M+m)}{a^4}\cdot \qquad (1)$$

Here M and m are the masses of the two components and a is their orbital separation.

In a binary with stable mass transfer the change of the radius of the donor exactly matches the change of its Roche lobe. This condition combined with an approximate equation for the size of the Roche lobe (Paczyński 1967),

$$R_{\rm L} \approx 0.46\,a\left(\frac{m}{M+m}\right)^{1/3} \qquad \text{for } m < 0.8M, \qquad (2)$$

may be used to derive the rate of mass transfer for a semi-detached binary in which the mass transfer is driven by GWR (Paczyński 1967)

$$\frac{\dot{m}}{m} = \left(\frac{\dot{J}}{J}\right)_{\rm GWR} \times \left[\frac{\zeta(m)}{2} + \frac{5}{6} - \frac{m}{M}\right]^{-1}. \qquad (3)$$

Here $\zeta(m)$ is the logarithmic derivative of the radius of the donor with respect to its mass ($\zeta \equiv {\rm d}\,\ln r/\,{\rm d}\,\ln m$). For the mass transfer to be stable, the term in brackets must be positive, i.e.

$$q \equiv \frac{m}{M} < \frac{5}{6} + \frac{\zeta(m)}{2}\cdot \qquad (4)$$

The mass transfer becomes dynamically unstable when this criterion is violated, probably causing the binary components to coalesce (Pringle & Webbink 1975; Tutukov & Yungelson 1979).

3. The nature of the mass donor: Two formation scenarios

3.1. Close double white dwarfs as AM CVn progenitors

From the spectra of AM CVn stars it is inferred that the transferred material consists mainly of helium. Faulkner et al. (1972) suggested two possibilities for the helium rich donor in AM CVn: (i) a zero-age helium star with a mass of 0.4 – 0.5 $M_\odot$ and (ii) a low-mass degenerate helium white dwarf. The first possibility can be excluded because the helium star would dominate the spectrum and cause the accretor to have large radial velocity variations, none of which is observed. Thus they concluded that AM CVn stars are interacting double white dwarfs. Their *direct progenitors* may be detached close double white dwarfs which are brought into contact by loss of angular momentum due to GWR within the lifetime of the Galactic disk (for which we take 10 Gyr). The less massive white dwarf fills its Roche lobe first and an AM CVn star is born, if the stars do not merge (see Sect. 3.2). We discussed the formation of such double white dwarfs in Nelemans et al. (2001b).

To calculate the stability of the mass transfer and the evolution of the AM CVn system, one needs to know the mass–radius relation for white dwarfs. This depends on the temperature, chemical composition, thickness of the envelope etc. of the white dwarf. However, Panei et al. (2000) have shown that after cooling for several 100 Myr the mass–radius relation for low-mass helium white dwarfs approaches the relation for zero-temperature spheres. As most white dwarfs that may form AM CVn stars are at least several 100 Myr old at the moment of contact (Tutukov & Yungelson 1996), we apply the mass–radius relation for cold spheres derived by Zapolsky & Salpeter (1969), as corrected by Rappaport & Joss (1984). For helium white dwarfs with masses between 0.002 and 0.45 $M_\odot$, it can be approximated to within 3% by (in solar units)

$$R_{\rm ZS} \approx 0.0106 - 0.0064\,\ln M_{\rm WD} + 0.0015 M_{\rm WD}^2. \qquad (5)$$

We apply the same equation for the radii of CO white dwarfs, since the dependence on chemical composition is negligible in the range of interest.

3.2. Stability of the mass transfer between white dwarfs

In Fig. 1 we show the limiting mass ratio for dynamically stable mass transfer (Eq. (4)) as the upper solid line, with $\zeta(m)$ derived from Eq. (5). The initial mass-transfer rates, as given by Eq. (3), can be higher than the Eddington limit of the accretor (Tutukov & Yungelson 1979). The matter that cannot be accreted is lost from the system, taking along some angular momentum. The binary system may remain stable even though it loses extra angular momentum. However heating of the transferred material, may cause it to expand and form a common envelope in which the two white dwarfs most likely merge (Han & Webbink 1999). Therefore, we impose the additional restriction to have the initial mass transfer rate lower than the Eddington accretion limit for the companion ($\sim$$10^{-5}\,M_\odot\,{\rm yr}^{-1}$). This changes the limiting mass ratio below which AM CVn stars can be formed to the lower solid line in Fig. 1. In this figure we over-plotted our model distribution of the current birthrate of AM CVn stars that form from close binary white dwarfs (see Sect. 4).

In the derivation of the Eq. (3) it is implicitly assumed that the secondary rotates synchronously with the orbital revolution and that the angular momentum which is drained from the secondary is restored to the orbital motion via tidal interaction between the accretion disk and the donor star (see, e.g. Verbunt & Rappaport 1988, and references therein).

However, the orbital separation when Roche lobe overflow starts is only about 0.1 $R_\odot$ and the formation of the accretion disk is not obvious; the matter that leaves the vicinity of the first Lagrangian point initially follows a ballistic trajectory, passing the accreting star at a minimum distance of ∼10% of the binary separation (Lubow & Shu 1975), i.e. at a distance comparable to the radius of a white dwarf (∼0.01 $R_\odot$). So, the accretion stream may well hit the surface of the accretor directly instead of forming an accretion disk around it (Webbink 1984).

The minimum distance at which the accretion stream passes the accretor is computed by Lubow & Shu (1975) (their $\widetilde{\omega}_{\rm min}$), which we fit with

$$\frac{r_{\rm min}}{a} \approx 0.04948 - 0.03815 \log(q) + 0.04752 \log^2(q) - 0.006973 \log^3(q). \qquad (6)$$

The value of q at which the radius of the accretor equals $r_{\rm min}$ is presented as a dotted line in Fig. 1; above this line the accretion stream hits the white dwarfs' surface directly and no accretion disk is formed.

In absence of the disk the angular momentum of the stream is converted into spin of the accretor and mechanisms other than disk–orbit interaction are required to transport the angular momentum of the donor back to the orbit. The small separation between the two stars may result in tidal coupling between the accretor and the donor which is in synchronous rotation with the orbital period. The efficiency of this process is uncertain (Smarr & Blandford 1976; Campbell 1984), but if tidal coupling between accretor and donor is efficient the stability limit for mass transfer is the same as in the presence of the disk (Eq. (4)). In the most extreme case all the angular momentum carried with the accretion stream is lost from the binary system. The lost angular momentum can be approximated by the angular momentum of the ring that would be formed in the case of a point-mass accretor: $\dot{J}_{\dot m} = \dot m \sqrt{GMar_{\rm h}}$, where $r_{\rm h}$ is the radius of the ring in units of a. This sink of angular momentum leads to an additional term $-\sqrt{(1+q)r_{\rm h}}$ in the brackets in Eq. (3). As a result the condition for dynamically stable mass transfer becomes more rigorous:

$$q < \frac{5}{6} + \frac{\zeta(m)}{2} - \sqrt{(1+q)r_{\rm h}}. \qquad (7)$$

This limit (with $r_{\rm h}$ given by Verbunt & Rappaport (1988) and again the additional restriction of a mass transfer rate below the Eddington limit) is shown in Fig. 1 as the dashed line.

Figure 1 shows that, with our assumptions, none of the AM CVn binaries which descend from double white dwarfs (which we will call the *white dwarf family*) forms a disk at the onset of mass transfer. After about 10^7 yr, when the donor mass has decreased below 0.05 $M_\odot$ (see Fig. 4) and the orbit has become wider a disk will form.

Only one of the currently known 14 close double white dwarfs possibly is an AM CVn progenitor; WD 1704+481A has $P_{\rm orb} = 3.48$ hr, $m = 0.39 \pm 0.05\,M_\odot$ and $M = 0.56 \pm 0.05\,M_\odot$ (Maxted et al. 2000). It is close to the limit for dynamical stability, but because the initial mass transfer rate is expected to be super-Eddington it may merge.

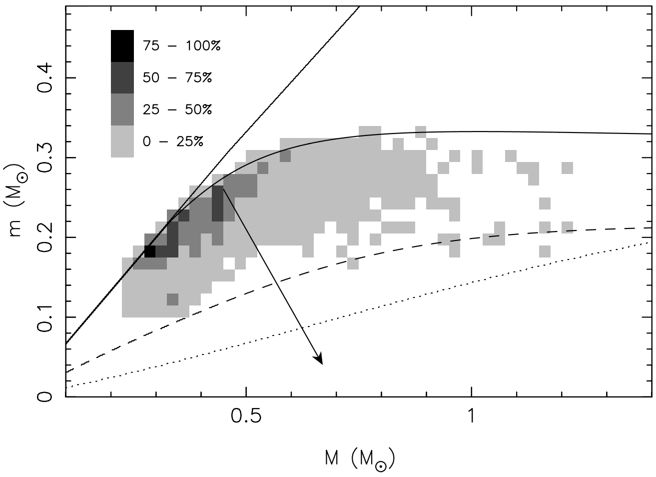

Fig. 1. Stability limits for mass transfer in close double white dwarfs. Above the upper solid curve the mass transfer is dynamically unstable. Below the lower solid line the systems have mass transfer rates below the Eddington limit in the case of an efficient tidal coupling between the accretor and the orbital motion. If the coupling is inefficient, this limit shifts down to the dashed line. Above the dotted line the stream hits the companion directly at the onset of the mass transfer and no accretion disk forms. As the evolution proceeds (parallel to the arrow) a disk eventually forms. The gray shades give the current model birthrate distribution of AM CVn stars that form from close binary white dwarfs. The shading is scaled as a fraction of the maximum birthrate per bin, which is 1.4 10^{-4} yr^{-1}

3.3. Binaries with low mass helium stars as AM CVn progenitors: A semi-degenerate mass donor

Another way to form a helium transferring binary in the right period range was first outlined by Savonije et al. (1986). They envisioned a neutron star accretor, but the scenario for an AM CVn star, with a white dwarf accretor, is essentially the same (Iben & Tutukov 1991). One starts with a low-mass, non-degenerate helium burning star, a remnant of so-called case B mass transfer, with a white dwarf companion. If the components are close enough, loss of angular momentum via GWR may result in Roche lobe overflow before helium exhaustion in the stellar core. Mass transfer is stable if the ratio of the mass of the helium star (donor) to the white dwarf (accretor) is smaller than ∼1.2 (Tutukov & Fedorova 1989; Ergma & Fedorova 1990)[1]. When the mass of the helium star decreases below ∼0.2 $M_\odot$, core helium burning stops and the star becomes semi-degenerate. This causes the exponent in the mass-radius relation to become negative and, as a consequence, mass transfer causes the orbital period to increase. The minimum period is ∼10 min. With strongly decreasing mass transfer rate the donor mass drops below 0.01 $M_\odot$ in a few Gyr, while the period of the system increases up to ∼1 hr; in the right range to explain the AM CVn stars. The luminosity of the donor drops below $10^{-4}\,R_\odot$ and its effective temperature to several

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[1] This stability limit is the same as for hydrogen-rich stars with radiative envelopes.

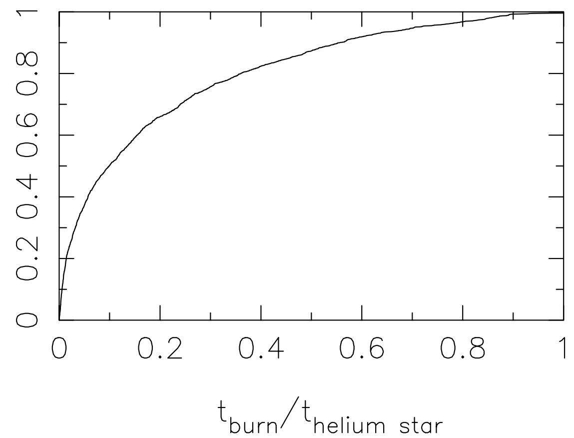

Fig. 2. Cumulative distribution of the ratios of the helium burning time that occurred *before* the mass transfer started and the *total* helium burning time for the model systems

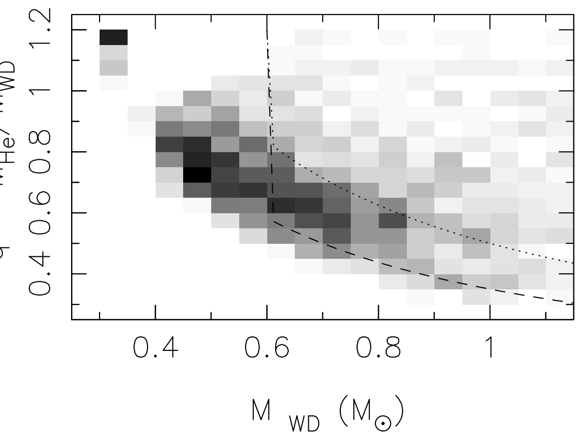

Fig. 3. Distribution of helium stars with white dwarf companions that currently start stable mass transfer. In the systems to the right of the dotted and dashed lines the white dwarfs accrete at least 0.3 $M_\odot$ and 0.15 $M_\odot$ before ELD, respectively, at high accretion rates. These lines show two choices of mass limits above which, with our assumption, the binaries are disrupted by edge-lit detonation before they become AM CVn systems. The systems in the top left corner are binaries with helium white dwarf accretors

thousand K. We will call the AM CVn stars that formed in this way the *helium star family*. Note that in this scenario a disk will always form because the orbit is rather wide at the onset of the mass transfer. The equations for efficient coupling thus hold.

The progenitors of these helium stars have masses in the range 2.3–5 $M_\odot$. The importance of this scenario is enhanced by the long lifetimes of the helium stars: $t_{\rm He} \approx 10^{7.15}\ M_{\rm He}^{-3.7}$ yr (Iben & Tutukov 1985), comparable to the main-sequence lifetime of their progenitors, so that there is enough time to lose angular momentum by gravitational wave radiation and start mass transfer before the helium burning stops.

Our simulation of the population of helium stars with white dwarf companions, suggests that at the moment they get into contact the majority of the helium stars are at the very beginning of core helium burning. This is illustrated in Fig. 2. Having this in mind, we approximate the mass-radius relation for semi-degenerate stars by a power-law fit to the results of computations of Tutukov & Fedorova (1989) for a 0.5 $M_\odot$ star, which filled its Roche lobe shortly after the beginning of core helium burning (their model 1.1). For the semi-degenerate part of the track we obtain (in solar units):

$$R_{\rm TF} \approx 0.043\, m^{-0.062}. \tag{8}$$

Trial computations with the relation $R \approx 0.029\, m^{-0.19}$ from the model of Savonije et al. (1986) which had $Y_{\rm c} = 0.26$ at the onset of the mass transfer reveals a rather weak dependence of our results on the mass–radius relation.

As noticed by Savonije et al. (1986), severe mass loss in the phase before the period minimum, increases the thermal time-scale of the donor beyond the age of the Galactic disk and thus prevents the donor from becoming fully degenerate and keeps it semi-degenerate.

Another effect of the severe mass loss is the quenching of the helium burning in the core. Tutukov & Fedorova (1989) show that during the mass transfer the central helium content hardly changes (especially for low mass helium stars). Therefore, despite the formation of an outer convective zone, which penetrates inward to regions where helium burning took place, one would expect that in the majority of the systems the transferred material is helium-rich down to very low donor masses. However, donors with He-exhausted cores at the onset of mass transfer ($Y_{\rm c} \lesssim 0.1$) may in the course of their evolution start to transfer matter consisting of a carbon-oxygen mixture (see Fig. 3 in Ergma & Fedorova 1990,who used the same evolutionary code as Tutukov & Fedorova). Figure 3 shows the population of low-mass helium stars with white dwarf companions which currently start mass transfer, (i.e. have $q \lesssim 1.2$), derived by means of population synthesis. It is possible that only a fraction of them evolves into AM CVn stars. Upon Roche lobe overflow, before the period minimum, most helium donors lose mass at an almost constant rate close to $3\ \ 10^{-8}\, M_\odot$ yr^{-1}. Accretion of He at such rates by a carbon-oxygen (CO) white dwarf may trigger a detonation in the layer of the accumulated matter (Taam 1980). This may further cause the detonation of the underlying CO dwarf, so-called edge-lit detonation, ELD (Livne 1990; Livne & Glasner 1991; Woosley & Weaver 1994; Livne & Arnett 1995). The conditions for ELD to occur: the mass of the white dwarf, the range of accretion rates, the mass of the accumulated layer, etc. are still actively debated.

However, examples computed by Limongi & Tornambè (1991) and Woosley & Weaver (1994) show that if $\dot{M} \gtrsim 10^{-8}\ M_\odot$ yr^{-1} the helium layer inevitably detonates if $\Delta M_{\rm He} \gtrsim 0.3\ M_\odot$ and $M_{\rm CO} \gtrsim 0.6\ M_\odot$. Therefore, as one of the extreme cases, we reject all systems which satisfy these limits from the sample of progenitors of AM CVn stars, assuming that they will be disrupted before the helium star enters the semi-degenerate stage. As another extreme, we assume that only 0.15 $M_\odot$ has to be accreted for ELD

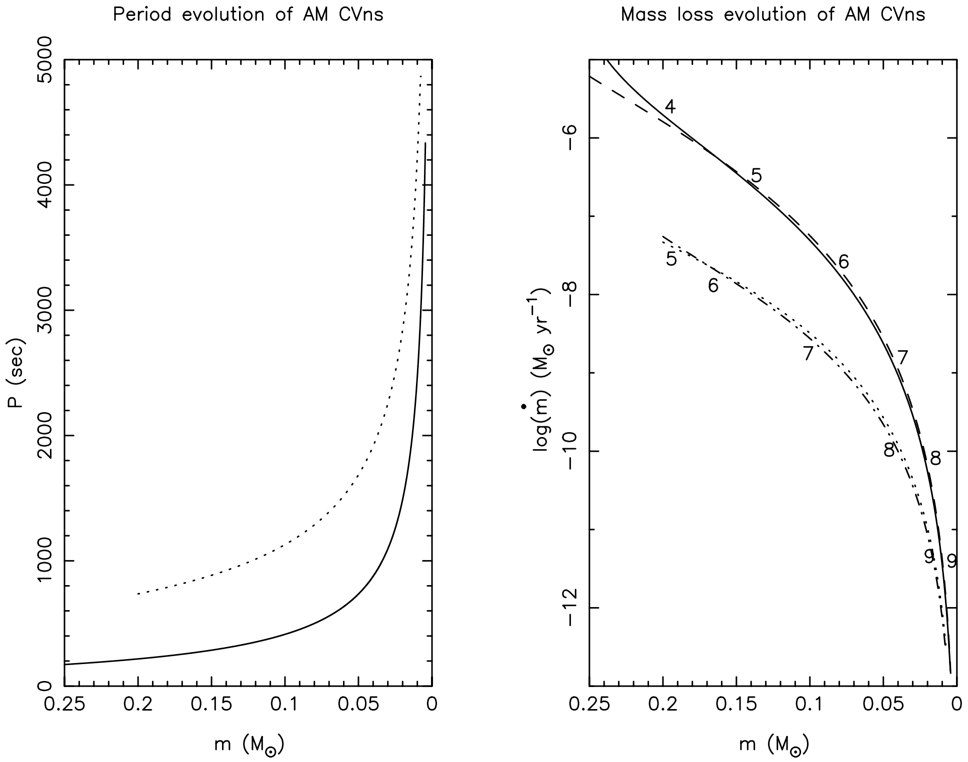

Fig. 4. Examples of the evolution of AM CVn systems. The left panel shows the evolution of the orbital period as function of the mass of the secondary (donor) star. The right panel shows the change in the mass transfer rate during the evolution. The solid and dashed lines are for white dwarf donor stars of initially 0.25 $M_\odot$ transferring matter to a primary of initial mass of 0.4 and 0.6 $M_\odot$, respectively, assuming efficient coupling between the accretor spin and the orbital motion. The dash-dotted and dotted line are for a helium star donor, starting when the helium star becomes semi-degenerate (with a mass of 0.2 $M_\odot$). Primaries are again 0.4 and 0.6 $M_\odot$. The numbers along the lines indicate the logarithm of the time in years since the beginning of the mass transfer

(as a compromise between results of Limongi & Tornambè 1991; Woosley & Weaver 1994). The relevant cut-offs are shown in Fig. 3.

For CO white dwarf accretors less massive than 0.6 $M_\odot$ we assume that accretion of helium results in "flashes" in which the He layer is ejected or lost via a common envelope formed due to expansion of the layer. Such events may be repetitive.

Limongi & Tornambè (1991) show that for accretion rates below 10^{-8} $M_\odot$ yr^{-1} more than ~0.4 $M_\odot$ helium has to be accreted before detonation. For systems in which the donor becomes semi-degenerate and the mass accretion rates are low we limit the accumulation of He only by adopting the Chandrasekhar mass as a maximum to the total mass of the accreting white dwarf.

3.4. Summary: Two extreme models for AM CVn progenitors

We recognise two possibilities for each family of potential AM CVn systems, which are: efficient or non-efficient tidal coupling between the accretor and the orbital motion in the white dwarf family, and two limits for the disruption of the accretors by ELD in the helium star family. We compute the populations for every possible solution and combine them into two models: model I, in which there is no tidal coupling and ELD is efficient in the destruction of potential progenitor systems (an "inefficient" scenario for forming AM CVn systems) and an "efficient" model II, in which there is an effective tidal coupling and ELD is efficient only in systems with the most massive donors. However, we give the birth rates and number of the objects for the four different solutions separately in Table 1. Figure 4 presents two examples of the evolution of the orbital period and the mass transfer rate for both families of AM CVn systems. Initially the mass transfer rate is very high but within a few million years it drops below 10^{-8} $M_\odot$ yr^{-1}. In the same time interval the orbital period increases from a few minutes, in the case of the white dwarf family, or from a little over 10 min, for the helium star family, to a few thousand seconds. The semi-degenerate donor systems have lower mass transfer rates and larger periods for the same donor mass due to their larger radii. The fact that the period is independent of the accretor mass (M) is a consequence of Eq. (2) and Keplers 3rd law leading to $P \propto (R^3/m)^{1/2}$.

4. The population of AM CVn stars

We used the population synthesis program SeBa, as described in detail in Portegies Zwart & Verbunt (1996),

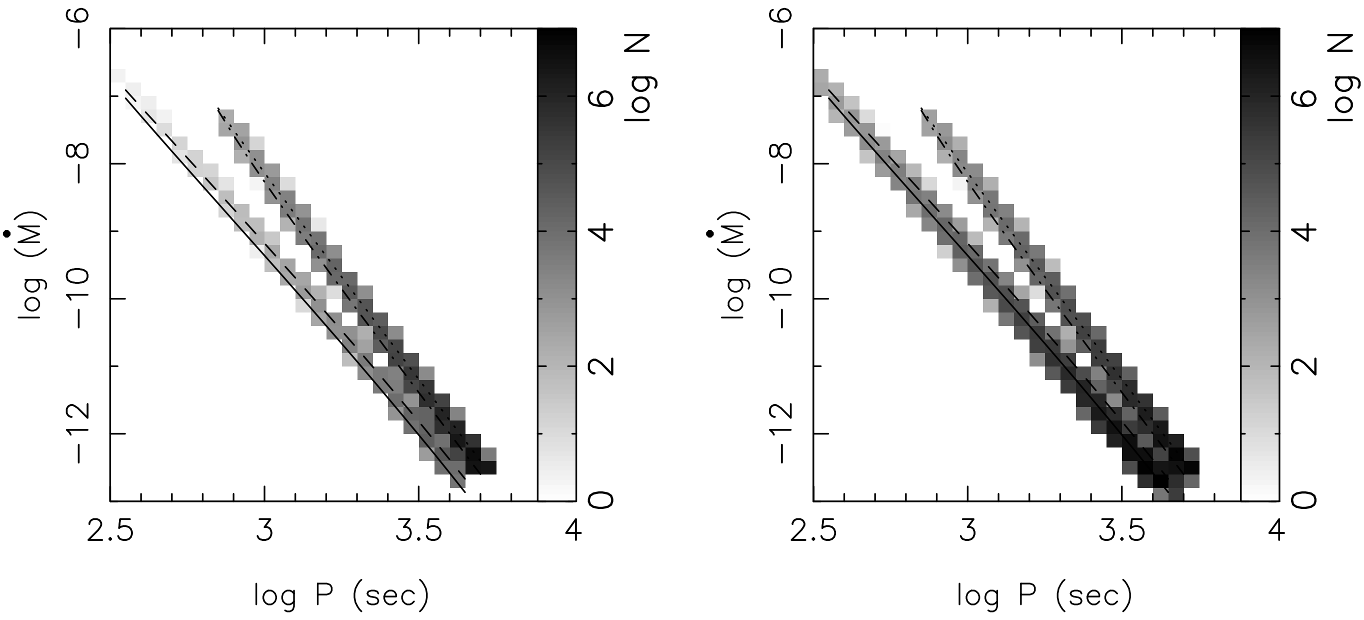

Fig. 5. The current Galactic population of AM CVn systems of the two families for models I (left) and II (right). The grey scale indicates the *logarithm* of the number of systems. The upper branch is the helium star family; the lower branch the white dwarf family. The lines show the orbital period and mass transfer rate evolution and correspond to the lines in Fig. 4

Portegies Zwart & Yungelson (1998) and Nelemans et al. (2001b) to model the progenitor populations. We follow model A of Nelemans et al. (2001b), which has an IMF after Miller & Scalo (1979) and flat initial distributions over the mass ratio of the components and the logarithm of the orbital separation and a thermal eccentricity distribution. We assume an initial binary fraction of 50% and that the star formation decreases exponentially with time, which is different from other studies of close double white dwarfs that assume a constant star formation rate. The mass transfer between a giant and a main-sequence star of comparable mass is treated with an "angular momentum formalism" which does not result in a strong spiral-in (Nelemans et al. 2000).

4.1. The total population

We generate the population of close double white dwarfs and helium stars with white dwarf companions and select the AM CVn star progenitors according to the criteria for the formation of the AM CVn stars as described above. We calculate the birthrate of AM CVn stars, and evolve every system according to the recipe described in Sect. 2, to obtain the total number of systems currently present in the Galaxy (Table 1) and their distribution over orbital periods and mass loss rates (Fig. 5).

The absence of an effective coupling between the accretor spin and the orbital motion (model I) reduces the current birth rate AM CVn stars from the white dwarf family by two orders of magnitude as compared to the case of effective coupling (model II). The fraction of close double white dwarfs which fill their Roche lobes and continue their evolution as AM CVn stars is 21% in model II but only 0.2% in model I (see also Fig. 1).

In model I the population of AM CVn stars is totally dominated by the helium star family. In model II where tidal coupling is efficient both families have a comparable contribution to the population. Increasing the mass of the critical layer for ELD from $0.15\,M_{\odot}$ to $0.3\,M_{\odot}$ almost doubles the current birth rate of the systems which are able to enter the semi-degenerate branch of the evolution. In the latter case almost all helium star binaries that transfer matter to a white dwarf in a stable way eventually become AM CVn systems (see Fig. 3).

In Fig. 5 we show the total current population of AM CVn systems in the Galaxy in our model. The evolutionary paths of both families are indicated with the curves (see also Fig. 4). Table 1 gives the total number of systems currently present in the Galaxy. The evolution of the systems decelerates with time and as a result the vast majority of the systems has orbital periods larger than one hour. The evolutionary tracks for the two families do not converge, since the mass loss of the helium stars prevents their descendants from recovering thermal equilibrium in the lifetime of the Galactic disk (see Sect. 3.3).

The minimum donor mass attainable within the lifetime of the Galactic disk is $\sim 0.005\,M_{\odot}$ for the descendants of the helium white dwarfs and $\sim 0.007\,M_{\odot}$ for the descendants of the helium stars. This is still far from the limit of $\sim 0.001\ M_{\odot}$ where the electrostatic forces in their interiors will start to dominate the gravitational force, the mass-radius relation will become $R \propto M^{1/3}$ (Zapolsky & Salpeter 1969), and the mass transfer will cease.

In Table 1 we give the local space density of AM CVn systems estimated from their total number and the Galactic distribution of stars, for which we adopt

$$\rho(R, z) = \rho_0\ \mathrm{e}^{-R/H}\ \mathrm{sech}(z/h)^2 \quad \mathrm{pc}^{-3} \tag{9}$$

Table 1. Birth rate and number of AM CVn systems in the Galaxy. The first column gives the model name (Sect. 3.4) followed by the current Galactic birthrate (ν in yr^{-1}), the total number of systems in the Galaxy (#) and the number of observable systems with $V < 15$ (# obs). The last column (σ in pc^{-3}) gives the local space density of AM CVn stars for each model. Due to selection effects the number of observable systems is quite uncertain (see Sect. 4.2)

	white dwarf family			He-star family			
Mod.	ν	#	# obs	ν	#	# obs	σ
	10^{-3}	10^{7}		10^{-3}	10^{7}		10^{-4}
I	0.04	0.02	1	0.9	1.8	32	0.4
II	4.7	4.9	54	1.6	3.1	62	1.7

as in Nelemans et al. (2001b). Here $H = 2.5$ kpc (Sackett 1997) and $h = 200$ pc, neglecting the age and mass dependence of h. These estimates can not be compared directly to the space density, estimated from the observations: 3 10^{-6} pc^{-3} (Warner 1995). In our model the space density is dominated by the long-period, dim systems, while Warner's estimate is based on the observed systems which are relatively bright. For a comparison of the observed and predicted populations we have to consider selection effects.

4.2. Observational selection effects: From the total population to the observable population

The known systems are typically discovered as faint blue stars (and identified with DB white dwarfs), as high proper motion stars, or as highly variable stars (see for the history of detection of most of these stars Ulla 1994; Warner 1995). The observed systems thus do not have the statistical properties of a magnitude limited sample.

Moreover, the luminosity of AM CVn stars comes mainly from the disk in most cases. Despite the fact that several helium disk models are available (e.g. Smak 1983; Cannizzo 1984; Tsugawa & Osaki 1997; El-Khoury & Wickramasinghe 2000) there is no easy way to estimate magnitude of the disk. Therefore, we compute the visual magnitude of the systems from very simple assumptions, to get a notion of the effect of observational selection upon the sample of interacting white dwarfs.

The luminosity provided by accretion is

$$L_{\rm acc} \approx 0.5\, G\, M \dot{m} \left(\frac{1}{R} - \frac{1}{R_{\rm L1}}\right). \tag{10}$$

Here R is the radius of the accretor and $R_{\rm L1}$ is the distance of the first Lagrangian point to the centre of mass of the accretor. We use an "average" temperature of the disk (see Wade 1984), which may be then obtained from $L = S\sigma T^4$, where S is the total surface of the disk:

$$S = 2\pi(R_{\rm out}^2 - R_{\rm WD}^2). \tag{11}$$

Table 2. Orbital periods, visual magnitudes and theoretical mass estimates for known and candidate AM CVn stars

Name	Period s	m_v	m (ZS)	m (TF)	Ref.
AM CVn	1028.7	14.1–14.2	0.033	0.114	1
HP Lib	1119	13.6	0.030	0.099	2
CR Boo	1471.3	13.0–18.0	0.021	0.062	3
V803 Cen	1611	13.2–17.4	0.019	0.054	2
CP Eri	1724	16.5–19.7	0.017	0.048	2
GP Com	2970	15.7–16.0	0.008	0.019	2
RX J1914+24	569	> 19.7	0.068	-	4
KL Dra		16.8–20			5

Theoretical mass estimates (in $M_{\odot}$) obtained from Eq. (5) are labelled by ZS, estimates from Eq. (8) by TF.
References: (1) Patterson et al. (1993), (2) Warner (1995), (3) Provencal et al. (1997), (4) Cropper et al. (1998), (5) Schwartz (1998).

We use $R_{\rm out} = 0.7 R_{\rm L1}$. The visual magnitude of the binary is then computed from the effective temperature and the bolometric correction (taken from Kuiper 1938), assuming that the disk is a black body. This allows us to construct a magnitude limited sample by estimating the fraction of the Galactic volume in which any system in our theoretical sample may be observed as it evolves.

We derive $P - \dot{M}$ diagrams for both models, similar to the ones for the total population, but now for the "observable" population, which we limit by $V = 15$. Changing $V_{\rm lim}$ doesn't change the character of graphs, since only the nearby systems are visible. The expected number of observable systems for the two families of progenitors is given in Table 1 and shown in Fig. 6. The observable sample comprises only one star for every million AM CVn stars that exists in the Galaxy. A large number of AM CVn stars may be found among very faint white dwarfs which are expected to be of the non-DA variety due to the fact, that the accreted material is helium or a carbon-oxygen mixture.

In the "inefficient" model I about one in 30 observed systems is from the white dwarf family. This is a considerably higher fraction than in the total AM CVn population where it is only one out of 100 systems. In the "efficient" model II, the white dwarf family comprises ∼60% of the total population and ∼50% of the "observable" one. The ratio of the total number of systems of the white dwarf family in models I and II is not proportional to the ratio of their current birthrates. This reflects the star formation history and the fact that the progenitors of the donors in model I are low mass stars that live long before they form a white dwarf. In model I the fraction of the observable systems which belong to the white dwarf family is higher than the fraction of the total number of systems that belong to this family. This is caused by the fact that the accretors in these systems are more massive (see Fig. 1), thus smaller, giving rise to higher accretion luminosities.

To compare our model with the observations, we list the orbital periods and the observed magnitude ranges for the known and candidate AM CVn stars in Table 2. For AM CVn we give $P_{\rm orb}$ as inferred by

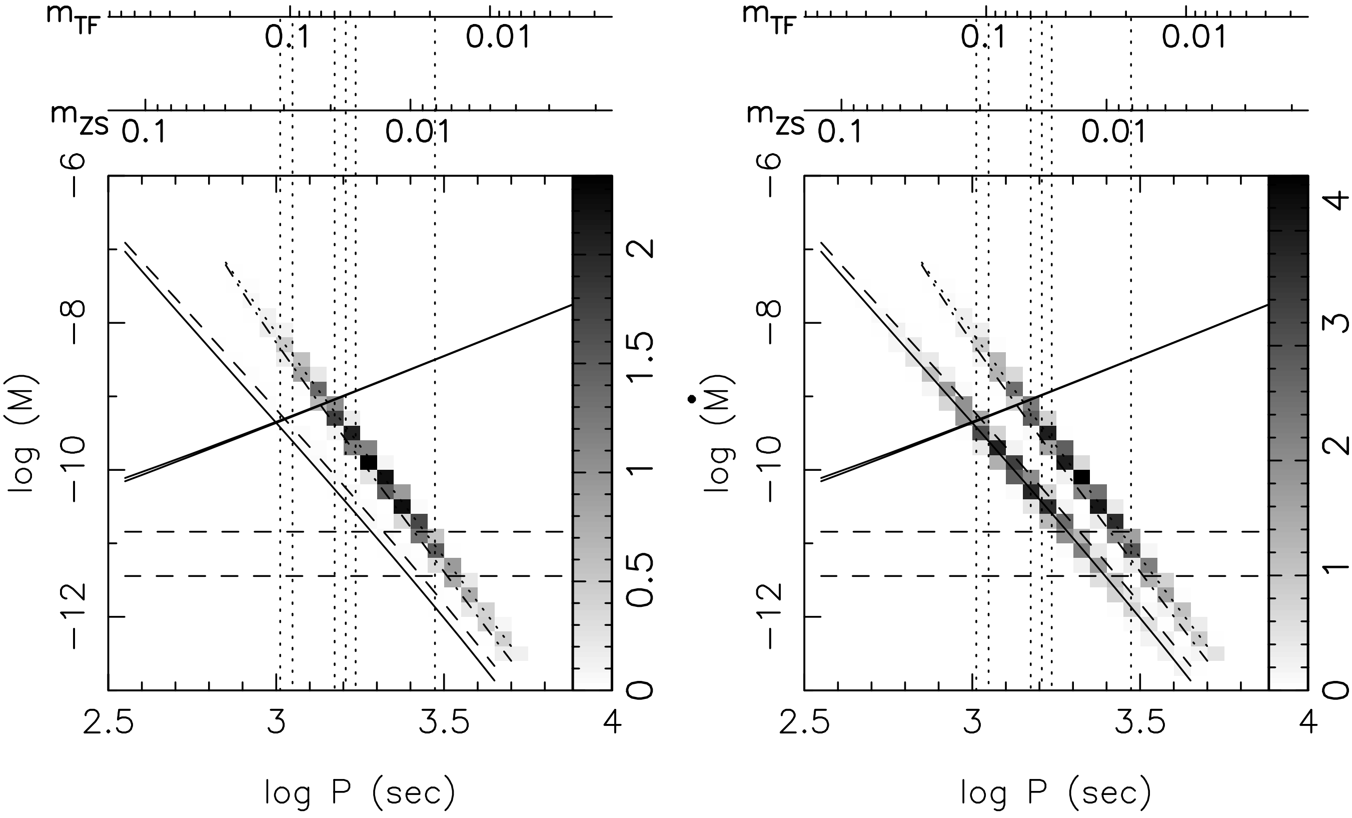

Fig. 6. Magnitude limited sample ($V_{\rm lim} = 15$) of the theoretical population of AM CVn stars for model I (left) and model II (right). The grey scale gives the number of systems, like in Fig. 5 but now on a linear scale (upper branch for the helium star family; lower branch for the white dwarf family). The selection criteria are described in Sect. 4.2. The periods of the observed systems (Table 2) are indicated with the vertical dotted lines. The stability limits for the helium accretion disk according to Tsugawa & Osaki (1997) are plotted as the solid slanted and dashed horizontal lines. Between these lines the disk is expected to be unstable. The upper dashed line is for an accretor mass of 0.5 $M_{\odot}$, the lower for a 1.0 $M_{\odot}$ accretor. The rulers at the top indicate the theoretical relation between the period and the mass for the mass-radius relations of the two AM CVn star families given by Eqs. (5) and (8)

Patterson et al. (1993) and confirmed as a result of a large photometry campaign (Skillman et al. 1999) and a spectroscopic study (Nelemans et al. 2001a). For the remaining systems we follow the original determinations or Warner (1995). Most AM CVn stars show multiple periods, but these are close together and do not influence our qualitative analysis. KL Dra is identified as an AM CVn type star by its spectrum (Jha et al. 1998), but still awaits determination of its period. The periods of the observed AM CVn stars are shown in Fig. 6 as the vertical dotted lines. The period of RX J1914+24 is not plotted because this system was discovered as an X-ray source and it is optically much fainter than the limit used here.

Figures 5 and 6 show that the uncertainty in both models and observational selection effects make it hard to argue which systems belong to which family. According to model I the descendants of close double white dwarfs are very rare. However, in that case one might not expect two observed systems at short periods (AM CVn and HP Lib). In both models I and II, systems with long periods (like GP Com) are more likely to descend from the helium star family. In the spectrum of GP Com, however, Marsh et al. (1991) found evidence for hydrogen burning ashes in the disk, but no traces of helium burning, viz. very low carbon and oxygen abundances. It is not likely that any progenitor of the helium star family completely skipped helium burning. More probably, this system belongs to the white dwarf family.

Most systems in the "observable" model population have orbital periods similar to the periods of the observed AM CVn stars that show large brightness variations; thus most modelled systems are expected to be variable. These brightness variations have been interpreted as a result of a thermal instability of helium disks (Smak 1983). In Fig. 6 we show the thermal stability limits for helium accretion disks as derived by Tsugawa & Osaki (1997): above the solid line the disks are expected to be hot and stable; below the horizontal dashed lines the disks are cool and stable and in between the disks are unstable. Note that the vast majority of the total Galactic model population (Fig. 5) is expected to have cool stable disks according to the thermal instability model, preventing them from being detected by their variability.

The period distributions of the "observable" population in our models agree quite well with the observed population of AM CVn stars. Better modelling of the selection effects is, however, necessary.

4.3. Individual systems

Table 2 gives theoretical estimates of the masses of the donor stars in the observed AM CVn stars, derived from the relation between the orbital period and the mass of the donor (see Sect. 3.4 and Fig. 6).

AM CVn stars may be subject to tidal instability due to which the disk becomes eccentric and starts precessing. Such instabilities are used to explain the superhump phenomenon in dwarf novae (Whitehurst 1988).

For AM CVn and CR Boo the observed 1051.2 s (Solheim et al. 1998) and 1492.8 s (Provencal et al. 1997) periodicities are interpreted as superhump periods. Following Warner (1995) we compute the mass ratio of the binary system using the orbital period ($P_{\rm orb}$) and the superhump period ($P_{\rm s}$) via:

$$\frac{P_{\rm s}}{P_{\rm s}-P_{\rm orb}} \approx 3.73\,\frac{1+q}{q}. \tag{12}$$

This results is $q = 0.087$ and 0.057 for AM CVn and CR Boo respectively. Assuming that they belong to the white dwarf family their accretor masses are $M = 0.38\,M_{\odot}$ and $M = 0.37\,M_{\odot}$. These values are at the lower end of the predicted distribution. If we apply the semi-degenerate mass–radius relation, the estimated masses of the accretors are high, even close to the Chandrasekhar mass for AM CV. The formation of systems with high-mass accretors has a low probability (see Fig. 3), which suggests that either Eq. (12) is not applicable for helium disks or alternatively that these binaries do not belong to the helium star family.

Maybe the most intriguing system is RX J1914.4+245; detected by *ROSAT* (Motch et al. 1996) and classified as an intermediate polar, because its X-ray flux is modulated with a 569 s period, typical for the spin periods of the white dwarfs in intermediate polars. Cropper et al. (1998) and Ramsay et al. (2000) suggest that it is a double degenerate polar with an orbital period equal to the spin period of the accreting white dwarf. The mass transfer rate in this system, inferred from its period ($\dot{m} \approx 1.8\ 10^{-8}\,M_{\odot}\ {\rm yr}^{-1}$) is consistent with the value deduced from the *ROSAT* PSPC data (Cropper et al. 1998) if the distance is ~100 pc.

Even though polars have no disk, the coupling between the accretor and donor is efficient due to the strong magnetic field of the accretor. We therefore anticipate that Eq. (4) applies without the correction introduced by Eq. (7). It may well be that magnetic systems in which the coupling is maintained by a magnetic field form the majority of stable AM CVn systems of the white dwarf family. We do not expect this system to belong to the helium star family, since its period is below the typical period minimum for the majority of the binaries in this family.

RX J0439.8-809 may be a Large Magellanic Cloud relative of the Galactic AM CVn systems. This system was also first detected by *ROSAT* (Greiner et al. 1994). Available X-ray, UV- and optical data suggest, that the binary may consist of two degenerate stars and have an orbital period < 35 min (van Teeseling et al. 1997, 1999).

RX J1914.4+245 and RX J0439.8-809 show that it is possible to detect optically faint AM CVn stars in supersoft X-rays, especially in other galaxies. The possibility of supersoft X-rays emission by AM CVn stars was discussed by Tutukov & Yungelson (1996). There are two probable sources for the emission: the accreted helium may burn stationary at the surface of the white dwarf if $\dot{m} \sim 10^{-6}\,M_{\odot}\ {\rm yr}^{-1}$ and/or the accretion disk may be sufficiently hot in the same range of accretion rates. However, the required high accretion rate makes such supersolf X-ray sources short-living (see Fig. 4) and, therefore, not numerous. Note that AM CVn, CR Boo, V803 Cen, CP Eri and GP Com are also weak X-ray sources (Ulla 1995).

The most recently found suspected AM CVn star, KL Dra, is also variable. Therefore we expect it to lie in the same period range as CR Boo, V803 Cen and CP Eri. Taking the limits for stability as given by Tsugawa & Osaki (1997) we expect the orbital period to be between 20 and 50 min (Fig. 6).

5. Discussion

A population synthesis study for AM CVn stars (and related systems) was done by Tutukov & Yungelson (1996), who considered only an "efficient" model. Their derived birthrate for the white dwarf family is $1.3\ 10^{-2}\ {\rm yr}^{-1}$, a factor three higher than the value in our "efficient" model. This difference can in part be explained by the different treatment of the mass transfer from a giant to a main sequence star of comparable mass (see Nelemans et al. 2000, 2001b). Most close double white dwarfs in our model have a mass ratio close to unity for which stable mass transfer is impossible (Sect. 2), while in the model of Tutukov & Yungelson (1996) they predominantly have $q \sim 0.5$–0.7 which is more favourable for stable mass transfer. Our higher integrated star formation rate only partly compensates for the loss of stable systems.

Another difference is that Tutukov & Yungelson (1996) conclude that the helium star family (non-degenerate helium stars in their terminology) do not contribute significantly to the AM CVn population. This is a consequence of their assumption that these systems, after the period minimum, live only for 10^8 yr. In contrast, our calculations show that their evolution is limited only by the lifetime of the Galactic disk. Tutukov & Yungelson estimate the total number of AM CVn stars from the helium star family as (1.9–4.6) 10^5 depending on the assumptions about the consequences of the accretion of helium. We find $2\ 10^7$ even when we let ELD destroy the systems which accrete only $0.15\ M_{\odot}$.

An additional complication is the possibility of the formation of a common envelope for systems where the accretion rate exceeds the rate of stationary helium burning at the surface of the accreting white dwarf ($\sim 10^{-6}\,M_{\odot}\,\mathrm{yr}^{-1}$). If such a common envelope forms the components of system may well merge. If it happens, it will occur directly after the Roche-lobe contact, when the highest accretion rate occurs. We do not consider this possibility in our model, because it involves too many additional (and unknown) parameters. Applying only the requirement that stable systems should accrete below the Eddington rate, we may overestimate the birthrate of AM CVn stars.

We did not discuss the Roche lobe overflow by low mass stars with almost exhausted hydrogen cores ($X_{\mathrm{c}} \sim 0.01$) which may also result in the formation of helium transferring systems with orbital periods ~10 min (Tutukov et al. 1987) because of its extremely low probability.

The prescription for ELD is related to the problem of SN Ia progenitors. In model I almost all accretors in the helium star family with initial $M \geq 0.6\,M_{\odot}$ "explode" and the ELD rate is close to 0.001 yr^{-1}. If ELDs really produce SNe Ia, they may contribute about 25% of their currently inferred Galactic rate. In model II $0.3\,M_{\odot}$ must be accreted prior to the explosion, and the ELD rate is only about $4\ \ 10^{-4}\ \mathrm{yr}^{-1}$. Even in model I we find a much lower ELD rate than Tutukov & Yungelson (1996, who find 0.005). This is partly due to a lower birthrate, but also to the different treatment of the mass transfer from a giant to a main sequence star of comparable mass (Nelemans et al. 2000), which causes the accretors in our model mainly to have masses below $0.6\ M_{\odot}$. Such systems probably never experience ELD.

In model II the accretors in both families may accrete so much matter that they reach the Chandrasekhar mass. The rates for the white dwarf and helium star families for this process are $3\ 10^{-6}$ and $5\ 10^{-5}\ \mathrm{yr}^{-1}$.

In both families the accretors can be helium white dwarfs (see Figs. 1 and 3). It was shown by Nomoto & Sugimoto (1977) that accretion of helium onto helium white dwarfs with $\dot{m} = (1\text{–}4)\ \ 10^{-8}\,M_{\odot}\,\mathrm{yr}^{-1}$ results either in a helium shell flash (at the upper limit of the accretion rates) or in central detonation which disrupts the white dwarf (for lower $\dot{m}$). The detonation occurs only when the mass of the accretor grows to $\sim 0.7\,M_{\odot}$. In our calculations this happens for the helium star family at a rate of $\sim 4\ 10^{-6}\ \mathrm{yr}^{-1}$. For the white dwarf family it happens only in model II, at a rate of $\sim 2\ 10^{-6}\ \mathrm{yr}^{-1}$.

6. Conclusions

We study the formation of AM CVn stars from (i) close detached double white dwarfs which become semi-detached and (ii) helium stars that transfer matter to a white dwarf and stop burning helium due to mass loss and become dim and semi-degenerate.

We find that, with our assumptions, in all cases where a double white dwarf potentially can form an AM CVn star no accretion disk will be formed in the initial phase of mass transfer. Normally the disk provides the feedback of angular momentum to the orbit, stabilising the mass transfer. In absence of a disk, the stability of the mass transfer in the semi-detached white dwarf binary depends critically on the efficiency of the coupling between the accretor and the donor. If this coupling is not efficient most systems merge, and the formation rate of AM CVn stars from double white dwarfs becomes very low. In this case it is possible that magnetically coupled systems are almost the only ones to survive. RX J1914.4+245 may be such a system.

In the second channel the formation of AM CVn stars may be prevented by explosive burning of the accumulated helium layer which may cause detonation of the CO white dwarf accretor and the disruption of the system.

We combine our population synthesis results into two models, an "efficient" model in which the stability of mass transfer is not affected by the absence of an accretion disk and the explosive helium burning disrupting the system happens when $0.3\ M_{\odot}$ is accumulated and an "inefficient" model in which the absence of an accretion disk is very important and the explosive helium disrupting the system happens already when $0.15\ M_{\odot}$ is accumulated. Applying very simple selection effects we estimate that in the "inefficient" model only one in 30 potentially observed systems descends from double white dwarfs. In the "efficient" model both families produce comparable numbers of observable systems. The observed systems fall roughly in the expected range of periods for a magnitude limited sample.

We conclude that to learn more about the AM CVn population both theory (stability of the mass transfer and helium accretion disks) and observations (especially the distances and the completeness of the sample) need to be improved.

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