REVIEW PAPER

Poroelasticity: parameters reviewed

H.-J. Kümpel

Institute of Geophysics, University of Kiel, Leibnizstr. 15, D-2300 Kiel 1, Germany

SUMMARY

Deformation of porous rocks is investigated by experts of various fields. This is probably the reason why poroelastic parameters are neither uniformly defined nor represented by standard symbols in the literature, and why inconsistencies have appeared in some publications. This paper is intended to clarify the use of material properties in order to facilitate application of poroelasticity theory. The key parameters referenced from a selection of mostly recent papers are: drained and undrained moduli and Poisson ratios, grain and pore compressibilities, the coefficient of effective stress, the Skemption ratio (pore pressure parameter), storage parameters, the Darcy conductivity, and the hydraulic diffusivity. The latter parameter governs the diffusional behaviour of poroelastic processes, as has been demonstrated by Rice & Cleary (1976). Herein, their formulation of the theory is used. References to Biot's (1941) parameters, which may be more familiar to some researchers, are given for completeness. A variety of extensions of the theory that have been elaborated by others is outlined in the last section.

Key words: Biot's theory, consolidation of saturated media, drained and undrained parameters, mechanical properties of porous rocks, poroelasticity, quasi-static deformation.

1 INTRODUCTION

Deformation of porous media is of fundamental interest in a variety of fields, like soil mechanics, fluid reservoir production, and crustal dynamics. It is also believed to play a major role in phenomena related to failure of rocks and sediments, e.g. in the tectonic earthquake cycle, in fluid pressure induced seismicity, or landslides. Since Biot (1941) formulated the 'General Theory of Three-Dimensional Consolidation', progress has been made in identifying various poroelastic material properties and in applying the theory to physical processes that are observed *in situ* or in laboratories. Probably because the behaviour of porous rocks is studied by experts of diverse branches, namely rock and soil engineers, hydrologists, and geophysicists, material parameters are neither uniformly defined nor represented by standard symbols in literature. For the non-expert, in particular, it is an arduous task to identify the various rock parameters from different references and to verify slight distinctions that occasionally occur. This article outlines poroelasticity in its most elementary form and attempts to expose the different uses of individual parameters in order to promote clarity and reduce confusion.

The resistance of rocks to deform under mechanical stress is characterized by various material parameters. If the entire volume of the rock consists of solid phase material, the rock is called compact, otherwise porous or cracked. All rocks known to occur in the Earth's crust are in fact (more or less) porous and cracked. Generally, the more complex are the constituents and the internal structure of the rock, the higher is the number of independent parameters necessary to identify its properties. As is well known, two mechanical parameters are required to describe the state of deformation of a compact medium if utmost simplifications are adopted; if porosity is accounted for, at least four parameters are needed. The subject of this paper is restricted to the latter type of media, i.e. to poroelasticity based on quasi-static, infinitesimal, isothermal, reversible deformation of fully saturated, macroscopically homogeneous and isotropic porous rocks with a connected pore structure. Whether these simplifications are suitable for evaluation of field tests has to be proven in practice.

Throughout the paper, the term 'rock' refers to both

consolidated and unconsolidated rocks. Thus, it also applies to sediments that are aggregates of solid grains. The poroelasticity concept used here is phenomenological and is independent from mixture theories. A priori, no restrictions are made as to the shape of pores or cracks. Isotropic behaviour of representative rock volumes, however, demands any voids to be arranged in some statistically balanced, omnidirectional order.

Each of the mechanical parameters to be discussed in subsequent sections allows the description of a state of deformation due to steady pressure conditions. As in elasticity theory, a fundamental prerequisite to this is the existence of equilibrium between stresses and deformation. In poroelasticity, the conservation of pore fluid mass is of similar importance. Biot (1941) succeeded in combining this principle with the elasticity equilibrium conditions to a conclusive theory. Previously, Terzaghi (1923) established a formulation for 1-D deformation.

In naming his work *Theory of Consolidation*, Biot (1941) applied the term 'consolidation' in its soil-mechanical sense, according to which a sedimentary rock consolidates through the loss of pore fluid. When using this term, it is often implied that the solid and fluid phases of the rock are incompressible. In its geological sense, consolidation also means solidification through diagenetic processes such as cementation and recrystallization (American Geological Institute 1976). To avoid confusion, the term *poroelasticity theory* is preferred (see also Geertsma 1973). Recent studies on poroelastic phenomena often apply Rice & Cleary's (1976) formulation of the theory. This makes use of rock parameters which are more easily interpreted than those introduced by Biot. Both the solid and the fluid phases are assumed to be compressible.

Sections 2, 3, and 4 form the main part of the paper. First, the governing equations for 3-D poroelastic deformation are given. Then, the mechanical rock parameters are described in some detail. Section 4 deals with the hydraulic diffusivity of poroelastic phenomena. In Sections 5 and 6, various extensions of the theory are outlined, and the more frequently used parameters are presented in a table. Further useful relations are noted in the Appendix. (Annotations regarding nomenclature or special conventions are inserted in square brackets.)

2 GOVERNING EQUATIONS

In continuum mechanics, grossly speaking, it is common to define stress normal to the surface (tensile stress) to be positive. This convention is also used here. Then, *compressive stress*, σ_{ii}, is negative and has a different sign than *pore pressure*, P, and *confining pressure*, P_c (Fig. 1). In fields like rock or soil mechanics, tectonics, and seismology, compressive stress is generally agreed to be positive, and thus has the same sign as pore or confining pressure. The sign of *normal strain*, e_{ii}, is almost always defined in the same sense as that of its generating *stress*. [An exception is e.g. the work of Narasimhan, Kanehiro & Witherspoon (1984).] The convention adopted here is the same as in articles of Biot (1941), Bredehoeft (1967), Rice & Cleary (1976), Rudnicki (1985), and others. Jaeger & Cook (1979), Turcotte & Schubert (1982), Van der Kamp & Gale (1983), and Hsieh, Bredehoeft & Rojstaczer (1988), for instance,
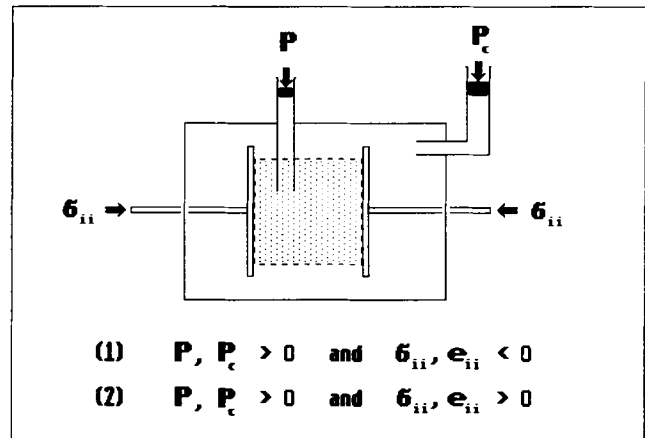

Figure 1. Sign conventions for pore pressure, P, confining pressure, P_c, normal stress, σ_{ii}, and normal strain, e_{ii}, due to σ_{ii}. The rock sample (shaded) is placed in a vessel completely filled with liquid. (1) Common in continuum mechanics, (2) common in rock/soil mechanics, tectonics and seismology. Convention (1) is adopted here.

note compressive stress and volume reduction to be positive. With this convention, the signs of the relevant expressions in the following equations alter accordingly.

If a (porous) rock is fully saturated, macroscopically both .isotropic and homogeneous, and if solid and fluid phases are chemically inert, and inertia forces and temperature changes negligible, the complete set of differential equations for variations of stress or pressure in regimes of constant rock parameters and constant hydraulic diffusivity reads

$$G\nabla^2 u_i + \frac{G}{1-2v}\frac{\partial \Delta_V}{\partial x_i} = \alpha \frac{\partial p}{\partial x_i} \tag{1}$$

$$Q^{-1}\frac{\partial P}{\partial t} + \alpha \frac{\partial \Delta_V}{\partial t} = \varkappa \nabla^2 P \tag{2}$$

with x_i = coordinate in orthogonal, spatially fixed system ($i = 1, 2, 3$), $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$ = Laplacian operator, t = time, u_i = displacement of a matrix particle parallel to the axis x_i, $\Delta_V = e_{11} + e_{22} + e_{33}$ = volume dilatation ($e_{ii} = \partial u_i/\partial x_i$), P = excess pore pressure (= excess pressure of the pore fluid), G = shear modulus, v = Poisson ratio for drained conditions, α = dimensionless coefficient of effective stress, Q^{-1} = compressibility introduced by Biot (1941), and $\varkappa$ = Darcy conductivity (in length^3 × time × mass^{-1}). The variables are the three displacements u_i and the excess pore pressure P with respect to an initially static situation. P does not comprise parts of the increase in hydrostatic pressure with depth. In Rice & Cleary's (1976) notation, α and Q^{-1} are expressed by v_u, the Poisson ratio for undrained conditions, and the Skempton ratio (or pore pressure parameter) B, where

$$\alpha = \frac{3(v_u - v)}{(1 - 2v)(1 + v_u)B}, \tag{3}$$

and

$$Q^{-1} = \frac{9}{2}\frac{(1 - 2v_u)(v_u - v)}{(1 - 2v)(1 + v_u)^2 GB^2}. \tag{4}$$

v_u and B are more easily obtained from laboratory experiments, e.g. from undrained tests, than Q^{-1}. The exact meanings of the parameters will become clear in Section 3.

Equation (1) results from combining the equilibrium conditions for volume elements,

$$\sum_{j=1}^{3} \partial\sigma_{ij}/\partial x_j = 0, \qquad i = 1, 2, 3, \tag{5}$$

with Hooke's generalized linear law, extended for poroelastic bodies,

$$2Ge_{ij} = \sigma_{ij} + 3v/(1+v)P_c\,\delta_{ij} + (1-2v)/(1+v)\alpha P\,\delta_{ij}, \tag{6}$$

where the σ_{ij} $(=\sigma_{ji}$ for $i \neq j)$ are shear strains, $P_c = -(\sigma_{11} + \sigma_{22} + \sigma_{33})/3$ is the increase in confining pressure or decrease in total normal stress, $2e_{ij} = \partial u_j/\partial x_i + \partial u_i/\partial x_j = 2e_{ji}$ for $i \neq j$ are shear strains, and δ_{ij} is the Kronecker symbol. Biot (1941) extended Hooke's Law for elastic bodies (a) by adding the pore pressure term to the stress–strain relations which include dilatational components and (b) by replacing the shear modulus and the Poisson ratio of compact bodies through the rock parameters G and v.

Appearance of P as an additional variable to the displacements u_i requires introduction of a fourth relation aside the three versions of equation (1); that is equation (2). It is obtained by substituting Darcy's Law for laminar flow through a hydraulically isotropic matrix (absence of body forces),

$$q_i = -\varkappa\,\partial P/\partial x_i, \tag{7}$$

into the linearized relation for fluid mass conservation (Rice & Clearly 1976),

$$(m - m_0)/\rho_0 = \alpha c(P/B - P_c), \tag{8}$$

i.e. by setting

$$\partial\left(\frac{m - m_0}{\rho_0}\right)\bigg/\partial t = -\sum_{i=1}^{3} \partial q_i/\partial x_i = \varkappa\nabla^2 P. \tag{9}$$

$m - m_0$ denotes the increment of fluid mass per unit rock volume, ρ_0 is the density of the pore fluid for zero excess pore pressure, q_i is the specific discharge or Darcy velocity of the pore fluid (i.e. the loss of fluid volume per time and per unit rock area normal to x_i), and $c = 3(1 - 2v)/[2(1 + v)G]$ is the compressibility of the matrix for drained conditions. P_c in equation (8) may be found to be $\alpha P - c^{-1}\Delta_v$ by summing up equation (6) for the normal components, and $\alpha c/B$ is equivalent to $Q^{-1} + \alpha^2 c$. $[(m - m_0)/\rho_0$ corresponds to the 'variation in water content', θ, of Biot (1941). He originally introduced new material parameters R and H in expressing the right-hand side of equation (8) as $P/R - P_c/H$.]

Equations (6) and (8) are the constitutive equations in poroelasticity. G, v, v_u, B (or G, v, α, Q) are the four independent poroelastic parameters which describe the behaviour of porous rocks within this theory. The fifth parameter in equations (1) and (2), the Darcy conductivity $\varkappa$, reflects a hydraulic property of the system. It enters through Darcy's Law and denotes the ratio of intrinsic

permeability, k, to dynamic viscosity of the pore fluid, η, e.g. in units Darcy/Poise (1 Darcy/Poise $= 0.98697 \times 10^{-8}\,\mathrm{cm^3\,s\,g^{-1}}$). The properties k and η do not appear as individual parameters in poroelasticity. [The term 'Darcy conductivity' is actually suggested here, because no other term seems to be agreed upon. Biot (1941) names $\varkappa$ (his k) 'coefficient of permeability'. This expression, however, is usually taken as an equivalent of the hydraulic conductivity, the dimension of which is length × time^{-1} (Davis & De Wiest 1966). In Rice & Cleary (1976), and Rice, Rudnicki & Simons (1978), $\varkappa$ (their κ) is named 'permeability'.]

Since Darcy's Law is only valid for saturated media, the above equations cannot simply be applied to rocks which are partially saturated or dry. Also, only in saturated media can the parameters for undrained conditions, v_u and B, assumed to be constant. Both are functions of the compressibility of the pore filling which strongly depends on pressure when gas is present. [Formulations of the theory regarding partially saturated media can be found in many text books of petroleum science. They usually ignore compressibility of the solid phase and of the matrix.]

Rice & Cleary propose to use the stresses σ_{ij} as unknowns alongside P, rather than the displacements u_i. Then, in addition to the equations below, the compatibility conditions for strains must be fulfilled for evaluating the altogether seven unknowns (e.g. Jaeger & Cook 1979). The six independent equations replacing equation (1) become

$$\nabla^2[(1 + v)\sigma_{ij} + 3vP_c\,\delta_{ij}] - 3\frac{\partial^2 P_c}{\partial x_i\,\partial x_j}$$

$$+ \frac{3(v_u - v)}{(1 + v_u)B}\left(\nabla^2 P\,\delta_{ij} + \frac{\partial^2 P}{\partial x_i\,\partial x_j}\right) = 0. \tag{10}$$

The seventh equation is equation (2) in P and P_c, that is

$$\frac{9(v_u - v)}{2(1 + v)(1 + v_u)GB}\frac{\partial}{\partial t}(P/B - P_c) = \varkappa\,\nabla^2 P, \tag{11}$$

whereby the useful relation

$$\Delta_v = c(\alpha P - P_c) \tag{12}$$

following from Hooke's generalized linear law and equations (3) and (4) have been applied.

Unlike static deformations of purely elastic media, poroelastic deformations are time-dependent phenomena. They show complete analogy with linear, fully coupled thermoelastic deformations. There, Fourier's law of heat flux takes a role similar to Darcy's Law (Biot 1956a; Rice & Cleary 1976). Formulations for plane strain and axisymmetric poroelastic conditions are given in Appendix A. Tables of equivalent expressions for the coefficients in equations (1) and (2) may be found in Appendix B (Tables B1 and B2). Analytical solutions of the governing equations only exist for simple geometrical configurations. Solutions for fluid injection through point sources, for instance, were published by Rice & Cleary (1976), Cleary (1977; but see Rudnicki, 1981), or Rudnicki (1986a). Many solutions have been calculated using numerical techniques (e.g. Christian & Boehmer 1970; Yokoo, Yamagata & Nagaoka 1971a,b; Safai & Pinder 1980; Lewis & Schrefler 1987; Shi & Wang 1988.)

3 MECHANICAL ROCK PARAMETERS

3.1 Regimes of validity

Some general notes on (poro)elastic parameters, the conditions of drainage, and consequences of static and dynamic evaluation techniques will help to clarify the circumstances under which the parameters are applicable.

Elastic and poroelastic parameters

The simplest model of a rock is a *compact*, macroscopically isotropic and homogeneous body. If exposed to small, quasi-static, mechanical forces, its state of deformation can be described by using any two of the five mechanical parameters of compact media, which are in common use, namely: E_*, the modulus of elasticity or Young's modulus, c_*, the compressibility or inverse of the bulk modulus, G_*, the shear modulus or modulus of rigidity, or first Lamé parameter, λ_*, the second Lamé parameter, and v_*, the Poisson ratio (e.g. Jaeger & Cook 1979). [The index '$*$' is set to expose the distinction between compact and porous medium parameters.] A table of all possible relations among these parameters is given by Gassmann (1951).

Generally, E_*, c_*, G_*, λ_*, and v_* are not constants but depend on the prevailing stress and the body's temperature. Yet, for minor pressure changes under isothermal conditions they can be regarded as approximately constant. The body's behaviour is then named 'linear'. The body itself is called 'elastic', and the five parameters 'elasticity parameters'. The relationship between pressure increments and deformation is reversible. In many problems, rocks react like compact, elastic bodies. A sufficiently accurate description of their state of deformation can be obtained when the confining pressure P_c and any stresses σ_{ij} on the surfaces are known.

In the Earth's crust, all rocks are *porous*. They comprise a solid phase, the rock matrix or skeleton, and a liquid and/or gaseous phase, the pore fluid. The rock is called saturated when the pore volume is entirely filled with pore liquid. We will assume that all constituents of the rock are chemically inert and that neither additional pores nor cracks are created by dissolution or stress changes. If V_n denotes the pore volume and V_s the volume of the solid phase, $V = V_s + V_n$ is the bulk volume, and $n = V_n/V$ is the (volume) porosity of the rock.

The pore volume of a rock may not be totally connected; in particular may individual pores be isolated from each other. Rocks of this type will not explicitly be considered here. Their behaviour can be described when the terms porosity and pore volume are redefined in that they only refer to pore space containing *mobile* pore fluid; i.e. to volume, which significantly contributes to the rock's permeability. In such case, the rock's matrix is inhomogeneous, and V_s is the volume of both the solid phase and the *immobile* fluid phase. However, equations (1) and (2) assume a connected pore structure (Rice & Cleary 1976).

In pressure and temperature ranges, in which the appropriate rock parameters can be regarded as being constant, they are named 'poroelasticity parameters'. Correspondingly, a porous body is termed 'poroelastic', when the relationship between increments of pore and confining pressures on the one hand, and deformation on the other, is reversible. A frequently used lax term is 'poroelastic parameters' (and 'elastic parameters', respectively).

Drained and undrained parameters

The behaviour of rocks, the porosity of which cannot be neglected in context of an investigated phenomenon, may be studied under drained and undrained conditions. Accordingly, two sets of mechanical rock parameters appear. The parameters will be denoted as E, c, G, λ, v for the drained case and E_u, c_u, G_u, λ_u, v_u for the undrained case. Again, somewhat lax but common terms are 'drained' and 'undrained' parameters. Within each group, the same dependences among individual parameters exist as for the parameters of compact media.

Both draining conditions are best understood from jacketed laboratory tests. Undrained conditions prevail if the rock sample is subjected to a change in confining pressure and the pore fluid is prevented from escaping or entering by a thin, impermeable skin (Fig. 2a). If, instead, the pore fluid is enabled to leave or enter the probe, e.g. via a drainage tube, so that after some time of adjustment the pore pressure attains its original value in all parts of the sample, conditions are drained (Fig. 2b). Immediately after a change in confining pressure, i.e. before the pore fluid starts to flow, undrained conditions also exist in this case. Obviously, pore pressure and confining pressure are independent variables. In poroelasticity, they are coupled through the equations (6) and (8).

As pointed out by Rice & Cleary (1976) and Rice & Rudnicki (1979), the term 'undrained deformation' can only be applied to a rock volume in which uniform (with respect to hydrostatic) pore pressure prevails, i.e. in which P can be represented by a single figure. Since our approach is macroscopic, we are not dealing with an arbitrarily small volume. Instead, we may only consider volumes that are large compared to the size of the largest pore contained therein.

A fundamental prediction of Biot's theory is that shear stresses are decoupled from pore pressure [see equation (6) for $i \neq j$] and, hence, the shear modulus does not depend on the conditions of drainage. Consequently, the drained shear modulus must equal the undrained shear modulus,

$$G = G_u. \tag{13}$$

From the interdependencies among the parameters follow

$$E_u = \frac{1 + v_u}{1 + v} E, \tag{14a}$$

$$c_u = \frac{(1 - 2v_u)}{(1 - 2v)} \frac{(1 + v)}{(1 + v_u)} c, \tag{14b}$$

$$\lambda_u = \frac{(1 - 2v)}{(1 - 2v_u)} \frac{v_u}{v} \lambda. \tag{14c}$$

Since the ratio between lateral extension and longitudinal shortening is always larger for undrained than drained conditions, we have $v \leq v_u \leq 0.5$, and $E \leq E_u$, $\lambda \leq \lambda_u$, $c \geq c_u$. Differences between values of drained and undrained parameters become smaller, the more the draining conditions
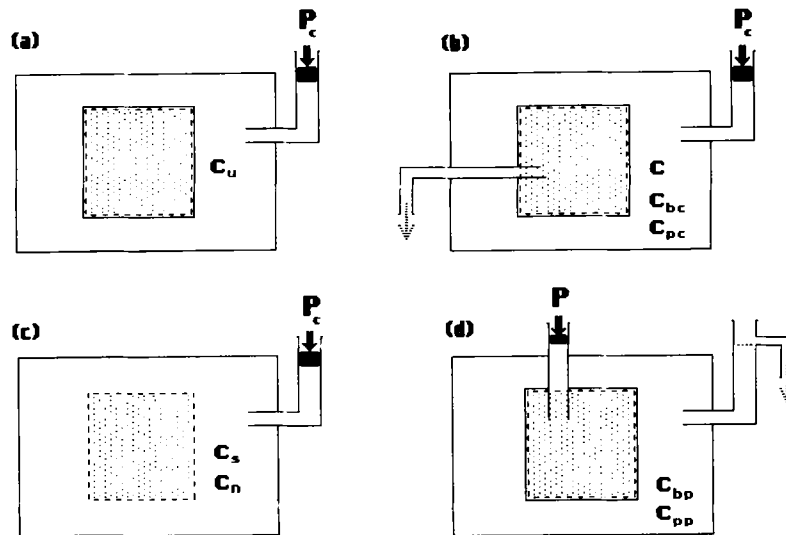
Figure 2. Schematics of compressibility measurements. Rock samples in (a), (b), (d) are sealed by thin, impermeable skin (jacketed tests), sample in (c) is unsealed (open pressure test). c_u = undrained compressibility, c = drained or matrix compressibility, c_s = grain compressibility, c_n = pore compressibility, c_{bc}, c_{pc}, c_{bp}, c_{pp} = compressibilities discussed by Geertsma (1957) and Zimmerman *et al.* (1986).

become irrelevant; that is, the less compressible the matrix is ($v \to 0.5$), the more compressible the pore filling is ($v_u \to 0$), the more the structure of the matrix prevents fluid flow, or the higher the viscosity of the pore filling is ($\varkappa \to 0$, respectively). [Skempton (1954) expresses a possible coupling between shear stresses and pore pressure by his dimensionless parameter A, whereby $A = 1/3$ denotes complete decoupling, equivalent to equation (13). In shear tests with unconsolidated sediments he showed that A can significantly differ from $1/3$, depending on the confining pressure or on the degree of deformation; thus, equation (13) appears to be a stringent condition for real rocks.]

Equation (6) reveals that internal pore pressure disturbances may also deform a rock. A pore pressure disturbance exists, if the pore pressure within the rock is not hydrostatically balanced, i.e., if in some part of the rock the pore pressure deviates from the product of the gravitational acceleration, the density of the pore fluid, and the distance to the relevant equipotential surface. In such a case, the excess pore pressure attempts to displace material to (re-) establish hydrostatic conditions. Since the pore fluid is more mobile than the rock matrix, relative movements occur, leading to friction. The greater the pore pressure gradient, the viscosity of the pore fluid, and the contact area between pore fluid and rock matrix, the stronger are frictional forces and rock deformation during pore pressure equalization. A comprehensive article on these phenomena is that of Rumer (1969).

Static and dynamic parameters

The compressibility and the shear modulus of a rock sample (likewise the other dependent parameters) are related to the seismic P-wave and shear-wave velocities, v_P, v_S, by $v_P = [(\tilde{c}_u^{-1} + 3/4\tilde{G})/\rho]^{1/2}$ and $v_S = (\tilde{G}/\rho)^{1/2}$. ρ is the bulk density of the rock. The tilde sign is used to indicate that both moduli are of seismic, i.e. dynamic origin, as opposed to static origin when deduced from conventional undrained

compression and shear tests. Comparison of values obtained from both techniques shows considerable scattering, in particular for unconsolidated sediments (Ohkubo & Terasaki 1977; Schön 1983). Depending on the frequency of the deforming forces, rock parameters can therefore be classified into 'static' and 'dynamic' parameters. Static and dynamic processes in this context generally differ in the drainage conditions, the relevance of turbulent flow in the pore space, the amplitudes of stresses and strains, and the significance of inertia terms. The first two points are inherent to porous media; the latter two equally apply to compact elastic bodies.

Clearly, drained conditions do not prevail during the passage of a seismic wave. Deformation from high-frequency (short wavelength) seismic waves may not meet the criteria of undrained conditions either, as only volumes of large extent (compared to the size of the largest pore or crack) are considered. According to Rice & Rudnicki (1979), an estimate of the drained rock parameters can nevertheless be obtained from seismic measurements, when the measurements are carried out on equivalent gas-saturated (dry) samples, because the gas offers little resistance to deformation forces. Similarly, resistance to pore fluid flow during slow loading of saturated rock is slight under drained conditions. Turbulent flow in any case is neglected when Darcy's Law is used in the form of equation (7). We merely point out here that discrepancies may occur when investigations are extended over different frequency regimes. All parameters discussed in the subsequent are meant to reflect rock properties under quasi-static conditions.

3.2 Compressibility parameters

The significance of different types of compressibilities is of fundamental importance in poroelasticity. It is therefore sensible to look at them in more detail.

Different ways exist to arrive at parameters denoting

relations between volume dilatation of a rock sample and the generating pressure change: a phenomenological one, from the analysis of laboratory tests, a physical one, from investigation of formal relationships between the involved quantities, and an analytical one, as from application of the mixture theory. Parameters derived from laboratory tests are widely used in literature and will be discussed in the following. Expressions obtained in the other ways will be mentioned for comparison reasons, only.

Phenomenological approach

The *undrained compressibility*, c_u, may be obtained from a jacketed test like that in Fig. 2(a). It is defined by

$$c_u = -\frac{1}{V}\left(\frac{\partial V}{\partial P_c}\right)_m. \tag{15}$$

Since the volume change is negative for increasing confining pressure, the minus sign ensures that numerical values of c_u will be positive. [The notation is consistent with that of Rice *et al.* (1978). Likewise, Palciauskas & Domenico (1982) refer to $K_u = c_u^{-1}$ as the 'undrained bulk modulus'. In Brown & Korringa (1975), c_u ($= \kappa^*$ in their paper) is named 'effective compressibility'. Other authors use the symbol K^{-1} (Thomsen 1985), or $\bar{\beta}$ (Rojstaczer & Agnew 1989).]

The *drained compressibility* or *matrix compressibility*, c, is defined by

$$c = -\frac{1}{V}\left(\frac{\partial V}{\partial P_c}\right)_P. \tag{16}$$

It may be measured in a drained jacketed test as in Fig. 2(b). [Some authors, like Narasimhan *et al.* (1984), use c with subscript '$_m$' to show that c is a parameter of the matrix, and not of the solid phase. If no subscript is written, the connection to c_u for 'undrained compressibility' is emphasized. Skempton (1960) uses C in such a way (C_c, in Skempton 1954). Accordingly, many authors use K for c^{-1} (Nur & Byerlee 1971; Cornet & Fairhurst 1974; Rice & Cleary 1976; Rudnicki 1985). Instead of c, one also finds κ (Biot & Willis 1957), κ_A (Brown & Korringa 1975), κ_b (Morland & Donaldson 1984), c_b (Geertsma 1957; Domenico 1977), $1/K^*$ (Thomsen 1985), β (Green & Wang 1986; Rojstaczer 1988), or α_b (Shi & Wang 1986, 1988). The subscript '$_b$' stands for 'bulk'.]

Another measure of compressibility, mostly referred to as *grain compressibility*, c_s, is defined by

$$c_s = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{P=P_c}. \tag{17}$$

It may be obtained from an open pressure test. In such a set-up, the rock sample is not jacketed so that incremental confining pressure may enter the rock via both the matrix and the pore fluid (Fig. 2c). Due to differing compressibilities, the liquid and solid phases reallocate the available space. After a period of adjustment, the change in pore pressure will be the same everywhere inside the sample and thus be identical to the change in confining pressure.

Unless the rock contains unconnected pores, c_s is identical to the compressibility of a compact medium solely made of the rock's solid phase, $c_* = -1/V_*(\partial V_*/\partial P_c)$. In fact, few of the authors dealing with deformation of porous media

distinguish between c_* and c_s. As noted above, equations (1) and (2) implicitly assume that the deformation of the solid material under various combinations of normal stress and pore pressure can be characterized by the single parameter c_s (Rice & Cleary 1976; Van der Kamp & Gale 1983).

[Skempton (1960) and Biot (1973) use the symbol C_s. The subscript '$_s$' stands for 'solid phase'. Nur & Byerlee (1971) use K_s for c_s^{-1}, naming it 'grain modulus', whereas Green & Wang (1990) note 'unjacketed bulk modulus'. Rice & Cleary (1976), and Rudnicki (1985), who reserve K_s for c_*^{-1}, use K_s' instead. Other symbols for grain compressibility found in the literature are δ (Biot & Willis 1957; Morland & Donaldson 1984), κ_M (Brown & Korringa 1975), K_i^{-1}, with '$_i$' for 'intrinsic' (Cornet & Fairhurst 1974), c_r, with '$_r$' for 'rock' (Geertsma 1957; Zimmermann, Somerton & King, 1986), α_s (Shi & Wang 1986, 1988), β_s (Hamilton 1971), and β_u (Green & Wang 1986; Rojstaczer 1988). The latter choose the subscript '$_u$' referring to '(drained) unjacketed test'.]

The compressibility of the pore space, or *pore compressibility* (Biot & Willis 1957; Brown & Korringa 1975), may be derived from the porosity change in an open pressure test (Fig. 2c). Its defining relation is

$$c_n = -\frac{1}{V_n}\left(\frac{\partial V_n}{\partial P}\right)_{P=P_c}. \tag{18}$$

In case the porosity does not vary for equal changes in P and P_c we find

$$c_n = -\frac{1}{nV}\left(n\frac{\partial V}{\partial P}\right)_{P=P_c} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{P=P_c} = c_s. \tag{19}$$

This applies when the immobile part of the rock is homogeneous. Then, identical pressure increments act on all internal and external surfaces of the matrix, producing homogeneous deformation, which reduces the matrix linearly, i.e. $n = $ constant. Values of c_n and c_s for real rocks are mostly in the same order of magnitude or even quite similar to one another (Rice & Cleary 1976). This explains why both compressibilities are frequently used as equivalents in literature. Green & Wang (1986) point out that considerable differences may also occur. [c_n does not explicitly appear in the paper of Biot & Willis (1957); instead, they express it as $c_f - \gamma/n$ ($= c - \gamma/f$ in their article), whereby γ is introduced as 'coefficient of fluid content'. In a more recent paper, Biot (1973) uses the symbol c_m (cf. Brown & Korringa 1975). Other authors use κ_ϕ (Brown & Korringa 1975) or β_ϕ (Green & Wang 1986), with '$_\phi$' being another common symbol for porosity, or K_p^{-1} (Thomsen 1985). Rice & Cleary (1976) and Rudnicki (1985) note K_s'' for c_n^{-1}.]

Neither the fluid compressibility $c_f = -1/V_f(\partial V_f/\partial P)$, nor the pore compressibility c_n, nor the porosity n are individual parameters in poroelasticity. Instead they appear as constituents of the expression

$$\gamma = n(c_f - c_n) \tag{20a}$$

(Biot & Willis 1957; Green & Wang 1986). We will name γ the 'Biot–Willis parameter' hereafter. When used as a

poroelastic parameter in equations (1) and (2),

$$\gamma = n(c_f - c_s) \qquad (20b)$$

is assumed.

In summary, three compressibilities can be used as poroelastic parameters: the undrained compressibility, c_u, the drained or matrix compressibility, c, and the grain (or solid phase) compressibility, c_s. They may be used to replace three of the parameters G, v, v_u, B in the governing equations. As a fourth parameter reflecting some volume strain property of saturated rock, the Biot–Willis parameter γ may be chosen. Conforming expressions are given in the Appendix (Table B3). Note that c and c_s exclusively reflect properties of the solid phase; c_u and γ also depend on the compressibility of the pore filling.

As mentioned in Section 2, Biot (1941) introduced the compressibilities H^{-1}, named 'a measure of the compressibility of the soil for a change in water pressure', and R^{-1}, which 'measures the change in water content for a given change in water pressure'. According to Biot & Willis (1957), Geertsma (1957), and Green & Wang (1986), these parameters are related to the aforementioned compressibilities by

$$H^{-1} = c - c_s \qquad (21a)$$

and

$$R^{-1} = c - c_s + \gamma. \qquad (21b)$$

Utilizing the relation $Q^{-1} = R^{-1} - \alpha H^{-1}$ of Biot (1941), Q^{-1} can be written as

$$Q^{-1} = \alpha c_s + \gamma. \qquad (22)$$

[In his classical paper, Biot (1941) assumed the pore fluid to be incompressible ($c_f = 0$), but did not explicitly make use of this. He later realized that the theory in its original form is applicable to saturated (!) porous media containing compressible fluid (e.g. Biot 1955; Biot & Willis 1957). Rice & Cleary (1976) erroneously stated $R^{-1} = H^{-1} - nc_n$, instead of equation (21b) (Green & Wang 1986).]

Other approaches

A more physical access to rock compressibilities has been presented by Geertsma (1957) and Zimmerman *et al.* (1986). They define four parameters, only one of which, the drained compressibility c, has already been considered. The parameters result from *the various relationships between changes in V, V_n and P, P_c*:

$$c_{bc} = -\frac{1}{V}\left(\frac{\partial V}{\partial P_c}\right)_P = c, \qquad (23a)$$

$$c_{bp} = \frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{P_c}, \qquad (23b)$$

$$c_{pc} = -\frac{1}{V_n}\left(\frac{\partial V_n}{\partial P_c}\right)_P, \qquad (23c)$$

$$c_{pp} = \frac{1}{V_n}\left(\frac{\partial V_n}{\partial P}\right)_{P_c}. \qquad (23d)$$

[Left subscripts denote the reference volume, i.e. '$_b$' for 'bulk volume' and '$_p$' for 'pore volume', right subscripts the

altered pressure, '$_c$' for 'confining pressure' and '$_p$' for 'pore pressure'. The first two compressibilities are referred to as 'bulk compressibilities' by Zimmerman *et al.* (1986), the latter two as 'pore compressibilities'.] Experimentally, the situations described in equations (23a,c) are established by drained jacketed tests (Fig. 2b), those described in (23b,d) are represented in Fig. 2(d).

From differential calculus follows

$$c_s = c_{bc} - c_{bp}, \qquad (24)$$

$$c_n = c_{pc} - c_{pp}. \qquad (25)$$

Zimmerman *et al.* (1986) also show, that in case of elasticity

$$c_{pc} = (c - c_s)/n, \qquad (26a)$$

$$c_{pp} = [c - (1+n)c_s]/n \qquad \text{for } c_s = c_n, \qquad (26b)$$

$$c_{pp} = (c - c_s)/n - c_n \qquad \text{for } c_s \neq c_n. \qquad (26c)$$

[Equation (26b) was also given by Brown & Korringa (1975) and by Morland & Donaldson (1984). The former use κ'/n instead of c_{pc}, the latter use κ_p. Morland & Donaldson call c_{pc} 'formation compaction'. Moreover, they use κ_e for nc_{pp}, naming it 'effective rock compressibility'. Together with c and c_s (κ_b and δ in their article), Morland & Donaldson (1984) thus use the same compressibilities as Geertsma (1957) and Zimmerman *et al.* (1986), which they, however, deduce from the mixture theory.]

An expression for rock compressibility based on the *mixture theory* is

$$c_{mix} = nc_f + (1-n)c_s \qquad (27)$$

(e.g. Hamilton 1971). Herein, the compressibilities of the fluid and solid phases are averaged according to their volume proportions. This neglects pore fluid mobility and a possible dependence of porosity on pressure. Obviously, c_{mix} expresses some sort of rock compressibility for constant porosity n, under undrained conditions. It equals Biot's Q^{-1} when $c \gg c_s = c_n$ (see equation 22).

3.3 Parameters of effective pressure, pore pressure and fluid storage

Other parameters often utilized to describe poroelastic phenomena are the coefficient of effective stress, α, the Skempton ratio, B, and various storage coefficients. Their significance in poroelasticity, like that of the compressibilities, deserves a closer look as well as notification of interrelations with other parameters.

The coefficient of effective stress

The principle of effective stress (or effective pressure) states that deformation and strength of porous bodies are neither proportional to the confining pressure nor to the pore pressure but to the difference between them, named 'effective pressure', P_e. The principle was established empirically by Terzaghi (1925) for saturated clay.

Geertsma (1957) and Skempton (1960) base the following relation on experiments with various rocks:

$$P_e = P_c - (1 - c_s/c)P = P_c - \alpha P. \qquad (28)$$

It does not contradict Terzaghi's results because, in case of

saturated clay, $c_s \ll c$. Nur & Byerlee (1971) have theoretically established this relation. Garg & Nur (1973) set out the importance of constant matrix compressibility c for the validity of equation (28), i.e. that it only holds for infinitesimal changes of P_c and P. The coefficient

$$\alpha = (1 - c_s/c) \qquad (29)$$

is also called 'effective stress coefficient' [Zimmerman et al. (1986); see their paper for a more general form of the effective stress concept], 'soil–water interaction coefficient' (Schiffmann 1970; Morland 1978) or 'coefficient of internal deformation' (Christensen & Wang 1985). Green & Wang (1990) apply the term 'effective stress' to any linear combination of stress and pore pressure, suggesting interpretation of this term within the context of the physical property being discussed. Then the coefficient of effective stress needs to be specified, respectively. Note that by application of the effective stress concept, the number of pressure variables can be reduced from two to one. For instance, from (12), the matrix compressibility may be written as $c = -\Delta_V/P_c$.

From equations (23a,c) and (26a) follows

$$\alpha = \frac{c - c_s}{c} = n \frac{c_{pc}}{c_{bc}} = \frac{nV}{V_n} \left(\frac{\partial V_n}{\partial V} \right)_P = \left(\frac{\partial V_n}{\partial V} \right)_P. \qquad (30)$$

Accordingly, α indicates the change in pore volume per unit change in bulk volume under drained conditions. Because $c \geq c_s$, α takes values between 0 and 1. Schiffmann (1970) found that values of α generally fall in the range between porosity n and 1. Given that the microstructure of the rock is isotropic (as e.g. for a collection of spherical grains), a more sharply defined lower limit for α, resulting from the Hashin–Shtrikman bound for porous material, is

$$n \leq \frac{3n}{2+n} \leq \frac{3n(1-v_s)}{2(1-2v_s)+n(1+v_s)} \leq \alpha \qquad (31)$$

[after Zimmerman et al. (1986), by rearranging their equation (15); see also Watt, Davies & O'Connell (1976)]. Herein, v_s is the Poisson ratio for the solid fraction of the rock. Equality of all terms only holds for $n = 1$. A value close to 1 ($c \gg c_s$) is typical for unconsolidated sediments. [Less frequently used symbols for α are n, χ and ζ.]

The Skempton ratio

Another rock property of fundamental interest is the 'pore pressure parameter', B, introduced by Skempton (1954). It is defined as the change in pore pressure per unit change in confining pressure under undrained conditions, i.e.,

$$B = \left(\frac{\partial P}{\partial P_c} \right)_m. \qquad (32)$$

Other terms for B are 'induced pore pressure parameter' (Cleary 1977), 'Skempton's pore pressure coefficient' (Segall 1985; Rudnicki 1986a; McTigue 1986), 'undrained pore pressure buildup coefficient' (Green & Wang 1986), 'three-dimensional loading efficiency' (Neuzil 1986), or 'Skempton's coefficient' (Rojstaczer & Angew 1989; Green & Wang 1990). Palciauskas & Domenico (1982) and Narasimhan et al. (1984) refer to B as the 'tidal efficiency', a term introduced by Jacob (1940). In view of an increasingly

wide use of B as a poroelastic parameter, the notion 'Skempton ratio' as a precise and seasonably short term is suggested here. [Van der Kamp & Gale (1983) note β for B; they also discuss the form B takes for 1-D deformation, naming it 'loading efficiency'.]

Like α, B takes values between 0 and 1. An exact relation may be derived from the elaboration of Brown & Korringa (1975), namely,

$$B = \frac{\alpha c}{\alpha c + \gamma} = \frac{c - c_s}{c - c_s + n(c_f - c_n)} \qquad (33)$$

[see Green & Wang (1986), for a detailed discussion]. For $c_s \ll c$ (or $\alpha \approx 1$) and $c_n \ll c_f$, (33) is equivalent to the empirically found relation of Skempton (1954), $B = (1 + nc_f/c)^{-1}$. The expression

$$B = (c - c_u)/(c - c_s), \qquad (34)$$

given by Brown & Korringa (1975), yields $c_u = (1 - B)c + Bc_s$ and $\alpha B = 1 - c_u/c$. Consequently, $c_u = c_s$ requires $B = 1$, and $c_u = c$ holds if $B = 0$. As a function of v, v_u, α, the Skempton ratio may be obtained from equation (3), given by Rice & Cleary (1976). B is related to Biot's compressibilities H^{-1}, R^{-1}, Q^{-1} through $B = R/H = \alpha c/(Q^{-1} + \alpha^2 c)$.

Since the Skempton ratio is less frequently used than other rock parameters, some of its values for real rocks may be of interest. Laboratory techniques for determining B are described by Green & Wang (1986). Skempton (1954) showed experimentally that B lies close to 1 for fully saturated, unconsolidated sediments. For increasing gas saturation, B falls rapidly to low values. Various authors found a marked dependence on the effective pressure, P_e. For $P_e \approx 0$ MPa, B values close to 1 have been observed for sandstones, limestone, dolomite, marble, granite, halite, and clay (Mesri, Adachi & Ullrich 1976; Green & Wang 1986; McTigue 1986). Effective pressures around 10 MPa yield values between 0.33 and 0.69 (not determined for halite and clay). Calculated B values for three sandstones, two granites and a marble, deduced from other rock parameters that were obtained for low to moderately high effective pressures, are found to lie between 0.51 and 0.88 (Rice & Cleary 1976), but yield values as low as 0.23 when P_e goes up to 200 MPa (Roeloffs 1988).

In situ techniques to determine B are not yet known. Estimates from well level fluctuations induced by atmospheric pressure changes or tidal strain require assumptions of other formation parameters, like the grain compressibility and the Poisson ratio, and of a fixed ratio of horizontal to vertical strains (Bredehoeft 1967; Rhoads & Robinson 1979; Rojstaczer & Agnew 1989). Interpretation of tilt signals observed in the vicinity of pumped wells give in situ values of the quantity $\bar{B} = (1 + v_u)/(1 - v_u)B/3$ (Kümpel 1989). By this technique, $\bar{B}$ values ranging from 0.08 to 0.7 were found for saturated glacial deposits at shallow depth, and a value close to 0.6 for a granodiorite at 400 m depth, respectively.

Storage parameters

The potential for storage capacity of pore fluid reflects a rock property that, like α and B, is best understood as a function of compressibility. The 'storage coefficient',

sometimes called 'hydraulic capacity', is traditionally used by hydrologists when making statements about the productivity of aquifers. It was introduced as a dimensionless quantity by Theis (1935). According to Jacob (1940), it corresponds to 'the volume of water of a certain density released from storage within the column of aquifer underlying a unit surface area during a decline in (piezometric) head of unity'. When this parameter is divided by the thickness of the aquifer, the so-called 'specific storage coefficient' results, in dimension length^{-1}.

Bodvarsson (1970) adopts as storage coefficient the volume of pore fluid which per unit rock volume and unit pore pressure increment is stored in the aquifer. In this form, the parameter takes the dimension pressure^{-1}. The relationship between the various definitions is

$$S_T = MS_s = \rho_f g MS, \tag{35}$$

whereby S_T represents the storage coefficient after Theis, M is the thickness of the aquifer, S_s is the specific storage coefficient, ρ_f the pore fluid density, g the gravitational acceleration, and S the 'storage coefficient' according to Bodvarsson. We herein use Bodvarsson's notion because of formal advantages when relating it to other compressibilities and to α and B; but we will name it *storage compressibility* to expose its dimension.

Expressions for the storage compressibility found in the literature are

$$S = c + nc_f \tag{36}$$

[e.g. Cooper (1966), Bredehoeft (1967), Krauß (1974), Bear & Corapcioglu (1981a,b), Hsieh *et al.* (1988), Shi & Wang (1988); also Bodvarsson (1970), but he erroneously places c_s, his 'c_r', instead of c], or

$$S = (1 - n)c + nc_f \tag{37}$$

(Davis & De Wiest 1966; Robinson & Bell 1971; Rhoads & Robinson 1979), or

$$S' = (1 - \beta)(c - c_s) + \gamma, \tag{38}$$

with $\beta = 2(1 - 2v)\alpha/[3(1 - v)]$ (Van der Kamp & Gale 1983). Equations (36) and (37) assume incompressible grains ($c_s = 0$) and differ in that equation (36) expresses storage compressibility for a deforming rock matrix whereas (37) refers to a stationary system (Cooper 1966; De Wiest 1966; Delcourt-Honorez 1988). Both equations are often written with the vertical matrix compressibility, $c' = \alpha(1 + v)c/[3(1 - v)]$ (Geertsma 1973), instead of c [which occasionally leads to confusion; see e.g. Narasimhan *et al.* (1984) and Hsieh *et al.* (1988)]. Equation (38) is derived from Rice & Cleary's (1976) work, assuming zero horizontal strain ($e_{11} = e_{22} = 0$) and constant vertical load ($\sigma_{33} = 0$). Such conditions are conventionally applied to aquifers in hydrology. S' reduces to the expression of (37), written with c', if the matrix is much more compressible than the grains ($c \gg c_s$, i.e. $\alpha \approx 1$). Rojstaczer & Agnew (1989) deduce a modified form of (38), which is applicable to vertical (e.g. barometric) loading of horizontally layered aquifers.

If no restrictions with regard to the geometry and boundary conditions are made, the storage compressibility

takes the '3-D' form

$$S = \alpha c + \gamma = c - c_s + n(c_f - c_s) \tag{39}$$

(Van der Kamp & Gale 1983; also used by Neuzil 1986), which is identical to Biot's R^{-1}, see equation (21b). [Note that Van der Kamp & Gale (1983) set S' for the 3-D specific storage coefficient, that is $\rho_f g S$ here, and S_s for the 1-D coefficient, which is $\rho_f g S'$ here.] To avoid ambiguity, Green & Wang (1990) suggest using solely S' as in (38) as the storage parameter, i.e. to restrict its definition to the conventional aquifer conditions.

When taken as a poroelastic parameter in the constitutive equations, expressions with S' are naturally more complex than those with S in the form of equation (39). Because in the latter form S is related to c, α and B by

$$S = \alpha c / B, \tag{40}$$

substitution in equation (8) yields

$$(m - m_0)/\rho_0 = S(P - BP_c) \tag{41}$$

(Brockhoff 1990). Thus, S can be regarded as the fluid mass (of density ρ_0) released from a unit rock volume for a pressure change of amount $P - BP_c$. Van der Kamp & Gale (1983) showed that (11) also takes a simple form with S, namely,

$$S \frac{\partial}{\partial t}(P - BP_c) = \varkappa \nabla^2 P. \tag{42}$$

Moreover, since the validity of the conventional aquifer conditions is questionable in many, even classical hydrological problems (like fluid reservoir production or aquifer recharge), it is felt that the storage compressibility in the solely general case, i.e. in its 3-D form, is most appropriate when using it as a poroelastic parameter.

In principle, S may be determined through well testing (e.g. Brown *et al.* 1972; Krusemann & De Ridder 1973; Mattheß & Ubell 1983), or by analysing tidally or atmospherically forced well level fluctuations (e.g. Jacob 1940; Bredehoeft 1967; Robinson & Bell 1971; Van der Kamp & Gale 1983; Narasimhan *et al.* 1984; Rojstaczer & Agnew 1989). The latter procedures are restricted to confined aquifers and require an estimate of its compressibility. As shown by Hsieh, Bredehoeft & Farr (1987), the sensitivity of natural well level fluctuations for determining S is, however, low.

4 HYDRAULIC DIFFUSIVITY

By simple manipulations it is possible to derive a diffusion equation for the term $P/B - P_c$ (Rice & Cleary 1976). Summing up equation (10) for $i = j$ yields

$$\nabla^2 \left[\frac{2(v_u - v)}{(1 - v)(1 + v_u)} \frac{P}{B} - P_c \right] = 0 \tag{43}$$

or rather

$$\nabla^2(P/B - P_c) = \frac{(1 + v)(1 - v_u)}{(1 - v)(1 + v_u)B} \nabla^2 P. \tag{44}$$

Substituting $\nabla^2 P$ of the right-hand side into equation (11) and combining the coefficients leads to

$$\partial/\partial t(P/B - P_c) = D\nabla^2(P/B - P_c), \tag{45}$$

whereby

$$D = \frac{2(1-v)(1+v_u)^2}{9(1-v_u)(v_u-v)} xGB^2. \tag{46}$$

Rice & Cleary apply equations (10) and (45) to calculate σ_{ij} and P. The solutions are dependent on the three rock parameters v, v_u, B, and on the hydraulic property D (or xG). Dependence on one of the moduli E, G, c, or λ appears only if displacements or strains are to be calculated.

In literature, the parameter D is named 'diffusion constant', 'hydraulic diffusivity', 'fluid diffusivity' (of the term $P/B - P_c$, respectively), 'consolidation constant', or 'consolidation coefficient'. [c is another frequently used symbol for D, but conflicts with the matrix compressibility here.] Biot (1941) derived it in the equivalent form

$$D = \frac{2(1-v)xG}{(1-2v)\alpha^2 + 2(1-v)GQ^{-1}}, \tag{47}$$

for the diffusion equation in P, that is $\partial P/\partial t = D\,\partial^2 P/\partial x_i^2$. This equation follows from (45), when the conventional aquifer conditions $e_{11} = e_{22} = 0$; $\sigma_{33} = 0$ hold (Van der Kamp & Gale 1983).

It is useful to examine the forms D takes for some particular cases: when the compressibilities of the solid and liquid phases are neglected ($c_s = c_f = 0$; $\alpha = B = 1$; $v_u = 1/2$), D reduces to the simple form

$$D = 2\frac{1-v}{1-2v} xG = x/c' \tag{48}$$

($c' =$ vertical compressibility). If in (47) Q^{-1} is eliminated by making use of equation (22), we obtain

$$D = x[\gamma + \alpha(c' + c_s)]^{-1}. \tag{49}$$

Neglecting grain and matrix compressibilities, given that the fluid compressibility is rather high (αc, αc_s, $nc_s \ll nc_f$), yields

$$D = x/(nc_f). \tag{50}$$

This form is often used in petroleum sciences, together with the diffusion equation in P (e.g. Dake 1978).

From equation (49) it is evident that the hydraulic diffusivity is not only influenced by x and γ, but also by the compressibilities c_s and c. In most cases, $(\alpha - n)c_s$ will be much smaller than $nc_f + \alpha c'$. Therefore, a widely applicable form of D, not simplifying as strongly as equation (50), is

$$D = x/(nc_f + \alpha c'). \tag{51}$$

If, at least, c' is of the same magnitude as c_f, the diffusivity of pressure anomalies is significantly lower than expected from (50): or, deformation of the rock matrix retards the settling of a pore pressure anomaly.

A frequently used, simplified diffusion equation in hydrology is

$$\frac{\partial P}{\partial t} = \frac{T}{S_T'} \frac{\partial^2 P}{\partial x_i^2} \tag{52}$$

(e.g. Davis & De Wiest 1966; Mattheß & Ubell 1983). Here, $T = KM$ denotes the 'transmissivity' of a formation of thickness M, $K = \rho_f xg$ is the medium's hydraulic conductivity [also named 'coefficient of permeability', 'effective permeability', or 'seepage coefficient' (Davis & De Wiest 1966)], and $S_T' = \rho_f gMS'$ is the dimensionless storage coefficient for 1-D deformation after Theis (cf. equations 35 and 38). Comparison of coefficients shows that in this case

$$D = KM/S_T' = x/S'. \tag{53}$$

The various measures of hydraulic properties of rocks together with the different notions in use are summarized in Table 1.

Under natural conditions, the values of D cover a wide range. This is not surprising, since the intrinsic permeability k of rocks ranges from 10^{-17} to 10^{-3} cm^2, that is from 10^{-9} to 10^5 Darcy (Davis 1969; Brace 1984; Schopper 1984), and k is directly proportional to D. In situ values D may be obtained from pump tests. They are mainly established from the ratio of the transmissivity to the storage coefficient, according to equation (52), and are representative for the region in which pore pressure changes are distinct. Estimates for larger rock complexes are given by Li (1984/85). These are obtained from observations of induced

Table 1. Parameters of hydraulic rock properties and pore fluid flow. The first of several notions (if any) has been used in this text.

Parameter	Dimension	Notions	Remarks
D	L^2T^{-1}	hydraulic diffusivity, fluid diffusivity, diffusion constant, consolidation constant, consolidation coefficient	coefficient in diffusion equation
x	L^3TM^{-1}	Darcy conductivity	coefficient in Darcy's Law
k	L^2	(intrinsic) permeability	$x\eta$, where $\eta =$ dynamic fluid viscosity
K	LT^{-1}	hydraulic conductivity, coefficient of permeability, effective permeability, seepage coefficient	$\rho_f xg$, where $\rho_f =$ fluid density, and $g =$ gravitational acceleration
q_i	LT^{-1}	Darcy velocity, specific discharge	loss of fluid volume per unit time and unit rock area (normal to x_i)
T	L^2T^{-1}	(aquifer) transmissivity	KM, where $M =$ aquifer thickness

seismicity—due to water injection into boreholes, or water level changes in reservoirs—of transient seismic v_P/v_S anomalies, and the migration of underground explosion aftershocks. The values reported by Li fall in the range from 100 to $3 \times 10^5 \, \mathrm{cm^2 \, s^{-1}}$, independent of the rock type.

5 EXTENSIONS

Biot's and Rice & Cleary's equations portray a strongly simplified model of natural processes. Indeed, many authors have worked on extensions of the poroelasticity theory. The purpose of such efforts is to account for as broad a spectrum of observed physical phenomena as possible. This section aims to outline the variety of extensions that have been elaborated.

5.1 Anisotropy

Elastic properties of the solid phase and flow conditions for the pore fluid will seldom be isotropic. In orthotropic (= 3-D) anisotropy, the number of independent mechanical parameters for compact elastic bodies increases from 2 to 21; that for poroelastic media increases from 4 to 28. If anisotropic flow conditions prevail, Darcy's generalized Law could be applied. Herein, the Darcy conductivity $\varkappa$ appears as a symmetrical tensor of six different components. As a result, the number of differential equations to be resolved increases by two. The three gradients $\partial P/\partial x_i$ may be introduced as unknowns, replacing the excess pore pressure, P (Biot 1955; Oda, Hatsuyama & Ohnishi 1987).

The principle of effective pressure, too, may be extended to meet anisotropic conditions. According to Carroll (1979), three coefficients of effective pressure are needed for orthotropic anisotropy, and two for transversal (= 2-D) anisotropy. The relations between these coefficients and the poroelastic parameters are particularly simple, if the solid fraction of the rock is isotropic, that is, if anisotropy is solely caused by the structure of the pore space.

5.2 Non-linearities

Deviations from linear stress–strain behaviour may be due to various causes. Accordingly, different approaches are suggested for the modelling. Walsh (1965), O'Connell & Budiansky (1974), and Mavko & Nur (1978) describe pressure-dependent parameters for rocks with 'penny-shaped' cracks. Walsh & Grosenbaugh (1979) derive an effective compressibility for rocks with cracks of arbitrary form. Zimmerman *et al.* (1986) examine the dependence of compressibilities on effective pressure, P_e. The dependence of permeability on P_e is studied by Walsh & Brace (1984). Narasimhan & Kanehiro (1980) consider the variability of the storage coefficient as function of pore and confining pressure. Non-linear effects associated with the long-term compaction of sedimentary basins are studied by Bethke & Corbet (1988). Rice (1975, 1979) and Rice & Rudnicki (1979) deal with non-linear deformation prior to failure. Non-linear phenomena due to partly gaseous pore fluids in porous rocks are covered in text books of petroleum science (for instance Dake 1978; see also Narasimhan & Witherspoon 1977).

5.3 Inelasticity

If viscoelasticity plays a significant role, Geertsma (1957) suggests to introduce three 'poroviscous' material parameters, in addition to the poroelastic ones. According to Biot (1973), viscoelastic rheology of 'semi-linear' media—the solid and liquid phases of which react elastically if separated—can be described by frequency-dependent operators. O'Connell & Budiansky (1977) obtain relations that can be used to derive viscoelastic moduli of cracked rocks, whereby dissipation occurs either by viscous shear relaxation within the cracks, or by fluid flow between them. Small, Booker & Davis (1976) report that the transition from undrained to drained deformation can be described as elastoplastic behaviour of the rock matrix. Cleary (1977) presents equations for plastic deformation in anomalous regions of otherwise poroelastic media. Dislocations in poroelastic rocks are treated, for instance, by Rice & Cleary (1976), Roeloffs & Rudnicki (1984/85), Rudnicki (1985, 1986b, 1987), Advani *et al.* (1987), and Rudnicki & Hsu (1988).

5.4 Rapid pressure changes

A multitude of publications deals with the propagation of seismic waves in porous media. When modelling dynamic pressure changes, the equilibrium equations are expanded by inertial forces, being proportional to the acceleration of the rock particles, and Darcy's Law is modified to account for turbulent flow. A small selection of the discourses in this field are the articles of Biot (1956b, 1962), Bodvarsson (1970), Thomsen (1985), Boutin, Bonnet & Bard (1987), and the books of Bourbié, Coussy & Zinszner (1987) and Stoll (1989). Cleary (1978) points out some inherent difficulties that occur with rapid deformations in poroelastic media.

5.5 Thermal effects

Thermal effects may be of great importance for specific problems such as heat steam injection for enhanced recovery of hydrocarbons, underground nuclear waste storage, exploitation of geothermal reservoirs, dewatering of subducted sediments, vulcanism, metamorphism, etc. They can be incorporated in poroelasticity when expanding the stress–strain relationship by the product of the bulk modulus, the thermal expansion coefficient, and the excess temperature. The temperature anomaly, as the fourth unknown, must fulfill the heat conduction equation (e.g. Carslaw & Jaeger 1959). Some recent papers about thermal effects in saturated porous media are those of Morland (1978), Bear & Corapcioglu (1981c), Palciauskas & Domenico (1982), Delaney (1982), Noorishad, Tsang & Witherspoon (1984), Booker & Savvidou (1985), Hart & John (1986), McTigue (1986), and those of Shi & Wang (1986, 1988).

6 CONCLUSIONS

Pore pressure phenomena have long since been investigated in soil mechanics and fluid reservoir engineering. They also receive increasing attention in geophysics. Precise notations

of rock parameters as well as usage of standard terms and symbols could help to reduce existing confusions and facilitate application of the poroelasticity theory. Table 2 resumes the parameters presented in some detail here. The Biot compressibility Q^{-1} and the Biot–Willis parameter γ have been included to allow for an easy reference to earlier papers in this field. Both Q^{-1} and γ are of less experimental interest than the other parameters. There are 330 possibilities to choose different sets of four independent poroelastic rock parameters out of the 11 noted in Table 2. This also documents that adding a single variable (i.e. pore pressure) to elasticity results into an amazing multitude of valid notations. Equivalent expressions for four different sets are listed in Appendix B (Table B3) as examples.

Irrespective of the feasibility and the necessity for extension in certain cases, the linear theory of poroelasticity for quasi-static pressure changes in isotropic media will continue to be widely applied. There are at least two reasons for this: first, models involving simple physical mechanisms, in general, can be calculated more easily than models based on complex theories. Linear theories have the advantage over non-linear ones in that fundamental solutions, such as of point or line sources, can be simply added up. In this way, even complicated boundary conditions may be approximated. Secondly, detailed theories are not suitable if applied to a region, the internal structure of which is not well known, for instance, when the course of internal boundaries or gradient layers is uncertain. Likewise, the obviousness of anisotropy by itself does not justify application of an anisotropic version of poroelasticity. The many rock parameters entering into the formulation will often have to be fixed rather arbitrarily, and control over essential features of the examined process might get lost.

Rice & Cleary (1976) point out that the potential of 'simple linear isotropic models seems by no means exhausted either as to identification, even of primary aspects, of porous media effects or as to availability of convenient analytical formulations'. Of course, the decision of whether dynamic, thermal or other effects can be disregarded has finally to be based on estimates of their (potential) influence on the variables.

Table 2. List of significant parameters of saturated porous media. Any one except c_f, c_n, D, n, $\varkappa$ may be used as one of four independent mechanical parameters in poroelasticity. (L, $L_\perp$ = length and width of rock sample, parallel and perpendicular to uniaxial stress.)

Parameter	Definition	Dimension	Notion	Remarks
B	$\left(\dfrac{\partial P}{\partial P_c}\right)_m$	—	Skempton ratio	Skempton's (1954) second pore pressure parameter, aside A
c	$-\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P_c}\right)_P$	$ML^{-1}T^{-2}$	matrix compressibility	no index denoting drained condition, as opposed to undrained
c_f	$-\dfrac{1}{V_f}\dfrac{\partial V_f}{\partial P}$	$ML^{-1}T^{-2}$	fluid compressibility	'$_f$' for fluid
c_n	$-\dfrac{1}{V_n}\left(\dfrac{\partial V_n}{\partial P_c}\right)_{P-P_c}$	$ML^{-1}T^{-2}$	pore compressibility	'$_n$' for porosity
c_s	$-\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P}\right)_{P-P_c}$	$ML^{-1}T^{-2}$	grain compressibility	'$_s$' for solid
c_u	$-\dfrac{1}{V}\left(\dfrac{\partial V}{\partial P_c}\right)_m$	$ML^{-1}T^{-2}$	undrained compressibility	'$_u$' for undrained
D	$\dfrac{\nabla^2(P-BP_c)}{\partial/\partial t(P-BP_c)}$	L^2T^{-1}	hydraulic diffusivity	(Rice & Cleary 1976)
G	$0.5\,\sigma_{ij}/e_{ij},\ i\neq j$	$ML^{-1}T^{-2}$	shear modulus	identical to undrained shear modulus G_u in poroelasticity
n	V_n/V	—	porosity	—
Q^{-1}	$\alpha c_s + \gamma$	$ML^{-1}T^{-2}$	Biot compressibility	coefficient of $\partial P/\partial t$ in equation (2)
S	$\dfrac{(m-m_0)/\rho_0}{P-BP_c}$	$ML^{-1}T^{-2}$	storage compressibility	'3-D' version, identical to R^{-1} of Biot (1941)
α	$(P_c-P_e)/P$	—	coeff. of effective stress	—
γ	$n(c_f-c_s)$	$ML^{-1}T^{-2}$	Biot–Willis parameter	$n(c_f-c_n)$ for inhomogeneous matrix (Biot & Willis 1957)
$\varkappa$	k/η	L^3TM^{-1}	Darcy conductivity	k = intrinsic permeability, η = dynamic fluid viscosity
ν	$-\dfrac{L}{L_\perp}\left(\dfrac{\partial L_\perp}{\partial L}\right)_P$	—	drained Poisson ratio	no index denoting drained condition, as opposed to undrained
ν_u	$-\dfrac{L}{L_\perp}\left(\dfrac{\partial L_\perp}{\partial L}\right)_m$	—	undrained Poisson ratio	'$_u$' for undrained

ACKNOWLEDGMENTS

REFERENCES

Advani, S. H., Torok, J. S., Lee, J. K. & Choudhry, S., 1987. Explicit time-dependent solutions and numerical evaluations for penny-shaped hydraulic fracture models, *J. geophys. Res.*, **92**, 8049–8055.

American Geological Institute, 1976. *Dictionary of Geological Terms*, Anchor Press, Doubleday, Garden City, New York.

Bear, J. & Corapcioglu, M. Y., 1981a. Mathematical model for regional land subsidence due to pumping 1. Integrated aquifer subsidence equations based on vertical displacement only, *Water Resour. Res.*, **17**, 937–946.

Bear, J. & Corapcioglu, M. Y., 1981b. Mathematical model for regional land subsidence due to pumping 2. Integrated aquifer subsidence equations for vertical and horizontal displacements, *Water Resour. Res.*, **17**, 947–958.

Bear, J. & Corapcioglu, M. Y., 1981c. A mathematical model for consolidation in a thermoelastic aquifer due to hot water injection or pumping, *Water Resour. Res.*, **17**, 723–736.

Bethke, C. M. & Corbet, T. F., 1988. Linear and nonlinear solutions for one-dimensional compaction flow in sedimentary basins, *Water Resour. Res.*, **24**, 461–467.

Biot, M. A., 1941. General theory of three-dimensional consolidation, *J. appl. Phys.* **12**, 155–164.

Biot, M. A., 1955. Theory of elasticity and consolidation for a porous anisotropic solid, *J. appl. Phys.*, **26**, 182–185.

Biot, M. A., 1956a. Thermoelasticity and irreversible thermo-dynamics, *J. appl. Phys.*, **27**, 240–253.

Biot, M. A., 1956b. Theory of propagation of elastic waves in a fluid saturated porous solid. I. Low frequency range. II. Higher frequency range, *J. acoust. Soc. Am.*, **28**, 168–191.

Biot, M. A., 1962. Generalized theory of acoustic propagation in porous dissipative media, *J. acoust. Soc. Am.*, **34**, 1254–1264.

Biot, M. A., 1973. Nonlinear and semilinear rheology of porous solids, *J. geophys. Res.*, **78**, 4924–4937.

Biot, M. A. & Willis, D. G., 1957. The elastic coefficients of the theory of consolidation, *J. appl. Mech.* **24**, 594–601.

Bodvarsson, G., 1970. Confined fluids as strain meters, *J. geophys. Res.*, **75**, 2711–2718.

Booker, J. R. & Savvidou, C., 1985. Consolidation around a point heat source, *Int. J. Num. Analyt. Meth. Geomech.*, **9**, 173–184.

Bourbié, T., Coussy, O. & Zinszner, B., 1987. *Acoustics of Porous Media*, Editions Technip, Paris.

Boutin, C., Bonnet, G. & Bard, P. Y., 1987. Green functions and associated sources in infinite and stratified poroelastic media, *Geophys. J. R. astr. Soc.*, **90**, 521–550.

Brace, W. F., 1984. Permeability of crystalline rocks: New in situ measurements, *J. geophys. Res.*, **89**, 4327–4330.

Bredehoeft, J. D., 1967. Response of well-aquifer systems to earth tides, *J. geophys. Res.*, **72**, 3075–3087.

Brockhoff, J., 1990. Modellrechnungen mit der Methode der finiten Elemente zu Verformungen in der Umgebung von Brunnen, *Diploma thesis*, University of Kiel.

Brown, R. J. S. & Korringa, J., 1975. On the dependence of the

elastic properties of a porous rock on the compressibility of the pore fluid, *Geophysics*, **40**, 608–616.

Brown, R. H., Konoplyantsev, A. A., Ineson, J. & Kovalevsky, V. S., 1972. Studies and reports in hydrology 7: Groundwater studies. An international guide for research and practice, *Unesco Publ. SC. 70 XX. 7*, Paris.

Carroll, M. M., 1979. An effective stress law for anisotropic elastic deformation, *J. geophys. Res.*, **84**, 7510–7512.

Carslaw, H. S. & Jaeger, J. C., 1959. *Conduction of Heat in Solids*, Oxford University Press, Oxford.

Christensen, N. I. & Wang, H. F., 1985. The influence of pore pressure and confining pressure on dynamic elastic properties of Berea sandstone, *Geophysics*, **50**, 207–213.

Christian, J. T. & Boehmer, J. W., 1970. Plane strain consolidation by finite elements, *J. Soil Mech. Found. Div.*, Proc. ASCE, **96**, SM4, 1435–1457.

Cleary, M. P., 1977. Fundamental solutions for a fluid-saturated porous solid, *Int. J. Solids Struct.*, **13**, 785–806.

Cleary, M. P., 1978. Elastic and dynamic response regimes of fluid-impregnated solids with diverse microstructures, *Int. J. Solids Struct.*, **14**, 795–819.

Cooper, H. H., 1966. The equation of groundwater flow in fixed and deforming coordinates, *J. geophys. Res.*, **71**, 4785–4790.

Cornet, F. H. & Fairhurst, C., 1974. Influence of pore pressure on the deformation behavior of saturated rocks, *Proc. 3rd Congr. Int. Soc. Rock Mechanics*, vol. 1, B, pp. 638–644, Nat. Acad. of Sciences, Washington, DC.

Dake, L. P., 1978. Fundamentals of reservoir engineering, *Developments in Petroleum Science*, vol. 8, Elsevier, Amsterdam.

Davis, S. N., 1969. Porosity and permeability of natural materials, in *Flow Through Porous Media*, pp. 53–89, ed. De Wiest, R. J. M., Academic Press, New York.

Davis, S. N. & De Wiest, R. J. M, 1966. *Hydrogeology*, Wiley, New York.

Delaney, P. T., 1982. Rapid intrusion of magma into wet rock: Groundwater flow due to pore pressure increases, *J. geophys. Res.*, **87**, 7739–7756.

Delcourt-Honorez, M., 1988. Sur la contribution des variations de la pression atmosphérique et des fluctuations du niveau des nappes aquiferes aux variations de la pesanteur, *PhD thesis*, Université de Louvain.

De Wiest, R. J. M., 1966, On the storage coefficient and the equations of groundwater flow, *J. geophys. Res.*, **71**, 1117–1122.

Domenico, S. N., 1977. Elastic properties of unconsolidated porous sand reservoirs, *Geophysics*, **42**, 1339–1368.

Garg, S. K. & Nur, A., 1973. Effective stress laws for fluid saturated porous rocks, *J. geophys. Res.*, **78**, 5911–5921.

Gassmann, F., 1951. Über die Elastizität poröser Medien, *Vierteljahresschrift d. Naturf. Ges. Zürich*, **96**, 1–24.

Geertsma, J., 1957. The effect of fluid pressure decline on volumetric changes of porous rocks, *Trans. AIME*, **210**, 331–340.

Geertsma, J., 1973. Land subsidence above compacting oil and gas reservoirs, *J. Petrol. Tech.*, **25**, 734–744.

Green, D. H. & Wang, H. F., 1986. Fluid pressure response to undrained compression in saturated sedimentary rocks, *Geophysics*, **51**, 948–956.

Green, D. H. & Wang, H. F., 1990. Specific storage as a poroelastic coefficient, *Water Resour. Res.*, **26**, 1631–1637.

Hamilton, E. L., 1971. Elastic properties of marine sediments, *J. geophys. Res.*, **76**, 579–604.

Hart, R. D. & John, C. M. St., 1986. Formulation of a fully-coupled thermal-mechanical-fluid flow model for non-linear geologic systems, *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, **23**, 213–224.

Hsieh, P. A., Bredehoeft, J. D. & Farr, J. M., 1987. Determination

of aquifer transmissivity from Earth tide analysis, *Water Resour. Res.*, **23**, 1824–1832.

Hsieh, P. A., Bredehoeft, J. D. & Rojstaczer, S. A., 1988. Response of well aquifer systems to Earth tides: Problem revisited, *Water Resour. Res.*, **24**, 468–472.

Jacob, C. E., 1940. The flow of water in an elastic artesian aquifer, *EOS, Trans. Am. Geophys. Union*, **II**, 574–586.

Jaeger, J. C. & Cook, N. G. W., 1979. *Fundamentals of Rock Mechanics*, Wiley, New York.

Krauß, I., 1974. Die Bestimmung der Transmissivität von Grundwasserleitern aus dem Einschwingverhalten des Brunnen-Grundwasserleitersystems, *J. Geophys.*, **40**, 381–400.

Kruseman, G. D. & De Ridder, N. A., 1973. *Untersuchung und Anwendung von Pumpversuchsdaten*, Vlg. R. Müller, Köln-Braunsfeld.

Kümpel, H.-J., 1989. Verformungen in der Umgebung von Brunnen, *Habil. thesis*, University of Kiel.

Lewis, R. W. & Schrefler, B. A., 1987. *The Finite Element Method in the Deformation and Consolidation of Porous Media*, Wiley, Chichester.

Li, V. C., 1984/85. Estimation of in-situ hydraulic diffusivity of rock masses, *Pure appl. Geophys.*, **122**, 545–559.

Mattheß, G. & Ubell, K., 1983. *Allgemeine Hydrogeologie—Grundwasserhaushalt*, Gebr. Bornträger Vlg., Berlin.

Mavko, G. & Nur, A., 1978. Effect of nonelliptical cracks on the compressibility of rocks, *J. geophys. Res.*, **83**, 4459–4468.

McTigue, D. F., 1986. Thermoelastic response of fluid-saturated porous rock, *J. geophys. Res.*, **91**, 9533–9542.

Mesri, G., Adachi, K. & Ullrich, C. R., 1976. Pore pressure response in rock to undrained change in all-round stress, *Geotechnique*, **26**, 317–330.

Morland, L. W., 1978. A theory of slow fluid flow through a porous thermoelastic matrix, *Geophys. J. R. astr. Soc.*, **55**, 393–410.

Morland, L. W. & Donaldson, E. C., 1984. Correlation of porosity and permeability of reservoirs with well oscillations induced by Earth tides, *Geophys. J. R. astr. Soc.*, **79**, 705–725.

Narasimhan, T. N. & Witherspoon, P. A., 1977. Numerical model for saturated-unsaturated flow in deformable porous media, 1. Theory, *Water Resour. Res.*, **13**, 657–664.

Narasimhan, T. N. & Kanehiro, B. Y., 1980. A note on the meaning of the storage coefficient, *Water Resour. Res.*, **16**, 423–429.

Narasimhan, T. N., Kanehiro, B. Y. & Witherspoon, P. A., 1984. Interpretation of Earth tide response of three deep, confined aquifers, *J. geophys. Res.*, **89**, 1913–1924.

Neuzil, C. E., 1986. Groundwater flow in low-permeability environments, *Water Resour. Res.*, **22**, 1163–1195.

Noorishad, J., Tsang, C. F. & Witherspoon, P. A., 1984. Coupled thermal–hydraulic–mechanical phenomena in saturated fractured porous rocks: Numerical approach, *J. geophys. Res.*, **89**, 10 365–10 373.

Nur, A. & Byerlee, J. D., 1971. An exact effective stress law for elastic deformation of rock with fluids, *J. geophys. Res.*, **76**, 6414–6419.

O'Connell, R. J. & Budiansky, B., 1974. Seismic velocities in dry and saturated cracked solids, *J. geophys. Res.*, **79**, 5412–5426.

O'Connell, R. J. & Budiansky, B., 1977. Viscoelastic properties of fluid-saturated cracked solids, *J. geophys. Res.*, **82**, 5710–5735.

Oda, M., Hatsuyama, Y. & Ohnishi, Y., 1987. Numerical experiments on permeability tensor and its application to jointed granite at Stripa mine, Sweden, *J. geophys. Res.*, **92**, 8037–8048.

Ohkubo, T. & Terasaki, A., 1977. Physical property and seismic wave velocity of rock, *OYO Technical Note*, TN-22, Tokyo (originally in *Soil and Foundation*, **19**, 1971, in Japanese).

Palciauskas, V. V. & Domenico, P. A., 1982. Characterization of drained and undrained response of thermally loaded repository rocks, *Water Resour. Res.*, **18**, 281–290.

Rhoads, G. H. Jr & Robinson, E. S., 1979. Determination of aquifer parameters from well tides, *J. geophys. Res.*, **84**, 6071–6082.

Rice, J. R., 1975. On the stability of dilatant hardening for saturated rock masses, *J. geophys. Res.*, **80**, 1531–1536.

Rice, J. R., 1979. Theory of precursory processes in the inception of earthquake and rupture, *Gerland's Beitr. Geophysik*, **88**, 91–127.

Rice, J. R. & Cleary, M. P., 1976. Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents, *Rev. Geophys. Space Phys.*, **14**, 227–241.

Rice, J. R. & Rudnicki, J. W., 1979. Earthquake precursory effects due to pore fluid stabilization of a weakening fault zone, *J. geophys. Res.*, **84**, 2177–2193.

Rice, J. R., Rudnicki, J. W. & Simons, D. A., 1978. Deformation of spherical cavities and inclusions in fluid-infiltrated elastic materials, *Int. J. Solids Struct.*, **14**, 289–303.

Robinson, E. S. & Bell, R. T., 1971. Tides in confined well-aquifer systems, *J. geophys. Res.*, **76**, 1857–1869.

Roeloffs, E. A., 1988. Fault stability changes induced beneath a reservoir with cyclic variations in water level, *J. geophys. Res.*, **93**, 2107–2124.

Roeloffs, E. A. & Rudnicki, J. W., 1984/85. Coupled deformation-diffusion effects on water level changes due to propagating creep events, *Pure appl. Geophys.*, **122**, 560–582.

Rojstaczer, S., 1988. Intermediate period response of water levels in wells to crustal strain: Sensitivity and noise level, *J. geophys. Res.*, **93**, 13 619–13 634.

Rojstaczer, S. & Agnew, D. C., 1989. The influence of formation material properties on the response of water levels in wells to Earth tides and atmospheric loading, *J. geophys. Res.*, **94**, 12 403–12 411.

Rudnicki, J. W., 1981. On 'Fundamental solutions for a fluid-saturated porous solid' by M. P. Cleary, *Int. J. Solids Struct.*, **17**, 855–857.

Rudnicki, J. W., 1985. Effect of pore fluid diffusion on deformation and failure of rock, in *Mechanics of Geomaterials—Rocks, Concretes, Soils*, pp. 315–347, ed. Bazant, Z. P., Wiley, Chichester.

Rudnicki, J. W., 1986a. Fluid mass sources and point forces in linear elastic diffusive solids, *Mech. Materials*, **5**, 383–393.

Rudnicki, J. W., 1986b. Slip on an impermeable fault in a fluid-saturated rock mass, in *Earthquake Source Mechanics*, Geophys. Monograph 37, Maurice Ewing vol. 6, pp. 81–89, eds Das, S., Boatwright, J. & Scholz, C. H., American Geophysical Union, Washington, DC.

Rudnicki, J. W., 1987. Plane strain dislocations in linear elastic diffusive solids, *J. appl. Mech.*, **54**, 545–552.

Rudnicki, J. W. & Hsu, T. C., 1988. Pore pressure changes induced by slip on permeable and impermeable faults, *J. geophys. Res.*, **93**, 3275–3285.

Rumer, R. R., 1969. Resistance to flow through porous media, in *Flow Through Porous Media*, pp. 91–108, ed. De Wiest, R. J. M., Academic Press, New York.

Safai, N. M. & Pinder, G. F., 1980. Vertical and horizontal land deformation due to fluid withdrawal, *Int. J. Numer. Anal. Meth. Geomech.*, **4**, 131–142.

Schiffmann, R. L., 1970. The stress components of a porous medium, *J. geophys. Res.*, **75**, 4035–4038.

Schön, J., 1983. *Petrophysik: Physikalische Eigenschaften von Gesteinen und Mineralien*, F. Enke Vlg., Stuttgart.

Schopper, J., 1984. Permeabilität der Gesteine, in *Landolt-Börnstein*, V/1a, pp. 278–303, Springer Verlag, Heidelberg.

Segall, P., 1985. Stresses and subsidence resulting from subsurface fluid withdrawal in the epicentral region of the 1983 Coalinga Earthquake, *J. geophys. Res.*, **90**, 6801–6816.

Shi, Y. & Wang, C.-Y., 1986. Pore pressure generation in

sedimentary basins: Overloading versus aquathermal, *J. geophys. Res.*, **91**, 2153–2162.

Shi, Y. & Wang, C.-Y., 1988. Generation of high pore pressures in accretionary prisms: Inferences from the Barbados subduction complex, *J. geophys. Res.*, **93**, 8893–8910.

Skempton, A. W., 1954. The pore-pressure coefficients A and B, *Géotechnique*, **4**, 143–147.

Skempton, A. W., 1960. Effective stress in soils, concrete and rocks, *Proc. Conference on Pore Pressure and Suction in Soils*, pp. 4–16, Butterworths, London.

Small, J. C., Booker, J. R. & Davis, E. H., 1976. Elasto-plastic consolidation of soil, *Int. J. Solids Struct.*, **12**, 431–448.

Stoll, R. D., 1989. *Sediment Acoustics*, Springer, Berlin.

Terzaghi, K., 1923. Die Berechnung der Durchlässigkeitsziffer des Tones aus dem Verlauf der hydrodynamischen Spannungserscheinungen, *Sitzungsber. Akad. Wiss. Wien Math.-Naturwiss. Kl.*, Abt. 2A, **132**, 105.

Terzaghi, K., 1925. *Erdbaumechanik auf bodenphysikalischer Grundlage*, F. Deuticke, Leipzig.

Theis, C. V., 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *EOS, Trans. Am. geophys. Un.*, **16**, 519–524.

Thomsen, L., 1985. Biot-consistent elastic moduli of porous rocks: Low frequency limit, *Geophysics*, **50**, 2797–2807.

Turcotte, D. L. & Schubert, G., 1982. *Geodynamics—Application of Continuum Physics to Geological Problems*, Wiley, New York.

Van der Kamp, G. & Gale, J. E., 1983. Theory of Earth tide and barometric effects in porous formations with compressible grains, *Water Resour. Res.*, **19**, 538–544.

Verruijt, A., 1969. Elastic storage of aquifers, in *Flow Through Porous Media*, pp. 331–378, ed. De Wiest, R. J. M., Academic Press, New York.

Walsh, J. B., 1965. The effect of cracks on the compressibility of rock, *J. geophys. Res.*, **70**, 381–389.

Walsh, J. B. & Brace, W. F., 1984. The effect of pressure on porosity and the transport properties of rock, *J. geophys. Res.*, **89**, 9425–9431.

Walsh, J. B. & Grosenbaugh, M. A., 1979. A new model for analyzing the effect of fractures on compressibility, *J. geophys. Res.*, **87**, 3532–3536.

Watt, J. P., Davies, G. F. & O'Connell, R. J., 1976. The elastic properties of composite materials, *Rev. Geophys. Space Phys.*, **14**, 541–563.

Yokoo, Y., Yamagata, K. & Nagaoka, H., 1971a. Finite element method applied to Biot's consolidation theory, *Soils and Foundations*, **11/1**, 29–46.

Yokoo, Y., Yamagata, K. & Nagaoka, H., 1971b. Finite element analysis of consolidation following undrained deformation, *Soils and Foundations*, **11/4**, 37–58.

Zimmerman, R. W., Somerton, W. H. & King, M. S., 1986. Compressibility of porous rocks, *J. geophys. Res.*, **91**, 12765–12777.

APPENDIX A: PLANE STRAIN AND AXIAL SYMMETRY FORMULATIONS

Governing equations for plane strain conditions

Introducing $e_{33} = e_{13} = e_{23} = 0$; e_{11}, e_{22}, $e_{12} \neq 0$ in equation (6) yields

$$\sigma_{33} = v(\sigma_{11} + \sigma_{22}) - (1 - 2v)\alpha P, \tag{A1}$$

$$\sigma_{13} = \sigma_{23} = 0. \tag{A2}$$

With equation (A1), the confining pressure is

$$P_c = [(1 - 2v)\alpha P - (1 + v)(\sigma_{11} + \sigma_{22})]/3. \tag{A3}$$

Since also $\partial P/\partial x_3 = 0$, insertion into the equations of stress–strain equilibrium and fluid mass conservation leads for plane strain conditions again to equations (1) and (2), whereby now $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$, $\Delta_V = e_{11} + e_{22}$, and $i = 1, 2$.

Governing equations for axial symmetry

In cylindrical coordinates (r, θ, z) and for $\partial u_r/\partial \theta = \partial u_\theta/\partial \theta = \partial u_z/\partial \theta = 0$; $\partial P/\partial \theta = 0$, the equilibrium equations become

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0, \tag{A4a}$$

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0, \tag{A4b}$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zr}}{\partial r} + \frac{\sigma_{zr}}{r} = 0 \tag{A4c}$$

(Verruijt 1969). Since $e_{\theta\theta} = (\partial u_\theta/\partial \theta + u_r)/r = u_r/r$, we obtain from equation (6)

$$\sigma_{\theta\theta} = v(\sigma_{rr} + \sigma_{zz}) + 2(1 + v)G u_r/r - (1 - 2v)\alpha P, \tag{A5}$$

and for the confining pressure

$$P_c = [(1 - 2v)\alpha P - (1 + v)(\sigma_{rr} + 2G u_r/r + \sigma_{zz})]/3. \tag{A6}$$

If we proceed as in Section 2, the resultant governing equations for axial symmetry become

$$G\left(\nabla^2 u_r - \frac{u_r}{r^2}\right) + \frac{G}{1 - 2v}\frac{\partial \Delta_V}{\partial r} = \alpha \frac{\partial P}{\partial r}, \tag{A7}$$

$$G\nabla^2 u_z + \frac{G}{1 - 2v}\frac{\partial \Delta_V}{\partial z} = \alpha \frac{\partial P}{\partial z}, \tag{A8}$$

$$Q^{-1}\frac{\partial P}{\partial t} + \alpha \frac{\partial \Delta_V}{\partial t} = \varkappa \nabla^2 P, \tag{A9}$$

with $\nabla^2 = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + \partial^2/\partial z^2$ and $\Delta_V = \partial u_r/\partial r + u_r/r + \partial u_z/\partial z$.

APPENDIX B: PARAMETER CONVERSION TABLES

Table B1. Simple equivalent expressions for the coefficients of the left-hand side of equation (1). See Section 3.1 for the meaning of the parameters.

	G	$G/(1-2v)$
G, λ	—	$\lambda + G$
λ, v	$\dfrac{\lambda(1-2v)}{2v}$	$\dfrac{\lambda}{2v}$
E, v	$\dfrac{E}{2(1+v)}$	$\dfrac{E}{2(1+v)(1-2v)}$
c, v	$\dfrac{3(1-2v)}{2(1+v)c}$	$\dfrac{3}{2(1+v)c}$
G, E	—	$\dfrac{G^2}{3G-E}$
G, c	—	$\dfrac{3+Gc}{3c}$
λ, c	$\dfrac{3(1-\lambda c)}{2c}$	$\dfrac{3-\lambda c}{2c}$
E, c	$\dfrac{3E}{9-Ec}$	$\dfrac{9}{(9-Ec)c}$

Table B2. Simple equivalent expressions replacing the Biot compressibility Q^{-1} in equation (2). c, c_u, S = drained, undrained and storage compressibility.

	Q^{-1}
α, B, c	$(B^{-1} - \alpha)\alpha c$
α, B, c_u	$\alpha c_u / B$
α, B, S	$(1 - \alpha B)S$
α, c, S	$S - \alpha^2 c$
α, c, c_u	$\alpha^2 \dfrac{cc_u}{c-c_u}$
c, c_u, S	$c_u S/c$
c, c_u, B	$\dfrac{(c-c_u)c_u}{cB^2}$
c, B, S	$(1 - B^2 S/c)S$

Table B3. Equivalent expressions for poroelastic parameters as functions of four mechanical parameters and of the Darcy conductivity, κ.

Parameter	$c, c_s, \gamma, \nu;\ \kappa$	$B, G, \nu, \nu_u;\ \kappa$	$c, c_s, c_u, \nu_u;\ \kappa$	$c, S, \alpha, \nu;\ \kappa$
B	$\dfrac{c-c_s}{c-c_s+\gamma}$	—	$(c-c_u)/(c-c_s)$	$\alpha c/S$
c	—	$\dfrac{3(1-2\nu)}{2(1+\nu)G}$	—	—
c_s	—	$\dfrac{3}{2(1+\nu)G}\left((1-2\nu)-\dfrac{3(\nu_u-\nu)}{(1+\nu_u)B}\right)$	—	$(1-\alpha)c$
c_u	$\dfrac{(c-c_s)c_s+\gamma c}{c-c_s+\gamma}$	$\dfrac{3}{2(1+\nu)G}\left((1-2\nu)-\dfrac{3(\nu_u-\nu)}{1+\nu_u}\right)$	—	$c-(\alpha c)^2/S$
D	$\dfrac{c\kappa}{(c-c_s)(c'+c_s)+c\gamma}$ †	$\dfrac{2}{9}\dfrac{(1-\nu)(1+\nu_u)^2}{(1-\nu_u)(\nu_u-\nu)}\kappa GB^2$	$\dfrac{(1+\nu_u)+2(1-2\nu_u)c/c_u}{3(1-\nu_u)}\dfrac{(c-c_u)\kappa}{(c-c_s)^2}$	$\dfrac{3(1-\nu)\kappa}{3(1-\nu)S-2(1-2\nu)\alpha^2 c}$
G	$\dfrac{3(1-2\nu)}{2(1+\nu)c}$	—	$\dfrac{3(1-2\nu_u)}{2(1+\nu_u)c_u}$	$\dfrac{3(1-2\nu)}{2(1+\nu)c}$
Q^{-1}	$(c-c_s)c_s/c+\gamma$	$\dfrac{9}{2}\dfrac{(1-2\nu_u)(\nu_u-\nu)}{(1-2\nu)(1+\nu_u)^2GB^2}$	$\dfrac{(c-c_s)^2 c_u}{(c-c_u)c}$	$S-\alpha^2 c$
S	$c-c_s+\gamma$	$\dfrac{9}{2}\dfrac{\nu_u-\nu}{(1+\nu)(1+\nu_u)GB^2}$	$(c-c_s)^2/(c-c_u)$	—
α	$1-c_s/c$	$\dfrac{3(\nu_u-\nu)}{(1-2\nu)(1+\nu_u)B}$	$1-c_s/c$	—
γ	—	$\dfrac{9}{2}\dfrac{(\nu_u-\nu)(1-B)}{(1+\nu)(1+\nu_u)GB^2}$	$(c-c_s)(c_u-c_s)/(c-c_u)$	$S-\alpha c$
ν	—	—	$\dfrac{(1+\nu_u)c_u-(1-2\nu_u)c}{2(1+\nu_u)c_u+(1-2\nu_u)c}$	—
ν_u	$\dfrac{3\nu c(\gamma+c-c_s)+(1-2\nu)(c-c_s)^2}{3c(\gamma+c-c_s)-(1-2\nu)(c-c_s)^2}$	—	—	$\dfrac{3\nu S+(1-2\nu)\alpha^2 c}{3S-(1-2\nu)\alpha^2 c}$

† $c' = \dfrac{(1+\nu)(c-c_s)}{3(1-\nu)}$ = vertical compressibility.