POROSITY VARIATION IN FILTER CAKE UNDER CONSTANT-PRESSURE FILTRATION

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To examine the propriety of the so-called "modern filtration theory", porosity variations in constant-pressure filtration cakes are measured electrically by means of filter equipment having six pairs of disk-type electrodes. Experimental results show good agreement with the theoretical values predicted by filtration theory on the basis of compression-permeability data. The so-called "retarded packing compressibility" phenomena previously presented by Rietema *et al.* are not recognized through all runs attempted in this study.

Since the compression-permeability cell technique was introduced by P.C. Carman[3] and B.F. Ruth[9], research work on filtration has been greatly focused on the internal states of filter cakes[5,6,11]. However, the classic theories originally presented by Carman[3] and Ruth[9] were theoretically correct only for incompressible cakes. It had been possible to deal only approximately with general compressible cakes until the apparent velocity variations of filtrate through cakes were analyzed by F.M.Tiller et al.[14].

Recently a new analytical method[13] of apparent velocity variations of both liquids and solids through filter cakes has been presented, and it has turned out that the effects of the velocity distributions of liquids and solids could not be theoretically neglected, especially for highly concentrated slurries. On the basis of the new theory, one can predict the dynamic internal conditions of filter cakes. It has been well known in industry that a denser cake of higher filtration resistance with less cracking tendency is formed from a more dilute slurry[4]. Therefore, diluting the filter feed frequently cures cake cracking problems in industrial filter operations[4]. The above operating tricks may easily be verified quantitatively or qualitatively on the basis of the new filtration theory.

In 1967 Baird et al.[2], using nine pairs of metal electrode-pins of 0.5 in. length, measured experimentally the variations of electric resistance through filter cake. Although the resistances were not converted into local porosity values in cake, they deduced that a collapse occurred in the filter cake when filtration had been going on for some time and a critical thickness had been reached. They concluded that porosity variation was not uniform, as had been conventionally assumed, and the minimum porosity in a filter cake was not always adjacent to the filter medium. This special type of packing compressibility was formerly called the phenomenon of "retarded packing compressibility" (R. P. C.) by K. Rietema[8].

In this study, the local porosity variations in constant-pressure filtration cake are measured by an electrical method. Contrary to the deduction presented by Baird et al.[2], the experimental results show the propriety of the fundamental postulates of the new theory[13].

Relation of Porosity and Electric Resistance

1. Effects of side-wall friction in compression cell

As mentioned in the previous paper[10], there is a considerable effect of friction between the side wall of a compression cell and a compressed cake. All results obtained by compression-permeability cell tests must be corrected in due consideration of the frictional effect.

Under an applied pressure p, the variation of the solid compressive pressure p_v may be approximately expressed by Eq.(1)[10], when the compression equilibrium has been reached.

$$p_v = \left(p + \frac{C}{k_o f}\right)\exp\left(-\frac{4k_o f}{D}z\right) - \frac{C}{k_o f} \qquad (1)$$

where k_o is the so-called "coefficient of earth pressure at rest" in soil mechanics, f the coefficient of friction, C the cohesive force between the side wall of cell and the compressed cake, D the inside diameter of the cell and z is the distance measured from the top of the compressed cake. $k_o f$ and C are assumed to be constants depending upon the materials of the side wall and the slurry. Values of $k_o f$ and C being known, the distribution of the solid compressive pressure p_v can be calculated by Eq.(1).

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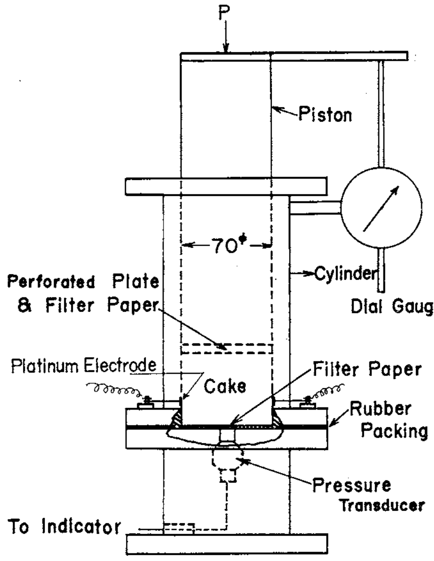

Fig. 1 Schematic view of compression cell

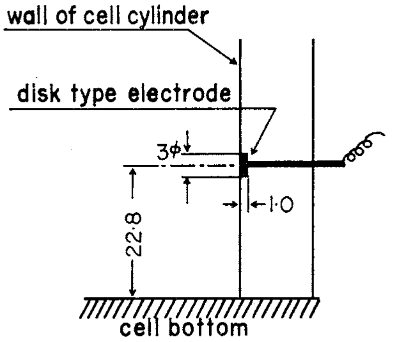

Fig. 2 Schematic view of electrode assembly

Table 1 Characteristics of Mitsukuri Gairome Clay

True density of solids, ρ_s: 2.824 [g/cm^3]

	microns diameter	wt% above stated size
Distribution of particle size (by Coulter Counter)	20	8
	10	31
	5	58
	3	95
	2.5	99.9

The solid compressive pressure transmitted to the bottom of the cake, p_T, may be written as[10]

$$p_T = \left(p + \frac{C}{k_o f}\right)\exp\left(-\frac{4k_o f}{D}Z\right) - \frac{C}{k_o f} \tag{2}$$

where Z is the total thickness of the cake in equilibrium. The transmitted pressure p_T and the cake thickness Z in equilibrium being measured under the various conditions of applied loads p and charged slurry volumes, the values of $k_o f$ and C could be evaluated by a numerical method mentioned in the previous paper[10].

The average solid compressive pressure p_s through the cake may be defined by[10]

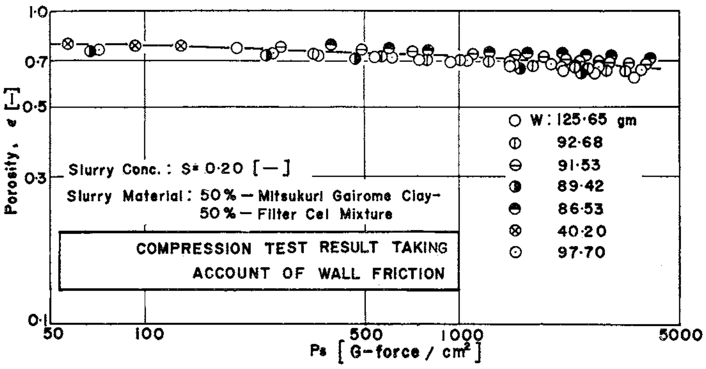

Fig. 3 Porosity vs. solid compressive pressure

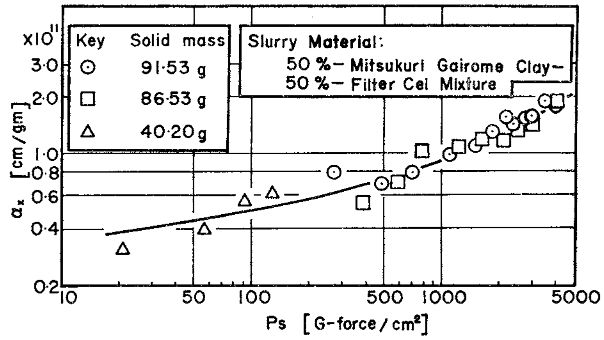

Fig. 4 Specific filtration resistance vs. solid compressive pressure

$$p_s = \frac{1}{Z}\int_0^Z p_v dz = \frac{\left(p + \frac{C}{k_o f}\right)}{\frac{4k_o f}{D}Z}\left\{1 - \exp\left(-\frac{4k_o f}{D}Z\right)\right\} - \frac{C}{k_o f} \tag{3}$$

In view of the existence of appreciable friction effects in the compression cell[7, 17], the average solid compressive pressure p_s defined by Eq.(3) should be used to analyze the compression cell data.

In **Fig. 1**, a compression test cell used is shown. The cell consists essentially of a piston with a porous end and a cylinder, both made of plexiglass which are electrical insulators. At the center of the cell bottom, a wire-strain-gauge pressure transducer is fixed to measure the transmitted pressure p_T. The cell cylinder has a pair of platinum electrodes, 3 mm in diameter and 1 mm in thickness. Instead of the pin-type electrodes which protrude into cake as attempted by Baird et al.[2], disk-type electrodes are employed which are flush with the wall of the cylinder as shown in **Fig. 2** to minimize interference with compressed cake. The experiments are carried out using 50 wt% Mitsukuri Gairome Clay-50 wt% Filter Cel mixture slurry, the characteristics of the Clay material being tabulated in **Table 1**. The solids concentration of the slurry is 20 wt% in 0.93 wt% solution of salt. **Figs. 3** and **4** illustrate the compression-permeability results corrected for the frictional effect by using Eq.(3).

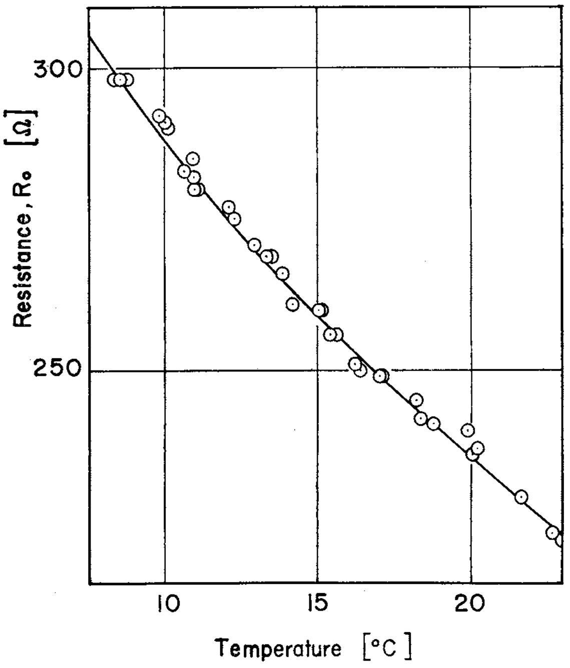

Fig. 5 R_0 vs. temperature

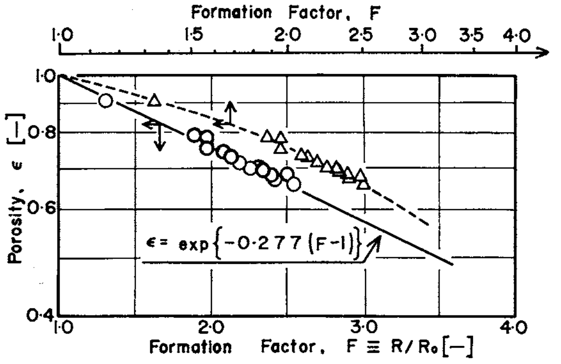

Fig. 6 Porosity vs. formation factor

The measurements of p_T and Eq.(2) give the values of $k_of=0.207$[–] and $C=66.0$ [G/cm^2], with about 20% of the root-mean-square deviation defined in previous paper10).

2. Electric resistance

In electrical measurements of the porosity of a porous medium, Archie1) showed empirically that the so-called "formation factor" F defined by

$$F=\frac{\text{Electric resistance of a saturated porous medium, } R}{\text{Electric resistance of the saturating fluid at the same temperature, } R_0} \tag{4}$$

could be related to the porosity ε of the medium by a relation of the form

$$\varepsilon = F^{\beta} \tag{5}$$

where β is a factor depending on the system concerned. However, Eq.(5) may not necessarily be proper for all cases and an attempt should be made to find the best correlation for each case.

Firstly, a saline solution of 0.93wt%NaCl is poured into the cell cylinder and the resistances, R_0, of the liquid at various temperatures are measured, the results being shown in **Fig. 5.** Then, compression tests of slurry are carried out. The resistance of the compressed cake R, the cake thickness Z and the temperature of liquid are measured when the compression equilibrium is reached, and a further load increment is applied. The formation factor is calculated from the electric resistance of the compressed cake and the curve shown in Fig. 5.

The value of the local solid compressive pressure p_{vE} at the electrodes being known from the following equation

$$p_{vE}=\left(p+\frac{C}{k_of}\right)\exp\left\{-\frac{4k_of}{D}(Z-z_E)\right\}-\frac{C}{k_of} \tag{6}$$

the value of the local porosity ε at the depth of electrodes can be evaluated by using the modified compression test results shown in Fig. 3. In Eq.(6), z_E is the height of the center of the electrode disk measured from the bottom of the cell.

As clearly indicated by **Fig. 6** of porosity ε vs. formation factor F, it is apparent that semilogarithmic plots of the following form

$$\varepsilon = \exp\{-0.277(F-1)\} \tag{7}$$

are more appropriate rather than logarithmic plot's of the form of Eq.(5).

Constant-Pressure Filtration

In view of the variation of liquid velocity q_x and solid migration rate r_x through filter cakes, the basic flow equation can be written as13)

$$q_x-\frac{\varepsilon_x}{1-\varepsilon_x}r_x=\frac{g_c}{\mu\alpha_x\rho_s(1-\varepsilon_x)}\cdot\frac{dp_x}{dx} \tag{8}$$

where ε_x is the local porosity, α_x the local specific resistance, x the distance from the medium, μ the viscosity of liquid, ρ_s the true density of solids and p_x the local hydraulic pressure which can be related to the local solid compressive pressure p_s in the form:

$$p_x+p_s=p \tag{9}$$

Solving the continuity equation14) under a constant-pressure condition yields the following equations of q_x and r_x through cake.

$$\frac{q_x}{q_1}=1-\frac{r_x}{q_1}=1-\frac{(\varepsilon_x-\varepsilon_{avx})(m-1)}{\varepsilon_{av}(1-ms)}\cdot s\frac{x}{L} \tag{10}$$

where q_1 is the value of q_x at the medium, i.e. the filtration rate, ε_{avx} the average porosity for the portion of the cake between the septum and a distance x, ε_{av} the average porosity of the entire cake, m the ratio of wet to dry cake mass, s the mass fraction of solids in slurry and L the thickness of cake.

It should be emphasized that Eq.(10) can be obtained on the basis of the assumption that under a constant-pressure filtration the porosity variations through filter cakes depend only on the normalized distance x/L. Based upon the results of the compression-permeability test, one can predict the filtration performance under any condition of filtration pressure and slurry concentration by combining

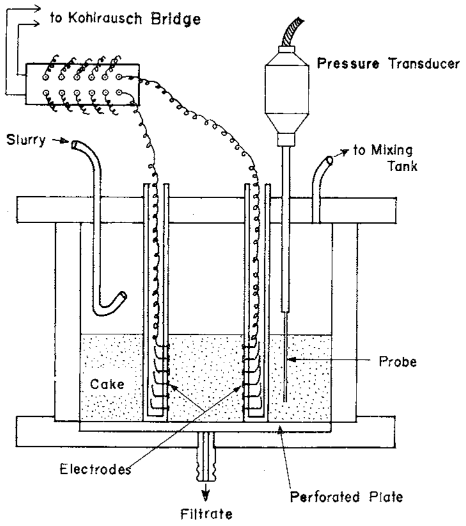

Fig. 7 Schematic view of the filter used

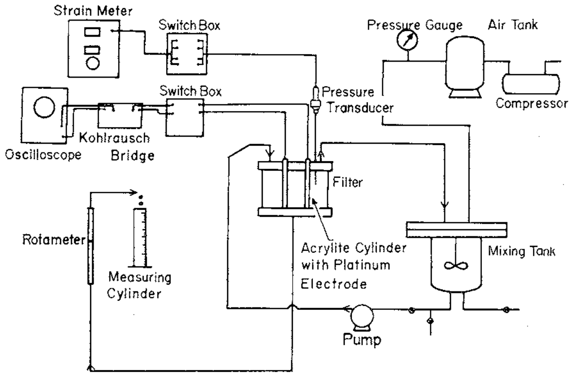

Fig. 9 Flow lines and wiring diagrams of measurement devices

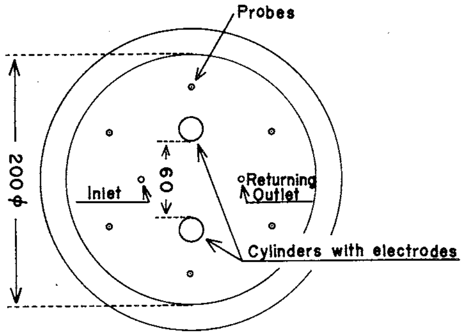

Fig. 8 Plan view of the arrangements of filter

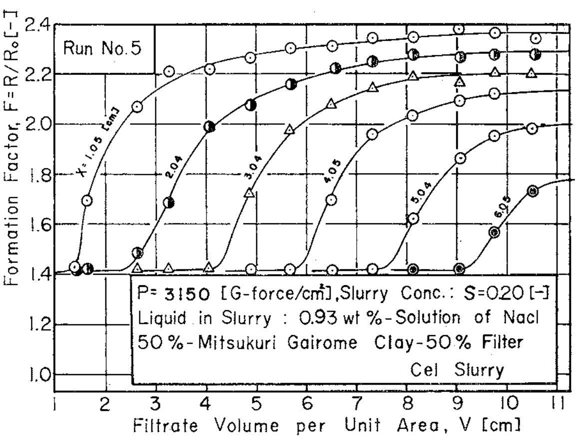

Fig. 10 Variations of formation factor

Eqs. (8), (9) and (10) and solving them numerically[12].

1. Equipment and procedure

To conduct constant pressure filtration experiments the filter shown in **Fig. 7** is used to give the distributions of electric resistance R and hydraulic pressure p_x in cake. The filter consists essentially of a plexiglass cylinder of inside diameter 200 mm, a stainless-steel top disk and a bottom plexiglass disk which supports a perforated plexiglass plate with a filter paper on it. The top disk is fitted with an inlet and a returning conduit of slurry, six probes for measurement of hydraulic pressure p_x and two vertical sealed plexiglass pipes of outside diameter 20.2 mm having six pairs of platinum disk-type electrodes. Each pair of electrodes faces each other at a distance of 6 cm. Fitting of the two vertical pipes to the top disk is done carefully so that their bottoms contact the filter paper when the filter equipment is assembled. The electrodes for measuring electric resistance R are arranged at distances of 10.5, 20.4, 30.4, 40.5, 50.4 and 60.5 mm and the probes for measuring p_x at distances of 1.8, 6.4, 19.3, 28.1, 39.4 and 53.2 mm from the medium. A plan view of the arrangements is shown in **Fig. 8**.

Filtrate is introduced to a rotameter and runs into a measuring cylinder.

To avoid sedimentation of solids in the flow line and the filter chamber, a portion of slurry is recirculated by a small centrifugal pump. The rotating blades of the pump are made of rubber so that the effects on the particles in slurry can be minimized. In **Fig. 9**, the flow lines and wiring diagrams of the measurement devices are shown schematically.

After the filtrate line from the medium to the rotameter is filled with 0.93% salt solution of the same concentration as that of filtrate, the filter equipment is assembled and then the filter chamber is filled up with slurry by the pump. Contsant-pressure filtration experiments are carried out by applying compressed air pressure. R-distributions are measured by using a kohlrausch bridge and an oscilloscope, p_x-variations by a pressure transducer and an indicator, and the volume of filtrate and its temperature are obtained at some interval of filtration time. Used slurry in these experiments is the same as that used in the compression cell test.

Experimental Results and Discussion

Fig. 10 illustrates the experimental results of the variations of the formation factor F defined by Eq. (4) during a constant-pressure filtration. Fig. 10 was

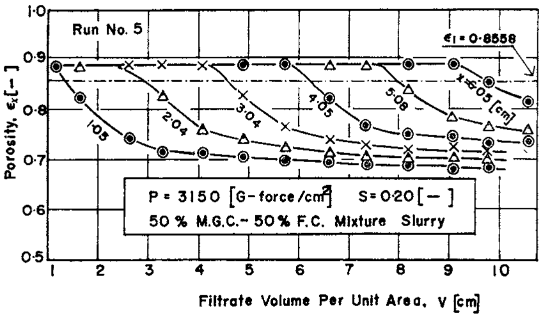

Fig. 11 Porosity variations

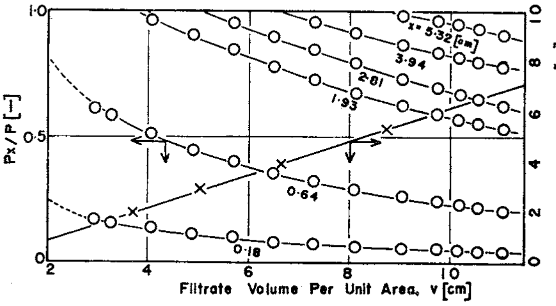

Fig. 13 Variations of local hydraulic pressure

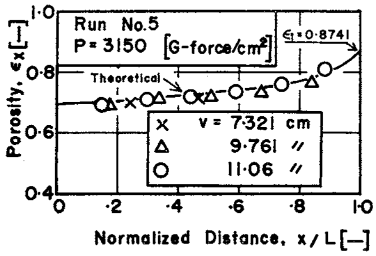

Fig. 12 Local porosity vs. normalized distance

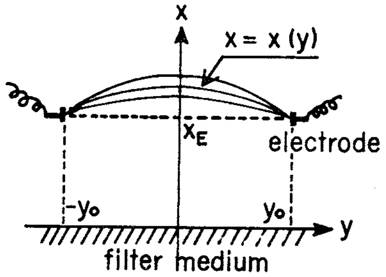

Fig. 14 Schematic view of electric current path

prepared by using the measured values R at various depths in cake and pre-determined temperature dependency of R_0 of each pair of electrodes, as typically shown in Fig. 5. (cf. Appendix)

By using Eq.(7), the formation factors in Fig. 10 can be converted into the porosities as shown in **Fig. 11**. Fig. 11 indicates that the porosity at any point in cake decreases monotonously with the filtration time and approaches a constant value depending on the filtration pressure. In this study, the so-called "retarded packing compressibility" is never recognized as shown in Fig. 11, and the porosity variation proves the plausibility of the so-called "modern filtration theory".

Contrary to the results obtained in this study, Rietema[8] and Baird et al.[2] reported that R.P.C. phenomena were observed, using metal pin-type electrodes which were 0.5 inch in length and ran parallel to each other at a horizontal distance of 0.5 inch. It might be concluded that abnormal phenomena had occurred, perhaps because the metal pins had supported the cake.

Fig. 12 shows the experimental data of porosity versus normalized distance x/L, where cake thickness L is determined from p_x-variations shown in **Fig. 13**.

In Fig. 12, the porosity distribution curve theoretically determined[13] by using compression permeability measurements is also pictured. In the modern filtration theory, there is a basic assumption that under constant-pressure filtration the local porosity depends upon the normalized distance x/L only when filtration has proceeded for some time. Fig. 12 may support exactly the propriety of this basic assumption.

Some doubt exists concerning the accuracy of measurements of local porosity by the electrical method as attempted in this paper. Because of a rather long distance between the electrodes, the flux of electric current may naturally bend towards the direction of smaller resistance in cake, i.e. of larger porosity as shown in **Fig. 14**. Therefore, one may obtain a little larger value of porosity than the actual local value and a necessary modification to the observed value of formation factor should be made. It certainly seems that good agreement shown in Fig. 12 rests on the fact that the cake thickness attempted in this study is very thick, up to 7 cm.

Conclusion

1. Local porosities at various depths in cake during constant-pressure filtration are determined experimentally by an electrical method and show good agreement with the results from filtration theory based upon compression-permeability data. The so-called "retarded packing compressibility" phenomena are never to be recognized through all runs in this study.
2. In the modern filtration theory, there exists a basic postulate that the local porosity ε_x depends only upon the normalized distance x/L under constant-pressure filtration. Experimental results in this study support the propriety of this postulate, as is clearly shown in Fig. 12.
3. The porosity can be expressed by a function of formation factor only, even if the temperature of liquid varies to some extent. The type of function may depend upon the system concerned.

Appendix

To imrove still further the accuracy of measurements of local porosity by the electrical method, the distance between electrodes should be minimized at the sacrifice of unfavorable effect on cake formation. Furthermore, one should make a possible modification for the electric path.

On the rough assumption that the electric current may mainly flow along the path of minimum resistance, electric resistance R_0 for filtrate may be written by

$$R_0=\frac{1}{\sigma}\frac{2y_0}{A_E} \tag{A1}$$

where y_0 is a half distance between electrodes, A_E the plate area of disk type electrode and σ is an empirical constant depending mainly on the electric conductivity of filtrate. The apparent electric resistance of cake, R_A, can be expressed by

$$R_A=\frac{1}{\sigma A_E}\int_{-y_0}^{y_0}\frac{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}{\varepsilon(x)}dy \tag{A2}$$

for the system in which the porosity changes in accordance with a continuous relation of $\varepsilon\equiv\varepsilon(x)$. The current-path function $x\equiv x(y)$ in Eq.(A2) means a stationary curve i.e. the minimum resistance curve for the functional16) expressed by

$$\Phi\{x\}=\int_{-y_0}^{y_0}\frac{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}{\varepsilon(x(y))}dy \tag{A3}$$

On the other hand, one should obtain the true local value of electric resistance in cake R_T for the straight path of current exactly at the height x_E of the electrode represented by

$$R_T=\frac{1}{\sigma A_E}\frac{2y_0}{\varepsilon(x_E)} \tag{A4}$$

Combination of Eqs.(A1), (A2) and (A4) yields the relation between the true and the apparent value of formation factors in the form

$$(R_T/R_0)=(R_A/R_0)\cdot f_E \tag{A5'}$$

or

$$F=F_A f_E \tag{A5}$$

where f_E is defined by

$$f_E\equiv R_T/R_A\equiv\frac{2y_0}{\varepsilon(x_E)}\int_{-y_0}^{y_0}\frac{\sqrt{1+\left(\frac{dx}{dy}\right)^2}}{\varepsilon(x)}dy \tag{A6}$$

Therefore, the apparent value of formation factor F_A may be modified by multiplying the correction factor f_E defined by Eq.(A6).

Practically, however, the correction factor f_E seems to be nearly unity with sufficient accuracy except for the special case where the electrode is extremely close to a discontinuous layer such as a cake surface. Although the correction factor f_E is assumed to be approximately unity in this study, experimental values of porosity show good agreement with those determined from the so-called modern filtration therory.

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Notation

A_E = plate area of disk-type electrode [cm^2]
C = cohesive force per unit area [G–force/cm^2]
D = diameter of cell cylinder [cm]
F = formation factor defined by Eq.(4) [—]
f = coefficient of internal friction [—]
f_E = correction factor defined by Eq.(10) [—]
g_c = conversion factor [dyne/G-force]
k_o = the so-called "coefficient of earth pressure at rest" in soil mechanics [—]
L = thickness of filter cake [cm]
m = ratio of wet to dry cake mass [—]
p = applied pressure [G–force/cm^2]
p_s = solid compressive pressure [G–force/cm^2]
p_T = transmitted pressure at $z = Z$ [G–force/cm^2]
p_v = solid compressive pressure at $z = z$ [G–force/cm^2]
p_{vE} = solid compressive pressure at $z = z_E$ [G–force/cm^2]
p_x = local hydraulic pressure [G–force/cm^2]
q_x = apparent velocity of filtrate at distance x from medium [cm/sec]
q_1 = value of q_X at $x = 0$, i.e. filtration rate [cm/sec]
r_x = apparent velocity of solids at a distance x from medium [cm/sec]
R = electric resistance of cake [ohm]
R_0 = electric resistance of filtrate [ohm]
$R\{x\}$ = the functional defined by Eq.(11) [ohm]
s = mass fraction of solids in slurry [—]
x = distance from filter medium [cm]
x_E = distance of electrode from the medium [cm]
y = horizontal co-ordinate shown in Fig. 14 [cm]
y_0 = half distance between electrodes [cm]
Z = thickness of compressed cake in equilibruim [cm]
z = distance from the cake surface in compression permeability cell [cm]
z_E = z-value at the position of electrode [cm]

〈Greek Letters〉

α_x = local specific filtration resistance [cm/gm]
β = an empirical constant [—]
ε = porosity [—]
ε_x = local porosity at x [—]
ε_{av} = average porosity for entire cake [—]
ε_{avx} = average porosity for cake between medium and distance x [—]
μ = viscosity of filtrate [gm/cm·sec]
ρ_s = true density of solids [gm/cm^3]
σ = an empirical constant [1/cm·ohm]

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