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Portfolio Diversification, Leverage, and Financial Contagion

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Abstract

Models of "contagion" rely on market imperfections to explain why adverse shocks in one asset market might be associated with asset sales in many unrelated markets. This paper demonstrates that contagion can be explained with basic portfolio theory without recourse to market imperfections. It also demonstrates that "Value-at-Risk" portfolio management rules do not have significantly different consequences for portfolio rebalancing and contagion than other rules. The paper's main conclusion is that portfolio diversification and leverage may be sufficient to explain why investors would find it optimal to sell many higher-risk assets when a shock to one asset occurs.

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I. INTRODUCTION

The Mexican peso crisis that began in late 1994 was an adverse shock not just to Mexico but to several Latin American countries and to other countries around the world. Likewise, the financial consequences of the collapse of the Thai baht in 1997 and the unilateral debt restructuring by Russia in 1998 were far-reaching and created turbulence in even the largest and most developed capital markets in the world. These recent episodes of market turbulence have generated interest in why and how local financial events can affect market dynamics and cause turbulence in financial markets in other countries. Several models of financial contagion have been developed that can explain why investors might sell many risky assets when an adverse shock affects just one asset. All of these new models associate financial contagion with market imperfections—most often asymmetric information.

This paper demonstrates that 'contagious selling' of higher-risk assets can be explained with the basic principles of portfolio theory without recourse to market imperfections. The paper examines the textbook portfolio allocation problem of an investor and focuses on two types of shocks considered by this new literature. The paper's implications for optimal portfolio rebalancing can be summarized as follows. First, an adverse shock to a single asset's return distribution can lead to a reduction in other risky asset positions. However, this result is sensitive to the properties of the portfolio manager's objective function and the characteristics of the joint distribution of asset returns.[2] Second, the consequences of an adverse shock to the realized return on the portfolio hinge mainly on whether or not the investor is leveraged. A leveraged investor will *always* reduce risky asset positions if the return on the leveraged portfolio is less than the cost of funding the portfolio. This result does not depend on margin calls: it applies to portfolios and institutions that rely on borrowed funds (whether margined or not). Thus, a loss on a specific position--such as a bond market position in Russia in the fall of 1998--may be sufficient to cause a leveraged investor to reduce risky positions in all markets. The paper quantifies optimal portfolio rebalancing responses under plausible assumptions about the magnitudes of adverse shocks and finds that the net reduction in risky positions is large for reasonably low degrees of leverage.

The paper also examines claims made recently that Value-at-Risk (VaR) rules produce contagion. One claim is that a *general* increase in asset-return volatility will cause a reduction in positions in all markets (that is, contagion). However, as demonstrated in this paper, this argument is a fairly general prediction of elementary portfolio theory and is not unique to VaR rules. Another claim is that VaR rules have very different,

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[2] Of course, the predictions of the various models of contagion discussed below are also generally dependent on the parameterization.

volatility-enhancing implications for financial markets. The analysis of this paper shows that VaR rules do not produce portfolio rebalancing dynamics that are very different from a variety of other portfolio management rules. In short, the emphasis by some on VaR rules as a particular or unique source of contagion or volatility in financial markets seems unwarranted. The main conclusion of this paper is that portfolio diversification and leverage may be sufficient to explain contagion.

The paper proceeds as follows. Section 2 briefly summarizes the existing literature on contagion and, in particular, the similarities and differences between the current paper and existing models of contagion. Section 3 describes the framework and the types of shocks that motivate portfolio rebalancing. Sections 4-5 analyze formally the consequences of these shocks, and section 6 discusses some numerical examples. The final section offers concluding comments.

II. Existing Literature on Contagion

As noted above, some of the recent 'contagion' literature is concerned with explaining why local events--in Mexico (1994), Thailand (1997), and Russia (1998)--might cause investors to decrease investment positions in a wide range of higher-risk markets, thereby transmitting the local event to markets in other countries.[3] Empirically, it has been difficult to disentangle how much of the spillover to other countries' financial markets is due to 'pure contagion' rather than to common fundamentals. Nevertheless, there appears to be substantial comovement in asset prices across countries that is not explained by common fundamentals.[4] Theoretical models of financial contagion have been developed to account for this "residual", all of which rely on market imperfections of one form or another.

Most often the nature of the market imperfections is information. Calvo and Mendoza (1999) use a standard mean-variance model to show that costs of verifying the validity of market rumors can lead to asset sales unrelated to fundamentals. Kodres and Pritsker (1999) study a model with four types of investors, some of which are not rational, and some of which have better information than others. The explanation for contagion that this paper emphasizes is 'cross-market hedging': if some asset returns are correlated, then an adverse shock in one market can lead to selling in another market.

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[3] The following discussion and the implicit definition of contagion is limited to models that seek to explain why investors decrease positions in risky assets other than the one which experiences an adverse shock. See Masson (1998) and Wolf (1999) for thorough discussions of the contagion literature.

[4] Some papers have argued that simultaneous deterioration in a sufficiently broad set of fundamentals can explain nearly simultaneous currency attacks across countries. See, for example, Agenor and Aizenman (1998) and Chan-Lau and Chen (1999).

Papers by Calvo (1999) and King and Wadhwani (1990) both emphasize a signal extraction problem. Assuming that asset prices depend on an idiosyncratic and a common factor, King and Wadhwani show that a shock to the idiosyncratic factor in one market will in general prompt investors to adjust positions in other markets because of uncertainty about the type of shock that has occurred. Calvo (1999) argues that if informed investors trade for reasons other than just information then uninformed investors may mimic informed investors even though *ex post* it turns out that no new information about fundamentals was revealed.

Calvo's explanation relies on a sufficiently important set of informed investors liquidating many positions simultaneously for reasons other than information. An important question is what type of shock would cause this. Calvo (1998,1999) suggests margin calls. Although Calvo does not formally model leverage and margin calls, his argument is simply that informed investors are by and large quite sophisticated and thus are most likely leveraged. A margin call in one market would therefore require that these investors liquidate various positions to satisfy the margin call.

All of the above models require market imperfections to explain contagion. This paper examines whether market imperfections are necessary to explain why an investor would reduce various risky positions when an adverse shock affects only one asset. It does so by adapting the textbook model of an investor's portfolio optimization problem to portfolio rebalancing due to local shocks. There are no market imperfections in this framework, and the analysis is confined to partial equilibrium exercises: the scope of the analysis is simply to examine how an individual investor might react, on impact, to an event such as Russia's *de facto* default on bond obligations. As in some of the other papers discussed above, `contagious selling' is defined simply as a withdrawal by an investor from many risky assets when an adverse shock occurs to only one of them.

III. Portfolio Management Rules and Rebalancing Events

This section of the paper describes the analytical framework and the portfolio management rules used to examine portfolio rebalancing and contagion, and it also formalizes the types of events that are examined in sections 4 and 5.

A. Portfolio Choice

Faithful to standard portfolio theory we study the current-period portfolio allocation problem of a 'portfolio manager'. For reasons that will be made clear, it will be useful to consider this portfolio allocation problem at different dates indexed by t. The purpose of introducing time in this limited way is simply to formalize an intertemporal link between the return on the portfolio at any date and available equity capital of the portfolio manager. This link forms the basis of the analysis in section 5 (and in the numerical examples in section 6) of consequences for portfolio rebalancing of a shock to the investor's equity capital.

In each period t the portfolio manager rebalances the portfolio based on a portfolio management rule (discussed below) and perceptions of the joint distribution of asset returns.

This portfolio may be leveraged. Denote the amount of capital in the portfolio in period t by V_t, and let W_t denote the magnitude of the position in risky assets. Thus, $W_t = V_t + B_t$, where B_t represents borrowing (or lending, if negative). We interpret leverage broadly as debt financing of investment positions, including margined positions (which are discussed below).

The borrowing/lending gross rate that the manager faces is denoted r, and the realized gross return, denominated in a numeraire currency, on risky asset i in period $t+1$ is $R_{i,t+1}$. Asset returns at $t+1$ have a conditional joint-normal distribution, based on the period t information set of the manager, with means $\mu_{i,t+1}$, variances $\sigma^2_{i,t+1}$, and covariances $c_t^{ij} = \rho_{t+1}^{ij}\sigma_{i,t+1}\sigma_{j,t+1}$, where ρ_{t+1}^{ij} is the conditional correlation between assets i and j.

The choice variables for the portfolio manager are the portfolio weights $\{w_{i,t}\}_{i=0}^{N}$, where $i = 0$ denotes borrowing/lending. Prior to the portfolio being rebalanced in period t, the fully-reinvested value of the position in asset i is $w_{i,t-1}V_{t-1}R_{i,t}$; after it is rebalanced, the position in asset i is $w_{i,t}V_t$. Note that if the portfolio is leveraged then $B_t = -w_{0,t}V_t > 0$ is the magnitude of leverage, and $W_t = (1 - w_{0,t})V_t > V_t$ is the position in risky assets.

B. Portfolio Management Rules

Portfolio rebalancing in any time period t can be driven by one of many 'portfolio management rules.' Consider first some portfolio management rules implied by elementary portfolio theory. The first rule is an *expected return benchmarking rule*: the manager chooses the least risky portfolio that attains a target expected return on equity capital during the period. Formally, if $\mu_{p,t+1}$ denotes the expected return per unit of capital and $\sigma_{p,t+1}$ the standard deviation of return, then the objective is:

$$\text{minimize } \sigma_{p,t+1}, \tag{1}$$

$$\text{subject to: } \mu_{p,t+1} \geq k. \tag{2}$$

A closely related portfolio management rule is a *volatility benchmarking rule*: the manager chooses the portfolio with the highest expected return, subject to the constraint that the level of risk does not exceed a threshold level.

Next, consider a rule that permits some flexibility in choosing both the expected return and risk of the portfolio, where the degree of flexibility is determined by an underlying risk tolerance parameter. Formally, the *tradeoff rule* considered is the well-known specification:

$$\text{maximize } \mu_{p,t+1} - \frac{1}{2}\tau\sigma^2_{p,t+1}, \tag{3}$$

where τ is the risk tolerance parameter. This objective function is utilized in several of the theoretical papers discussed in section 2.

The return and volatility benchmarking rules are equivalent in a mean-variance framework: any rule that benchmarks expected returns has an equivalent volatility benchmarking rule because the constraint in each rule defines a single point on the efficient set. The implications of these two rules for portfolio rebalancing are the same, and we therefore restrict the discussion below to the expected return benchmarking rule. Note that the tradeoff rule locates the portfolio at the point of tangency between the efficient set and the map of indifference curves defined by (3).

Also examined below is a class of portfolio management rules that quantify and constrain the downside risk of a portfolio. These rules have been popularized in the 'Value at Risk' approach to risk management, but the essential idea underlying them was developed long ago by Telser (1955), who labeled them 'Safety-First Rules'.

Under these rules, in each period t the portfolio manager seeks to maximize the expected return on equity, subject to a maximum probability of potential losses exceeding a specified threshold level. Formally:

$$\text{maximize } \mu_{p,t+1}, \tag{4}$$

subject to:

$$\text{Prob}\left[R_{p,t+1} < \hat{R}\right] \le m, \tag{5}$$

where $R_{p,t+1}$ is the gross rate of return on equity capital. In words, (5) states that there is at most an m percent chance of incurring losses between t and $t+1$ that exceed $(1-\hat{R})V_t$ dollars. If asset returns are normally distributed, this constraint can be written:

$$E_t R_{p,t+1} \ge \hat{R} + n\sigma_{p,t+1}, \tag{6}$$

where n is uniquely determined by m.[5] For example, if $\text{m} = 0.025$, then $n = 1.96$.

This constraint is equivalent to the following more common formulation in the literature discussing Value at Risk portfolio management rules:

$$\text{Prob}\left[V_{t+1} \le \hat{V}\right] \le m, \tag{7}$$

where there is an m percent chance of losing capital exceeding $\hat{V}$, and where $\hat{V}$ is the "value at risk". Since $R_{p,t+1} = V_{t+1}/V_t$, then defining $\hat{R} = \hat{V}/V_t$ yields the first version of the constraint, as presented in Telser (1955).

The mechanics of this portfolio selection rule can be understood by referring to the usual diagram depicting the opportunity set of available portfolios in mean-standard deviation space (Figure 1). With borrowing/lending rate r, all mean-variance efficient

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[5] Telser (1955) shows that the normality assumption is inessential: for an arbitrary distribution of asset returns, the Tchebycheff inequality yields a similar constraint.

portfolios lie on a straight line with vertical intercept r and slope $\left(\mu^*_{p,t+1} - r\right)/\sigma^*_{p,t+1}$. Portfolio '*' is the 'tangency portfolio': it is the portfolio comprised entirely of risky assets that is defined by the point of tangency between a ray from the vertical axis (with intercept r) and the set of feasible portfolios comprised of just risky assets.[6] The constraint (6) traces out a straight line in mean-standard deviation space, with intercept $\hat{R}$ and slope n. The permissible portfolios must lie on or above this line. This portfolio selection problem has an interior solution (that is, finite borrowing/lending) only when both of the following parametric restrictions are satisfied: $\hat{R} < r$, and $n > \left(\mu^*_{p,t+1} - r\right)/\sigma^*_{p,t+1}$. Under these assumptions, there exists an unique optimal portfolio, defined by the intersection of the constraint and the linear efficient set. Consequently, the optimal portfolio is a linear combination of borrowing/lending and portfolio '*'.

C. Volatility Events and Capital Events

Conditional on these portfolio management rules, the paper focuses on the optimal portfolio rebalancing response to one of two types of events. The first event is a shock to a single asset's return distribution. Calvo and Mendoza (1999) and Kodres and Pritsker (1999) consider this type of shock. The second type of event is an *ex post* shock to returns, such as the realization of losses on an investor's position in an asset (or several assets if there are common fundamentals). Calvo (1998,1999) is concerned with this type of shock in tandem with margin calls on informed investors.

The two basic types of events considered below can be defined more formally as follows:

Definition 1. *A 'volatility event' at time t is an increase in the (conditional) variance of an asset's return at time* $t+1$.

Definition 2. *A 'capital event' at time t is a decrease in* V_t.

A volatility event as defined here is a very narrow experiment, but it is a useful one because it isolates the effect of increased volatility in one market (one asset) on portfolio rebalancing. Below, more general experiments are considered that allow also for changes in other moments—namely the expected return on the same asset and cross-correlations with other assets—and for the simultaneous occurrence of volatility and capital events.

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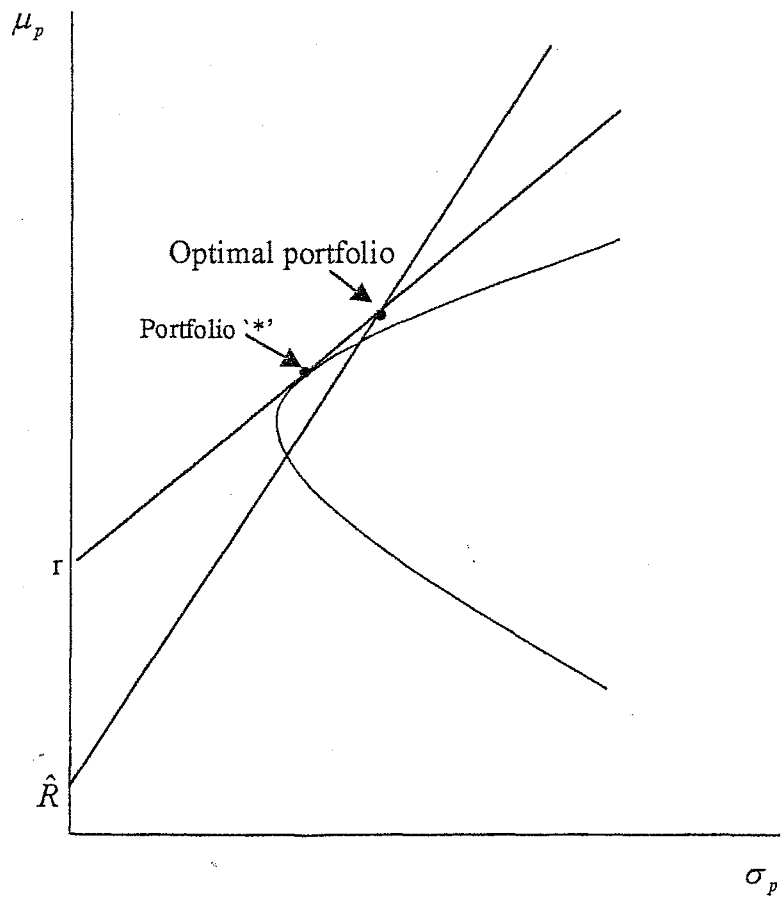
[6] The findings discussed below do not hinge on the availability of the debt finance. However, we are interested in the role of leverage in some of what follows, and for that reason we focus on this case.

Figure 1: Loss-Constraint Rule



IV. Volatility Events

In considering the impact of a volatility event, the amount of capital, V_t, is taken as given at the moment the event occurs, because the event relates to a change in the conditional distribution of asset returns in the future.[7] The analysis assumes $N = 2$ (that is, there are only two risky assets). This permits an explicit characterization of the optimal portfolio under all portfolio management rules, and allows one to examine analytically (rather than numerically) the effects of shocks to asset return distributions. This is accomplished largely without a loss of generality because no additional structure has been imposed on the joint distribution of asset returns beyond normality.[8]

The following result characterizes portfolio rebalancing for the return-benchmark and tradeoff rules when the portfolio optimally has long positions in both assets.[9]

Proposition 1. *If the optimal portfolio has long positions in both risky assets and there is positive covariance between asset returns, then for both the return-benchmark and tradeoff rules a volatility event in asset 2 necessarily decreases the position in asset 2 and increases the position in asset 1. If the covariance between asset returns is negative, then these same predictions hold for the return-benchmark rule, but under the tradeoff rule the optimal holdings of both risky assets decrease.*

This result can be interpreted in terms of 'income' and 'substitution' effects. An increase in the risk of asset 2 effectively raises the relative price of asset 2, and thus creates an incentive to tilt the portfolio away from asset 2 and toward other assets (a substitution effect). On the other hand, any given basket of risky assets is now riskier or 'more expensive' and this creates the incentive to reduce demand for risky assets generally (an income effect). The return-benchmark rule permits no flexibility in trading off risk and return on the portfolio, which ensures the portfolio readjustment is driven entirely by the substitution effect. Under the tradeoff rule, the substitution effect is weaker when asset returns are

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[7] It is implicitly assumed in what follows that if the source of leverage is margin then the initial margin constraint is not binding (*i.e.* $V_t/W_t \geq \iota$, where ι is the initial margin requirement).

[8] To permit analytical results, one could instead allow for an arbitrarily large number of risky assets, but impose additional structure on the joint distribution of asset returns. For instance, Calvo and Mendoza (1999) assume that all but one risky asset have normal iid distributions, which implies that the optimal weight on all of these risky assets is identical. This model maps directly into a two risky asset model.

[9] Derivations of the optimal portfolios for each of the rules as well as all technical proofs are contained in the appendix.

negatively correlated because diversification opportunities are greater, so the income effect dominates.

Several assumptions underly the formal results in Proposition 1. It is reasonable to ask whether it is these assumptions or the two portfolio management rules themselves that account for the fact that the rules examined in Proposition 1 will not generally imply the kind of portfolio rebalancing that is associated with contagion. That is, would a more general and less confining set of underlying assumptions imply that a volatility event would be associated with contagion under the return-benchmark and tradeoff rules?

First, only in the case of a positive covariance between asset returns does a volatility event increase the position in other assets under both the return-benchmark and the tradeoff rule. But is this the most interesting and relevant case and assumption? Because asset returns are generally positively correlated across countries, and particularly between emerging markets in the same region, it would appear that this is a reasonable assumption.[10]

Second, proposition 1 assumes that the covariance between asset returns is not affected by a volatility event. However, there are other possibilities. One is that the covariance between asset returns increases for a positive correlation when $\sigma_{2,t+1}$ increases via the mechanism $c_{t+1} = \rho_{t+1}\sigma_{1,t+1}\sigma_{2,t+1}$. For instance, this appears to be the argument advanced by some for Value-at-Risk models.[11] This link would appear to be a possible mechanism for producing contagion, but it is straightforward to show that this effect is not significant enough to produce contagion—proposition 1 holds in this case also. Another possibility is that the event is associated with a change in the correlation ρ_{t+1}. It can be shown, however, that this type of event alone cannot result in both asset demands decreasing, except in the case of $\rho_{t+1} < 0$, and then only for the tradeoff rule, which is qualitatively exactly the same result as proposition 1 for a volatility event.

Third, proposition 1 assumes that expected returns are constant. It might be interesting to examine an 'expected return event', defined as a decrease in the expected return on asset 2, perhaps in addition to a volatility event. This would not affect proposition 1 because asset 1 would become an even more favorable investment opportunity than asset 2.

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[10] For instance, during the period December 1991–December 1996, of 84 pairwise correlations between dollar-denominated daily returns for 14 emerging equity markets, Kaminsky and Reinhart (1999) show that 70 are positive; and when Russia is excluded, none exceed -.10. See also International Monetary Fund (1997).

[11] See Folkerts-Landau and Garber (1998), for example. They argue that increased volatility in one market will lead to upward reassessments of risk in correlated markets.

Fourth, as stated explicitly in proposition 1, the parameterization is assumed to be such that the optimal portfolio involves long positions in both risky assets. As shown formally in the appendix, under the return-benchmark and tradeoff rules a volatility event can produce selling of long positions in other assets by investors that have short positions in the event asset. In such cases, the volatility event causes short sellers to reduce short positions (as the asset is riskier to short sell), which *ceteris paribus* will tend to reduce the size of the long positions in other assets. This possibility is of some interest, but it does not explain why an investor would sell assets in all markets: closing out short positions requires *purchasing* the event asset.

In summary, under the return-benchmark and tradeoff rules, the fairly narrow predictions for portfolio rebalancing in the presence of a volatility event appear to reflect the characteristics of the portfolio management rules themselves. The next result shows that the loss-constraint rule has richer predictions than these other rules.

Proposition 2. *For the loss-constraint rule, a volatility event in asset 2 necessarily reduces the optimal position in asset 2, but has an ambiguous effect on the position in asset 1. Specifically, the event decreases (increases) the position in asset 1 if the following inequality is satisfied (is not satisfied):* [12]

$$n < \frac{2(\mu_{1,t+1} - r)(\mu_{2,t+1} - r)\sigma^*_{p,t+1}}{[(\mu_{2,t+1} - r)\sigma^2_{1,t+1} + (\mu_{1,t+1} - r)c_{t+1}][(\mu_{1,t+1} - r)\sigma^2_{2,t+1} + (\mu_{2,t+1} - r)c_{t+1}]}. \tag{8}$$

Drawing on the previous discussion, the reason why the loss-constraint rule can cause selling of all risky assets is that this rule can produce greater 'income effects.' The magnitude of the income effect is determined by the parameter n, which depends on the risk tolerance of the portfolio manager--the parameter m as defined in equation (5). When n is a relatively small number, then the risk tolerance of the portfolio manager is higher and the portfolio management rule requires only a fairly small increase in expected return on the portfolio to compensate for increased risk (see equation (6)). Loosely, this implies that the demand for risky assets is very sensitive to changes in the expected return per unit of risk on available portfolios of risky assets. In effect, the income effect of a change in the price of risk is large when n is low. Consequently, a volatility event, which amounts to a reduction in the expected return per unit of risk from choosing any given portfolio available in the market, produces a large income effect that results in a reduction in the demand for risky assets generally.

When asset returns are positively correlated, the loss-constraint rule is unique among the three rules considered here in explaining why an investor might reduce both risky asset positions when the volatility of the return on one asset increases. One can draw two conclusions from this observation. First, the portfolio management rule matters greatly for the response of an investor to a simple shock like a volatility event. Second, there are

[12] An explicit expression for σ^*_p is provided in the appendix.

plausible portfolio management rules for which portfolio managers would reduce all risky asset positions when volatility in one asset increases.

V. CAPITAL EVENTS

If 'increased volatility' at time t is associated also with a negative rate of return on the event asset at time t and also possibly on assets that are correlated with the event asset due to common fundamentals, then it may not be innocuous to assume that capital, V_t, is unaffected by the 'event'. In fact, that assumption would be a reasonable approximation only for investors that do not have positions in the event asset, or in assets that are correlated with the event asset because of common fundamentals.[13]

At the beginning of period t the amount of equity capital is V_t, where $V_t = V_{t-1}\left[r + \sum_{i=1}^{N} w_{i,t-1}\left(R_{i,t} - r\right)\right]$. A capital event at t is $V_t < V_{t-1}$, or equivalently $R_t^p < 1$, where $R_t^p = \left[r + \sum_{i=1}^{N} w_{i,t-1}\left(R_{i,t} - r\right)\right]$. A capital event could be the result of a significant loss on a position in one asset or, if there are common fundamentals, by losses on more than one asset. The next result isolates the consequences of a capital event on the optimal scale of investment in all risky assets.

Proposition 3. *Suppose the conditional distribution of asset returns is the same at time* $t-1$ *and t and a capital event occurs in period t. Then, for all portfolio management rules:*
(a) If the portfolio is not leveraged, then the optimal amount invested in risky assets at time t *is greater than the (fully-reinvested) value of the portfolio prior to rebalancing. Thus, there are net purchases of risky assets during period* t.
(b) If the portfolio is leveraged, then the optimal amount invested in risky assets at time t *is less than the (fully-reinvested) value of the portfolio prior to rebalancing. Thus, there are net sales of risky assets during period* t.

The assumption that the conditional asset return distribution is the same on both dates implies that the optimal portfolio weights are also the same on both dates. Thus, a reduction in capital affects only the scale of investment in risky assets and the amount of leverage. If the portfolio is not leveraged, then some (non-negative) fraction of capital is invested in the riskless asset. Proposition 3 implies that optimal portfolio rebalancing would shift some of the capital invested in riskless assets toward risky assets in order to re-establish optimal portfolio weights.This occurs because, prior to rebalancing the portfolio in period t, the total

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[13] One has to be careful at the event date to differentiate correlation between asset returns caused by common fundamentals from correlation caused by selling pressures–'pure contagion' effects. Of course, this is the fundamental issue in empirical studies that attempt to measure the latter by controlling for common fundamentals.

risky asset position falls by more than the reduction in capital—the riskless asset position yields a positive return, so the reduction in capital is equal to the loss on risky asset holdings less the income on riskless asset holdings. This implies that, on balance, there will be positive inflows in period t to risky assets.

In comparison to the case of an unleveraged portfolio, the optimal response to a capital event for a leveraged portfolio is to reduce leverage and reduce the total position in risky assets. Unlike in some of the other studies cited earlier, this has nothing to do with margin calls, because by assumption maintenance margin requirements are not important. When a portfolio is leveraged, optimal portfolio rebalancing can be associated with investors withdrawing capital from many higher-risk markets when losses are initially encountered in only one market.

Proposition 3 states that there will be a net inflow (outflow) to risky assets when the portfolio is unleveraged (leveraged). The next proposition identifies the net change in individual asset positions.

Proposition 4. *Assume that the conditional distribution of asset returns is the same at dates $t-1$ and t and a capital event occurs in period t. For all portfolio management rules, the optimal scale of investment in each asset (including borrowing/lending) is lower at t than at $t-1$. The optimal scale of investment in any asset i is less than (greater than) the fully-reinvested value of the position at the beginning of period t if the realized return on i is greater (less than) than the weighted-average return on the overall portfolio:*

$$R_{i,t} > (<)\ r + \sum_{i=1}^{N} w_{i,t-1}\left(R_{i,t} - r\right).$$

The first part of the proposition follows from the fact that the optimal scale of investment (for all portfolio management rules described above) in any asset is proportional to capital: $w_{i,t}V_t$. Lower capital at t means that the optimal scale of risky asset positions is lower than at $t-1$. Leverage is important insofar as the portfolio weights $w_{i,t}$ are (on average) larger than is the case for an unleveraged portfolio. Thus, a capital event of any given magnitude will produce larger reductions (on average) in the optimal scales of investment in individual assets.

The second part of the proposition compares investment positions after the portfolio has been rebalanced in period t to the value of positions just prior to rebalancing, assuming that returns are fully reinvested.[14] Optimal rebalancing involves reducing higher-yielding positions and investing the proceeds in lower-yielding assets in order to restore optimal portfolio weights. It is tempting to conclude that a capital event can not, therefore, lead to the reduction of positions in all risky assets. As discussed next, however, this depends on leverage.

[14] The case of zero reinvestment is equivalent to the comparison made in the first part of the claim.

If the portfolio is not leveraged, then a capital event will lead to an *increase* in positions in the assets with the largest losses. In the absence of changes in the relative future outlook for asset returns, portfolios will simply be rebalanced to re-align portfolio weights. In comparison, leverage can lead to sales of all risky assets, and a corresponding reduction in leverage. To see why, note that there are two consequences of leverage. First, leverage generally increases the sensitivity of V_t to risky asset returns because individual risky asset weights are necessarily larger (on average) than for an unleveraged portfolio. Thus, a given loss on a portfolio of risky assets generates a larger capital event. Second, for a capital event of a given magnitude, larger portfolio weights on risky assets implies that the optimal scale of investment in any asset, $w_{i,t}V_t$, is reduced by a larger amount when the portfolio is rebalanced. These two consequences of leverage can easily lead to large reductions in all risky asset positions when a capital event occurs, as is shown in the numerical exercises in section 6 below.

To illustrate some of the above ideas, consider the following simple example. Suppose that $R_{i,t} = 1$ for $i = 1,\ldots,N$. That is, the value of all risky asset positions have not changed from $t-1$ to t. Recalling that $R_t^P = \sum_{i=1}^{N} w_{i,t-1}R_{i,t} + w_{0,t}r$, note that for an unleveraged portfolio $V_t \geq V_{t-1}$, with strict inequality if there is a positive position in the riskless asset (*i.e.* $w_{0,t} > 0$). However, for a leveraged position, $V_t < V_{t-1}$, because $w_{0,t} < 0$. Thus, there would be net sales of each and every risky asset during period t, and a corresponding reduction in the scale of leverage. In this example, the presence of leverage itself is responsible for the capital event because the returns on risky asset positions are not sufficiently high to finance the position.

A. Margin Calls

In the preceding discussion, maintenance margin constraints are not important. There are two circumstances in which this is realistic. The first is when the portfolio manager directly takes a margined position and the leverage ratio, B_t/V_t, does not increase sufficiently that maintenance margin constraints are binding, *after* the portfolio has been rebalanced. As shown in section 6 below, rebalancing associated with some events can lead to substantial deleveraging, and thus this case is not as restrictive as it may appear. The second circumstance is when leverage arises for reasons other than buying securities *directly* on margin. This would be the case for trading portfolios of commercial and investment banks. It would also be reasonable for other institutional investors (such as hedge funds) that leverage their securities market positions through bank credit, repurchase agreements, and off-balance sheet transactions.[15] We next discuss briefly the role of maintenance margin requirements in those circumstances where they may become binding due to a capital event.

[15] The importance of non-margined leverage in particular is discussed at length in President's Working Group on Financial Markets (1999).

Maintenance margin requirements stipulate that the amount of capital underlying a long position in risky assets must not be less than some minimum percentage of the market value of the long position. For example, in the United States, margin calls on long positions in equity securities occur when capital falls below a specified percentage of the portfolio value, typically 25 percent of the market value of the position.[16]

It is straightforward to incorporate a maintenance margin requirement in the above framework. Recall that W_{t-1} denotes the size of risky asset position established in period $t-1$, financed by V_{t-1} dollars of capital and B_{t-1} in margin. Then a maintenance margin requirement at the beginning of period t before the portfolio is rebalanced – is a constraint:

$$\frac{V_t}{V_t + B_{t-1}} \geq \beta, \tag{9}$$

where $\beta \in [0,1]$ is the minimum percentage of the position that must be supported by capital. As has been implicitly assumed above, this assumes that any loss or gain on the net position is absorbed by capital. This is consistent with existing practice.

A *margin call* in period t is triggered by the event $V_t/(V_t + B_{t-1}) < \beta$. A decrease in portfolio value totaling $W_{t-1} - W_t = V_{t-1} - V_t > 0$ leads to a margin call totaling $\beta B_{t-1}/(1-\beta) - V_t$. The interesting case occurs when the margin call must be satisfied by reducing the size of the position, that is by *deleveraging*.[17] Specifically, a margin call totaling x would require the liquidation of $(1-\beta)x/\beta$ of the portfolio. As discussed above, even in the absence of maintenance margin requirements, the portfolio manager will optimally wish to reduce leverage when capital falls. Thus, a margin call will only be significant when it leads to a greater reduction in leverage than is optimal from the portfolio manager's perspective.

Consider an example. Let $\beta = 0.25$. Suppose that the size of the risky-asset position is 100 in period $t-1$, and the maintenance margin requirement is just satisfied in that period. Further, suppose the portfolio incurs a loss from $t-1$ to t of 10 percent. Assuming (for illustration purposes) that the margin call occurs before the portfolio is rebalanced, this

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[16] The Federal Reserve Board sets the initial margin currently at 50 percent (Regulation T). They do not, however, impose lower limits on equity capital as a percentage of the portfolio value subsequent to the date the position is initially established. These so-called maintenance margin requirements are imposed by brokers, and are typically 25 percent for long positions in equity markets. Maintenance margin requirements can be substantially higher or lower than this in other markets and for transactions other than long positions in primitive securities.

[17] The alternative is simply that the manager comes up with new capital—*i.e.*, capital injected into the portfolio from an external source—equal to the margin call.

would lead to a 10 unit margin call in period t, which would in turn require selling 30 units worth of the position; fully one-third of the total value of the position for a 10 percent reduction in capital. This assumes that margin requirements are consolidated—they apply to the total leveraged position in all assets. In practice, this may not be the case. For instance, a margined position by a foreign investor in Russia, for example, may be through a Russian broker, and thus the margin requirement would not be consolidated with the investor's accounts held elsewhere. In that case, a margin call could occur simply because the Russian position deteriorates—rather than the value of the total position—and would in general optimally need to be financed through liquidation of all positions.

B. Do Value-at-Risk Portfolio Management Rules Have Unique Consequences?

Some have claimed recently that the loss-constraint rule, and specifically, the VaR methodology, is a major source of 'contagion' and thus market volatility (see for example, Folkerts-Landau and Garber (1998), The Economist (1999)). The numerical exercises studied in section 6 below will provide some insight into differences in portfolio management rules. Nonetheless, three observations are noteworthy from the analysis above. First, the suggestion (*e.g.*, The Economist (1999)) that VaR portfolio management rules are somehow different because they imply that positions should be reduced when volatility rises overstates the uniqueness of VaR rules in this regard. If an asset's return volatility increases, then *ceteris paribus* most portfolio management rules will call for the position to be reduced. Second, a volatility event alone can lead to a general reduction in higher-risk positions under VaR rules, whereas for the other rules considered above this is only possible if there is either negative covariance between asset returns or there are short positions in the event asset. Third, a capital event has the same qualitative effect in all of the portfolio management rules. Most importantly, a capital event will tend to produce 'contagion' under a wide variety of portfolio management rules when the investor is leveraged.

Loss-constraint portfolio management rules could have fundamentally different predictions for the nature of the rebalancing if a capital event is associated also with a change in a parameter governing the rule, namely $\hat{R}$ or m.[18] But changes in the parameters $\hat{R}$ or m

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[18] For instance, write the constraint (7) at time $t-1$ in terms of equity capital as $\text{Prob}[R_{p,t}V_{t-1} < (1-\beta)V_{t-1}] \le m$, where $\beta \in (0,1)$, and βV_{t-1} is the maximum amount of capital that can be lost by the manager; in terms of the previous statement of this constraint, $\hat{R} = (1-\beta)$. Suppose that coming into period t the portfolio incurs a loss of $\gamma \le \beta$ percent of capital. If the total capital available in t is therefore only $(1-\gamma)V_{t-1}$, then to maintain an m percent chance of losing the allocated capital would require in period t that $\text{Prob}[R_{p,t+1}V_{t-1}(1-\gamma) < (1-\beta)V_{t-1}] \le m$, or $\text{Prob}[R_{p,t+1} < (1-\beta)/(1-\gamma)] \le m$. Since $(1-\beta)/(1-\gamma) > (1-\beta)$, it follows that the manager must have a higher cutoff return $\hat{R}$. That would imply a shift to less risky portfolio. An alternative interpretation that has the same implications is that the magnitude m falls in light of losses, implying that with lower

(continued...)

are, in rules based on standard portfolio theory, tantamount to a change in the target expected return for the return-benchmark rule or a change in the risk tolerance parameter for the tradeoff rule. Thus, there is nothing unique about loss-constraint rules in how they interact with a capital event.

VI. Numerical Exercises

To illustrate the propositions discussed above, some numerical exercises are presented. As a benchmark, suppose that in period $t-1$, capital is $V_{t-1} = 100$ and the joint asset-return distribution is as follows:[19] $\mu_{1,t} = 112$, $\mu_{2,t} = 115$, $r = 106$, $\sigma_{1,t} = 8$, $\sigma_{2,t} = 12$, and $\rho_t = 0.1$. This parameterization results in identical 'Sharpe ratios' for both assets equal to 0.75.[20] In practice, (*ex post*) Sharpe ratios range between 0.2 and 2.0, depending on the asset (or portfolio), the time period used to compute means and standard deviations, and the frequency of the assumed holding period. The value of the Sharpe ratio in our example is in line with estimates for emerging equity markets and for higher-yielding bonds.[21] The consequences of alternative parameterizations are not particularly important (as discussed below).

The experiment considered has two features. First, at time t the value of $\sigma_{2,t+1}$ increases by 50 percent (to a value of 18). This increase is substantial, but it is less than observed increases in bond and equity return volatility at the onset of the crisis in Asia as well as the Mexico crisis.[22] It is likely that selling pressures due to the event further raise price volatility, and thus we use a conservative measure of the volatility event.

The second feature of the experiment is that a capital event occurs at time t. Specifically, we assume a loss of ten percent per unit invested in risky assets. We do not make any additional assumptions about how this loss is spread between the two risky assets; one can readily consider different possibilities by comparing the changes in asset positions between the two dates to an assumed loss on the positions in each asset. The magnitude of

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probability can the manager accept the same size losses. That too would imply a shift toward a less risky portfolio.

[19] Gross returns are expressed in percent.

[20] The Sharpe ratio here is defined in the standard way as the mean return minus the riskless rate, divided by the standard deviation. See Sharpe (1994) for a thorough discussion.

[21] See, for example, International Monetary Fund (1997,1999).

[22] Using weekly returns, the standard deviation of many Asian equity markets in 1997 and Latin American equity markets in 1995 roughly doubled (see International Monetary Fund (1998,1999)).

the capital event caused by the ten percent loss on the risky asset portfolio will depend on the amount of leverage in the portfolio. Specifically, capital at time t is given by $V_t = V_{t-1}\left[\left(w_{1,t-1}R_{1,t} + w_{2,t-1}R_{2,t}\right) + w_{0,t-1}r\right]$. A loss of ten percent per unit invested implies $\left(w_{1,t-1}R_{1,t} + w_{2,t-1}R_{2,t}\right)/\left(w_{1,t-1} + w_{2,t-1}\right) = 0.90$. Thus, capital at time t is calculated as $V_t = V_{t-1}\left[0.90\left(w_{1,t-1} + w_{2,t-1}\right) + w_{0,t-1}r\right]$.

In addition to the parameters discussed above, there are parameters specific to the various portfolio management rules—k for the return benchmark rule, τ for the tradeoff rule, and $\{n, \hat{R}\}$ for the loss-constraint rule. We choose $\{k, \tau, \hat{R}\}$ to yield specific degrees of leverage; the additional parameter for the loss-constraint rule, n, is set at 1.96. We consider three levels of leverage: no leverage, 'modest leverage' (a one-to-one leverage ratio), and 'high leverage' (a three-to-one leverage ratio). The largest of these leverage ratios is reasonably high leverage for an investor that has a margined position. However, these leverage ratios are very low compared with some major financial institutions that, at times, may have significant positions in emerging markets.[23]

Turning to the results, note first that in period $t-1$, conditional on a given degree of leverage, all portfolio management rules result in nearly identical portfolios of risky assets (Tables 1-3). The rebalanced portfolio is also very similar for two of the three rules considered. In fact, in all examples discussed, the loss-constraint rule and the tradeoff rule yield very similar asset positions before and after the event, whereas rebalancing under the return-benchmark can be quite different because of the rigidity of this rule—the manager has no flexibility to tradeoff risk and return on the overall portfolio. Note that the change in optimal positions in individual assets between the two dates is broken down into the individual effects of the volatility event and the capital event.[24] One can therefore identify the consequences of just one of these two events occurring.

There are four main observations from this exercise. First, the capital event alone reduces each asset position by precisely the magnitude of the capital event–which is −10 with zero borrowing/lending, and a proportionately larger amount with leverage—times the period $t-1$ optimal asset weight; this is proposition 4. Note also that the magnitude of the capital event, and thus the magnitude of the rebalancing, increases by a multiple of the leverage

[23] Long Term Capital Management's balance sheet leverage was 28-to-1 at end–1997; it also had off-balance sheet OTC derivatives positions with notional value totaling 1.3 trillion at end-1997. The average balance sheet leverage of the five largest U.S. commercial bank holding companies at end-1998 was 14-to-1, while the five largest investment banks' average leverage ratio was 27-to-1. The source for these figures is President's Working Group on Financial Markets (1999).

[24] The effect of the volatility event is calculated using period – $t-1$ capital, while the effect of the capital event is calculated using period $t-1$ optimal asset weights $\{w_{1,t-1}, w_{2,t-1}\}$.

ratio: a 3-to-1 leverage ratio increases the magnitude of the capital event almost six-fold, and leads to reductions in asset positions more than twenty times larger than without leverage.

Second, as discussed in section 4, the volatility event alone results in a tilting of the portfolio toward the non-event asset. Leverage simply magnifies the magnitudes involved in this rebalancing of the portfolio. Recall that there are circumstances in which a volatility event produces reductions in both risky assets; evidently, these circumstances do not arise in this first exercise, but they will in some exercises discussed below.

Third, as the formal results above suggest, the volatility and capital events complement each other in producing a sharp decrease in the position in the event asset. In the absence of leverage, this reduction is roughly 50 percent of the original position in the event asset, and equivalent to about twice the magnitude of the reduction in capital. This implies that, even if all losses on the portfolio were due to losses on the event asset, the manager would sell some of this position in period t. As noted above, leverage results in proportionately larger consequences of both the volatility and capital events. Taken together, even with modest leverage the total reduction in the position in the event asset more than doubles.

The fourth observation is that it is certainly *not difficult to generate substantial reductions in the optimal position in the non-event asset*. Most striking is how large risky asset sales are with leverage: for a 3-to-1 leverage ratio, sales of the non-event asset are generally in excess of half the initial position, and well in excess of all of the initial capital in the portfolio. Thus, supposing that the capital event occurs because of losses largely on the event asset position, this suggests that roughly half of the position in the non-event asset is liquidated. In addition, while there are some differences in portfolio rebalancing under the three rules (particularly for the return-benchmark rule), when there is leverage these differences are of second-order importance. In the presence of a capital event, a leveraged portfolio will be rebalanced in such a way that there would be large reductions in all risky asset positions under all portfolio management rules (propositions 3-4).

The numbers at the bottom of each panel in the tables show the ratios of total risky asset positions to capital at the two dates, as well as the inverse of this ratio—which is (9), the ratio that is important for maintenance margin requirements. As discussed in section 5, a main reason for the large asset reductions during period t (when leverage is present) is due to a reduction in the *scale* of borrowed funds. In addition, the volatility event changes optimal portfolio weights, which could produce a further reduction in the scale of leverage by reducing the optimal leverage ratio. Indeed, for two of the three portfolio management rules (the return-benchmark rule is the exception) the leverage ratio falls quite sharply. This implies that, to the extent that leverage is due to margined positions, a margin call in this example would produce additional deleveraging only for one of the three rules (the return-benchmark rule).

Consider the effect of alternative correlations between asset returns. A higher positive correlation (Tables 4-5) or a negative correlation (Tables 6-7) does not yield substantially different conclusions, especially when the portfolio is leveraged. Recall that a volatility event can reduce the optimal position in the non-event asset only in two cases: for the tradeoff rule when there is negative correlation between asset returns, and it is also possible in the case of the loss-constraint rule (for any correlation). Both of these possibilities arise in Tables 6-7 which reports the results for a negative correlation. Note that in none of the numerical examples discussed above for a positive correlation does the loss-constraint rule result in the volatility event reducing the position in the non-event asset. As proposition 4 suggests, this is more likely for a lower value of n. Reducing n from 1.96 to 1.5 (results are not reported in a table) is in fact sufficient to produce this result in the case of a positive correlation, but this effect is really quite small. Again, the consequences of deleveraging are much more importantly quantitatively than differences in portfolio management rules.

Finally, consider the consequences of a different borrowing/lending rate r. This changes the Sharpe ratios for both assets. Again, the results are reasonably robust to changes in r, particularly when leverage is present because the magnitude of the changes in positions from deleveraging due to the capital event dominate all other effects. For instance, Table 8 reports the results for modest leverage and a reduction in r by 2 percentage points.

VII. Conclusion

There are two main conclusions of this paper. First, a shock to a single asset's return distribution may lead to a reduction in other risky asset positions. This result is sensitive to the portfolio management rule and the parameterization of the joint distribution of asset returns. Second, the impact of a capital event on optimal portfolio rebalancing hinges mainly on whether or not the portfolio (or institution) is leveraged. The general conclusion is simple, but fundamental: an investor with a leveraged portfolio will reduce risky asset positions if the return on the leveraged portfolio is less than the cost of funding. This conclusion is independent of whether the leverage is margined or not; that is, it is independent of whether margin calls occur.

As Calvo (1998,1999) and others have argued, a relatively high degree of leverage helps explain why the Russian event had greater and geographically wider financial effects than other recent events.[25] The point emphasized in this paper is that in the presence of leverage, elementary portfolio theory leads to the conclusion that it is optimal to deleverage and reduce risky positions. That it is optimal to reduce exposures to risky assets is just as relevant for financial institutions that do not take outright leveraged positions, but instead simply finance their activities with borrowed funds, as is the case for banks and other financial institutions (see President's Working Group on Financial Markets (1999)).

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[25] The President's Working Group on Financial Markets (1999) discusses how leverage, at both the institutional level and in terms of margined positions, were a major factor underlying the turbulence in international financial markets during the summer and fall of 1998.

The paper considers a variety of simple portfolio management rules, and evaluates claims that VaR rules cause contagion. One claim is that contagion occurred after the Russian unilateral restructuring because financial institutions use VaR rules. As shown in this paper, contagion is a fairly general prediction of elementary portfolio theory under reasonable conditions and is not at all unique to VaR rules. Another claim is that VaR rules have different, volatility-enhancing implications for financial markets. This paper finds that, for the most part, VaR rules do not produce portfolio rebalancing dynamics that are very different from the other portfolio management rules considered. The claim that VaR rules are the source of contagion or market volatility in recent crises and turbulence seems unwarranted. According to the analysis presented in this paper, financial contagion could be a side-effect associated with textbook-type optimal rebalancing of diversified, leveraged portfolios.

Table 1. No Leverage

(Return correlation 0.1)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions: Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	60.0	76.1	24.6	-6.0	16.1
Asset 2	40.0	21.2	-16.4	-4.0	-18.8
Borrow/lend	0.0	-7.4	-8.2	0.0	-7.4
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	1.08			
V/(V+B)	1.00	0.92			
B. Tradeoff Rule					
Asset 1	60.0	57.0	3.3	-6.0	-3.0
Asset 2	40.0	15.9	-22.3	-4.0	-24.1
Borrow/lend	0.0	17.1	19.0	0.0	17.1
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.81			
V/(V+B)	1.00	1.23			
C. Loss-Constraint Rule					
Asset 1	60.0	57.6	4.0	-6.0	-2.4
Asset 2	40.0	16.1	-22.1	-4.0	-23.9
Borrow/lend	0.0	16.3	18.1	0.0	16.3
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.82			
V/(V+B)	1.00	1.22			

Table 2. Modest Leverage

(Return correlation 0.1)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions		
			Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	120.0	125.2	49.2	-31.2	5.2
Asset 2	80.0	34.9	-32.8	-20.8	-45.1
Borrow/lend	-100.0	-86.1	-16.4	26.0	13.9
Totals	100.0	74.0	0.0	-26.0	-26.0
(V+B)/V	2.00	2.16			
V/(V+B)	0.50	0.46			
B. Tradeoff Rule					
Asset 1	120.0	93.8	6.7	-31.2	-26.2
Asset 2	80.0	26.2	-44.6	-20.8	-53.8
Borrow/lend	-100.0	-45.9	37.9	26.0	54.1
Totals	100.0	74.0	0.0	-26.0	-26.0
(V+B)/V	2.00	1.62			
V/(V+B)	0.50	0.62			
C. Loss-Constraint Rule					
Asset 1	120.0	94.7	8.0	-31.2	-25.3
Asset 2	80.0	26.4	-44.3	-20.8	-53.6
Borrow/lend	-100.0	-47.2	36.2	26.0	52.8
Totals	100.0	74.0	0.0	-26.0	-26.0
(V+B)/V	2.00	1.64			
V/(V+B)	0.50	0.61			

Notes: See Table 1.

Table 3. High Leverage

(Return correlation 0.1)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions: Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	240.0	142.1	98.4	-139.2	-97.9
Asset 2	160.0	39.7	-65.6	-92.8	-120.3
Borrow/lend	-300.0	-139.8	-32.8	174.0	160.2
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	4.33			
V/(V+B)	0.25	0.23			
B. Tradeoff Rule					
Asset 1	240.0	106.4	13.4	-139.2	-133.6
Asset 2	160.0	29.7	-89.3	-92.8	-130.3
Borrow/lend	-300.0	-94.1	75.9	174.0	205.9
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	3.24			
V/(V+B)	0.25	0.31			
C. Loss-Constraint Rule					
Asset 1	240.0	107.5	16.1	-139.2	-132.5
Asset 2	160.0	30.0	-88.5	-92.8	-130.0
Borrow/lend	-300.0	-95.6	72.5	174.0	204.4
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	3.28			
V/(V+B)	0.25	0.31			

Note: See Table 1.

Table 4. No Leverage

(Return correlation 0.5)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions: Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	60.0	84.0	33.3	-6.0	24.0
Asset 2	40.0	16.0	-22.2	-4.0	-24.0
Borrow/lend	0.0	-10.0	-11.1	0.0	-10.0
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	1.11			
V/(V+B)	1.00	0.90			
B. Tradeoff Rule					
Asset 1	60.0	70.9	18.7	-6.0	10.9
Asset 2	40.0	13.5	-25.0	-4.0	-26.5
Borrow/lend	0.0	5.6	6.2	0.0	5.6
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.94			
V/(V+B)	1.00	1.07			
C. Loss-Constraint Rule					
Asset 1	60.0	72.5	20.5	-6.0	12.5
Asset 2	40.0	13.8	-24.7	-4.0	-26.2
Borrow/lend	0.0	3.7	4.1	0.0	3.7
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.96			
V/(V+B)	1.00	1.04			

Notes: Parameter values are as in Table 1, with asset correlation 0.5.

Table 5. High Leverage

(Return correlation 0.5)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions: Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	240.0	156.8	133.3	-139.2	-83.2
Asset 2	160.0	29.9	-88.9	-92.8	-130.1
Borrow/lend	-300.0	-144.7	-44.4	174.0	155.3
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	4.44			
V/(V+B)	0.25	0.23			
B. Tradeoff Rule					
Asset 1	240.0	132.3	75.0	-139.2	-107.7
Asset 2	160.0	25.2	-100.0	-92.8	-134.8
Borrow/lend	-300.0	-115.5	25.0	174.0	184.5
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	3.75			
V/(V+B)	0.25	0.27			
C. Loss-Constraint Rule					
Asset 1	240.0	135.3	82.2	-139.2	-104.7
Asset 2	160.0	25.8	-98.6	-92.8	-134.2
Borrow/lend	-300.0	-119.1	16.5	174.0	180.9
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	3.84			
V/(V+B)	0.25	0.26			

Note: Parameter values are as in Table 1, with asset correlation 0.5

Table 6. No Leverage

(Return correlation -0.25)

	Period t-1	Period t	Effects on Asset Positions		
	Positions	Positions	Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	60.0	72.0	20.0	-6.0	12.0
Asset 2	40.0	24.0	-13.3	-4.0	-16.0
Borrow/lend	0.0	-6.0	-6.7	0.0	-6.0
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	1.07			
V/(V+B)	1.00	0.94			
B. Tradeoff Rule					
Asset 1	60.0	46.3	-8.6	-6.0	-13.7
Asset 2	40.0	15.4	-22.9	-4.0	-24.6
Borrow/lend	0.0	28.3	31.4	0.0	28.3
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.69			
V/(V+B)	1.00	1.46			
C. Loss-Constraint Rule					
Asset 1	60.0	43.4	-11.8	-6.0	-16.6
Asset 2	40.0	14.5	-23.9	-4.0	-25.5
Borrow/lend	0.0	32.1	35.7	0.0	32.1
Totals	100.0	90.0	0.0	-10.0	-10.0
(V+B)/V	1.00	0.64			
V/(V+B)	1.00	1.56			

Note: Parameter values are as in Table 1, with asset correlation -0.25.

Table 7. High Leverage

(Return correlation -0.25)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions: Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	240.0	134.4	80.0	-139.2	-105.6
Asset 2	160.0	44.8	-53.3	-92.8	-115.2
Borrow/lend	-300.0	-137.2	-26.7	174.0	162.8
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	4.27			
V/(V+B)	0.25	0.23			
B. Tradeoff Rule					
Asset 1	240.0	86.4	-34.3	-139.2	-153.6
Asset 2	160.0	28.8	-91.4	-92.8	-131.2
Borrow/lend	-300.0	-73.2	125.7	174.0	226.8
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	2.74			
V/(V+B)	0.25	0.36			
C. Loss-Constraint Rule					
Asset 1	240.0	81.0	-47.1	-139.2	-159.0
Asset 2	160.0	27.0	-95.7	-92.8	-133.0
Borrow/lend	-300.0	-66.0	142.8	174.0	234.0
Totals	100.0	42.0	0.0	-58.0	-58.0
(V+B)/V	4.00	2.57			
V/(V+B)	0.25	0.39			

Note: Parameter values are as in Table 1, with asset correlation -0.25.

Table 8. Modest Leverage: Lower Value of r

(Return correlation 0.1)

	Period t-1 Positions	Period t Positions	Effects on Asset Positions Vol. Event	Cap. Event	Total Effect
A. Return Benchmark Rule					
Asset 1	125.0	128.7	44.3	-30.0	3.7
Asset 2	75.0	32.5	-32.2	-18.0	-42.5
Borrow/lend	-100.0	-85.2	-12.1	24.0	14.8
Totals	100.0	76.0	0.0	-24.0	-24.0
(V+B)/V	2.00	2.12			
V/(V+B)	0.50	0.47			
B. Tradeoff Rule					
Asset 1	125.0	99.8	6.3	-30.0	-25.2
Asset 2	75.0	25.2	-41.8	-18.0	-49.8
Borrow/lend	-100.0	-49.0	35.6	24.0	51.0
Totals	100.0	76.0	0.0	-24.0	-24.0
(V+B)/V	2.00	1.64			
V/(V+B)	0.50	0.61			
C. Loss-Constraint Rule					
Asset 1	125.1	91.9	-4.0	-30.1	-33.1
Asset 2	75.0	23.2	-44.4	-18.0	-51.8
Borrow/lend	-100.0	-39.2	48.4	24.1	60.8
Totals	100.0	75.9	0.0	-24.1	-24.1
(V+B)/V	2.00	1.52			
V/(V+B)	0.50	0.66			

Note: Parameter values are as in Table 1, with r=104.

Appendix I

Sketch of Proof of Proposition 1.

First note that optimal risky asset shares under the expected return benchmarking rule are:

$$w_{1,t} = \frac{(k-r)(\widetilde{\mu}_{1,t+1}\sigma^2_{2,t+1} - \widetilde{\mu}_{2,t+1}c_{t+1})}{\widetilde{\mu}_{1,t+1}(\widetilde{\mu}_{1,t+1}\sigma^2_{2,t+1} - \widetilde{\mu}_{2,t+1}c_{t+1}) + \widetilde{\mu}_{2,t+1}(\widetilde{\mu}_{2,t+1}\sigma^2_{1,t+1} - \widetilde{\mu}_{1,t+1}c_{t+1})}, \tag{A1}$$

$$w_{2,t} = \frac{(k-r)(\widetilde{\mu}_{2,t+1}\sigma^2_{1,t+1} - \widetilde{\mu}_{1,t+1}c_{t+1})}{\widetilde{\mu}_{1,t+1}(\widetilde{\mu}_{1,t+1}\sigma^2_{2,t+1} - \widetilde{\mu}_{2,t+1}c_{t+1}) + \widetilde{\mu}_{2,t+1}(\widetilde{\mu}_{2,t+1}\sigma^2_{1,t+1} - \widetilde{\mu}_{1,t+1}c_{t+1})}, \tag{A2}$$

where $\widetilde{\mu}_i = \mu_i - r$. Asset demands are necessarily positive if *both*:

$$\widetilde{\mu}_{1,t+1}\sigma_{2,t+1} - \rho_{t+1}\widetilde{\mu}_{2,t+1}\sigma_{1,t+1} > 0, \tag{A3}$$

and

$$\widetilde{\mu}_{2,t+1}\sigma_{1,t+1} - \rho_{t+1}\widetilde{\mu}_{1,t+1}\sigma_{2,t+1} > 0, \tag{A4}$$

or both of these inequalities are reversed. If both are reversed then that implies: $\widetilde{\mu}_{2,t+1}/\sigma_{2,t+1} < \rho_{t+1}\widetilde{\mu}_{1,t+1}/\sigma_{1,t+1}$, and $\widetilde{\mu}_{1,t+1}/\sigma_{1,t+1} < \rho_{t+1}\widetilde{\mu}_{2,t+1}/\sigma_{2,t+1}$, which is impossible for $\rho \in [0,1]$. In addition, note that the denominators of (A1)-(A2) can be written:

$$\widetilde{\mu}_{1,t+1}\sigma^2_{2,t+1} + \widetilde{\mu}^2_{2,t+1}\sigma^2_{1,t+1} - 2\widetilde{\mu}_{1,t+1}\widetilde{\mu}_{2,t+1}c_{t+1},$$

which is necessarily greater than the magnitude $(\widetilde{\mu}_{1,t+1}\sigma_{2,t+1} - \widetilde{\mu}_{2,t+1}\sigma_{1,t+1})^2 > 0$. This implies that the denominators of (A1)-(A2) are positive. Thus, either (A3) *and* (A4) are satisfied, *or* only one of them is reversed. The former case corresponds to long positions in both risky assets, and the latter case corresponds to a short position in one asset. It follows that $w_{1t} > 0$ if and only if (A3) is satisfied, and $w_{2t} > 0$ if and only if (A4) is satisfied. It is easily shown that the sign of $dw_{1,t}/d\sigma_{2,t+1}$ depends on whether or not inequality (A4) is satisfied. Thus, for long positions in both assets, $dw_{1,t}/d\sigma_{2,t+1} > 0$. Similarly, $dw_{2,t}/d\sigma_{2,t+1} < 0$ when inequality (A4) is satisfied.

For the tradeoff rule asset demands are:

$$w_{1,t} = \frac{(\widetilde{\mu}_{1,t+1}\sigma^2_{2,t+1} - \widetilde{\mu}_{2,t+1}c_{t+1})}{\tau(\sigma^2_{1,t+1}\sigma^2_{2,t+1} - c^2_{t+1})}, \tag{A5}$$

$$w_{2,t} = \frac{(\widetilde{\mu}_{2,t+1}\sigma^2_{1,t+1} - \widetilde{\mu}_{1,t+1}c_{t+1})}{\tau(\sigma^2_{1,t+1}\sigma^2_{2,t+1} - c^2_{t+1})}. \tag{A6}$$

Again, it is easily shown that (A3)-(A4) are necessary for asset demands to both be positive. It is also straightforward to show that $dw_{1,t}/d\sigma_{2,t+1} < 0$ for $c_{t+1} > 0$ and (A4) satisfied, and $dw_{1,t}/d\sigma_{2,t+1} > 0$ if $c_{t+1} < 0$. Finally, $dw_{2,t}/d\sigma_{2,t+1} < 0$ (regardless of the sign of c_{t+1}) when (A4) is satisfied.

Derivation of Optimal Portfolio Under the Loss-Constraint Rule.

The first step in characterizing the optimal portfolio is to calculate portfolio '*'. Equating the slopes of the opportunity set of risky asset portfolios with the slope of the linear efficient set implies (dropping time subscripts) that:

$$\mu_p^* = r + \tilde{\mu}_i \left(\frac{(\sigma_p^*)^2}{c_{i*}} \right), \quad i = 1,2, \tag{A7}$$

where c_{i*} is the covariance between the return on asset i and the portfolio '*'. We therefore have two equations in two unknowns—the two weights corresponding to the two risky assets in portfolio '*'. Let the weight on asset 1 in portfolio '*' be denoted by α, with the weight on asset 2 being $1-\alpha$ by the adding up constraint. Dividing the two expressions given by (A7) and rearranging gives the following expression for α:

$$\alpha = \frac{\tilde{\mu}_1 \sigma_2^2 - \tilde{\mu}_2 c}{\tilde{\mu}_2 \sigma_1^2 + \tilde{\mu}_1 \sigma_2^2 - (\tilde{\mu}_1 + \tilde{\mu}_2) c}. \tag{A8}$$

Note that $(\mu_p^* - r) = (\mu_1 x + \mu_2 y)/(x+y)$ and $\sigma_p^* = (x^2\sigma_1^2 + y^2\sigma_2^2 + 2xyc)^{1/2}/(x+y)$, where $x = \tilde{\mu}_1\sigma_2^2 - \tilde{\mu}_2 c$ and $y = \tilde{\mu}_2\sigma_1^2 - \tilde{\mu}_1 c$.

It is straightforward to show that $d\alpha/d\sigma_2 > 0$ so long as $\tilde{\mu}_2\sigma_1^2 - \tilde{\mu}_1 c > 0$, which is true if portfolio '*' is comprised of only long positions in risky assets. Also, regarding the second effect discussed in the text, notice that conditional on any basket of risky assets, an increase in σ_2 reduces the slope of the opportunity set—by increasing the denominator of $(\mu_p - r)/\sigma_p$—associated with linear combinations of this basket of risky assets and borrowing/lending.

The second step in characterizing the optimal portfolio is to find the weights $\{w_0, w_1, w_2\}$ such that $w_0 = 1 - w_1 - w_2$ and the following two equations hold simultaneously:

$$\mu_p = r + z\sigma_p, \tag{A9}$$

$$\mu_p = \hat{R} + n\sigma_p, \tag{A10}$$

where $\mu_p = w_0 r + w_1\mu_1 + w_2\mu_2$ and $z = (\mu_p^* - r)/\sigma_p^*$. The portfolio that solves both of these equations is the optimal portfolio. It can be shown that the above two equations yield a quadratic equation in w_1 (or equivalently, in w_2) of the form $\bar{a}w_1^2 + \bar{b}w_1 + \bar{c} = 0$. Normally, by the quadratic formula, there would be two roots of this equation, but as shown below it is the case that $\bar{b}^2 - 4\bar{a}\bar{c} = 0$. This implies that there is a unique root, $w_1 = -\bar{b}/2\bar{a}$, where:

$$\bar{a} = \sigma_1^2 + \left(\frac{\tilde{\mu}_1}{\tilde{\mu}_2} \right)^2 \sigma_2^2 - 2\left(\frac{\tilde{\mu}_1}{\tilde{\mu}_2} \right) c, \tag{A11}$$

$$\bar{b} = -\left(\frac{2z(r-\hat{R})}{\tilde{\mu}_2(n-z)}\right)\left(\frac{\tilde{\mu}_1\sigma_2^2}{\tilde{\mu}_2} - c\right). \tag{A12}$$

This solution implies:

$$w_2 = \frac{z(r-\hat{R})}{\tilde{\mu}_2(n-z)} - w_1\left(\frac{\tilde{\mu}_1}{\tilde{\mu}_2}\right). \tag{A13}$$

Proof that $\bar{b}^2 - 4\bar{a}\bar{c} = 0$.

The expressions for the coefficients $\bar{b}$ and $\bar{a}$ are given above. The expression for $\bar{c}$ is (dropping time subscripts):

$$\bar{c} = \left(\frac{r-\hat{R}}{n-z}\right)^2\left(\frac{z^2\sigma_2^2}{(\mu_2 - r)^2} - 1\right).$$

Using these it follows that $\bar{b}^2 - 4\bar{a}\bar{c}$ is proportional to:

$$\frac{z^2}{\tilde{\mu}_2^2}\left(\frac{\tilde{\mu}_1\sigma_2^2}{\tilde{\mu}_2} - c\right)^2 - \left(\sigma_1^2 + \frac{\tilde{\mu}_1^2\sigma_2^2}{\tilde{\mu}_2^2} - \frac{2\tilde{\mu}_1 c}{\tilde{\mu}_2}\right)\left(\frac{z^2\sigma_2^2}{\tilde{\mu}_2^2} - 1\right), \tag{A14}$$

Expanding (A14) and canceling terms yields:

$$\frac{z^2}{\tilde{\mu}_2^2}\left(c^2 - \sigma_1^2\sigma_2^2\right) + \left(\sigma_1^2 + \frac{\tilde{\mu}_1^2\sigma_2^2}{\tilde{\mu}_2^2} - \frac{2\tilde{\mu}_1 c}{\tilde{\mu}_2}\right). \tag{A15}$$

Next, note that $z = (\tilde{\mu}_1 x + \tilde{\mu}_2 y)/(x^2\sigma_1^2 + y^2\sigma_2^2 + 2xyc)^{1/2}$. Using the definitions of x and y in z, substituting this in (A15), multiplying the resulting expression by the denominator of z, and expanding this expression yields a very large number of terms, all of which cancel. Thus, $\bar{b}^2 - 4\bar{a}\bar{c} = 0$

Proof of Proposition 2.

First consider $dw_{1,t}/d\sigma_{1,t+1}^2$ (which is, of course, identical to $dw_{2,t}/d\sigma_{2,t+1}^2$). Dropping time subscripts, w_1 can be written:

$$w_1 = \frac{(r-\hat{R})x}{nq^{1/2} - \tilde{\mu}_1 x - \tilde{\mu}_2 y}, \tag{A16}$$

where $q = (x^2\sigma_1^2 + y^2\sigma_2^2 + 2xyc)$.

Notice that σ_1^2 only affects the denominator of w_1. Focusing on the denominator which we henceforth denote by D we have:

$$\frac{d(D)}{d\sigma_1^2} = \frac{1}{q^{1/2}}\left(\frac{n}{2}\left(x^2 + 2y\tilde{\mu}_2\sigma_2^2 + 2x\tilde{\mu}_2 c\right) - \tilde{\mu}_2^2 q^{1/2}\right). \tag{A17}$$

Recall that $n > (\mu_p^* - r)/\sigma_p^*$. Since the right side of the above derivative is strictly increasing in n, if it can be shown to be positive for the smallest possible value of n, then that is sufficient to establish the claim. Thus, let $n = (\mu_p^* - r)/\sigma_p^*$. Substituting this into the right side of the above derivative and rearranging yields:

$$\text{sign}\left(\frac{d(D)}{d\sigma_1^2}\right) = \text{sign} - \left[2(\sigma_p^*)^2 \frac{d(\tilde{\mu}_1 x + \tilde{\mu}_2 y)}{d\sigma_1^2} - (\tilde{\mu}_1 x + \tilde{\mu}_2 y)\frac{d(\sigma_p^*)^2}{d\sigma_1^2}\right]. \tag{A18}$$

Next, notice that

$$\text{sign}\left(\frac{d[(\mu_p^* - r)/\sigma_p^*]}{d\sigma_1^2}\right) = \text{sign}\left[2(\sigma_p^*)^2 \frac{d(\tilde{\mu}_1 x + \tilde{\mu}_2 y)}{d\sigma_1^2} - (\tilde{\mu}_1 x + \tilde{\mu}_2 y)\frac{d(\sigma_p^*)^2}{\sigma_1^2}\right]. \tag{A19}$$

It follows that if $d((\mu_p^* - r)/\sigma_p^*)/d\sigma_1^2 < 0$ then it has been shown that $d(D)/d\sigma_1^2 > 0$ and thus $dw_1/d\sigma_1^2 < 0$.

To complete the proof then we need to establish that $d((\mu_p^* - r)/\sigma_p^*)/d\sigma_1^2 < 0$. In the two risky asset case, any point on the opportunity set of risky assets is given by the standard programming problem that minimizes portfolio variance subject to the portfolio having an expected return equal to some number *k*, where *k* pins down each point on the opportunity set. In the two asset case, the constraint uniquely defines the asset proportions in the portfolio. Thus, since the asset proportions for each point on the opportunity set do not depend on σ_1, it follows that the effect of an increase in σ_1 is that it causes a horizontal (rightward) shift in each point on the opportunity set. Consequently, it must be the case that a ray with intercept *r* and which is tangent to the opportunity set must have lower slope for higher σ_1. This completes the proof of the first part of the claim.

To establish the second part of the claim, note that:

$$\text{sign}\left(\frac{dw_1}{d\sigma_2^2}\right) = \text{sign}\left[\tilde{\mu}_1\left(\sigma_p^* n - (\tilde{\mu}_1 x + \tilde{\mu}_2 y)\right) - x\left(\frac{n}{2\sigma_p^*}\left(2x\tilde{\mu}_1\sigma_1^2 + y^2 + 2\tilde{\mu}_1 yc\right) - \tilde{\mu}_1^2\right)\right].$$

Substituting in the expressions for *x* and *y*, simplifying, and rearranging yields the expression in the claim.

Restatement of Proposition 1 in the Case of Short Positions.

The following result parallels proposition 1 for the case of short positions in one of the two risky assets.

Proposition A1: Given a positive covariance between asset returns, if the optimal portfolio has a short position in asset 2 and a long position in asset 1, then for both the return-benchmark and tradeoff rules a volatility event in asset 2 necessarily reduces the short position in asset 2 and reduces the long position in risky asset 1. If the covariance *between asset returns is negative then these same predictions hold for the return benchmarking rule, but under the tradeoff rule the long position in asset 1 increases. Finally, if the optimal portfolio has a short position in asset 1 and a long position in asset 2, then the qualitative consequences of a volatility event are exactly the same as proposition 1.*

Sketch of Proof of Proposition A1.

As discussed in the proof of proposition 1, the optimal portfolio for the return-benchmark rule has a short position in asset 2 if inequality (A4) is violated, and a long position in asset 1 if inequality (A3) is satisfied. Thus, $dw_{1,t}/d\sigma_{2,t+1} < 0$ and $dw_{2,t}/d\sigma_{2,t+1} > 0$ because (A4) is violated by assumption.

For the tradeoff rule, the sign of $dw_{1,t}/d\sigma_{2,t+1}$ depends on the sign of $\rho_{t+1}(\tilde{\mu}_{2,t+1}\sigma_{1,t+1} - \tilde{\mu}_{1,t+1}\rho_{t+1}\sigma_{2,t+1})$, and the sign of $dw_{2,t}/d\sigma_{2,t+1}$ depends on the sign of $-(\tilde{\mu}_{2,t+1}\sigma_{1,t+1} - \tilde{\mu}_{1,t+1}\rho_{t+1}\sigma_{2,t+1})$. The term in parentheses is negative when (A4) is violated. The first part of the claim follows.

For the case in which the manager has a short position in asset 1 and a long position in asset 2, the claim follows because (A4) is now satisfied, as in proposition 1.

Proof of Proposition 3.

The size of the risky asset position in period *t* after the portfolio is rebalanced is $W_t = \sum_{i=1}^{N} w_{i,t-1}V_t$, since $w_{i,t} = w_{i,t-1}$ if the conditional distribution of asset returns is the same at the two dates. Thus, $dW_t/dV_t = \sum_{i=1}^{N} w_{i,t}$. If the portfolio is not leveraged then $w_{0,t} > 0$, and thus $\sum_{1=1}^{N} w_{i,t} < 1$, implying that the position in risky assets falls by less than the reduction in capital. Similarly, if $w_{0,t} < 0$, then $\sum_{1=1}^{N} w_{i,t} > 1$, and the risky asset position falls by more than the reduction in capital. In this case, leverage is also reduced since leverage is equal to $w_{0,t}V_t$, which is increasing in V_t.

Proof of Proposition 4.

The optimal position in any asset *i* in any period *s* is $w_{i,s}V_s$. If the conditional distribution of asset returns is the same at dates $t-1$ and *t*, then $w_{i,t-1} = w_{i,t}$. Thus, $w_{i,t}V_t < w_{i,t-1}V_{t-1}$ if $V_t < V_{t-1}$, which is true by assumption.

At the start of period t, before the portfolio has been rebalanced, the fully-reinvested position in any asset is $w_{i,t-1}R_{i,t}V_{t-1}$. In contrast, after rebalancing, the desired amount invested in this asset is $w_{i,t}V_t$. Thus, the position in asset i is reduced in period t if $w_{i,t-1}R_{i,t}V_{t-1} > w_{i,t}V_t$. Next, note that $V_t = \sum_{i=0}^{N} w_{i,t-1}R_{i,t}$ (where asset 0 denotes borrowing/lending), and also that, if the conditional distribution of asset returns is identical at $t-1$ and t, then $w_{i,t-1} = w_{i,t}$. In this case, we have that the position in asset i is reduced if:

$$R_{i,t} - r > \sum_{i=0}^{N} w_{i,t-1}(R_{i,t} - r).$$

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