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Portfolio Optimization for Defence Applications

KYLE ROBERT HARRISON[1], (Member, IEEE), SABER ELSAYED[1], (Member, IEEE), IVAN GARANOVICH[2], TERENCE WEIR[2], MICHAEL GALISTER[2], SHARON BOSWELL[2], RICHARD TAYLOR[2], AND RUHUL SARKER[1], (Member, IEEE)

[1]School of Engineering and Information Technology, University of New South Wales at Canberra, Australian Defence Force Academy, Canberra, ACT 2600, Australia
[2]Joint and Operations Analysis Division, Defence Science and Technology Group, Department of Defence, Canberra, ACT 2610, Australia

Corresponding author: Kyle Robert Harrison (kyle.harrison@unsw.edu.au)

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ABSTRACT The problem of designing an effective future defense force is quite complex and challenging. One methodology that is often employed in this domain is portfolio optimization, whereby the objective is to select a diverse set of assets that maximize the return on investment. In the defense context, the return on investment is often measured in terms of the capabilities that the investments will provide. While the field of portfolio optimization is well established, applications in the defense sector pose unique challenges not seen in other application domains. However, the literature regarding portfolio optimization for defense applications is rather sparse. To this end, this paper provides a structured review of recent applications and identifies a number of areas that warrant further investigation.

INDEX TERMS Future force design, portfolio optimization, defense planning, project selection, project prioritization, uncertainty, robustness.

I. INTRODUCTION

Future Force Design (FFD) is an important planning task undertaken by defense organizations to assist in making critical investment decisions pertaining to the development of a future defense force. Among many other difficulties associated with FFD, defense organizations face the problem of selecting an optimal portfolio of investments that provide the most appropriate balance of capabilities. The investments that constitute the portfolios are often quite varied and can include components such as research, training procedures, and equipment procurement, upgrade, or maintenance. The selection of an optimal portfolio of investments is certainly not a new problem and has attracted a large amount of research effort since the development of the portfolio optimization theory in the 1950s [1]. Applications of portfolio optimization arise in a number of different contexts, most commonly in financial markets [1]–[4] and R&D contexts [5]–[9], but can be found even in areas such as quantum computing [10].

While each application domain of portfolio optimization comes with its own unique challenges and nuances, the applications of portfolio optimization in the defense sector face a number of distinctive characteristics [11]. Firstly, in contrast to traditional financial applications, defense applications are often faced with the task of optimizing many different objectives. These objectives generally cannot be reduced to a single quantifiable value and are often in conflict. Common objectives include, but are not limited to, defending the nation and its citizens, defending allies, defending against cyber-security threats, maintaining and fostering political allies, and providing humanitarian aid in disaster areas. Each of these situations involves many specific mission needs and uncertain aspects.

This uncertainty poses a second unique challenge associated with defense applications of portfolio optimization. Not only is the future global landscape uncertain, but also there are many uncertain aspects associated with simply selecting projects in the defense sector. The most common, and most easily addressed, forms of uncertainty arise in aspects such as the cost, available budget, and threat scenarios. However, as stated by Gray [12], defense is often tasked with providing answers to questions that have not even been posed yet. Technical, governmental, and geopolitical changes can all have a dramatic impact on FFD. Despite their impacts, it is impossible to foresee these changes and their effects. Nonetheless, strategies to hedge against these potential events must be considered during the planning phase.

Planning in the defense sector is also characterized by complex inter-dependencies among the capability alternatives. Furthermore, these complex inter-dependencies and capability synergies may not be fully understood and

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may arise as a byproduct of the environment in which they are realized. Constraints also pose an important challenge for defense planning. Often, there will be many constraining factors, such as budgetary limitations, different "colors of money" [13], scheduling and manufacturing requirements, and available personnel. Defense planning process also often considers long-planning horizons whereby decisions can have long-lasting implications on both a national and global scale.

Due to these challenges and nuances, defense planning should not be considered strictly an optimization process. Rather, it should be considered as an administrative tool with political repercussions [14]. Despite these known challenges and the importance of FFD, it is often reported that the literature regarding portfolio optimization for defense applications is rather scant [15]–[19]. This is especially true for applications that consider capability-oriented approaches.

Therefore, in this paper, recent applications of portfolio optimization applied to defense-oriented planning are surveyed. The review is presented in two broad sections. The first section considers the application areas and describes the modeling and problem formulations. The second section discusses the main challenges associated with portfolio optimization in the defense sector and explores how these challenges have been addressed in the literature.

The remainder of this paper is structured as follows. Section II provides relevant background information. Section III presents the methodology used to conduct this review. Sections IV and V provide the main body of the review, discussing the applications of portfolio optimization in the defense sector and the major challenges associated with this domain, respectively. Section VI provides a discussion of the current state as well as some avenues for future research. Finally, concluding remarks are given in Section VII.

II. BACKGROUND

This section provides background information on portfolio optimization, the knapsack problem formulation, and the methodology known as Capability-Based Planning (CBP).

A. PORTFOLIO OPTIMIZATION

Portfolio optimization, in the general sense, is a process by which an optimal portfolio (i.e., distribution of assets) is selected according to some objective measure, with the caveat that the associated risk should also be minimized [1]. In the traditional, financial formulation, portfolio optimization is concerned with maximizing the expected return of a set of investments while also minimizing the associated risks, such as stock-market volatility. Markowitz [1] argued that the characteristics of a particular asset should not be viewed independently. Rather, one should consider how an asset affects the risk and return of the entire portfolio. Based on this theory of portfolio optimization, it is possible to formulate a multi-objective problem that simultaneously maximizes the return and minimizes risk [20]. Using multi-objective optimization, a set of non-dominated solutions, which represent optimal trade-offs between the return and risk, is attained. The decision maker can select the portfolio that suits their level of risk aversion and expectation of returns.

By the late 1950's, project selection was recognized as an important problem in the field of operations research [21]. Project selection problems were formulated by the need to determine the combinations of project proposals that an organization should fund to maximize the attainment of their objectives. Clearly, these problems had a high degree of similarity to the financial applications considered by Markowitz and, therefore, the underlying theories have been applied to more broad application areas than the financial markets for which they were developed. The next section discusses the classic mean-variance model proposed by Markowitz.

1) THE MEAN-VARIANCE MODEL

The main contribution of Markowitz [1] was the view that including risk as a secondary objective was a necessary component in portfolio optimization. This led to the development of modern portfolio analysis and is referred to as the mean-variance model. In such a model, the objective is to maximize the expected return (i.e., the mean), while minimizing the volatility of the investment (i.e., the variance).

To illustrate this concept, assume that there exist N assets, whose rates of return are given by the random variables $r_1, r_2, \ldots, r_N$, and that the proportions of an investment allocated to each asset are given by $w_1, w_2, \ldots, w_N$, such that $\sum_{i=1}^{N} w_i = 1$. The return for a portfolio P can then be calculated as

$$R_P = \sum_{i=1}^{N} w_i r_i. \tag{1}$$

The expected value of the return, and hence the investment portfolio, can then be calculated as the first moment of R_P,

$$\mu_P = \mathrm{E}[R_P] = \sum_{i=1}^{N} \mathrm{E}[w_i r_i] = \sum_{i=1}^{N} w_i \mu_i \tag{2}$$

where $\mu_i = \overline{r_i}$ is the average return for asset i.

Classical portfolio optimization focused solely on maximization of (2). The work of Markowitz [1] revolutionized the field of portfolio theory by adding the minimization of risk as an additional objective. The risk, in this context, can be calculated as the second moment of R_P (i.e., the variance), given by

$$\sigma_P^2 = \mathrm{Var}[R_P] = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \operatorname{cov}(r_i, r_j) \tag{3}$$

In this model, it is typically assumed that μ_i and σ_{ij} are known quantities.

While the variance provides a measure of risk, one critical issue is that it degrades when the distribution of possible returns is asymmetric. Specifically, this calculation of variance would place equal weight on both the upward and

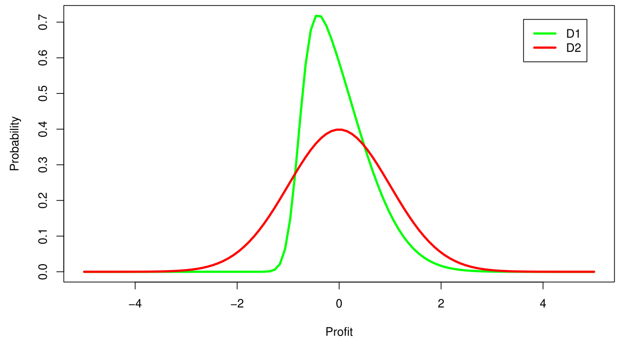

FIGURE 1. Example demonstrating two distributions with the same mean but different levels of upside and downside risk.

downward risks. For example, consider the two distributions given in Fig. 1, which each have a mean value of 0. Despite having the same mean, and thus expected value, the two distributions have significantly different risk portfolios. Note that, D1 has a variance of ≈ 0.39 while D2 has a variance of 1 and that D2 is symmetric about the mean while D1 is not. Thus, a measure of variance for D1 will be influenced to a greater degree by values above the mean given their larger spread. However, financial investors are typically only concerned with the downside risk and thus a measure of risk that can be skewed by the upside risk may be misleading. Furthermore, the traditional mean-variance model of Markowitz is known to overestimate the true return [22], [23]. Thus, it was necessary to devise an alternative measure of risk that was only influenced by the region of interest.

It should be noted that the subsequent discussion of alternative risk measures is not meant to cover the state-of-the-art. Rather, these measures are discussed only because they have arisen in the course of this review. For a more comprehensive review of risk measures in portfolio optimization, the reader is referred to [24] and [25].

2) DOWNSIDE RISK

In his early work, Markowitz recognized the inefficiencies associated with asymmetrical distributions when considering the mean-variance model and thus suggested an alternative measure of risk, known as Downside Risk (DSR) [26]. DSR is a measure of downward volatility, specifically the variance among returns that fall below some threshold τ, as given by:

$$DSR_P = \mathrm{E}[(R_P - \tau)^2 I_\tau], \tag{4}$$

where I_τ is an indicator function that returns 1 when $R_P \leq \tau$, and 0 otherwise. Note that, when $\tau = \mu_P = \mathrm{E}[R_p]$, this measure is known as the semi-variance. It is straightforward to see that when a distribution is symmetric, the DSR and variance are equivalent. However, in the case of an asymmetric distribution, the DSR value provides a more relevant measure of risk given that an investor is typically only concerned with minimizing the downward volatility.

3) CONDITIONAL VALUE-AT-RISK

A more recent measure of risk is known as the value-at-risk, which represents the predicted maximum loss with a specified probability level over a known time horizon. A further evolution of this measure is known as the Conditional Variance-at-Risk (CVaR) and represents a weighted average between the value-at-risk and the losses that exceed the value of the value-at-risk measure. It is noted that protecting against the value-at-risk does not limit exposure to worst-case scenarios – CVaR addresses this limitation.

The CVaR assumes that each asset has an associated linear loss function. The linear optimization problem can then be formulated as

$$\min_{x,y,\gamma} \left[\gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s \right] \tag{5a}$$

$$\text{subject to}: \; z_s \geq \sum_i (b_i - y_{is})' x_i - \gamma \tag{5b}$$

$$\sum_i \mu_i x_i \geq R \tag{5c}$$

where γ is the value-at-risk term, α is the specified probability level, b_i is the expected return, y_{is} is the stochastic simulated return scenario(s) for asset i, S is the number of stochastic scenarios to be generated using a Monte Carlo approach, and R is the expected minimum rate of return. Optimizing this model will provide a frontier depicting the best tradeoff between expected return and CVaR.

4) ROBUSTNESS

The primary objective of all portfolio optimization applications is to maximize some form of value while minimizing some form of cost or risk. In an ideal situation where all variables and external factors are known, selecting the best portfolio (or set of equivalent portfolios) is relatively straightforward. However, when faced with uncertainty, decision-making becomes significantly more difficult as one cannot necessarily assign a strict value to an asset nor the associated risk. For example, consider a financial investment context where the profit associated with one particular asset is given by the range [-50%, 50%] – the value of this asset drastically changes if the actual, realized return is -50% versus 50%. To make an informed decision, a decision-maker should then have information about what a plausible future might look like. Moreover, selecting a portfolio that is only minimally affected by such future uncertainties would be ideal.

One strategy to deal with future uncertainty is to make assumptions about how the future will unfold. However, decisions made under one set of assumptions may not hold under different assumptions. Thus, it would be beneficial to examine a wide variety of plausible future states and quantify how well a particular portfolio performs under the various possibilities. The robustness of a solution can then be defined as the degree to which a particular solution is stable under a set of plausible futures [27]. The simplest strategy for measuring robustness of an asset is to vary the uncertain values (within a reasonable range), then count the frequency that this asset appears in the optimal solution. Intuitively, an asset that appears in the optimal portfolio under a wide variety of scenarios is preferable to one that only appears under specific conditions. While robustness is certainly a useful metric, accurately calculating the value can be problematic. Specifically, the reliability of a robustness measure is inherently tied to how accurate the future states are represented.

B. THE KNAPSACK PROBLEM

A well-known problem in combinatorial optimization is the knapsack problem, which is the process of selecting a set of items, each with a given weight and value, that maximizes the total value while adhering to the specified weight limit of the knapsack. It is clear to see how this problem is related to portfolio optimization and FFD, as one can view the investment options as the ''items'' and the available budget as the ''weight limit.'' Therefore, the knapsack problem can be analogously formulated as maximizing the benefit associated with implementing projects that adhere to a set budget.

The most common formulation of the knapsack problem is the 0-1 (or binary) knapsack problem, which is formulated as

$$\max \sum_{i=1}^{n} v_i x_i \tag{6a}$$

$$\text{subject to } \sum_{i=1}^{n} w_i x_i \leq W \quad \text{and } x_i \in \{0, 1\} \quad \forall i \in 1, \ldots, n \tag{6b}$$

where v_i is the value of item i, x_i is the quantity, and w_i is the weight. The constraints given by (6b) stipulate that the total weight of the items must be less than the total capacity W, and that the quantity of item x_i is exactly zero or one, i.e., there is exactly one of each item that can be either included or excluded.

The knapsack problem is known to be NP-hard and, as such, there is currently no polynomial-time algorithm that can solve an arbitrary instance of the knapsack problem. However, an approximate solution with a bounded error term (ϵ) can be generated in polynomial time – a provably rare occurrence – by limiting the number of significant bits in the value terms [28].

Alternative formulations of the knapsack problem also exist where x_i can be any non-negative integer (i.e., $x_i \in \mathbb{Z}^+$), or x_i is a bounded non-negative integer (i.e., $x_i \in [0..c]$). Furthermore, one can consider formulations where x_i is a real-valued variable such that fractional portions of an item can be included, in both bounded and unbounded formulations.

Other notable formulations of the knapsack problem include the multi-objective knapsack problem, whereby one wishes to optimize more than just the total value of the items, and the multi-dimensional knapsack problem, whereby the items' weights and knapsack capacity are given by vectors such that the capacity constraint in all dimensions must be satisfied. Similarly, one can impose additional constraints on the solutions such that all constraints must be satisfied for a solution to be feasible.

C. CAPABILITY-BASED PLANNING

CBP provides an analytical paradigm that addresses the problem of planning under uncertainty, where the underlying goal is to provide capabilities that are suitable for a wide variety of challenges while operating within the confines of an economic framework that necessitates choice [29]. CBP arose as a successor to ''threat-based planning'' or ''point-scenario planning,'' whereby planning would revolve around meeting the requirements of countering specific threats [30]. In contrast, CBP encourages the adoption of solutions that address a wide variety of plausible future scenarios by focusing on the development of *capabilities* rather than direct countermeasures [31]. A capability, in this context, refers to the ability to achieve an operational effect and includes integral components such as doctrine, training, and leadership [31]. In a sense, CBP adopts a ''building-block'' methodology, such that a robust set of (generic) lower-level capabilities (such as battalions, resources, infrastructure, equipment, etc.) are developed in a manner whereby they can be composed to meet the requirements of the foreseeable future. Thus, the overall goal of CBP is to derive a capability development plan that is able to meet the overall strategic objectives of an organization.

The CBP paradigm consists of a few major components. Firstly, there must be a set of high-level objectives that are desired. For example, the 2016 Defence White Paper (DWP) [32], published by the Australian Department of Defence (DoD), lists three, equally-weighted strategic defence objectives: 1) deter, deny, and defeat threats against Australia or its national interests, 2) make effective military contributions to support the security of maritime South East Asia, and 3) contribute military capabilities to coalition operations that support Australia's global interests. These high-level objectives can then be further decomposed into sub-objectives. Consider objective 1) above, which can then be decomposed into numerous sub-capabilities, such as defence of the territory from attack, protection of offshore infrastructure, performing domestic counter-terrorism operations, defeating cyber attacks, etc.

Secondly, there must be an analytical framework that assesses such capabilities, with respect to both their benefits and risks, at an operational level. Essentially, there must be a mechanism whereby a quantitative score is assigned to each capability with respect to each objective. However, this poses a significant challenge as it is often difficult to quantify the benefits or risks associated with defence-related projects. It is often necessary to incorporate knowledge from Subject Matter Experts (SMEs) in the assessment phase. Consider, again, the strategic objective of the Australian Defence Force (ADF) to defer or defeat threats to Australia and suppose that an upgrade to an aircraft is programmed – to what degree does this address the strategic objective? Does it address the objective to a higher degree than a similar upgrade to a naval ship? The usefulness of CBP depends heavily on how adequately questions of this sort can be answered.

Finally, there is an inherent deep uncertainty associated with CBP. In the context of national defence, many uncertain factors arise such as changes in government or national security policies, local and global threat scenarios, technological advances, and budget availability. Decision makers thus need to account for these various uncertainties while making their planning decisions. Specifically, a plan that is robust to the volatility of all of these factors is far more valuable than a plan that would be a catastrophic failure if, for example, the available budget decreases in the next year. However, choosing the most robust plan does not necessarily provide the greatest benefit – how likely is it that a major change in *all* of these factors will be observed in the foreseeable future? Is a plan that provides a greater average value better than a plan that provides greater value in all but the most unlikely scenarios? Take, for example, the results from Fisher *et al*. [33], where it was found that withholding budget from one year was preferable to "frivolously" spending all available funds to minimize lapsed funding. While this seems illogical, the justification was that by withholding a portion of the budget to carry forward (i.e., "slippage"), there was greater planning space available for when a high-value project materializes; by maximizing the immediate benefit, the capacity for long-term benefit was diminished. However, it should be noted that slippage can lead to unexpected consequences in the future. Thus, there is an inherent complexity that deep uncertainty adds to this problem.

III. REVIEW METHODOLOGY

To identify relevant articles for this review, a search was conducted using Scopus[1] on 21/10/2019 for articles published within the last 20 years (i.e., after the year 1999) with either of the following patterns in the title or abstract:

- (defence OR military OR air force OR navy OR army OR armed forces OR weapon?) AND (portfolio optimization OR portfolio selection OR portfolio investment OR portfolio prioritization OR portfolio planning OR project optimization OR project selection OR project investment OR project prioritization OR project planning OR balance of investment OR cost?benefit analysis OR cost-efficiency analysis OR fleet-optimization OR capability planning OR acquisition decisions OR funds allocation OR allocation of funds)
- (weapon selection OR weapon planning OR weapon project) AND optimization

Note that, the '?' character permits the replacement of a single character. Hence, "cost?efficiency" would match with "cost efficiency," "cost-efficiency," or any other variation that had a single character between the terms "cost" and "efficiency." The search automatically accounted for British and American spelling variations such as "optimization" versus "optimisation." This preliminary search resulted in 212 articles. Given the relatively small number of papers returned, articles were manually filtered for relevance. Furthermore, additional articles found via the references contained in these articles were also included in this review. In total, 54 application-based articles were identified as relevant and included in Section IV. To the best of the authors' knowledge, articles from predatory journals were not included in this review.

[1]Available at: https://www.scopus.com/search/form.uri

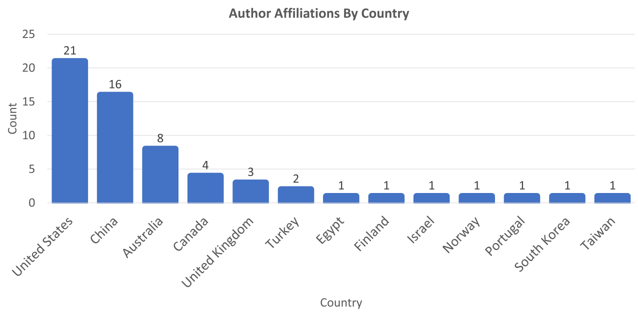

FIGURE 2. Country of author affiliations. Countries are counted only once per publication, regardless of the number of authors affiliated with the country.

Regarding the 54 application-oriented papers, Fig. 2 presents the countries of affiliation associated with each paper. Note that, a country was only counted once per publication, regardless of the number of authors affiliated with a particular country. It is evident that a majority of the work on defence applications comes from either the United States or China, with Australia at a distant third. Interestingly, from a pool of only 54 papers there were 13 countries represented, which indicates that portfolio optimization for defence applications is attracting global attention.

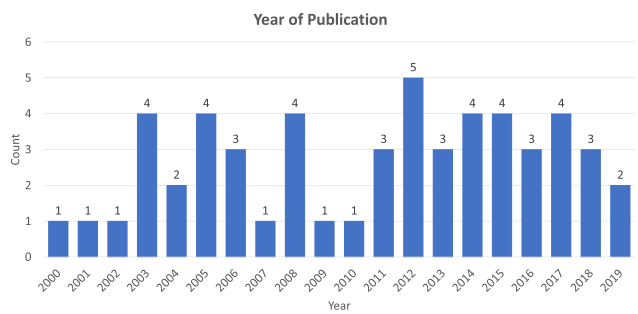

FIGURE 3. Number of publications by year.

Fig. 3 visualizes the year of publication for the pool of 54 papers used in the applications section of this review. Evidently, this application area is generally attracting more attention in recent years. Nonetheless, the number of publications has been relatively stable over the entire examined period.

TABLE 1. Summary of literature regarding portfolio optimization for defence applications. Legend: MC – multi-criteria, MO – multi-objective, MOO – multi-objective optimization, IP – integer programming, LP – linear programming, MILP – mixed-integer linear programming, NIP – nonlinear integer programming, MIQP – mixed-integer quadratic programming.

Ref.	Year	Application	Multi-period	Uncertainty	MC/MO	MOO	Optimizer
[34]	2000	Laboratory equipment purchsing			✓		N/A
[6]	2001	R&D for weapons-complex clean-up			✓		N/A
[35]	2002	Cyber risk minimization			✓		LP
[36]	2003	Space system investments			✓		ILP
[37]	2003	Aerospace project selection	✓	✓			Commercial application (OptQuest)
[38]	2003	Air force weapon system selection			✓		0-1 IP (Excel)
[39]	2003	Vehicle procurement		✓	✓		MILP (XpressMP)
[40]	2004	Vehicle procurement		✓	✓		MILP (XpressMP)
[41]	2004	Air force research lab resource planning			✓		Nonlinear multi-attribute value function
[42]	2005	Information system selection			✓		Unspecified
[43]	2005	Army modernization	✓		✓	✓	MILP (ADBASE)
[44]	2005	Anti-terrorism resource allocation			✓		IP (LINGO)
[45]	2005	Anti-ballistic missile defence	✓				Pontryaguine principle
[46]	2006	Air force research resource allocation			✓		Nonlinear multi-attribute value function
[47]	2006	Procurement and balance of investment		✓	✓		N/A
[48]	2006	Military production plant planning			✓		LP (LINGO)
[49]	2007	Vehicle fleet planning	✓	✓	✓	✓	EA
[50]	2008	Defence technology management		✓	✓		N/A
[51]	2008	Vehicle fleet structure		✓	✓	✓	NSGA-II
[52]	2008	Resource planning	✓	✓	✓	✓	EA
[53]	2008	Humanitarian project selection			✓	✓	LP
[54]	2009	Low-expenditure project selection					IP (unspecified)
[55]	2010	Missile system selection			✓		LP (LINGO)
[56]	2011	Dynamic weapon-target assignment	✓				Novel heuristic
[57]	2011	Military investment assets	✓		✓		GA and Tabu Search
[58]	2012	Infrastructure resource allocation		✓	✓	✓	Latin hypercube sampling
[59]	2012	Countermeasure resource allocation	✓				IP (MOSEK)
[60]	2012	Weapon systems portfolio selection		✓	✓	✓	Novel heuristic (exhaustive)
[15]	2012	Capability planning	✓		✓	✓	NSGA-II (custom)
[61]	2012	Weapon system planning			✓		DE
[62]	2013	Uncertainty reduction in flight testing		✓	✓		Exhaustive generation
[63]	2013	Future force design	✓	✓			IP (CPLEX)
[64]	2013	Optional project selection		✓			NIP (CPLEX)
[65]	2014	Project selection		✓	✓		LP
[16]	2014	Project scheduling	✓	✓	✓	✓	NSGA-II (custom)
[66]	2014	Project selection			✓		DE (custom)
[67]	2014	Naval warfare SoS evolution		✓			LP (YALMIP)
[68]	2011	Weapon system of systems		✓	✓		N/A
[33]	2015	Navy project selection	✓	✓			IP (CPLEX) / robustness heuristic
[69]	2015	Weapon system selection			✓	✓	MO IP
[70]	2015	Investment planning	✓		✓		IP (GMPL)
[71]	2015	SoS military capability planning		✓	✓		MIQP (YALMIP)
[72]	2016	Military planning			✓	✓	MO MILP / EA
[73]	2016	Weapon system selection	✓				Adaptive immune GA
[74]	2016	Project selection	✓	✓	✓	✓	0-1 LP
[75]	2017	Weapon system selection			✓		Unspecified (exhaustive)
[17]	2017	Future force design	✓	✓	✓	✓	MOEA/D
[18]	2017	Weapon selection and planning	✓	✓	✓	✓	NSGA-II
[76]	2017	Weapon equipment planning	✓		✓		GA
[19]	2018	System selection		✓	✓	✓	NSGA-II
[77]	2018	High-end weapon selection			✓		Novel heuristic
[78]	2018	Submarine acquisition	✓		✓	✓	NSGA-II
[79]	2019	Robust weapon planning	✓	✓	✓	✓	NSGA-II
[80]	2019	System selection		✓	✓	✓	NSGA-II

The review is presented in two main components, applications and challenges. In the first component, an overview of the problem formulations and proposed models are given. The second component of this review discusses the main difficulties associated with portfolio optimization in the defence sector and examines how the relevant literature has addressed these difficulties.

IV. APPLICATIONS OF PORTFOLIO OPTIMIZATION IN THE DEFENCE SECTOR

Table 1 provides a brief summary of the overall application areas and methodological aspects of the studies considered in this review, with further details about the specific contributions from each paper given below. Studies are presented in chronological order.

Baker *et al*. [34] employed value-focused thinking to provide an analytical framework that assisted in decision-making for US Air Force laboratory equipment purchasing. A hierarchical model was formulated to ensure that the educational outcomes were the foremost concern in the decision-making process. This model was subsequently used to assign weights to the various identified attributes via a survey completed by SMEs. Projects that attained a minimum valuation were funded immediately whereas the remaining projects had to compete for funding.

Jones *et al*. [6] examined the prioritization of research and development (R&D) for nuclear weapon complex cleanup for the US Department of Energy. The prioritization was done using three independent goals. Specifically, the objectives considered were the minimization of the overall risk at the end of a ten-year planning period, minimization of the risk in each year during the planning period, and maximization of the cost-effectiveness. Despite having multiple goals, this study did not employ multi-objective optimization principles. Rather, a hypothetical analysis was conducted for each goal, reflecting on what the outcome may be if this goal were implemented. No optimization was performed to provide empirical evidence to support the hypothetical outcomes.

Hamill *et al*. [35] considered value-focused thinking to mitigate the risks associated with cyber attacks on the US Department of Defense. Various threats were assigned an overall risk score according to their effects on the network if successful, such as the number of affected users and duration as well as their respective probabilities of being used. Multiple criteria were combined using a simple additive value function. Linear programming was employed to optimize the allocation of resources towards minimizing the risk. No experimental results were reported.

Brown *et al*. [36] considered long-term (24 years) space system investments for the US Air Force Space Command. The problem was considered in five sub-steps, namely mission area assessment, mission needs analysis, mission solution analysis, portfolio optimization, and portfolio refinement. The first two phases were primarily oriented towards the formalization of the strategic goals and requirements. The solution analysis phase provided a valuation and cost analysis for each project. The optimization phase used mixed-integer linear programming to select a portfolio according to the system constraints and budget. Finally, the refinement phase allowed the decision-maker to account for various preferences and constraints that could not be stated mathematically. A case study using 200 candidate systems for adoption during the 2002-2025 planning horizon was considered, with 74 systems being considered components of the best portfolio.

The study of Crawford *et al*. [37] focused on the description of a software package for addressing the generic problem of project selection over multiple years. To provide a worst-case analysis, it was assumed that every project could immediately begin and could last the entire duration of the planning period. A software package, referred to as Crystal Ball,[2] was employed to carry out uncertainty analysis by facilitating the input of multiple possible values for a particular variable. A Monte Carlo method was then used to indicate the likelihood of specific outcomes. Another software package, referred to as OptQuest,[3] was employed to perform optimization. No experimental results were provided in this study.

Greiner *et al*. [38] proposed a hybrid decision support methodology for screening weapon system development projects for the US Air Force using a 0-1 integer portfolio optimization model (i.e., a knapsack problem). Decision making was done in two main phases. First, an Analytic Hierarchy Process (AHP) [81] component was used to formulate a hierarchical criteria structure that would assist in the derivation of a relative weighting scheme. In the experimental section, 15 projects were considered and the optimal portfolio was found using the integer programming facilities in Microsoft Excel. Further experiments considered a partial allocation of funds and categorical budget constraints. In all experiments, the optimized results provided significantly higher valuation scores than the Air Force's recommended portfolio.

Walmsley and Hearn [39] aimed to balance an investment regarding armoured support vehicles using mixed-integer programming. Specifically, the objective was to minimize the production and supply costs while satisfying budget, resource, and minimum supply constraints. The study considered 7 vehicle types and formulated three distinct optimization problems:

1) Minimize the cost for a 100% compliant fleet.
2) Maximize the number of compliant roles with respect to a given budget.
3) Maximize the number of compliant vehicles with respect to a given budget.

Note that, a compliant fleet was defined as a fleet where there existed no shortfall in any compliance category, a compliant role was defined as a role in which a fully compliant vehicle was allocated, and a fully compliant vehicle was defined as a vehicle that fully satisfied the requirements of the roles to which it had been allocated. A linear programming implementation (XpressMP[4]) was used for optimization. A sensitivity analysis was conducted by repeating the optimization process again using under- and over-estimates of the capabilities/requirements. Results of the sensitivity analysis indicated various areas where significant costs savings could be achieved for only minimal reductions in the compliance constraints, which led to a re-evaluation of the requirements. Conversely, increasing the requirement estimates by up to 15% led to no significant increase in cost.

[2]While not explicitly stated, this is assumed to be Oracle Crystal Ball, available at https://www.oracle.com/applications/crystalball/

[3]Again, this software was not explicitly referenced but is assumed to be the package provided by OptTek Systems, Inc., available at https://www.opttek.com/products/optquest/

[4]FICO® Xpress Optimization, available at: https://www.fico.com/en/products/fico-xpress-optimization

A subsequent study by the same authors [40] provided no notable methodological changes.

Parnell *et al*. [41] described a system developed by the US Air Force Research Laboratory to assist in the selection of science and technology projects that best met future warfighting capabilities for their prospective clients. This system was reported to be an alternative to the technique proposed by Brown *et al*. [36], and was specifically designed to lower the level of detail as the yearly cost analysis found in [36] was not needed for this use-case. The proposed methodology consisted of ten steps that considered the entire planning life cycle from the compilation of research documents through to project initiation. Their proposed model used a non-linear, aggregated weighting scheme to combine the 16 evaluation criteria and optimization was performed using FrontlineSolver®[5] via Microsoft Excel. Uncertainty in the valuation was addressed via triangular distribution ranges that were sampled using a Monte Carlo approach. Experiments were then repeated using increasingly restrictive budget constraints. No further details regarding the data were provided.

Buckshaw *et al*. [42] described a value-focused risk and design analysis methodology used by the US Department of Defense to determine optimal resource allocation for information systems operating in a hostile environment. Their proposed methodology employed a linear additive model to aggregate the value of multiple measures. Modeling of the adversary was done using their attack preferences, rather than using direct probabilities of events occurring. In other words, the study did not consider the probability of an adversary employing a specific attack, rather it was assumed that the adversary would perform attacks that had a maximal effect according to four objectives. In this context, the objective of the study was to maximally mitigate the effect of the adversary attacks. No specifics on data were provided. However, a brief summary of seven previous applications of their methodology was provided. The largest application considered had 200 projects.

Chan *et al*. [43] examined a goal-setting model to address a time-dependent variant of the project selection problem in the context of a vehicle modernization process for the US Army. In their study, a bi-objective formulation was developed to maximize support for two different assault missions, thereby providing a direct trade-off situation over a four-year time period. The optimization was subject to a number of constraints, namely budgetary, capability, production, and technology-needs constraints. Using an exact solver software, ADBASE,[6] two optimal portfolios were identified and were reported to represent different weightings of the mission types, respectively. The analysis showed that the second and third funding cycles (i.e., the middle phase) were critical to the success of the planning, irrespective of the weights and vehicle types. A small amount of effort was applied to produce a non-linear implementation of the model, but was unsuccessful [43].

Haynes *et al*. [44] examined the problem of allocating US Marine Corps anti-terrorism resources using a utility score derived as a function of the cost, benefit, various utility factors, and facility prioritization weights. The optimization component was a straightforward 0-1 knapsack formulation whereby the objective was to maximize the utility score subject to budgetary constraints. Much of this study focused on the implementation of their model using a web-service framework with no empirical data nor details regarding the optimizer being given.

Tsaganea [45] examined the allocation of funds towards missile defence capabilities over a fixed planning horizon. This study took a substantially different approach than the others considered in this study. Specifically, the problem was modeled as a dynamical system such that offensive and defensive systems were considered as mathematical variables. The performance metric was calculated as the number of incoming missiles that could not be destroyed by the defence systems, which was to be minimized. The resulting model was then solved analytically using dynamic systems theory rather than an optimization technique. Two hypothetical case studies that had different initial states, which were arbitrarily-designed but assumed to be plausible, were considered and a 10-year plan was derived for each. It was concluded that different funding policies were required when different scenarios were considered.

Preiss *et al*. [46] considered a strategic investment model for the US Air Force Research Laboratory and argued that a value-model approach goes beyond that of a typical capability model by assisting in identifying the potential payoff of technology investments. It should be noted that this study was effectively a continuation of the study in [41]. In the proposed model, technology scoring was implemented via a criteria-based evaluation carried out by SMEs, such that performance was scored according to engineering estimates that described the current capability, minimum expected performance, and the best expected performance. To address uncertainties, the SMEs were encouraged to provide both best- and worst-case estimates that would be taken into account during the portfolio risk analysis phase. A case study was conducted using 70 projects from 8 different groups. However, no optimization was performed and only a high-level summary of the aggregate score across all projects was provided. Nonetheless, it was concluded that the scoring method was more conservative, accurate, and efficient than the method used in [41].

Dodd *et al*. [47] considered the generalized application of non-linear multi-attribute utility theory to conflict scenarios in order to facilitate a wider set of subjective attributes that could be specified by the decision-maker. Specifically, this work proposed a framework for representing command and control decision-making scenarios characterized by threats, uncertainty, and conflicting objectives. The model employed two utility functions, which considered the local

[5] Available at https://www.solver.com/

[6] The authors were unable to locate this software package and is thus assumed to be defunct.

and global effects of the strategic decisions but was proposed in a generalized manner such that additional utility functions could be included. Local effects included the potential for escalation of threat, loss of civilian life, theft of assets, taking of hostages, etc. whereas the global effects included higher-level considerations such as regional tension. An uncertainty vector was also used to quantify the amount of uncertainty a decision-maker had in the situation. Criteria weights were used to reflect the importance of achieving specific attribute-related goals. The objective for the decision-maker was then to choose the course of action that maximized the expected value of the aggregated utility functions.

Fu [48] considered project selection in military production plants using fuzzy goal programming. Piece-wise linear membership functions were given for each of three objective types, referred to as "around," "at most," and "at least." These three objective formulations facilitated the imposition of fuzzy constraints using (approximate) equality, less-than, and greater-than relationships. The objective of this formulation was to maximize the total degree of the fuzzy membership functions subject to a number of constraints. In the experimental section, the task was to optimize the schedule for a production plan that had 16 projects across four categories. Each project had an associated cost, income (i.e., value), number of working hours, and required completion date (specified as a particular month). An exact solver, LINGO,[7] was used to design the schedule for an entire year, using months as the time steps. Four case studies were considered using various fuzzy objective/constraint formulations. An optimal schedule according to each of these scenarios was given, with no subsequent analysis or discussion.

Baker *et al.* [49] examined the problem of optimizing vehicle fleet mixes in the context of military deployments. The problem had three objectives, namely to minimize the cost of the vehicles, minimize the variance among the number of vehicles of each type, and minimize the space that the vehicles occupied in strategic transport vessels. It was assumed that additional quantities of vehicles could be purchased *ad hoc*. A Multi-objective Evolutionary Algorithm (MOEA) was compared against two (integer) linear programming models that used a branch-and-bound technique. A case study consisting of three different vehicle types was used to exemplify the proposed MOEA. Different acceptance rates, defined as the probability that a decision made by an agent would be accepted, and population sizes[8] were examined for sensitivity purposes. It was observed that varying the cost function led to a noticeable impact on the spread and variety of optimal solutions that were attained whereas the population size had only a moderate impact on the variations attained in the final population.

[7]Lindo Systems Inc., available at https://www.lindo.com/index.php/products/lingo-and-optimization-modeling

[8]It was remarked that the population size was not fixed. Rather, the population size referred to the number of solutions that were initially generated.

Bizkevelci and Çakmak [50] proposed a technology management model for a general defence system consisting of three main phases in the integrated life-cycle, namely the formalization, development, and utilization phases. A case study was considered using two scenarios, peace-keeping and wartime operations, and the evaluation of both user and technological requirements. To determine critical technologies for each scenario, the mission needs and technologies were prioritized on a scale of 0 (inessential) to 5 (essential), with a weighted prioritization calculated by multiplying the two scores. An average value across each type of technology was calculated and was subsequently used to determine the technologies that were most effective in each scenario. No optimization was performed in this study.

Abbass *et al.* [51] proposed a scenario-based planning methodology referred to as computational scenario-based capability planning. The proposed computational planning model involved three stages, as follows:

1) Scenario Generation: identify deep uncertainties and design a database of future scenarios
2) Resource Planning under Time Constraints (RPTC) Sampling: parameterize the scenarios and employ a simulation methodology to generate tasks for the scenario. Formulate the overall problem, perform an optimization process to generate non-dominated solutions, then group and evaluate solutions over all scenarios
3) Recommendation: Use k-centroid clustering to cluster the list of non-dominated solutions

In the experimental section, a resource planning task with time constraints was considered over a 25-year planning horizon. This problem was a bi-objective problem with minimization of cost and maximization of robustness as the (conflicting) objectives. The robustness objective was simplified to the task of creating a balanced fleet and therefore was replaced with minimization of the variance among different forces. This process was applied to a hypothetical land mobility capability planning process to demonstrate how the scenario templates could be used to generate thousands of scenario instantiations. Six different vehicles and seven resource types were considered such that each vehicle could deliver various quantities of each resource. It was assumed that a wargaming simulation environment was available to provide the resource requirement values, each vehicle could work on a single task at a time, and that there were costs associated with not fulfilling the resource requirements of a task. A multi-objective optimization algorithm, Non-dominated Sorting Genetic Algorithm II (NSGA-II) [82], was used to generate non-dominated solutions, which were analyzed using clustering techniques.

Whitacre *et al.* [52] considered a scenario-based computational approach to resource planning problems using a steady-state MOEA. The study outlined three criteria that a solution to such problems should have, namely that it should have good (expected) performance on a wide variety of plausible future scenarios, have low-cost modifications that can be made to adapt to changes (adaptiveness), and be able to adequately address unforeseen problem conditions (robustness). The primary objective in the problem formulation was to satisfy all capability requirements. The uncertainty of future capability requirements was addressed via random sampling from normal distributions. In the experimental section, the number of time steps was set to 10, the number of asset types was set to 5, and the number of capability types was 4.

Vander Schaaf *et al.* [53] considered the selection of humanitarian infrastructure projects to be conducted by US military organizations in various countries. This study highlighted the importance of decision-support systems given that the success and impact of the selected projects would have significant geo-political implications for the US. The goal of this model was to maximize the cost-effectiveness while also meeting broad foreign policy objectives. Short-term objectives were to improve the lives of local civilians and to support and stabilize the local government. The overall, long-term objective was to reduce terrorism through the stabilization of a region and strengthening relationships between the US and the host nation. It was stated that the study aimed to test the hypothesis that a greedy multi-objective strategy could be employed to improve the selection process. An optimization problem was then formulated with four objectives, namely maximization of the value to the US, maximization of the local support, maximization of the training value, and minimization of the hazard to soldiers. A number of problem-specific constraints were also enforced. Optimization was performed by way of linear programming where the objective functions were converted into constraints. A form of goal programming was employed such that the optimization objective was to minimize the weighted difference among all objectives. This study was rather comprehensive in its experimental analysis and considered the sensitivity to two different budgets. However, the constraints imposed were very context-specific and therefore, the model proposed in this study can not be easily generalized to other contexts. Furthermore, while formulated as a multi-objective problem, a true multi-objective optimization process was not performed. Rather, a weighting scheme was employed to facilitate different trade-offs among the objectives.

Hurley [54] examined the problem of fixed-budget allocation for miscellaneous-requirement projects in the Canadian Air Force. Traditionally, an SME would allocate the available budget by selecting projects in order of priority, without regard for their cost. This study was premised on improving that particular approach. To this end, a mapping function was devised to define a relationship between project rank and project value. This allowed the conversion from (ordinal) project rankings to (cardinal) values that could be used for optimization. A standard knapsack approach, solved exactly with an unspecified commercial integer programming software, was used to find the optimal set of projects to implement from a set of 142 projects. Using a greedy priority-based selection as a baseline, 84 projects were selected. Using their proposed optimization approach, 134 projects were selected to be implemented. The optimized portfolio had 50 additional projects and led to approximately 25% more value for an increase of only \$76 (relative to the \$20 million budget). A further experiment examined the value that could be added for a small increase in budget and ultimately concluded that a small increase in budget would not lead to a substantial increase in value.

Lee *et al.* [55] considered the problem of selecting optimal weapon systems for the Armed Forces in the Republic of Korea. A hybrid composed of AHP and Principal Component Analysis (PCA) [83], [84] was used to determine the weights for each sub-criteria used in the decision-making process. The overall objective was then formulated using a goal programming approach. A case study was performed using six candidate missile systems that were evaluated against three main criteria and 19 sub-criteria. Five SMEs were consulted during the AHP phase and 5 principle components, accounting for 95% of the variability, were selected using PCA. A synthesized weight, calculated using both the AHP and PCA scores, was used to determine the objective score for each candidate missile system. Various constraints were implemented but no description nor justification was provided regarding the constraints. Note that, this study did not consider portfolio optimization *per se*, rather it considered the usage of decision-making techniques to assist in the selection of a single missile system according to multiple criteria.

Xin *et al.* [56] proposed a rule-based constructive heuristic to optimize the dynamic assignment of weapons to targets in a defensive context. The optimization objective in this study was to maximize the expected value of the surviving assets whereby the defender would adopt an optimal policy to assign their countermeasures to the weapons of their adversaries. This problem was iterated over a fixed planning horizon and included a dynamic component whereby future states of the environment would be dependent on the decisions made in previous iterations. Constraints were enforced such that weapons could fire only at a single target at a time, there was a limited amount of ammunition available, and there was time-dependent feasibility for certain weapon systems. A knowledge-based heuristic was proposed to solve the problem formulation. Specifically, the heuristic was aware that a higher priority should be assigned to targets that can cause higher damage, a higher priority should be assigned to targets that can be most effectively mitigated at a particular time step, and that assigning a weapon to a target will reduce its threat level. Furthermore, an explicit constraint handling mechanism was implemented to ensure that only feasible solutions were produced. To provide empirical evidence in support of the proposed heuristic, 13 randomized test cases were generated and their proposed heuristic was compared against a pure Monte Carlo approach. It was reported that the heuristic approach outperformed the Monte Carlo approach in all cases, especially in cases where the ratio of available weapons to targets was relatively low.

Yang *et al.* [57] proposed a semi-variance portfolio selection model that was applied to military investment assets over a 16 year planning period. This study claimed to be the first usage of a semi-variance measure to quantify risk in the military portfolio selection domain. The study considered 10 investment asset options using 12 years of historical budget data from the Taiwan Ministry of National Defence. The remaining four years of budget data were approximated by the Holt-Winters forecasting method [85]. For each asset option, a formula was derived to provide a corresponding measure of effectiveness, which was used to determine the return on investment. It was assumed that at each time step, assets would be purchased and the optimization objective was minimization of the semi-variance associated with the return on investment that would have been attained had the assets been purchased earlier. Cardinality and proportional constraints were also enforced. The experimental section employed a Genetic Algorithm (GA) and Tabu Search to optimize the risk-return trade-off for 12 investment assets, with no significant difference in performance observed between the two algorithms. However, the study did note that the experimental results and corresponding analysis were rather limited in depth.

Teague *et al.* [58] proposed an agent-based simulation to assess the regional stability attained from infrastructure project development by the US Army Corps of Engineers and the International Security Assistance Force in Afghanistan. The key objective in this model was to maximize the benefits associated with developing health-related infrastructure in the Jalalabad region in Afghanistan. Potential projects included infrastructure components such as the design of one or more water wells, building some number of hospitals, and the training of soldiers. Agent-based modeling was employed to assess the impact of the infrastructure decisions. The agents' behavior in the model was directly influenced by the status of their health. Portfolios were evaluated in terms of the resulting number of outpatient hospital visits, the total number of intensive care visits, and the total number of deaths. Latin hypercube sampling was used to sample 16 potential portfolios that had varying levels of well coverage, hospital coverage, and security profiles. Three portfolios were determined to be non-dominated with respect to both the health metrics and cost and a further analysis led to an ultimate decision on which portfolio was best to select.

Golany *et al.* [59] considered a countermeasure prioritization problem such that the objective was to optimize the development of countermeasures under limited resources. In this model, the defender was assumed to be aware of the attackers capabilities, intentions, and activities. The primary objective was then to develop countermeasures to maximally mitigate the effect of the attacker's weapons under temporal budgetary constraints. This model also included a scheduling component whereby the decision-maker had to decide when to start development of the countermeasures. Feasible development schedules were modeled using a network structure that was constructed in such a manner that the problem of selecting the optimal policy was reduced to finding the shortest path through the network that also adhered to the budgetary constraints. The construction of this network structure was exemplified under various assumptions regarding the number of countermeasures and their effectiveness scores. Numerical simulations were conducted using ten enemy weapons, ten possible countermeasures, three levels of intensity, and four time periods. This formulation leads to integer programs with approximately 38,000 variables and 73,000 constraints. Despite the size, solutions took an average of 15 minutes, though times ranging from a few seconds to about four hours were reported. Furthermore, multiple simulations were conducted to assess the robustness of solutions to small perturbations in the development times. Dynamic scenarios were explicitly mentioned as being left for future work.

Kangaspunta *et al.* [60] developed a portfolio methodology with a specific focus on addressing cost-efficient analysis across entire portfolios. In their work, being cost-efficient was simply defined as being a non-dominated portfolio. While not present in their work, it was noted that compatibility constraints could be added to ensure that, for example, mutually exclusive project constraints were respected, or to assert that certain systems must be deployed together. An important aspect of this study was the incorporation of project synergies, which permitted the introduction of non-linear costs associated with various sets of projects if they were implemented together. A pairwise comparison algorithm was devised to generate sets of non-dominated portfolios. The proposed approach was compared against three other approaches to generate the list of non-dominated solutions. Only one of the other approaches considered required fewer comparisons. However, it was noted that, despite requiring fewer comparisons, this competitor was only applicable when all solutions had unequal values with respect to one criterion, such that they can be sorted into a strictly decreasing order. Kangaspunta *et al.* also listed the recursive definition of this approach as a weakness given that some programming languages may not support recursion, but this argument seems largely unsubstantiated given that an overwhelming majority of programming languages support recursion. An experimental case study that had 290 portfolios was examined, 95 of which were identified as being non-dominated. One limitation that was explicitly mentioned was that cost should be treated as uncertain in subsequent studies. Furthermore, it was highlighted that the cost aspect should incorporate more than just monetary costs, such as loss of life, which would add an additional level of complexity to the problem.

Xiong *et al.* [15] proposed a multi-objective approach to address capability planning problems. Specifically, the problem was modeled as a multi-mode resource investment project scheduling problem. This formulation had two minimization objectives, the makespan (i.e., total schedule duration) and the cost. Two constraints were implemented to ensure that a predecessor task was always completed before any of its successors could be started and that the resource constraints were met at each timestep. Additionally, activities could be executed in different modes, which represented alternative resource requirements and durations for the same task. To optimize the proposed multi-objective problem, a preference-based variant of the NSGA-II algorithm was used. A synthetic test case with 16 tasks and 4 resource types was created to empirically validate the proposed approach. Each project had a maximum of four operating modes, that each had an independent probability of being selected. Various costs and precedence relations (with a maximum depth of 7) were synthesized to create a challenging optimization environment. A two-stage optimization process was conducted. The first phase was used to identify an approximate Pareto front. At this point, a decision-maker would intervene and select a region of interest such that the second phase of the search would then focus only on the sub-region of interest. The proposed, preference-based NSGA-II variant was shown to outperform the standard NSGA-II algorithm when the attained Pareto fronts were compared.

Yu *et al*. [61] proposed a differential evolution algorithm to optimize the selection of weapon system portfolios in the context of a system-of-systems (SoS) architecture [86]. Evaluation of candidate solution was done using Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [87] and was calculated using the cost and a measure referred to as the synthetic satisfactory degree of capability requirements. Various constraints were implemented to account for minimal allowable capability requirements, the budget, scheduling requirements, and the level of risk. A case study was conducted using seven potential weapon systems to examine the effects of different algorithmic configurations. Details regarding the specifics of this study were scarce and very little analysis of results was performed.

Bjorkman *et al*. [62] examined the optimization of resource allocation in the context of designing equipment tests for the US Department of Defense. This study was particularly interesting as the primary objective was to reduce the uncertainty associated with the testing process rather than strictly improving some measure of value. Specifically, this study quantified the value of a particular test as the estimated amount of uncertainty reduction that the test was expected to provide. This uncertainty reduction value was then used to formulate a traditional knapsack problem. This methodology was applied to five particular tests, with one example given in full detail – upgrading the brakes of an airplane to facilitate shorter stop distances. The primary objective of testing was to determine the stop distance. Sub-objectives of the problem were to determine the best braking technique and to determine the maximum landing distance with a confidence level of 0.99. Various uncertain aspects, such as the braking coefficients and accuracy of the testing instrumentation, were present. The optimization objective was to select an optimal set of tests such that the uncertainty in the resulting measurements was minimized. Monte Carlo methods were used to evaluate four proposed test options. As a baseline comparison, actual tests described in the literature were assumed to be tests that SMEs were likely to use. It was concluded that the optimized portfolios had a value of 4.3% to 8.4% higher than the portfolios selected by SMEs, for the same cost. Furthermore, it was highlighted that the largest difference was obtained with the lowest-valued portfolio, which could indicate that the optimization process was most effective for resource-constrained environments [62].

Fleischer Fauske *et al*. [63] proposed an optimization model for the Norwegian Defence Research Establishment to provide quantitative analysis for long-term defence planning. The single, primary objective of this model was to optimize the expected capabilities of the force structure by considering uncertainty in the future budget over a 20-year planning horizon using cost as a proxy for value. To account for uncertainty in the future budget, the budget was assumed to either increase by a specific percentage or not increase at all in each year. A scenario would then be constructed by selecting one of these options at each year. The probability of each event was assumed to be equal but could be specified as a parameter. A balanced sampling technique was used such that complementary scenarios (e.g., "increase, increase, no increase" and "no increase, no increase, increase") would always be selected in pairs. To prevent unnecessary delays in project initiation, the value of a project was adjusted according to the year in which the material was acquired, thereby penalizing delays in project initiation once the material was available. A number of realistic constraints were also taken into account by this study, such as constrained start dates, prerequisite projects, and mutually-exclusive projects. In their experimental results, the number of sampled budget scenarios was set to either 16, 32, or 64. However, due to the confidential nature of the data, very little additional detail was given about the experimental procedure and results.

Hurley *et al*. [64] proposed two risk-analytic approaches to allocate operating funds for the Canadian Department of National Defence. The primary motivation was to reduce the amount of overspending and "slippage," which is the amount of allocated funding that was unused in a given year. Note that, slippage doesn't necessarily imply that a project was left unfinished. Rather, it can be introduced when the maximum cost was overestimated, leading to funds that were allocated but unspent at the completion of a project. Therefore, this formulation was derived under the hypothesis that over-programming, while assuming that some activities will create slippage, could better optimize the budget. An analytic process was used to derive the probability of overspending if the first n activities, when sorted by priority, were programmed. Considering a set of 60 projects, the level of risk associated with implementing a varying number of projects was provided. Moreover, the experimentation was repeated using different levels of correlation between the project priorities and slippage probability. A further experiment formulated the problem as a knapsack problem, whereby the objective was to maximize the value of the portfolio while adhering to a given probability of overspending. This problem was solved using the CPLEX commercial solver.[9] Assuming that the probability of overspending should be no more than 10%, it was observed that the knapsack approach would often select lower-valued, lower-cost projects that were not considered using a greedy priority selection method.

Bakirli *et al.* [65] examined the use of a fuzzy multi-objective multiple-knapsack problem for the selection of defence projects. This study made use of fuzzy goal programming whereby an achievement function was used to quantify the weighted deviation from the target in multiple objectives. Despite the multi-objective formulation, this study did not employ multi-objective optimization. Rather, an objective weighting scheme was employed. The experimental section considered a hypothetical case study examining the technology needs of a defence organization. This case study consisted of 13 capabilities, eight scenarios, and 16 potential projects. Each project had an associated cost, benefit, risk, and environmental impact. The optimization objectives were to maximize benefit, minimize risk, and minimize the environmental impact while adhering to the total budget. Four different weight value profiles for the goal programming aspect were examined. Furthermore, these weight profiles were applied to each of the six different orderings of the objectives, thereby producing 24 possible weight schemes for the objective function. To determine the sensitivity to the budget, experiments were run using the entire budget, 2/3 of the budget, 1/2 of the budget, and 1/3 of the budget.

Xiong *et al.* [16] proposed a knowledge-based MOEA for stochastic extended-resource investment project scheduling problems, which are characterized by having flexible project start times and resource usages. In this formulation, project completion times were defined as a function of the allocated resources, such that increasing the amount of allocated resources could decrease the completion time. Moreover, there was a stochastic component to address various avenues of uncertainty, such as the duration, resource breakdown, and alteration of preferences. A multi-objective problem was formulated with three objectives, namely minimization of the makespan, minimization of the cost, and maximization of the robustness. Solutions were subject to both precedence and resource constraints. Scenarios were used to account for the aforementioned uncertainties.

To optimize the project schedule, Xiong *et al.* used a variant of NSGA-II with an additional knowledge mechanism. Two variants of the proposed approach were examined and compared against a standard NSGA-II, an MOEA with neighborhood restarting inspired by [88], and a bi-objective Tabu Search [89] referred to as MOTS. A synthetic test case was formulated in a military context and consisted of 16 activities to be scheduled, each with various precedence relationships, along with four types of resources. Three experiments examined the results with varying levels of uncertainty; experiment 1 addressed only the duration perturbation, experiment 2 examined duration perturbation alongside resource breakdown, whereas experiment 3 included all three types of uncertainty. A robust schedule was defined as one that is expected to perform well in a stochastic environment characterized by uncertainties. For the robustness measure, 50 scenario samples were taken. Various parameter settings were also investigated for sensitivity analysis. It was noted that the robustness measure decreased when the uncertainty increased, as can be expected. Regarding the algorithmic comparison, it was reported that MOTS performed worst whereas the proposed NSGA-II variants performed best.

Zhou *et al.* [66] considered the problem of optimizing defence project portfolios using goal programming. The portfolios were optimized using a modified variant of differential evolution. The simulation results indicated that the proposed approach outperformed both a standard genetic algorithm and particle swarm optimization. However, there was no mention of the data used nor the experimental design and thus, the scale of the problem considered is unknown. Moreover, the manuscript lacked many critical details needed to fully understand the proposed model.

Davendralingam and DeLaurentis [67] considered optimization of the CVaR associated with a naval warfare case study using agent-based simulation in a SoS context. The objective was minimization of the CVaR (as described in Section II-A3) subject to a number of SoS network-related constraints. In the experimental section, 16 projects were considered and various values for the minimum level of performance required were examined. Evidently, as the minimum level of performance required was increased, the CVaR also increased. No other analysis or conclusions were formed based on the experimental results.

Fisher *et al.* [33] proposed a defence project selection heuristic based on approximate dynamic programming. This study considered a year-over-year planning scenario such that projects could be added intermittently, but decisions had to be made in near real-time. Their approach was inspired by the need to minimize under-spending, which traditionally led to frivolous selection of low- to mid-valued projects at the end of the year to ensure the entirety of the annual budget was allocated. However, due to the multi-year cost associated with the projects, selection of these low-valued projects often meant that there was no remaining budget to initiate high-valued projects if they were to be originated in the near future. As a means to penalize the delay of initiating a project, the value of a project deteriorated over time. The delay meant that simply "waiting" for the available budget was a sub-optimal decision and thus immediate selection of high-valued projects was of utmost importance.

The optimization model of Fisher *et al.* was subject to two primary constraints, namely that a project could be initiated only once and that the budget had to be respected at all time steps. To address the aforementioned problem of frivolous year-end spending, the study examined whether lapsing (i.e., not using) a fixed portion of the available budget,

[9]CPLEX Optimizer, available at: https://www.ibm.com/analytics/cplex-optimizer

in the anticipation of a high-valued project being originated, would lead to better project selection over a multi-year planning period. The model first reduced the available budget, B_t, according to a parameter α, such that the updated (available) budget at time t was given by $B'_t = (1-\alpha)B_t$. At each time step in the planning period, a knapsack approach was used to find the optimal selection of projects using B'_t as the available budget. However, if there existed a high-valued project that had previously been delayed, the value of α was instead taken as the smallest value such that the high-valued project could be implemented immediately.

To select the best value for α, Fisher *et al.* used a Monte Carlo approach. Randomly generated projects were created according to probability distributions built from historical data to represent the set of projects that would be originated over a 25 year period. Various values of α were compared based on how well they performed relative to an offline version of the problem, where one plausible future was assumed to be completely certain. From the relatively small set of α values examined ($\{0.00, 0.30, 0.35, 0.40, 0.45, 0.50\}$), $\alpha = 0.40$ attained the best score but it was noted that the results were not highly sensitive to the choice of α. Note that, $\alpha = 0.40$ corresponds to lapsing (i.e., saving) 40% of the available budget each year, except when a high-valued project was originated. Therefore, it was concluded that not spending the entire budget each year was preferable. However, this study assumed that the lapsing and preservation of funds was permitted, which may not be a realistic assumption in many cases. Moreover, even if lapsing were permitted, 40% would likely be considered rather high. Interestingly, this study took an opposite approach to the study of Hurley *et al.*, which explicitly attempted to minimize slippage.

Dou *et al.* [69] proposed a multi-objective integer programming approach to selecting weapon system portfolios. Notably, this study explicitly used Pareto analysis to find non-dominated solutions. The valuation was done from two different perspectives, namely the technology push and requirement pull. The technology pushing aspect incorporated five aspects of readiness: technology and integration, function, system, capability, and portfolio. The requirement pull valuation included two measures that indicated the overall satisfaction level of the system at the individual system and portfolio levels, respectively. These satisfaction levels were considered uncertain and, as such, were supplied as probabilistic intervals based on SME consultation. A case study was conducted using 10 candidate weapon systems, which encompassed 20 technologies and 21 integrations. Some of the technologies and integrations were considered together as higher-level constructs, referred to as functions. Solutions were required to adhere to manufacturing and capability requirement constraints. Much of the analysis focused on what functions were provided by the various weapon systems and what functions were required to support each capability requirement. Of the 1022 feasible portfolios, 55 were identified as being non-dominated and thus warranted further inspection by the decision-maker.

Rempel and Young [70] developed a decision support system to assist decision-makers at the Canadian Department of National Defence with project portfolio selection. The system combined optimization, visualization, and manual revision features to assist decision-makers in their choices. Note that, this was the only study that explicitly mentioned an *a posteriori* manual refinement stage; this study was inherently focused on the usability of the system in a real-world context rather than as an academic endeavor. The system was designed to construct template plans that contained a viable 20-year schedule of major projects and planned expenditures as well as realistic plans for the introduction, maintenance, or divestment of capabilities. Moreover, a number of visualizations provided additional insight to the decision-makers.

Rempel and Young's optimization model was formulated as a multi-dimensional knapsack problem that represented two distinct funding sources. This problem was solved using the GNU Linear Programming Kit.[10] To account for the long-term funding implications of projects, budgetary constraints were examined for 40 years (i.e., 20 years longer than the planning horizon). Additional constraints that were enforced were an annual organizational capacity constraint and various dependency constraints. However, the system explicitly facilitated the implementation of arbitrary user-defined constraints. Interestingly, this model contained a time-limited solver whereby the user could supply a time limit and have the best, but possibly sub-optimal, solution returned after the specified amount of time had lapsed. Additionally, seven interactive visualization techniques were implemented to provide further insight into the provided solutions. It is noted that this study described a relatively comprehensive software solution with a number of interesting ideas, but offered no empirical results.

Davendralingam and DeLaurentis [71] proposed a modeling scheme for military capabilities using an SoS architecture. The study was premised on optimizing a portfolio using the classic mean-variance approach via mixed-integer quadratic programming. The optimization objectives were to maximize the overall capability while minimizing the risk in development time. Uncertainty in the expected returns was addressed via interval data, thereby producing linear margins in the calculated capabilities. Note that, this model definition was similar to, and presumed to be a continuation of, a previous study by the same authors [67].

Davendralingam and DeLaurentis then used a synthetic test case with 22 systems to exemplify the proposed approach. Various mutual exclusion constraints were enforced. For example, a requirement that the selected *communications package* system could interact with a maximum of four other systems was enforced as a constraint. Values for the risk aversion parameter were taken between

[10] GLPK, available at http://www.gnu.org/software/glpk/

0 and 1, in increments of 0.1, and represented various penalties associated with taking risks. They purported that the major benefit to their system was that it presented the decision-maker with portfolios that were relatively robust to the prescribed levels of uncertainty.

Konur *et al*. [72] proposed a military mission planning problem in the context of an SoS architecture using both multi-objective mixed-integer optimization and an evolutionary algorithm. A decomposition approach was proposed to alleviate the computational burden associated with solving this problem formulation. The decomposition approach would decompose the overall problem into multiple sub-problems, such that the true Pareto front was included in the union of the Pareto fronts for the sub-problems. The overall objectives of their model were specified as maximizing the total performance and minimizing the completion time. To demonstrate the proposed approach, a search and rescue mission planning scenario, consisting of eight capabilities and six systems, was considered. This study also considered both inflexible and flexible systems, where a flexible system was one that permitted online engineering design changes that could incorporate additional capabilities in the finished project. A number of empirical analyses were conducted and indicated that the proposed decomposition approach was able to decrease the running time for the exact solver. Regarding the evolutionary algorithm, the decomposition approach was reported to both reduce computational time and improve performance. The results also indicated that increased flexibility in the constituent systems led to overall better solutions.

Yang *et al*. [73] considered the problem of weapon system portfolio optimization where the objective was to determine an optimal assignment of weapon units to maximize the expected damage to hostile targets. This study considered a dynamic, target-based weapon system portfolio problem whereby the defender would detect a fixed number of hostile targets and their attack mechanisms at each time step. The goal was to then to maximize the defensive capabilities to mitigate these attacks. Constraints were used to ensure that the number of assigned units was not greater than the number of available units, each target was assigned at least one weapon from each system, the assignments were feasible, and the decision variables were positive integers. A case study consisting of five offensive units and four defensive units was considered in the experimental section using an adaptive immune genetic algorithm. Additionally, a random test-case generator was used to examine the scalability of the proposed methodology.

Zhang *et al*. [74] considered a portfolio selection problem with incomplete information, addressed via scenario generation, that contained both project dependencies and synergies. Project dependencies and synergies were addressed through the use of constraints, which could alter the cost and/or benefit, define that certain projects must be selected together, or prevent certain projects from being selected as part of the same portfolio. Regarding the optimization aspect, two objectives were defined, namely maximization of the return value and minimization of the risk. The return value was defined as the expected return among different scenarios whereas the risk was defined as the variance of the returns over some fixed time period. This problem was then solved as a binary linear program with multiple objectives. A set of non-dominated portfolios was generated exhaustively by examining all feasible portfolios and keeping only those that were non-dominated with respect to the other feasible portfolios. An illustrative example consisting of 20 projects was used to demonstrate their approach. 27 candidate portfolios were identified, with very little subsequent analysis conducted.

Cheng *et al*. [75] examined the optimization of a weapon system portfolio via combat network modeling in an SoS framework. In this study, the system was represented by a network such that the nodes denoted various weapon systems whereas the edges represented functions, missions, or tasks and denoted either the flow of reconnaissance, communication, or influence. The experimental section considered a hypothetical case with 10 projects across three categories with two enemy targets. Note that, there was no mention of the optimization approach that was used in this study, though it is assumed to have been an exhaustive approach. The results indicated that 129 feasible portfolios were found. Further analysis examined the cost-effectiveness ratio and total combat capability, and ultimately resulted in two recommended portfolios.

Shafi *et al*. [17] proposed a scenario-based, multi-period optimization technique for CBP in the context of the ADF using an evolutionary algorithm. Given the uncertainty of future scenarios, a Monte Carlo approach using reinforcement learning was employed to provide a measure of robustness against uncertain future states.

Firstly, Shafi *et al*. formulated a single-period, multi-objective optimization problem as minimization of the cost while also minimizing the strategic risk (or, equivalently, maximizing the effectiveness) across K planning scenarios. Thus, the problem was formulated as having $K+1$ objectives. The single-period formulation was extended to address a multi-year planning period, which was taken as 10 years, whereby different instances of the same problem (arising due to changes in scenarios) were to be solved at each instance of time.

To address the issue of deep uncertainty, Shafi *et al*. employed a Monte Carlo approach using reinforcement learning. At each iteration, the non-dominated set was first attained by solving the multi-objective, single-period optimization problem using the Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) [90]. Q-learning [91] was then used to assign a score to the selected portfolio according to how well it performed in the current scenario. Specifically, one action (i.e., portfolio) was selected from the non-dominated set, either randomly or using the highest Q-value score as determined by the probability parameter. A future state was then generated based on the selected action. The non-dominated set for the single-period optimization problem corresponding to this generated future state was determined and a reward was calculated for the selected portfolio based on the performance in the generated state. This process was continued for each time period, then repeated in its entirety for the specified number of simulation runs. The empirical results of this approach were compared against two heuristic techniques, random selection and greedy selection. It was observed that the proposed technique outperformed the heuristic approaches over the 10-year planning window. Furthermore, to justify the usage of Q-learning, the proposed approach was compared with a variant that did not use Q-learning. Results indicated that using Q-learning consistently improved the results. It is noted that the study of Shafi *et al*. [17] was very comprehensive when compared to the majority of the other studies considered in this review.

Xiong *et al*. [18] examined the problem of weapon selection and planning in dynamic environments. The effectiveness of a weapon was considered as a combination of both the quantity and operational time of a weapon. Furthermore, a synergistic effect could occur between various weapon systems. The synergy effect would be realized when all weapons within a given set were operational and had quantities within specific proportions. The primary objectives of the model were defined as maximization of the net present value and an overall effectiveness score. As an additional complexity, both the effectiveness and synergistic effects were modified after the completion of an enemy countermeasure, which occurred at predefined time steps, and was meant to model a decrease in effectiveness resulting from an enemy countermeasure. This element added a form of dynamism to the problem and would force the decision-maker to either continue with their current solution or determine an appropriate adaption.

In the experimental section, Xiong *et al*. used NSGA-II to optimize a hypothetical scenario that consisted of 20 weapon types and a planning horizon of 10 years (i.e., 120 months). A time-discount rate of 0.01 per month (selected arbitrarily) was used to modify values over time. The experimental section was largely used as a proof-of-concept to depict various aspects of the algorithmic performance.

Wang *et al*. [76] considered the problem of future weapon planning under uncertain capacity demands. This approach broke the overall planning cycle into several, shorter planning cycles and employed a multi-stage stochastic programming model to maximize the long-term benefit using budget, development times, and short-term benefits as constraints. This approach considered a long-term benefit but used short-term constraints. A genetic algorithm was designed and employed to optimize the selection process. In the empirical analysis, a planning cycle of 15 years was broken into three shorter periods, each 5 years, with 15 different equipment types, five capability categories, and a budget of 60 units. A single optimal solution was provided, with no subsequent analysis or discussion of the experimental results.

Li *et al*. [19] proposed a portfolio optimization approach based on an SoS architecture. Solutions were evaluated with respect to both qualitative measures of effectiveness and quantitative measures of performance, variations of which were accounted for through the usage of scenarios. The objectives were defined as maximization of the measures of effectiveness and performance and minimization of the cost. Completeness and connectivity constraints regarding the SoS architecture were enforced. In the experimental phase, NSGA-II, ϵNSGA-II, and ϵMOEA were compared. An experimental case study was conducted using randomly generated data with 30 weapon systems, 30 measures of effectiveness, and ten measures of performance. Solutions were analyzed in terms of their flexibility, survivability, resilience, and robustness. It was concluded that ϵNSGA-II exhibited the best performance of the examined algorithms.

Li *et al*. [77] proposed a capability-oriented approach for the selection of high-end weapon equipment portfolios. A novel measure of effectiveness, referred to as the operational capability evaluation index, was proposed and maximization of this metric was taken as the optimization objective. The proposed measure was a weighted sum of the operational capabilities for each weapon in the portfolio – an extension of a measure proposed in earlier work by the same authors [80]. A custom algorithm was proposed to address this problem formulation. This algorithm was reported to have a high computational cost. A case study, which included 10 potential weapon systems, three capability requirements, and a budget constraint was considered. 15 candidate portfolios were considered and analyzed using the proposed capability evaluation index on four different combat scenarios. The same portfolio was identified as being the best choice in all four scenarios. An additional experiment was constructed using 50 randomly generated weapon systems. The proposed approach was compared against, and found to outperform, two baseline heuristics that selected the lowest cost and maximum sum of capabilities, respectively.

Moallemi *et al*. [78] considered a resource-constrained submarine acquisition planning problem. The objectives of the proposed model were to maximize the availability of submarines while minimizing the waiting time associated with licensing and maintenance over time, assuming an uncertain future. A solution to this problem had to account for the initial quantities of the submarines and their crews as well as the quantities and schedules for new acquisitions. This study claimed to be the first instance that combined portfolio optimization and fleet mixing for both short-term and long-term planning. In the experimental case study, a planning horizon of nine years, with weekly time steps, was considered. This was further broken into three epochs, each consisting of 156 weeks (i.e., 3 years). A time-dependent scenario generation technique was employed to address uncertainties using transition logic rather than a range of values. The usage of transition logic allowed the uncertainty to be dependent upon other factors, such as the length of the planning horizon. NSGA-II was used as the optimizer and was executed on 9000 experiments to examine the robustness of the Pareto optimal solutions.

Xia *et al.* [79] considered the problem of weapon project planning under uncertainty. Each of the weapons were categorized into one of four categories as follows: 1) candidates in the development list, 2) weapons that can be produced, 3) weapons in service, and 4) retired weapons. Weapons could transition from one category to the next as appropriate. For example, a weapon that was in the development list could be transitioned to one that can be produced if it was selected for development. The primary objective was to decide both the start and end time of development, along with quantities, in each time frame. Four constraints were enforced such that

1) Projects in the development stage cannot be manufactured.
2) The retirement time must be later than the start time.
3) The acquisition amount must be 0 for all time periods after retirement.
4) The cost cannot exceed the budget.

This study used interval data to account for uncertainties in the required capabilities. A set of plausible future states were then selected using orthogonal design to create a representative set of scenarios. Note that, further details were not provided as Xia *et al.* did not consider the scenario design to be central to the study. Two minimization objectives, referred to as the total capability gap, which measured the gap between the capabilities and their corresponding requirements, and the total capability dispersion, which measured the variance in capability, were used in this study. These two objectives were claimed to be largely influenced by the mean-variance model of Markowitz, as described in Section II-A1.

To exemplify the proposed model, Xia *et al.* constructed a 20-year planning task. Each time period lasted five years and it was assumed that all information was known in advance. The study consisted of 24 capability requirements, 32 weapon projects (14 in service, 18 candidates), with 132 scenarios and a fixed budget in each period. NSGA-II was used as the optimizer and various analyses were conducted on the resulting Pareto optimal solutions. However, like many other studies considered in this review, there was only limited subsequent analysis.

V. ADDRESSING THE DIFFICULTIES ASSOCIATED WITH THE DEFENCE SECTOR

Defence applications have their own set of challenges that are different from those in other application domains. Specifically, applications in the defence sector face hostile and adaptive adversaries, which adds an element of game theory to the problem, various levels of approval and oversight during the decision making process, and complexity [92]. Furthermore, defence-related portfolio optimization tasks are characterized by a number of key difficulties, namely uncertainty, valuation, multiple conflicting criteria and/or objectives, and dynamism. Moreover, often there are a number of constraints, such as a budget or scheduling requirements, that are imposed on candidate solutions. These challenges, and some associated solutions that were identified in this review, are summarized in Fig. 4. In this section, the manner in which studies have addressed these concerns is discussed.

A. UNCERTAINTY

One important aspect that must be considered in portfolio optimization in the defence sector is uncertainty [93]. Many things about the future are unknown, such as the budget, the political landscape, the capability requirements, etc. Note that, uncertainty and risk are different aspects in that risk can generally be assigned a probability, while uncertainty cannot [31]. Therefore, planning and decision making should be done with robustness to a number of plausible futures in mind. However, it was found that accounting for uncertainty is surprisingly rare in a defence context [92].

Burk and Parnell [92] listed three broad techniques to address uncertainty, namely Monte Carlo approaches (useful when there are a large number of independent uncertainties), scenarios (useful when the uncertainties span the planning space), and decision trees (useful when there are only a few dependent uncertainties). Scenario-based approaches have become more prevalent in recent years, thereby forcing planners to account for uncertainties [31], [94]. Specifically, scenarios are commonly used for robustness calculations rather than as a direct valuation technique.

In support of Monte Carlo analysis, Mun and Housel [95] stated that using a point-estimate, akin to examining only a single scenario, is ineffective as it specifies only a single event that will occur with near-zero probability. Hence, Monte Carlo approaches should be employed to examine a wide variety of plausible future scenarios. Similarly, Maier *et al.* [94] noted that traditional, distribution-centric models of "best-guess" uncertainty are unlikely to be appropriate in a changing society. Rather, a model of plausible futures, i.e., scenarios, is required and should be used to assess the robustness. In the remainder of this section, the mechanisms used to address uncertainty are examined in chronological order. It should be noted that nearly all studies that considered uncertainty employed the usage of scenarios.

Crawford *et al.* [37] generated random trees to synthesize plausible technology roadmaps, which were used during the testing phase of their project selection software package. However, the generated trees were not used in the evaluation of candidates.

Parnell *et al.* [41] used Monte Carlo sampling to account for the uncertainty of risk associated with projects. The values were sampled from triangular distributions and used to estimate the distribution of values at the project and portfolio levels.

Barlow *et al.* [96] proposed a temporal risk assessment framework for future force design that contained three main components. Firstly, a simulation system that quantified the risk associated with a particular force structure. The second component was a set of constraints that governed the possible transformations regarding the force composition. Finally, a graph structure where edges represented possible transitions from one force to another with respect to time and resource

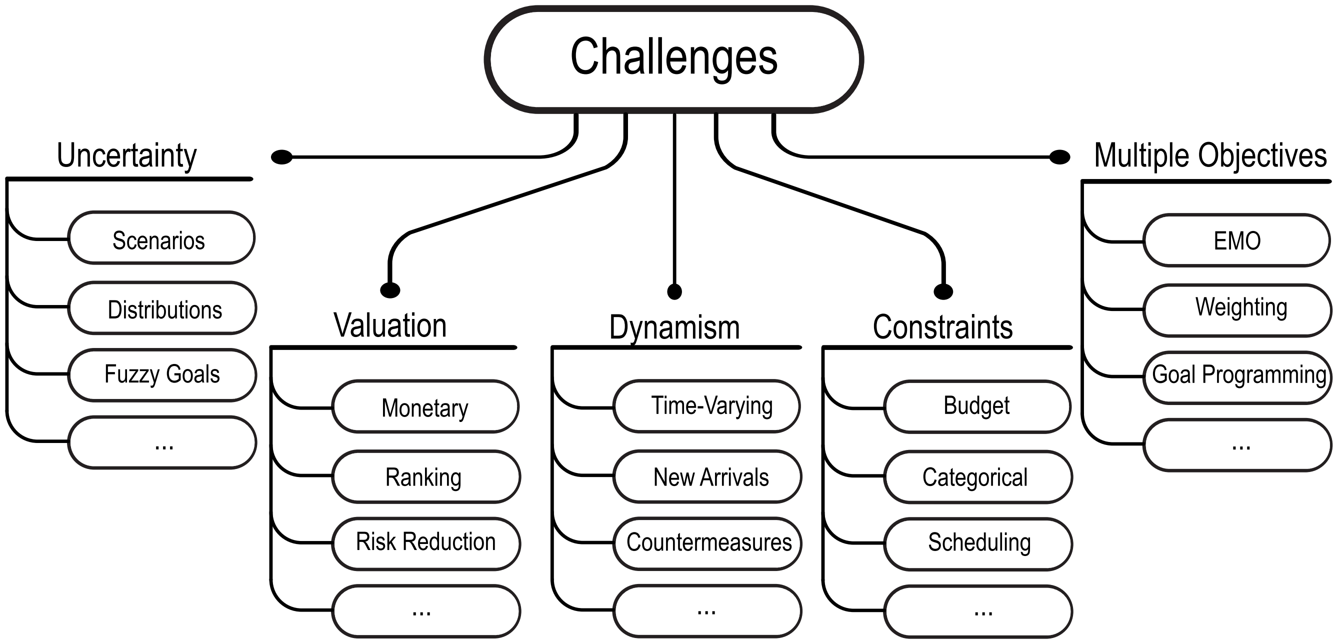

FIGURE 4. Summary of the main challenges associated with portfolio optimization for defence applications.

constraints. The graph is then traversed in such a manner that the risk is minimized, thereby providing insight regarding the best possible force transformations with respect to risk.

Abbass *et al.* [51] first argued that planning problems do not fall directly under the scope of dynamic or uncertain optimization problems. Rather, scenario planning differs from traditional prediction problems by the way in which uncertainty is handled. In optimization and/or traditional approaches, uncertainty is typically handled by associating events with probability distributions. It was argued that this approach has severe limitations: 1) that a massive amount of data is required to build and maintain accurate distributions, 2) there is inability to account for complex human behavior, 3) the inability to accurately account for sudden "shocks" or "surprises," and 4) the underlying assumption of the continuity of past trends.

Based on these limitations, Abbass *et al.* [51] argued that examining a planning problem as a pure optimization problem in a dynamic or uncertain environment can be misleading. To address this issue, SMEs were consulted to identify and assign a prevalence level to the factors underpinning future operations. As a result, three scenarios were identified. These scenarios were further parameterized using an agent-based simulation model to generate, for example, different sets of tasks and their associated durations.

Bizkevelci and Çakmak [50] examined two scenarios, namely peace keeping and wartime operations, in the context of prioritizing military technologies.

Whitacre *et al.* [52] employed scenarios to examine the robustness of their scheduling approach under uncertain capability requirements.

Xin *et al.* [56] considered four scenarios in the context of assigning weapons to targets in a dynamic environment. The scenarios accounted for whether a weapon could be used only once, used a prescribed number of times, used during all stages, and a hybrid case. Furthermore, Monte Carlo sampling was used to determine whether a particular attack would destroy its target.

Malmi *et al.* [97] employed a wargaming simulation to simulate the outcome of military battles. Their simulations made use of one or more scenarios that were derived from threat models and various assumptions on how the enemy will use their forces. It was noted that the combat calculations in their chosen modeling tool were performed using Markov chains rather than Monte Carlo sampling.

Bjorkman *et al.* [62] used Monte Carlo sampling to estimate the baseline uncertainty in the prediction of airplane landing roll distances under various test conditions.

Fleischer Fauske *et al.* [63] constructed a binary tree to generate plausible future budget decisions, then performed a balanced sampling to generate scenarios for evaluation. Note that, this was the only example of the decision tree uncertainty technique that was identified in this review.

Xiong *et al.* [16] employed randomly-generated scenarios to account for three types of uncertainties, namely the duration, resource breakdown, and precedence relations, which were subsequently used to assess the robustness of the generated schedules.

Zhang *et al.* [68] considered three future mission types as scenarios in the context of evaluating weapon systems, namely tactical assault, defence and protection, and occupy and control. The attributes associated with each scenario, as well as their ideal values, were assigned by SMEs.

Davendralingam and DeLaurentis [67] used Monte Carlo sampling to generate values for capability coefficients and covariances as well as to generate random portfolios for comparison.

Bakirli *et al.* [65] used scenarios to encompass the aspects of whether a coalition exists (true or false), the type of warfare (regular or irregular), and the type of threat (material or information). All eight combinations of these aspects were considered in their study. Furthermore, positive and negative market trends were considered as distinct scenarios when assessing risk factors and environmental policies were considered in both tighter and more relaxed scenarios.

Konur *et al.* [72] used three scenarios representing the level of flexibility in an SoS architecture design problem where flexibility was defined as the ability of individual systems to permit engineering design changes. Hence, inflexible systems had a fixed set of capabilities associated with them whereas a flexible system was one where the capabilities can be decoupled, thereby providing only a subset of its capabilities at a reduced cost and time commitment.

Zhang *et al.* [74] considered two types of scenarios, those with complete information and those with incomplete information, in the context of army engineering and manufacturing project selection. The main distinguishing factor was that scenarios with complete information had associated probability distributions whereas only interval data was available for those with incomplete information. The experimental results considered scenarios with incomplete information to address uncertainty in the return value of different projects.

Shafi *et al.* [17] employed Monte Carlo sampling to generate different operating scenarios, which consisted of assigning effectiveness scores to each project. The list of projects and their associated costs were also generated randomly according to distributions extracted from the 2012 Australian Defence Capability Plan (DCP) [98].

Moallemi *et al.* [78] employed randomly generated scenarios to account for the uncertainty associated with their acquisition strategy model. The scenarios were used to evaluate the robustness of solutions.

Xia *et al.* [79] employed the use of scenarios to examine the robustness of weapon planning solutions. However, little detail about the scenario generation process was given as it was not considered to be the main focus of the paper. Furthermore, Xia *et al.* made the claim that future scenarios should be provided by SMEs, which can then be used to describe uncertainties.

Despite their argument that scenario-based modeling may be inadequate for cyber warfare applications, Rowe *et al.* [99] employed Monte Carlo sampling to estimate the proportion of different types of attacks that were expected to be successful.

While it is clear that scenario-based modeling is prevalent in the field of portfolio optimization for defence applications, there are some valid criticisms that should be highlighted. Watson and Kasprzyk [100] argues that by specifying scenarios *a priori*, the ability to determine the relative importance of each scenario is limited; scenarios should be adaptive and change according to the online results of the simulation. Gray [12] argued that predicting the future is inherently biased as the future is predicted given the current scenario, which limits perspective and leads to an "undesired element of prophecy." Moreover, Gray [12] argued that the challenges associated with uncertainty are often overstated and proposed using trend-spotting as an alternative. Filinkov and Dortmans [101] stated that scenario-based analyses tend to focus on generic conflicts that may arise, but would likely be better focused on identifying and mitigating the conditions that give rise to these conflicts. Rowe *et al.* [99] argued that scenario-based modeling isn't well suited to address cyber warfare problems and proposed risk-based analysis as a suitable alternative. Despite these criticisms, scenario-based approaches are by far the most common approach to address uncertainty.

The only other notable examples of uncertainty were in the form of potential project failures, whereby it is stated that over-programming can hedge against potential project failures [17], [64], [70]. Additionally, Teague *et al.* [58] used an agent-based simulation to anticipate the long-term effects of implementing different infrastructure projects, such as water wells and hospitals, in post-conflict Afghanistan.

B. QUANTIFICATION OF VALUE

It is well known that defining a mechanism to valuate potential assets is one of the most challenging aspects associated with portfolio optimization in the defence sector and, furthermore, that many traditional measures are senseless in this context [31], [41], [42], [93], [101], [102]. In fact, the usage of misguided valuation schemes can be problematic. For example, Angstrom [103] argued that the US has often focused primarily on death and destruction, which directly influences how their future planning is conducted. This focus on damage output is further reinforced through the ease in attaining funding for weapons in contrast to the difficulty in justifying the allocation of assets towards developing adaptive and anticipatory forces. Moreover, it was argued that a narrowly-focused view of project valuation causes a biased view of the future, thereby limiting the capacity to balance short-term and long-term planning objectives [14], [103]. Given the complexity of assigning quantitative values to defence projects and/or assets, this section discusses the valuation schemes found in the literature.

In a general sense, the value of an asset is typically defined in relation to its cost or return on investment while the utility of an asset is a measure of its overall usefulness. In some cases, measures of utility and value can be conflicting. It was found in a 2011 review that most defence-oriented applications used a value metric, specifically an additive value model, rather than a utility metric [92]. Thus, many early studies were not focused on the utility of the selected assets, but rather attention was focused on the direct value the assets could provide. This is problematic given that measures such as investment costs and military equipment numbers offer little insight into the true value of defence capabilities such as peace, protection, conflict deterrence, and stability [93]. Similarly, Wall *et al*. [102] noted that traditional monetary valuation schemes are senseless in a defence context. Rather, there must be a notion of effectiveness, which is not straightforward and there exists many different views on how to measure effectiveness. As a result, many studies have struggled to provide a measure that fully captures effectiveness.

In contrast to simple valuation schemes, capability-based analysis can assist in addressing questions of whether the defence force provides an adequate level of capability [93]. However, simply defining and valuing capabilities in broad environment-based categories, such as air, land, and marine leads to a number of dysfunctional consequences such as duplication and gaps [31]. Rather, it was claimed that functional partitioning of capabilities into categories such as mobility and partnership building is a promising future direction [31].

As another alternative, Filinkov and Dortmans [101] argued that CBP approaches have an inherent inadequacy when applied to defence planning as they primarily focus on developing an investment strategy. Rather, this study argued that an insurance-based approach, referred to as hedging-based planning, better reflects the totality of a defence system by capturing both force structure (capability development and acquisition) and force generation (transformation of latent capability into operational capability over time). Essentially, this study claimed that hedging-based planning can account for two types of investments, namely those that relate to acquisition of physical assets (Type I) and those that integrate assets into usable capabilities (Type II). A measure of risk can then be defined based on whether or not a desired capability portfolio is met according to various constraints. The primary objective of an optimization process would then be to find the optimal way to invest in both Type I and Type II assets to achieve a force design that is able to defend against various scenarios over a given time frame. Note that, this study was purely focused on the proposal of a new methodology and was not experimental in nature. Nonetheless, it proposed an alternative viewpoint on how to address the problem of portfolio optimization in the defence sector.

Regarding the valuation schemes employed in the surveyed articles, arguably the simplest valuation scheme was that of [63], where the value of a project was defined to be the sum of its costs over its life-cycle. The most common form of valuation was quantification of effectiveness in one or more scenarios, arising in nearly every study that was considered – this is unsurprising given that the primary objective is always to maximize the return on investment and the most logical form of ''return'' is the effectiveness that implementing a project will provide. Quite often this measure of effectiveness was formulated in consultation with SMEs.

While cost was typically considered in the form of one or more budgetary constraints, a few studies explicitly added the minimization of cost as an optimization objective [6], [15]–[18], [40], [43], [49], [51], [52], [74]. Another relatively common approach to valuation was to consider the balance, or variance, associated with the project selection as an objective [16], [19], [49], [51], [57], [74]. Balance/variance can also be considered a measure of robustness. Alternatively, balance could be imposed as a constraint, as seen in [70]. In contrast, Davendralingam and DeLaurentis [67] argued that the concept of variance, which is typically assumed to follow a normal distribution, does not easily extend to the risk associated with defence applications where the complex inter-dependencies result in complicated joint distributions among the interacting agents.

Considering cost and risk, [95] argued that an optimization objective should not be simply to reduce cost as a means to reduce risk, especially in the case of multiple mutually-exclusive projects with different cost-benefit-risk profiles. Furthermore, it was argued that any measure of the return on investment should explicitly account for the associated risks rather than this being an afterthought.

A few other notable valuation schemes were present in the literature. In two studies that also considered a scheduling component, the makespan was used as an objective [15], [16]. Hurley *et al*. [64] provided a mapping from an ordinal priority-based rank to a cardinal value for each project. The mapping was defined using a monotonically decreasing function for projects of successively lower priority. Dodd *et al*. [47] used probability distributions to specify the likelihood of specific outcomes with respect to a specific goal to formulate parametric, marginal utility functions, which were combined to formulate the objective function with respect to both short-term and long-term considerations. Bizkevelci and Çakmak [50] derived a prioritization scheme based on the needs of a particular mission and the areas of technology that were required. The study of Bjorkman *et al*. [62] considered the entropy associated with testing measurements as a measure of value. While an interesting approach, this technique is not widely applicable to portfolio optimization. While not considered in their study, Li *et al*. [19] noted that survivability and resilience should be included as objectives.

To further complicate matters regarding project valuation, there was typically more than one primary objective in the examined articles such that many of the approaches dealt with multiple conflicting criteria or objectives. Therefore, a single valuation scheme would hardly be effective in these cases. The next section thus examines how multiple criteria and/or objectives were addressed in various studies.

C. MULTIPLE CRITERIA AND MULTIPLE OBJECTIVES

Multi-Criteria Decision Making (MCDM) is a general methodology for addressing complex decision-making problems that involve multiple, often conflicting, criteria against which the alternatives are evaluated. MCDM methods evaluate the performance of various alternatives with respect to different criteria and (subjective) opinions regarding the relative importance of each criterion. One particular tool used in MCDM is Multi-objective Optimization (MOO), which is concerned with finding a diverse set of alternatives that do not impose a relative weighting on each objective. Such solutions are referred to as non-dominated or Pareto optimal. However, the two terms are often used interchangeably in the literature, despite being subtly different in their methodology and goals.

1) MULTI-CRITERIA DECISION MAKING VERSUS MULTI-OBJECTIVE OPTIMIZATION

To illustrate the difference between MCDM and MOO, consider the example data provided in Table 2. In this example, there are three vehicle options (A, B, and C) that are to be evaluated against three criteria, namely speed, safety, and fuel consumption. This is an example of a problem that can be addressed via MCDM. In this context, one wishes to select the best vehicle according to these criteria. One technique to facilitate this decision-making is to assign a rating between 1 (lowest) and 10 (highest) for each vehicle against each constraint. A (subjective) weight can then be assigned to each criterion based on the preferences of the decision-maker(s). Both the rating and weighting steps often involve the consultation of SMEs. Using the assigned weights, a score can be calculated for each vehicle and criterion such that the total score is given by the summation of the scores for each criterion.

TABLE 2. Example multi-criteria decision problem for cars. Each criterion is assigned a score between 1 (lowest) and 10 (highest), then weighted, and a score assigned.

	Rating				Score		
Criterion	A	B	C	Weight	A	B	C
Speed	5	8	4	6	30	48	24
Safety	5	3	8	8	40	24	64
Fuel	5	4	7	4	20	16	28
Total					**90**	**88**	**116**

In Table 2, the decision-maker(s) prioritized the safety of the vehicle, followed by the speed, and then the fuel consumption by assigning weights of 6, 8, and 4 to the speed, safety, and fuel consumption criteria, respectively. Therefore, according to their preferences, vehicle C is determined to be the best choice. However, if the decision-maker decided to prioritize speed by assigning weights of 8, 4, and 3, respectively, then vehicle B would be assigned the highest total score. Note that, the choice of aggregating function is arbitrary and many alternatives exist.

If this problem were viewed in the context of MOO, each of the vehicle options would be considered equivalent as each are non-dominated with respect to the others,[11] i.e., no vehicle is objectively worse with respect to all of the criteria/objectives. The decision-maker would then consider each of the alternatives in more detail, ultimately making their decision using the set of non-dominated alternatives. In summary, the critical difference between MCDM and MOO is that MCDM is concerned with making a decision with respect to multiple criteria, whereas MOO is concerned with finding a diverse set of optimal, and hence equivalent, options with respect to multiple criteria, thereby deferring the decision making process.

[11] Note that, it is not typical for all alternatives to be non-dominated with respect to each other – this is simply a result of having only a few alternatives in this example.

2) WEIGHTING AND SCORING METHODS

One of the simplest strategies to address MOO is to reduce the problem to a single-objective problem. The most straightforward approach is to weight each objective according to the decision maker's preferences, then sum across all objectives. This is known as the weighted-sum approach. However, it should be noted that this approach is incapable of specifying complex preference information and can limit the ability to find all Pareto-optimal points [104]. Despite this limitation, if all the assigned weights are positive and greater than 0, then the weighted-sum approach is guaranteed to provide a Pareto-optimal solution [104]. Then, by varying the weight vector, multiple non-dominated solutions can be attained.

There were a number of studies that employed a weighted-sum approach. In [35], 15 weights were assigned to different severity and impact criteria to ascertain a risk valuation for cyber attacks. In [36], weights were assigned to tasks to indicate priority and the score of a candidate was then taken as the weighted sum of its contribution for each measure. [47] employed criteria weights to reflect the importance of achieving various operational goals. Reference [105] used a weighted sum of ten measured values to quantify the effectiveness of counter IED initiatives. [70] used an additive measure function to compute the overall value of each project according to three criteria, namely the alignment with national policy, the alignment with institutional capability needs, and the relative importance placed on the project by its sponsor.

While the weighted-sum approach is certainly the most straightforward approach, alternative weighting schemes are also possible. Parnell *et al.* [41] used a non-linear, multi-attribute function using relative weights assigned by SMEs. No further specifics were given apart from the fact that it was not simply an additive function. The follow-up study [46] does not explicitly state the objective function used and is thus assumed to have also used the non-linear function mentioned in [41]. Bizkevelci and Çakmak [50] used a multiplicative weight of priorities assigned to the overall mission needs and technological needs, respectively, to define a weighted priority.

Another prominent scoring technique that was found in the literature was AHP. AHP is an MCDM technique whereby the problem is decomposed into a hierarchical structure based on the factors used to influence the decision-making process. In AHP, these factors are selected and arranged hierarchically according to increasing levels of granularity such that the lowest level represents the decision-making environment. Pairwise comparisons are then used to derive relative priority weightings for each of the alternatives at the lowest level, rather than arbitrarily assigning these weights. These weights can then be used to score and rank individuals in an optimization context.

Greiner *et al.* [38] employed AHP to decompose the portfolio selection problem into seven main criteria, namely external factors, industrial base, capability, user needs, risk, standards, and funding. Each of these main criteria was then further decomposed into 22 sub-criteria, which were each assigned a weight via AHP. AHP was then used to derive priority weights for each criteria, which were subsequently used to select projects to fund.

Lee *et al.* [55] combined AHP and PCA [83], [84] to determine prioritization weights. The weights were combined according to

$$w_s = \frac{w_{Ai} w_{Pi}}{\sum_{i=1}^{m} w_{Ai} w_{Pi}}, \tag{7}$$

where w_{Ai} and w_{Pi} are the weights assigned to criterion i via AHP and PCA, respectively.

Bakirli *et al.* [65] employed AHP as a mechanism to assign usage probabilities of capability areas in various scenarios by surveying SMEs.

Another notable MCDM technique is known as goal programming [106], [107], which is a sub-branch of MOO whereby each objective is assigned a goal value that should be achieved. Objective fitness scores are then assigned based on deviations from these goal values, which may be weighted according to the priority levels of the objectives. The weighting can also facilitate direct comparison or summation of objectives that are otherwise not comparable. Goal programming thus facilitates the specification of a minimum level of capability in various objectives without resorting to the use of constraints. A number of studies in this review were found to employ goal programming [43], [48], [55], [65].

In a similar fashion to goal programming, TOPSIS is a MCDM method that stipulates that the best alternative is the one that has the shortest geometric distance from an ideal solution as well as the furthest geometric distance from the worst solution [87]. Two studies that used TOPSIS to assign values to candidate solutions were identified, namely [61] and [68].

3) EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION

Evolutionary Multi-Objective Optimization (EMO) [108] refers to a class of evolution-inspired computational intelligence techniques that aim to simultaneously optimize multiple conflicting objectives. Typically, EMO paradigms strive to find a diverse approximation of the true set of Pareto optimal solutions. It should be noted that EMO approaches, while relatively efficient computationally, are heuristic methodologies that are not guaranteed to find optimal solutions. Given the complexity and scale of many defence-related portfolio optimization problems, exact approaches are often infeasible and thus there has been a significant amount of research that employed EMO for defence-related portfolio optimization [15]–[19], [49], [51], [52], [72], [78], [79].

By far, the most common EMO methodology used in the defence literature is NSGA-II, which was used in the following studies: [15], [16], [18], [19], [51], [78], [79]. NSGA-II employs non-dominated sorting to hierarchically arrange the population based on an ordering imposed by Pareto dominance. Furthermore, NSGA-II uses a crowding metric during selection to promote diversity.

Regarding other EMO methodologies, [49], [52], and [72] made use of MOEA, which is a generic multi-objective algorithm based on evolutionary principles, whereas [17] employed MOEA/D, which is premised on decomposing the multi-objective problem into multiple single-objective problems solved simultaneously.

D. MULTI-PERIOD AND DYNAMIC ASPECTS

Despite the portfolio optimization problem being inherently dynamic, very few studies actually considered a model formulation where the problem changes over time. Rather, most studies that considered a multi-period planning horizon considered only a static context [15], [37], [43], [52], [59].

Crawford *et al.* [37] acknowledged that after each year, a project was either completed or one year closer to completion, thus concluding that the problem was inherently dynamic. Despite this acknowledgment, no mechanism to address the dynamism was present in their study. Chan *et al.* [43] considered planning over multiple periods, referred to as cohort years, such that initial and subsequent acquisition decisions can be differentiated and treated differently. Whitacre *et al.* [52] considered a 10-year, static planning period for resource allocation. Golany *et al.* [59] considered a multi-period approach for resource allocation in developing military countermeasures. Xiong*et al.* [15], [18] considered scheduling problems, which were inherently multi-period. Rempel and Young [70] considered a 20-year planning horizon along with an additional 20 years for the budget. Wang *et al.* [76] considered a 15-year planning horizon, broken into three 5-year cycles. Moallemi *et al.* [78] considered a 3-year planning horizon, in weekly periods, for submarine acquisition and crew decisions. Xia *et al.* [79] considered a multi-stage weapon planning problem, such that the time component imposed additional constraints regarding the transition of projects between different states (such as development and retirement).

While each of the aforementioned studies considered a multi-period approach, the problem remained static over time. In more comprehensive studies, a dynamic or time-variant aspect was added such that the problem would change over time. Brown *et al.* [13] considered that pairwise interactions could occur during one or more time periods in the future. Tsaganea [45] modeled a missile defence problem using dynamic systems theory. Specified as a dynamical system, the state of the system was directly dependent upon the previous state. Baker *et al.* [49] considered a dynamic multi-agent simulation to evaluate candidate fleet scheduling solutions. Xin *et al.* [56] incoporated a dynamic aspect in the form of enemy attacks. Given that it was unknown whether an attack will be successful or not, their approach to the weapon-target assignment problem directly addressed this form of dynamism. In some studies, the cost or value associated with a project would depend on the time [33], [63]. In a similar fashion, Zhang *et al*. [74] incorporated synergistic effects between projects by discounting their cost if they were implemented together. Xiong *et al*. [18] considered the budget available at each period according to a discount factor that was dependent upon the number of months that had passed. In the work of Shafi *et al*. [17], the list of available projects, the budget, and the effectiveness scores would vary in each period.

VI. DISCUSSION AND FUTURE DIRECTIONS

In the previous sections, a number of interesting application areas and approaches to overcome the associated challenges have been identified. However, most of these studies considered only one specific application, often in a manner that was specific to only the formulation they proposed, thus the work is typically not generalizable. Moreover, there are other aspects that have largely been ignored or not adequately addressed in the literature. This section discusses a number of open research areas that were identified through this review.

A. DEVELOPMENT OF BENCHMARK INSTANCES

From this review, it is evident that the applications of portfolio optimization in the defence sector are often too context-specific. Each application was developed independently, which left no coherent "best-choice" solution readily available. Furthermore, this makes the comparison of different studies nearly impossible. Therefore, one critical avenue of future research must be regarding the standardization of this research domain.

One specific task that should be carried out is the development of benchmark project instances that are readily available to the public. Such instances should account for, at minimum, various different characteristics pertaining to the difficulties described in Section V. These benchmark cases can provide a common ground for future studies to examine the effect of different portfolio optimization strategies. Moreover, a proper set of benchmark problems will help alleviate the issue of data confidentiality, which has often led to either *ad hoc* synthetic data generation or omitting a detailed description of the data altogether. Nonetheless, future studies should continue to examine real-world problem instances alongside the benchmark instances as it is acknowledged that benchmark instances cannot reasonably account for the nuances of all possible real-world instances.

B. TIME-DISCOUNTING

One important aspect that is not considered adequately in the literature is time-discounting. In most studies, value is considered only with respect to the present, not the future [109]. Specifically, costs incurred in the present tend to be weighted more heavily than those in the future given that costs are not time-discounted. Consider, for example, having \$100 in cash. If this cash is spent today, it has a value of \$100. Instead, if the cash is placed into a savings account that earns interest at an annual rate of r, it will be worth $\$100(1+r)^n$ after n years. Consider also that if one is set to receive \$100 in n years, the present value of that sum is worth $\frac{\$100}{(1+r)^n}$.

This same principle can be applied to non-financial decisions. In such applications, r is more commonly referred to as the discount rate as it is unlikely to coincide with any particular interest rate. This adds an additional, important dimension to the portfolio optimization problem as it incorporates the depreciation of value over time. However, determining an appropriate and realistic value for r, even for currency, isn't straightforward and no general consensus was found via a survey of 2160 economists [110]. In this survey, the most common response was a rate of 2%, while the mean response was 4% with a standard deviation of 3% [110]. To further complicate matters, Weitzman [110] concluded that for long-term planning, a decreasing discount rate is appropriate. In contrast, it was argued in a more recent study that for long-term planning, an increasing discount rate that tends towards the largest possible value as the horizon tends to infinity is more appropriate [111].

Regardless of the discount rate, if one is applied, the value that an asset has to a defence organization is not strictly monetary and will depend on capability gaps and the rate at which adversaries can adapt to new capabilities [11]. This adds additional difficulty to the problem. However, despite the uncertainty associated with selecting an appropriate value for the discount rate, this is an aspect of the portfolio optimization problem that should be considered in future research, especially in the context of the long planning horizons seen in the defence sector.

C. PROBLEM INSTANCE SIZE

Despite the exponential complexity, many of the considered studies examined only a small number of projects, thus allowing exact solution techniques, such as linear programming, to be a feasible choice. Linear integer programming, where both the objective function and constraints are linear, is known to be NP-hard and thus there is currently no known polynomial-time algorithm to solve such problems in the general case [112]. Deriving exact solutions becomes even more challenging when the objective function and constraints are no longer linear. Moreover, real-world applications in the defence sector are likely to contain a large number of projects to consider, thereby rendering exact solvers infeasible. For example, the Canadian Navy Level 1 business planner has over 1200 projects [64].

While a significant number of the studies examined the usage of heuristic methods, which are far more scalable than exact approaches, most studies considered only a small number of assets, typically less than 25. The largest number of projects considered by a study in this review was around 200 while only a few others considered more than 100. Therefore, an avenue that must be explored further is the scalability of portfolio optimization approaches when the number of assets is large.

D. MEASURING DEFENCE OUTPUT

While there is no debate that meeting strategic objectives should be a primary objective of defence planning, there are other aspects that can, and likely should, be considered when measuring the value of a defence project [93], [96]. Perhaps there should be a consideration of the economic and political implications, among others, regarding the selection of particular projects. For example, implementation of certain projects can impact the local and global economies and, in this regard, projects that benefit the national economy are likely more valuable than those that do not. Similarly, projects that foster the development of foreign policy and political allies should be considered more favourably than projects that may be viewed negatively by the global community. However, with such considerations comes ethical questions that must be addressed. For example, *is a project that defends a nation more valuable than one that benefits an ongoing conflict*? *How can combat effectiveness, economic impact, and political impact be compared*? *Should the immediate safety of citizens be prioritized over long-term regional stability*? Questions like these, and others, must be taken into account when making long-term decisions that afford the capacity for major global implications. Therefore, analyzing exactly what considerations are made when examining alternatives, and how such trade-offs are evaluated, is certainly a topic that should be given careful consideration in the future.

E. SELECTING HIGH-COST PROJECTS

The studies of both Rempel and Young [70] as well as Greiner *et al*. [38] observed that high-cost projects were often not selected by portfolio optimization techniques. This begs the question of whether the valuation models being used are inadequate or whether these high-cost projects were simply not worth the investment. One suggested approach to overcome this issue is to enforce a cardinality constraint, such that the number of projects is limited, thus preventing a large number of low-valued projects from being selected in place of fewer high-cost projects. While the selection of only low-valued projects may seem like an issue, Rempel and Young [70] also highlighted the fact that some of the benefit to automated approaches is that they *would* select portfolios that would otherwise be ignored. Hence it was explicitly decided not to address this concern in their solution. Nonetheless, further investigation should be made into the reasoning behind why these high-cost projects are not being selected.

In a sense, this concern is directly related to the transparency of the optimization process. Often, not only is the solution of importance, but also the rationale behind *why* this particular portfolio was selected is useful. Therefore, future research should also examine how to increase the transparency of the decisions made by automated approaches. For example, facilitating the direct comparison of two projects, in the context of an entire portfolio, would allow a decision maker to directly see the implications associated with choosing one project versus the other. Additionally, this transparency would also serve to confirm whether the mathematical formulation of the problem and its objective(s) does, in fact, align with the intended behavior of the system.

F. USER-FRIENDLINESS OF IMPLEMENTATIONS

Rempel and Young [70] highlighted a case where a sophisticated optimization approach was developed, yet largely abandoned in favor of more labour-intensive approaches, such as a ranked list, with greater familiarity. Thus, it is of utmost importance that portfolio optimization techniques used in the defence sector are mindful of end-users; a complex portfolio optimization framework that is not understandable by the decision makers will not be employed. Moreover, the current approaches do not facilitate the adjustment of portfolios once they have been selected. Authors of future studies are thus reminded that an non-intuitive system is unlikely to be adopted in the defence sector and that usability of the developed systems is paramount.

VII. CONCLUSION

In this manuscript, recent applications of portfolio optimization applied to the defence sector were reviewed. The review revealed a relatively small number of publications in the domain. Despite this, the application areas were broad and covered a plethora of unique scenarios. It was found that this particular application domain is rife with challenges, largely created by the unique environment in which defence organizations must make decisions. For example, the long-term investments and the lengthy operational time of projects is a feature not seen in most other applications of portfolio optimization. Moreover, the quantification of value, addressing uncertainty, handling multiple criteria and/or objectives, and dynamic environments were identified as the key challenges associated with selecting a robust portfolio of projects. Mechanisms to address each of these challenges were discussed. Finally, this paper identified and discussed a number of areas that warrant further investigation.

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KYLE ROBERT HARRISON (Member, IEEE) received the B.Sc. and M.Sc. degrees in computer science from Brock University, St. Catharines, Canada, in 2012 and 2014, respectively, and the Ph.D. degree in computer science from the University of Pretoria, Pretoria, South Africa. He is currently a Research Associate with the School of Engineering and Information Technology, University of New South Wales at Canberra, Australian Defence Force Academy, Canberra, ACT, Australia. From January 2019 to August 2019, he was a Postdoctoral Fellow of the Department of Electrical, Computer, and Software Engineering, University of Ontario Institute of Technology, Oshawa, ON, Canada. His research interests include computational intelligence, optimization, operations research, complex networks, and fitness landscape analysis.

SABER ELSAYED (Member, IEEE) received the Ph.D. degree in computer science from the University of New South Wales at Canberra, Australian Defence Force Academy, Canberra, Australia, in 2012, where he is currently a Senior Lecturer with the School of Engineering and Information Technology. His research interests include evolutionary algorithms, constraint-handling techniques for evolutionary algorithms, scheduling, big data, and cybersecurity using computational intelligence. Since 2019, he has been serving as the Chair of the IEEE Computational Intelligence Society (ACT Chapter). He was the winner of different IEEE-CEC competitions. He serves as an organizing committee member of different conferences in the evolutionary computation field.

IVAN GARANOVICH is a senior analyst specializing in strategic analysis of national security problems.

TERENCE WEIR is a senior analyst specializing in defence resource management and national security problems.

MICHAEL GALISTER has a broad experience across many areas of defence operations analysis as applied to force design, experimentation, and simulation in the context of both single service and joint forces.

SHARON BOSWELL is a senior mathematician with research interest in strategic decision analysis, particularly discrete optimization problems.

RICHARD TAYLOR is the Head of joint force analysis with the Australian Defence Science and Technology Group. He has extensive experience in the applications of network analysis methods to mathematical modeling and operations research problems.

RUHUL SARKER (Member, IEEE) received the Ph.D. degree from Dalhousie University, Halifax, NS, Canada, in 1992. He is currently a Professor with the School of Engineering and Information Technology and the Director of the Faculty of Postgraduate Research, University of New South Wales at Canberra, Canberra, ACT, Australia. He has authored the book entitled *Optimization Modelling: A Practical Approach* (Boca Raton, FL, USA: CRC, 2007) and published over 250 refereed articles in international journals, edited books, and conference proceedings. His current research interests include evolutionary optimization and applied operations research. He is currently an Associate Editor of *Memetic Computing* journal, the *Journal of Industrial and Management Optimization*, and *Flexible Service and Manufacturing Journal*.