Theoretical and Applied Economics
Volume XVIII (2011), No. 2(555), pp. 75-88

Portfolio Risk Analysis using ARCH and GARCH Models in the Context of the Global Financial Crisis*

Oana Mădălina PREDESCU
Bucharest Academy of Economic Studies
predescu_oana85@yahoo.com
Stelian STANCU
Bucharest Academy of Economic Studies
stelian_stancu@yahoo.com

Abstract. *This paper examines both the benefits of choosing an internationally diversified portfolio and the evolution of the portfolio risk in the context of the current global financial crisis. The portfolio is comprised of three benchmark indexes from Romania, UK and USA. Study results show that on the background of a global economic climate eroded strongly by the effects of the current financial crisis, international diversification does not reduce risk. Moreover, using ARCH and GARCH models shows that the evolution of portfolio volatility is influenced by the effects of the current global financial crisis.*

Keywords: global financial crisis; diversification; volatility; ARCH model; GARCH model.

JEL Code: G01.
REL Code: 11B.

---

* Ideas in this article were presented at the Symposium „The global crisis and reconstruction of economics?", 5-6 November 2010, Faculty of Economics, Bucharest Academy of Economic Studies.

1. Introduction

The sub-prime credit problem that started in the United States during 2007 affected the financial sector in other countries, especially Europe. Deterioration of this sector led to the collapse of national financial systems in different parts of the world, the result being a severe global financial crisis. The magnitude of the recent financial crisis is considered to have no precedent since the *Great Depression*, the International Monetary Fund (IMF) referring to this global recession as "*The Great Recession*". A study developed by the IMF in 2009 stated that the recent financial crisis has "*revealed important flaws in the current global architecture*". The IMF managing Director, Dominique Strauss-Kahn, has underlined, in May 2009, the necessity of a "*new global framework*" that can ensure a better coordination of national policies (Moshirian, 2010, pp. 5-6). Thus, any national policy must be strongly related with global policies in order to avoid another global financial crisis. However, not few have been those that questioned if a global financial system is desirable. In order to clarify this debate we must first understand the role of the financial system for the economy. The financial system represents the link between investors and private or public entities. Through this system the economy grows, the consumers also grow their capacity to consume, risks are shared and individuals confront with a smaller volatility on the market. All these correlations extend to the level of the global financial system, a system that works as a powerful and efficient mechanism when the economy is characterized by stability. Whereas the potential of the financial system certainly exists, words like *moral hazard* or *adverse selection* remind us that the economic theory has identified the reasons why financial markets do not always work perfectly, the failure of one or more markets generating losses for the other ones under the conditions of a global financial system. The recent financial crisis, as well as past episodes, teaches us very clearly that the capital flows guided by financial markets can represent something very different from an efficient and optimal allocation of savings towards the right investment projects. Increasing interconnections of financial institutions and markets lead to highly correlated financial risks, liquidity pressures and depletion of bank capital. Although the recent crisis originated in the United States, due to a highly integrated global economy, the financial markets around the world collapsed and the use of sophisticated financial instruments along with massive international capital flows facilitated its rapid spread across markets and borders (Claessens, Kose, Terrones, 2010).

Modern portfolio theory relies on the study developed by Markowitz (1952). Rubinstein (2002) appreciated that Markowitz's research represents the first mathematical formalization of the diversification concept of investments, emphasizing the fact that even though diversification reduces risk, it can not eliminate it completely. So, through diversification risk can be reduced without having any effects on the portfolio expected return. Thus, sub-perfectly correlated securities represent the "*right candidates*" to be included in a portfolio. Solnik (1974), among others, extended the initial CAPM (Capital Asset Pricing Model) and suggested that international diversification leads to better results than domestic diversification. However, financial integration leads to a significant correlation of security returns, the benefits of international diversification being greatly reduced (Aloui, 2010).

Taking all these aspects into consideration, the present paper aims to analyze the evolution of the risk of an internationally diversified portfolio in the context of the current financial crisis. The remainder of the paper is organized as follows: Section 2 presents theoretical aspects related to the volatility measurement of financial time series using ARCH and GARCH models; in Section 3 we report the empirical results of our study and in Section 4 we provide a summary of our conclusions.

2. Measuring volatility in financial time series: the ARCH and GARCH models

Philip Fransens (1988) noted that in the case of financial time series "various sources of news and other exogenous economic events may have an impact on the time series pattern of asset prices. Given that news can lead to various interpretations, and also given that specific economic events like an oil crisis can last for some time, we often observe that large positive and large negative observations in financial time series tend to appear in clusters" (Gujarati, 2004, p. 856).

In practice, linear time series models are incapable to explain a number of important features common to financial data, such as:

- *Leptokurtosis* – the tendency for financial asset returns to have distributions that exhibit fat tails;
- *Volatility clustering* or *volatility pooling* – the tendency for volatility to appear in bunches on financial markets. Thus large returns (of either sign) are expected to follow large returns, and small returns (of either sign) to follow small returns. One of the explanations for this phenomenon, which seems to characterize financial return series, would be the fact that the information arrivals which drive price changes occur in bunches.
- *Leverage effects* – the tendency for volatility to rise more following a large price fall, rather than following a price rise of the same magnitude.

Campbell, Lo and MacKinlay (1997) defined a non-linear data generating process as one where the current value is related non-linearly to current and previous values of the error term:

$$y_t = f(e_t, e_{t-1}, e_{t-2}, ...) \quad (1)$$

where e_t represents an independent and identically distributed (*iid*) error term, and f is a non-linear function. According to the three researchers, a more specific form of the non-linear model is given by the following equation:

$$y_t = g(e_{t-1}, e_{t-2}, ...) + e_t \sigma^2 (e_{t-1}, e_{t-2}, ...) \quad (2)$$

where g is a function of past error terms, and σ^2 is the variance term. Campbell, Lo and MacKinlay characterize models with non-linear g as being non-linear in mean and those with non-linear σ^2 as being non-linear in variance. Models can be linear in mean and variance (the classic regression model, ARMA models) or linear in mean, but non-linear in variance (GARCH models) (Brooks, 2010, pp. 380). The most commonly used financial models to measure volatility are the non-linear ARCH and GARCH models.

2.1. The autoregressive conditional heteroscedasticity model (ARCH)

One of the fundamental hypotheses of the classical regression model is the *homoscedasticity* or the hypothesis of constant error variance: $\text{var}(e_t) = \sigma^2(e_t)$, where $e_t \sim N(0, \sigma^2)$. The opposite case is known as *heteroscedasticity*. In the case of financial time series it is unlikely that the variance of the errors will be constant over time and hence it is preferred to consider a model that does not assume constant variance and which can describe how the variance of the errors evolves. As we mentioned earlier another important feature of financial series is known as *volatility clustering* or *volatility pooling*. This characteristic shows that the current level of volatility tends to be positively correlated with its level during the immediately preceding periods. Using the ARCH model (Engle, 1982) represents one of the modalities through which a phenomenon of this nature can be parameterized. In order to understand how this model works, a definition of the conditional variance of a random variable e_t is necessary. Thus, the conditional variance of e_t, denoted σ_t^2 has the following form:

$$\sigma_t^2 = \text{var}(e_t / e_{t-1}, e_{t-2}, ...) = E[(e_t - E(e_t))^2 / e_{t-1}, e_{t-2}, ...] \quad (3)$$

Since $E(e_t) = 0$, equation (3) becomes:

$$\sigma_t^2 = \text{var}(e_t / e_{t-1}, e_{t-2}, ...) = E[e_t^2 / e_{t-1}, e_{t-2}, ...] \quad (4)$$

Equation (4) states that the conditional variance of a zero mean normally distributed random variable e_t is equal to the conditional expected value of the square of e_t. In the case of the ARCH model, the autocorrelation in volatility is modeled by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \times e_{t-1}^2 \quad (5)$$

The above model is known as ARCH(1) and it shows that the conditional variance of the error term σ_t^2, depends on the immediately previous value of the squared error. Equation (5) represents only a part of the model, since nothing has been specified about the conditional mean. Under the conditions of the ARCH model, the conditional mean equation (which describes how the dependent variable y_t varies over time) can take almost any form. One example of a full model would be the following one:

$$y_t = \beta_1 + \beta_2 \times x_{2t} + \beta_3 \times x_{3t} + \beta_4 \times x_{4t} + e_t \quad (6)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \times e_{t-1}^2 \quad (7)$$

where $e_t \sim N(0, \sigma^2)$.

So the model given by equations (6)-(7) can be extended to the general case, where the error variance depends on q lags of squared errors, a model known as ARCH(q):

$$y_t = \beta_1 + \beta_2 \times x_{2t} + \beta_3 \times x_{3t} + \beta_4 \times x_{4t} + e_t \quad (8)$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \times e_{t-1}^2 + \alpha_2 \times e_{t-2}^2 + ... + \alpha_q \times e_{t-q}^2 \quad (9)$$

where $e_t \sim N(0, \sigma^2)$.

Since σ_t^2 represents the conditional variance, its value must be strictly positive (a negative variance at any point in time is meaningless). So all the coefficients in the conditional variance equation must be positive: $\alpha_i \geq 0, (\forall) i = 0,1,2,..., q$. A natural extension of the ARCH(q) model is the GARCH model.

2.2. The generalized autoregressive conditional heteroscedastic model (GARCH)

The GARCH model has been developed independently by Bollerslev (1986) and Taylor (1986). This model allows the conditional variance to be dependent upon previous own lags, so that the simplest equation form of the conditional variance is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \times e_{t-1}^2 + \beta \times \sigma_{t-1}^2 \qquad (10)$$

This is a GARCH(1,1) model and the conditional variance can be interpreted as a weighted function of a long term average value (dependent on α_0), of the information related to the volatility during the previous period ($\alpha_1 \times e_{t-1}^2$) and of the variance during the previous period ($\beta \times \sigma_{t-1}^2$). The general form of the GARCH(q, p) model, where the conditional variance depends on q lags of the squared error and p lags of the conditional variance is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \times e_{t-1}^2 + \alpha_2 \times e_{t-2}^2 + ... + \alpha_q \times e_{t-q}^2 + \beta_1 \times \sigma_{t-1}^2 + \\ + \beta_2 \times \sigma_{t-2}^2 + ... + \beta_p \times \sigma_{t-p}^2 \qquad (11)$$

or

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \times e_{t-i}^2 + \sum_{j=1}^{p} \beta_j \times \sigma_{t-j}^2 \qquad (12)$$

In academic literature a GARCH(1,1) model is considered to be sufficient in capturing the evolution of the volatility. A GARCH(1,1) model is equivalent to an ARCH(2) model and a GARCH(q, p) model is equivalent to an ARCH ($q+p$) model (Gujarati, 2004, p. 862).

The unconditional variance of the error term e_t is constant and given by the following equation:

$$\mathrm{var}(e_t) = \frac{\alpha_0}{1-(\alpha_1 + \beta)} \qquad (13)$$

as long as $\alpha_1 + \beta < 1$. For $\alpha_1 + \beta \geq 1$, the unconditional variance of the error e_t is not defined (non-stationarity in variance), and $\alpha_1 + \beta = 1$ represents Integrated GARCH or IGARCH (unit root in variance).

3. Empirical results

3.1. Data and descriptive statistics

We selected three benchmark indexes from 3 different countries, namely Romania (BET), UK (FTSE100) and USA (S&P500). We chose these indexes because according to the Pearson correlation coefficient the 3 analyzed markets are not perfectly correlated, representing thus the "*right candidates*" to be included in the portfolio (Table 1). However, Aloui (2010) underlined in a recent paper that this coefficient is not the best indicator for measuring market interdependence. He stated that this coefficient can not make a clear distinction between large and small returns, or between positive and negative returns. Moreover, the Pearson correlation estimate is constructed on the basis of the hypothesis of a linear association between the financial return series under consideration, whereas their linkages may well take non-linear causality forms. The solutions that can solve these problems are found in the transformations that can be applied to non-linear models (logarithms) or by using GARCH models. Thus, the study developed in this paper aims to analyze both the benefits of choosing an internationally diversified portfolio and the evolution of the portfolio risk in the context of the current global financial crisis.

Table 1

	BET	FTSE100	S&P500
BET	1.00	0.26	0.43
FTSE100	0.26	1.00	0.58
S&P500	0.43	0.58	1.00

We computed a database containing daily returns over the period January 4, 2005 to May 5, 2010, being registered 1,297 observations using the following formula:

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right)\times 100\% \tag{14}$$

where r_{it} represents the continuously compounded return of security i at time t, p_{it} represents the price for security i at time t, $i=\overline{1,n}$ and $t=\overline{1,T}$ ($n=3$ and $T=1297$). We chose a 0.5 weight for the investment in the BET index, a 0.25 weight for the investment in the FTSE100 and a 0.25 weight for the investment in the S&P500, being preferred a passive strategy of portfolio management.

Table 2

Descriptive statistics for the daily index returns

	BET	FTSE100	S&P500	Portfolio
Min	-0.135461	-0.092646	-0.094695	-0.088843
Max	0.105645	0.093842	0.109572	0.087974
Mean	0.000011	0.000049	0.000080	-0.000003
Standard Deviation	0.021893	0.014304	0.015155	0.014761
Skewness	-0.665793	0.062484	-0.029192	-0.446707
Kurtosis	8.364870	11.03812	12.24093	9.151124
Jarque-Bera	1651.241 Prob. 0.000000	3492.551 Prob. 0.000000	4615.061 Prob. 0.000000	2087.874 Prob. 0.000000

The results show that the riskiest market is the national capital market, the less risky being the UK market. The portfolio risk is moderate in comparison to the risks registered on the markets. The biggest return is obtained in the case of the S&P500 index, the lowest one being obtained by the chosen portfolio. According to the skewness and kurtosis indexes, all data series are asymmetrical and exhibit excess kurtosis. The Jarque-Bera statistics are highly significant for all return series for a significance level of 1%, being confirmed the assumption that the series are not normally distributed.

In Figure 1 is illustrated the variation of the daily returns over the period January 4, 2005 to May 5, 2010. We can observe from the graphics that the returns were fairly stable over the period January 2005 to September 2008. After this date all return series manifested instability especially due to the effects of the global financial crisis. Moreover, we can observe that the series present two specific features of non-linear models (*volatility clustering* and *leverage*).

In order to analyze portfolio risk we first estimated a non-linear model with the capacity to capture the evolution of portfolio volatility over the specified time horizon, and the model most commonly used in financial applications of this nature is GARCH(1,1). Before estimating the model we had to detect any ARCH effects in the portfolio return series. Thus we performed the Engle (1982) test. We chose a number of five lags and using the "*least squares method and ARMA*" we estimated an ARMA(1,1) model in order to perform afterwards the heteroscedasticity test. According to the test results there are ARCH effects in the portfolio return series (Table 3).

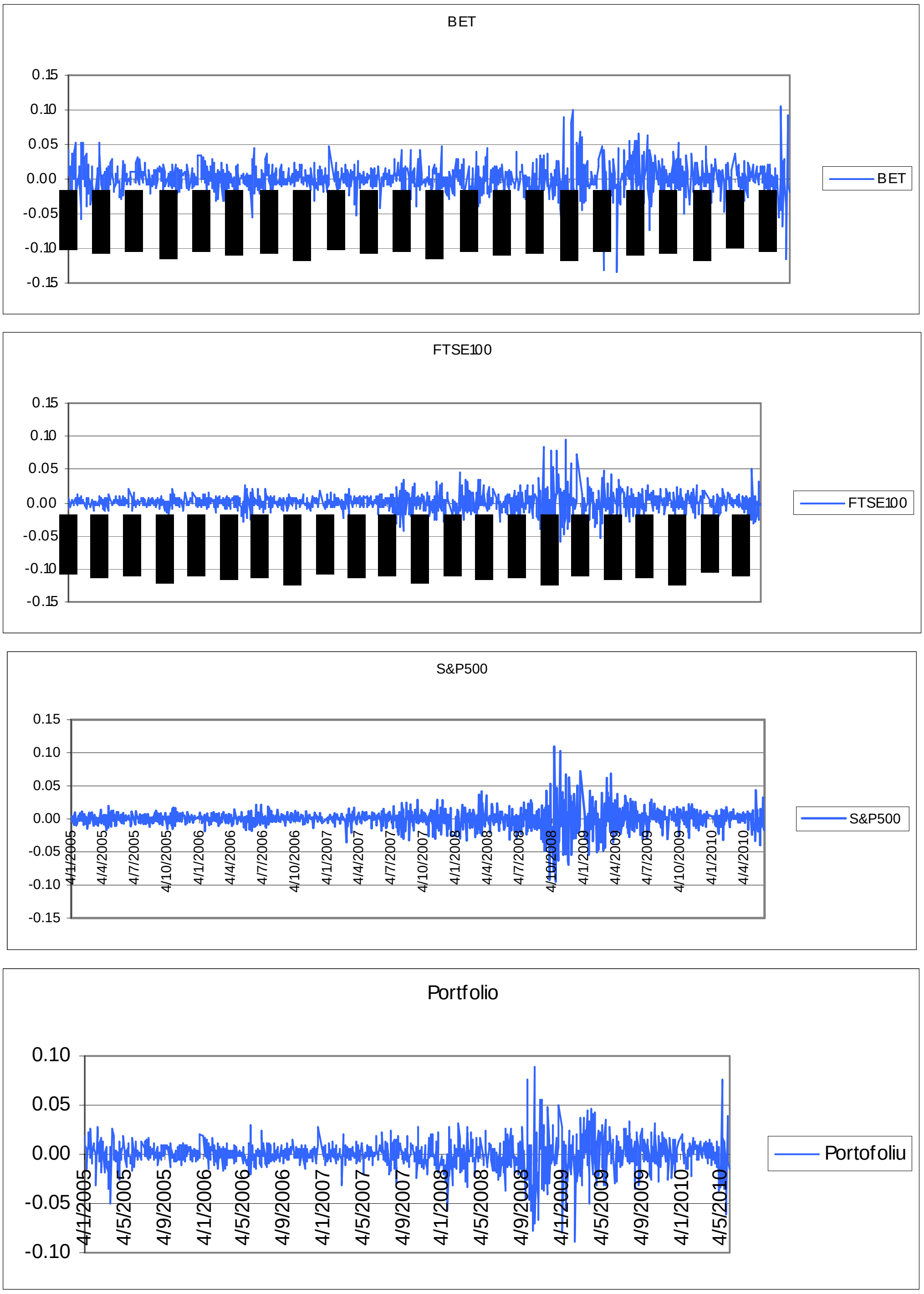
BET
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
BET
FTSE100
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
FTSE100
S&P500
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
4/1/2005
4/4/2005
4/7/2005
4/10/2005
4/1/2006
4/4/2006
4/7/2006
4/10/2006
4/1/2007
4/4/2007
4/7/2007
4/10/2007
4/1/2008
4/4/2008
4/7/2008
4/10/2008
4/1/2009
4/4/2009
4/7/2009
4/10/2009
4/1/2010
4/4/2010
S&P500
Portfolio
0.10
0.05
0.00
-0.05
-0.10
4/1/2005
4/5/2005
4/9/2005
4/1/2006
4/5/2006
4/9/2006
4/1/2007
4/5/2007
4/9/2007
4/1/2008
4/5/2008
4/9/2008
4/1/2009
4/5/2009
4/9/2009
4/1/2010
4/5/2010
Portofoliu

Figure 1

Table 3

Heteroskedasticity Test: ARCH			
F-statistic	49.01776	Prob. F(5,1285)	0.0000
Obs*R-squared	206.7917	Prob. Chi-Square(5)	0.0000

3.2. Estimation results

Using the "*ARCH method*" we estimated the GARCH(1,1) model (the results are shown in Table 4). The coefficients of the squared error and of the conditional variance are highly statistically significant (for a significance level of 1%, 5% and 10%). As expected, in a typical GARCH model for financial data the sum of the coefficients is close to 1. The coefficient of conditional variance is almost 0.9 and this implies that the shocks to the conditional variance are persistent and that large changes in the conditional variance are followed by other large changes and small changes are followed by other small changes. The variance intercept coefficient is very small and the coefficient of the squared error is 0.1.

Table 4

Dependent Variable: PORTFOLIO				
Method: ML - ARCH				
Sample: 1 1297				
Included observations: 1297				
Convergence achieved after 14 iterations				
Presample variance: backcast (parameter = 0.7)				
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)				
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000879	0.000269	3.263635	0.0011
	Variance Equation			
C	1.44E-06	3.74E-07	3.860082	0.0001
RESID(-1)^2	0.108449	0.010648	10.18496	0.0000
GARCH(-1)	0.891154	0.008593	103.7114	0.0000
R-squared	-0.003570	Mean dependent var		-2.57E-06
Adjusted R-squared	-0.003570	S.D. dependent var		0.014761
S.E. of regression	0.014787	Akaike info criterion		-6.060756
Sum squared resid	0.283384	Schwarz criterion		-6.044818
Log likelihood	3934.400	Hannan-Quinn criter.		-6.054775
Durbin-Watson stat	1.806960			

In order to validate this model we had to verify whether the squared errors presented ARCH effects. Thus, we analyzed both the correlogram of the squared error series and the results of the ARCH test. According to the correlogram there are no additional ARCH terms, a result confirmed also by the ARCH test for the significance levels of 1% and 5% (Tables 5 and 6).

Table 5

Heteroskedasticity Test: ARCH			
F-statistic	1.895668	Prob. F(5,1286)	0.0922
Obs*R-squared	9.452893	Prob. Chi-Square(5)	0.0923

Table 6

Sample: 1 1297						
Included observations: 1297						
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
|* |	|* |	1	0.129	0.129	21.486	0.000
| |	| |	2	-0.009	-0.026	21.587	0.000
| |	| |	3	0.018	0.023	22.029	0.000
| |	| |	4	0.023	0.018	22.741	0.000
| |	| |	5	0.006	0.002	22.792	0.000
| |	| |	6	0.021	0.021	23.355	0.001
| |	| |	7	0.039	0.034	25.363	0.001
| |	| |	8	0.016	0.007	25.707	0.001
| |	| |	9	0.021	0.019	26.297	0.002
| |	| |	10	0.027	0.020	27.238	0.002

Based on the estimated volatility equation, we generated the historical series of conditional volatility. Volatility can be measured through variance or standard deviation. So, the portfolio volatility (measured through standard deviation) is presented in the graphic below:

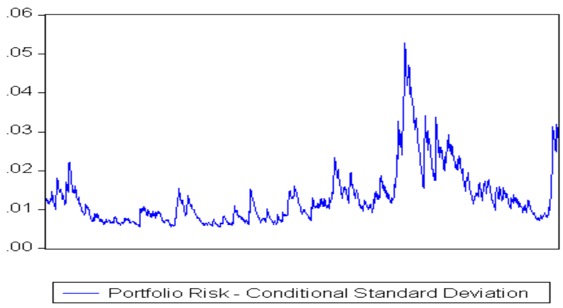

Figure 2

According to Figure 2 there are more volatile periods than others over the analyzed time period. The most pronounced volatility can be noted between 2008 and 2009 (the highest peak), being observed a slight fall in the next period and then in May 2010 (the end of the chosen time horizon) it can be observed another abrupt increase in the volatility. This evolution of the portfolio volatility is attributed to the effects of the current financial crisis that has put a print on the financial markets around the world. In order to confirm this conclusion we analyzed the daily returns of the indexes over the period 2008-2009 (Figure 3).

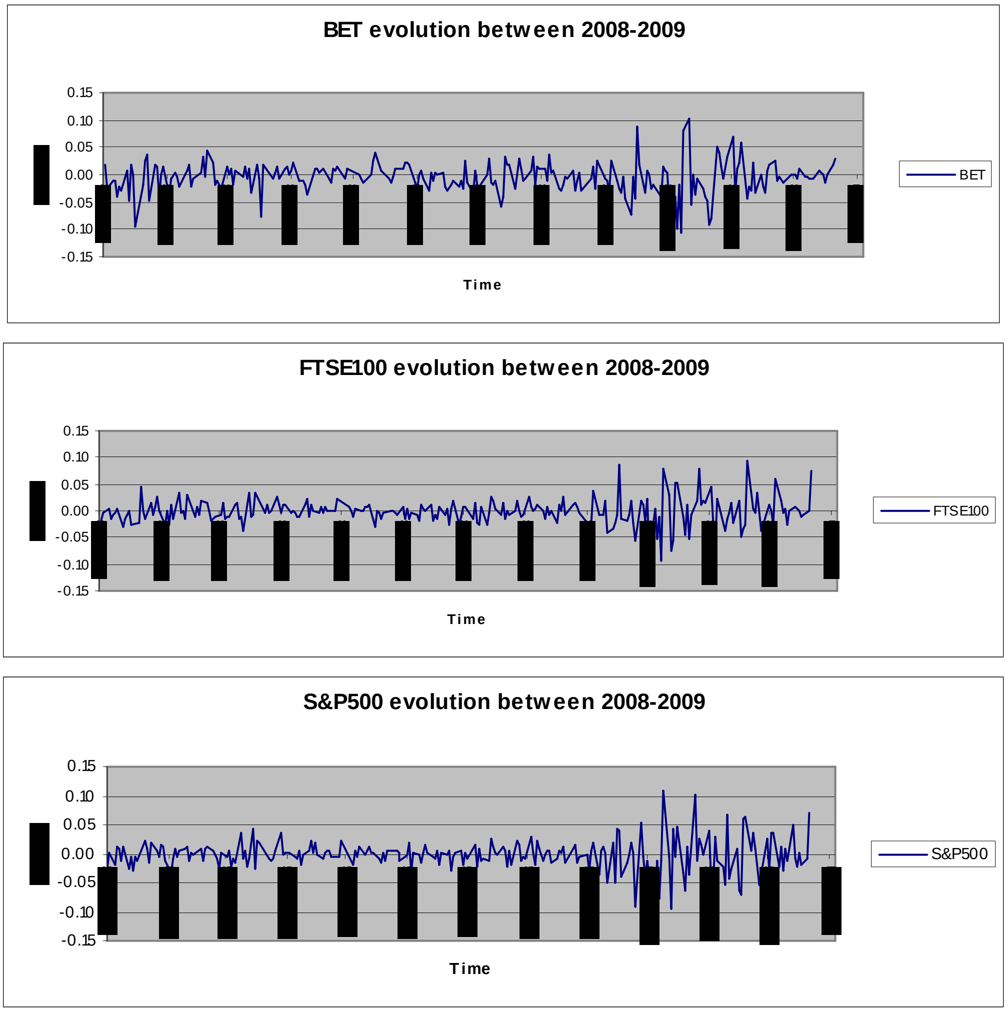

Figure 3

As it can be observed in September 2008 (20 September-12 October 2008 the speculative crisis in the US) represents the moment when the evolution of the return series starts to present considerable fluctuations. Moreover, the fall-winter of 2008 represents also the period when the crisis extended in Europe.

4. Conclusions

Studies of the transmission of return and volatility shocks from one market to another as well as studies of cross-market correlations are essential in finance, because they present numerous implications for capital allocation. As it was emphasized in this paper, at first sight some markets may seem slightly correlated, so international portfolio diversification can be in this case an optimal solution. However, on the background of a highly integrated global financial system, eroded strongly by the effects of the current crisis, international diversification does not reduce portfolio risk.

Using in this paper the ARCH and GARCH models we have been able to analyze the evolution of the risk of an internationally diversified portfolio, being chosen three benchmark indexes from three different countries, namely Romania (BET), UK (FTSE100) and USA (S&P500) to be included in the portfolio. We chose a 0.5 weight for the investment in the BET index, a 0.25 weight for the investment in the FTSE100 and a 0.25 weight for the investment in the S&P500. Starting from this portfolio we used the Engle (1982) test in order to track any ARCH effects in the portfolio return series. Taking into consideration that these effects were detected, we were able to estimate a GARCH(1,1) model. The estimation results showed that the volatility is persistent because the coefficient of the conditional variance is 0.9 and this means that the shocks to the conditional variance are persistent and that large changes in the conditional variance are followed by other large changes and small changes are followed by other small changes. On the basis of the estimated volatility equation we generated the historical series of the conditional volatility. The graphic of this series shows that there are more volatile periods than others, the most pronounced volatility being observed over the period 2008-2009. This evolution of the portfolio volatility is attributed to the current financial crisis, a fact confirmed by the graphics of the daily index returns over the period 2008-2009.

References

Aloui, R., „Global financial crisis, extreme interdependences, and contagion effects: The role of economic structure?”, *Jornal of Banking & Finance*, 2010, pp. 1-11

Bollerslev, T., „Generalized autoregressive conditional heterosceasticity”, *Journal of Econometrics*, 1986, pp. 307-327

Brooks, C., (2008). *Introductory Econometrics for Finance*, Cambridge University Press, New York, pp. 379-451

Claessens, C., Ayhan Kose, M., Terrones, M.E., „The global financial crisis: How similar? How different? How costly?”, *Journal of Asian Economics*, 21, 2010, pp. 247-264

Engle, R.F., „Autoregressive conditional heteroscedasticity with estimator of the variance of United Kindom inflation”, *Econometrica*, 1982, pp. 987-1008

Fama, E.F. (1976). *Foundations of Finance*, Basic Books, pp. 3-132, pp. 212-270

Gujarati, R.N. (2004). *Basic Econometrics*, Mc Graw-Hill, New York, pp. 856-867

Markowitz, H., „Portfolio Selection”, *The Journal of Finance*, 7(1), 1952, pp. 77-91

Moshirian, F., „The global financial crisis and the evolution of markets, institutions and regulation”, *Journal of Banking & Finance*, 2010, pp. 5-6

Rubinstein, M., „Markowitz’s Portfolio Selection: A Fifty-Year Retrospective”, *The Journal of Finance*, 57(3), 2002, pp. 1041-1045

Solnik, B., „Why do not diversify internationally rather than domestically”, *Financial Analyst Journal*, 1974, pp. 48-54

Stancu, S., Predescu, M.O., „The Choice of an Optimal Portfolio on the Romanian Capital Market under Uncertainty and Risk Terms, in the Actual Era of E-Business", *The Proceedings of the ninth international conference on informatics in economy*, Bucharest, Romanian, 2009, pp. 206-211

Stancu, S., Predescu, M.O., „Genetic Algorithm for the Portfolio Selection Problem on the Romanian Capital Market”, *Proceedings of International Conference on Engineering an Meta-Engineering*, Orlando, USA, 2010, pp. 57-60

*** http://www.bvb.ro
http://www.kmarket.ro
http://finance.yahoo.com