Progress of Theoretical Physics, Vol. 67, No. 5, May 1982

Position Operators for Extended Objects in Quantum Field Theory

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(Received December 14, 1981)

The operator structure of the collective coordinate associated with extended objects in quantum field theory is discussed in the context of renormalized perturbation theory.

§ 1. Introduction

In recent years, considerable effort has been devoted to the understanding of the structure of quantum field theories with extended objects.[1] A remarkable difference of such theories from the conventional field theory of homogeneous systems is the appearance of quantum mechanical degrees of freedom in addition to the usual particle-like modes.[2),3] These quantum mechanical modes describe the quantum mechanical motion of an extended object. Particularly important operators are the position operators which take care of the quantum fluctuation of the position of the extended object as well as the translational invariance of the theory.

The analysis of a quantum field theory with extended objects must take into account the presence of these position operators. Two methods have been proposed so far. One is the collective coordinate method,[4] in which the Heisenberg field operator and its canonical conjugate are decomposed into a new set of Heisenberg operators, namely the collective coordinate operator $X(t)$ and field operator $\chi(x, t)$ and their conjugates $p(t)$ and $\pi(x, t)$, respectively. This decomposition is accompanied by certain constraints which define $X(t)$ and $p(t)$. It is required that $X(t)\to X(t)+a$ induces the space translation of the Heisenberg operators and that $X(t)$ and $\chi(x, t)$ are independent as Heisenberg operators. The other method expresses the Heisenberg operators in terms of the physical operators or asymptotic fields which construct the physical Hilbert space.[3] This expression is called the dynamical map.

In this method it has been shown that the set of physical operators consists of two mutually commuting sets (q, p) and $(\alpha, \alpha^\dagger)$, where q is the quantum mechanical position-operator (quantum coordinate) and p is its canonical conjugate while α and $\alpha^\dagger$ stand for the annihilation and creation operators of particle-like modes respectively.[3,5] Thus, the Hilbert space is found to be a direct product of the Fock space of the particle-like modes and the quantum mechanical realization of (q, p). A remarkable fact is that, when P is the total momentum of the Heisenberg field and when it is expressed in terms of physical operators, the result is the simple relation $P=p$, implying that P commutes with α and $\alpha^\dagger$. In the dynamical map, q always appears in the combination $(x-q)$, and the translation of the Heisenberg operator is induced by $q \to q+a$. It was shown in Ref. 3) that it is always possible to find a representation where the quantum coordinate is given (in $(1+1)$-dimensions) by

$$q=\frac{1}{2}[H^{-1}, M]_+ ,$$

where M is the generator of Lorentz transformations and H is the Hamiltonian. This position operator is similar to the Newton-Wigner position operator[6] which represents the center of mass in configuration space of a quantum system.

The purpose of this paper is to make a comparison of the above two methods and to see the relation between the collective coordinate $X(t)$ and the quantum coordinate q by an explicit calculation of the dynamical map of $X(t)$. These two operators have a similarity in that they always appear in the combination $x-X(t) \sim x-q$. This however does not immediately imply that the dynamical map of $X(t)$ is q. Indeed, a central result of this paper is that, when the Heisenberg operators $\{X(t), p(t)\}$ and $\{\chi(x, t), \pi(x, t)\}$ are expressed in terms of physical operators, $X(t)$ contains not only (q, p) but also the annihilation and creation operators $(\alpha, \alpha^\dagger)$ of the particle-like modes, while $p(t)$ does not contain $(\alpha, \alpha^\dagger)$. This means that $X(t)$ does not commute with $(\alpha, \alpha^\dagger)$, and therefore that $X(t)$ cannot be used as an operator to separate the quantum mechanical part from the Hilbert space. The dependence of $X(t)$ on α and $\alpha^\dagger$ remains even when the quantum mechanical operators (q, p) are disregarded.

The computation of the dynamical map of the Heisenberg operators is quite complicated. The essential procedure is that the dynamical map of ψ is constructed in such a way that it is consistent with the Heisenberg equation and the equal time commutation relations in a renormalized perturbative expansion. The dynamical maps of the operators $\{X(t), p(t), \chi(x, t), \pi(x, t)\}$ of the collective coordinate method are then computed using $\psi(x, t)$ and the constraints which define $X(t)$ and $p(t)$. In this perturbative computation the lowest order term of $X(t)$ is found to be q. This fact makes $X(t)$ useful in perturbative calculations since one can identify the first order part of $X(t)$ as $q(t)$.

The following is the program of this paper. In the next section, the dynamical map of the Heisenberg operator in a one-component scalar field theory in $(1+1)$-dimensions is constructed. This dynamical map was already computed in the tree approximation in Ref. 5). In this paper that computation is extended to include one-loop corrections. At this stage the realization of the Heisenberg operators in the physical Hilbert space is shown to be consistent with the Heisenberg field equation and the equal time commutation relations in a renormalized perturbation expansion. In § 3 using the dynamical maps constructed in § 2 and the definitions of the Heisenberg operators of the collective coordinate method, the dynamical maps of $\{X(t), p(t), \chi(x, t), \pi(x, t)\}$ are constructed. Section 4 is devoted to the conclusion.

§ 2. The dynamical map

Consider a one-component scalar field in (1+1)-dimensions with field equation

$$(\partial^2+m^2)\psi(x)=F[\psi(x)] \tag{2·1}$$

and equal time canonical commutation relation

$$[\psi(x), \dot{\psi}(y)]\delta(x^0-y^0)=i\delta^2(x-y). \tag{2·2}$$

Consider a power counting parameter which is introduced by the substitution $\mathcal{L}[\psi(x)]\to\lambda^{-2}\mathcal{L}[\lambda\psi(x)]$ where $\mathcal{L}$ is the Lagrangian density which leads to Eq. (2·1). Then, the field equation becomes

$$(\partial^2+m^2)\psi(x)=\lambda^{-1}F[\lambda\psi(x)] \tag{2·3}$$

and the Heisenberg field, $\psi(x)$, can be expanded as

$$\psi(x)=\sum_{n=-1}^{\infty}\lambda^n\psi_n(x). \tag{2·4}$$

The quantity $\psi_{-1}(x)$ is a classical field satisfying the classical field equation

$$(\partial^2+m^2)\psi_{-1}(x)=F[\psi_{-1}(x)]. \tag{2·5}$$

In the following, the case where $\psi_{-1}(x)$ is a static topological soliton solution of Eq. (2·5) will be considered.

Substitution of Eq. (2·4) into (2·3) leads to

$$(\partial^2+m^2)\psi(x)=\sum\frac{\lambda^{l+\alpha_1+\cdots+\alpha_l-1}}{l!}F_l[\psi_{-1}(x)]\psi_{\alpha_1}(x)\cdots\psi_{\alpha_l}(x), \tag{2·6}$$

where

$$F_l[\psi]=\frac{\partial^l}{\partial\psi^l}F[\psi].$$

The quantity $F[\psi]$ must contain counterterms which are necessary for renormalization. These counterterms are of higher order in λ and are assumed to be the same as those for the quantum field theory of Eqs. (2·1) and (2·2) with no extended objects[7] (i.e., when $\psi_{-1}=0$). They contribute to $F_l[\psi_{-1}]$ by terms of higher order in λ,

$$F_l[\psi_{-1}(x)]=\sum_m \lambda^m F_l{}^m[\psi_{-1}(x)]. \tag{2·7}$$

Since the quantum corrections arise from the contractions of pairs of fields, m in Eq. (2·7) will always be even. Combining Eqs. (2·6) and (2·7) leads to

$$(\partial^2+m^2)\psi_n(x)=\sum \frac{1}{l!}F_l{}^m[\psi_{-1}(x)]\psi_{\alpha_1}(x)\cdots\psi_{\alpha_l}(x), \tag{2·8}$$

where $l+m+\alpha_1+\cdots+\alpha_l=n+1$; $\alpha_1, \cdots, \alpha_l\geq 0$. The first few orders of Eq. (2·8) are

$$\{\partial^2+m^2-F_1{}^0[\psi_{-1}(x)]\}\psi_0(x)=0\,, \tag{2·9}$$

$$\{\partial^2+m^2-F_1{}^0[\psi_{-1}(x)]\}\psi_1(x)=F_0{}^2[\psi_{-1}(x)]+F_2{}^0[\psi_{-1}(x)]\psi_0{}^2(x)/2!\,, \tag{2·10}$$

$$\{\partial^2+m^2-F_1{}^0[\psi_{-1}(x)]\}\psi_2(x)=F_1{}^2[\psi_{-1}(x)]\psi_0(x)+F_3{}^0[\psi_{-1}(x)]\psi_0{}^3(x)/3!$$
$$+F_2{}^0[\psi_{-1}(x)]\frac{1}{2}[\psi_0(x), \psi_1(x)]_+\,. \tag{2·11}$$

In Ref. 5) it was shown that the solution of Eq. (2·9) must be taken as

$$\psi_0(x)=-\left(q+\frac{p}{M}t\right)\psi'_{-1}+\tilde{\psi}_0(x), \tag{2·12}$$

where q is the quantum coordinate, p is the total momentum of the system and $\tilde{\psi}_0(x)$ represents the particle-like excitations of the system,

$$\tilde{\psi}_0(x)=\sum_i \frac{1}{\sqrt{2\omega_i}}(u_i(x)e^{-i\omega_i t}\alpha_i+u_i{}^*(x)e^{i\omega_i t}\alpha_i{}^\dagger)$$
$$+\int\frac{dk}{\sqrt{4\pi\omega_k}}(u_k(x)e^{-i\omega_k t}\alpha_k+u_k{}^*(x)e^{i\omega_k t}\alpha_k{}^\dagger), \tag{2·13}$$

where $u_i(x)$ and $u_k(x)$ are the bound and scattering state wavefunctions respectively. The physical operators $\{q, p, \tilde{\psi}_0(x), \dot{\tilde{\psi}}_0(x)\}$ obey the algebra

$$[q, p]=i\,, \tag{2·14}$$

$$[\tilde{\psi}_0(x), \dot{\tilde{\psi}}_0(y)]_{x^0=y^0}=i\left\{\delta(x-y)-\frac{1}{M}\psi'_{-1}(x)\psi'_{-1}(y)\right\}=i\,\mathcal{P}(x, y) \tag{2·15}$$

with all other combinations commuting and

$$\int dx\psi'_{-1}(x)\tilde{\psi}_0(x)=0\,, \tag{2·16}$$

$$M=\int dx\psi'_{-1}(x)\psi'_{-1}(x). \tag{2·17}$$

The primes in the above equations denote space derivatives and the dots denote time derivatives. In the following, $\tilde{\psi}(x)$ denotes the field $\psi(x)$ when q and p are disregarded.

The computational technique can be summarized in the following steps:

(i) Having computed the dynamical map to some given order, use Eq. (2·8) to compute the dynamical map to the next order.

(ii) At each order, add solutions of the homogeneous equation, $\{\partial^2+m^2-F_1{}^0[\psi_{-1}(x)]\}\psi_n=0$, until the following relation is satisfied:

$$-\frac{\partial}{\partial q}\psi_n(x)=\frac{\partial}{\partial x}\psi_{n-1}(x). \tag{2·18}$$

This condition guarantees that x and q always appear in the combination $x-q$ and therefore that q is the quantum coordinate for position.

(iii) Add q-independent solutions of the homogeneous equation until the total canonical momentum calculated using the equation

$$P_{n+1}=-\frac{1}{2}\int dx\ \sum_k[\dot{\psi}_k(x)\psi'_{n-k}(x)+\psi'_{n-k}(x)\dot{\psi}_k(x)] \tag{2·19}$$

satisfies

$$P_0=p\,, \qquad P_{n>0}=0\,.$$

A straightforward calculation following the above scheme and using Eqs. (2·9)~(2·12) leads to the following dynamical map:

$$\begin{aligned}
\psi(x)=&\Big\{1-\Big(q+\frac{p}{M}t-\frac{1}{4}[(x-q),\,p^2/M^2]_+-\frac{p\tilde{\mathcal{H}}_0}{M^2}t+\cdots\Big)\frac{\partial}{\partial x}\\
&+\frac{1}{2!}\Big(\Big(q+\frac{p}{M}t\Big)^2-\Big[q+\frac{p}{M}t,\,p^2/2M^2\Big]_+x+\cdots\Big)\frac{\partial^2}{\partial x^2}\\
&-\frac{1}{3!}\Big(\Big(q+\frac{p}{M}t\Big)^3+\cdots\Big)\frac{\partial^3}{\partial x^3}+\cdots\Big)\psi_{-1}(x)\\
&+\Big\{1-\Big(q+\frac{p}{M}t-\frac{1}{2}\frac{p^2}{M^2}x+\cdots\Big)\frac{\partial}{\partial x}\\
&-\Big(\frac{1}{2}\Big[\frac{p}{M},\,x-q\Big]_+-\frac{p^2}{2M^2}t+\cdots\Big)\frac{\partial}{\partial t}\\
&+\frac{1}{2!}\Big(\Big(q+\frac{p}{M}t\Big)+\cdots\Big)^2\frac{\partial^2}{\partial x^2}+\frac{1}{2!}\Big(\frac{p}{M}x+\cdots\Big)^2\frac{\partial^2}{\partial t^2}\\
&+\Big(\frac{1}{2}\Big[\frac{p}{M},\,q+\frac{p}{M}t\Big]_+x+\cdots\Big)\frac{\partial^2}{\partial x\partial t}+\cdots\Big\}\tilde{\psi}_0(x)\\
&+\Big\{1-\Big(\Big(q+\frac{p}{M}t+\cdots\Big)\frac{\partial}{\partial x}-\Big(\frac{p}{M}x+\cdots\Big)\frac{\partial}{\partial t}+\cdots\Big\}\tilde{\psi}_1(x)
\end{aligned}$$

$$+\{1+\cdots\}\tilde{\psi}_2(x)+\cdots, \tag{2·20}$$

where

$$\begin{aligned}\tilde{\psi}_1(x)=&\alpha_0(t)\psi'_{-1}(x)\\&+\int d^2y\tilde{g}(x,y)\{F_0{}^2[\psi_{-1}(y)]+F_2{}^0[\psi_{-1}(y)]\tilde{\psi}_0{}^2(y)/2!\},\end{aligned} \tag{2·21}$$

$$\begin{aligned}\tilde{\psi}_2(x)=&\alpha_1(t)\psi'_{-1}(x)\\&+\int d^2y\tilde{g}(x,y)\{F_1{}^2[\psi_{-1}(y)]\tilde{\psi}_0(y)+F_3{}^0[\psi_{-1}(y)]\tilde{\psi}_0{}^3(y)/3!\\&+F_2{}^0[\psi_{-1}(y)]\frac{1}{2}[\tilde{\psi}_0(y),\tilde{\psi}_1(y)]_+\},\end{aligned} \tag{2·22}$$

$$\alpha_0(t)=\frac{1}{M}\int dx\ x\left\{\frac{1}{2}(\dot{\tilde{\psi}}_0{}^2(x)-\tilde{\psi}_0(x)\ddot{\tilde{\psi}}_0(x))+V_0{}^2[\psi_{-1}(x)]\right\}, \tag{2·23}$$

$$\begin{aligned}\alpha_1(t)=\frac{1}{2M}\int dx\Big\{x\Big(&[\dot{\tilde{\psi}}_1(x),\dot{\tilde{\psi}}_0(x)]_+-\frac{1}{3}[\ddot{\tilde{\psi}}_1(x),\tilde{\psi}_0(x)]_+\\&-\frac{2}{3}[\tilde{\psi}_1(x),\ddot{\tilde{\psi}}_0(x)]_+-\frac{4}{3}F_0{}^2[\psi_{-1}(x)]\tilde{\psi}_1(x)\Big)-\frac{1}{3}[\tilde{\psi}_1(x),\tilde{\psi}_0{}'(x)]_+\Big\},\end{aligned} \tag{2·24}$$

$$\{\partial^2+m^2-F_1{}^0[\psi_{-1}(x)]\}\tilde{g}(x,y)=\delta(x^0-y^0)\mathscr{P}(x,y) \tag{2·25}$$

and

$$\tilde{\mathscr{H}}_0=\int dx\left\{\frac{1}{2}(\dot{\tilde{\psi}}_0{}^2(x)-\tilde{\psi}_0(x)\ddot{\tilde{\psi}}_0(x))+V_0{}^2[\psi_{-1}(x)]\right\}. \tag{2·26}$$

A detailed account of the calculational method used to arrive at Eq. (2·20) can be found in Ref. 5). The solution of the zero mode problem consists of the computation of the quantities $\alpha_0(t)$ and $\alpha_1(t)$ and is also outlined in Ref. 5). It can be shown that, given equations (2·14) and (2·15), the commutation relation (2·2) is satisfied by the above solution. In Eqs. (2·23) and (2·26), $V_0{}^2[\psi_{-1}(x)]$ is the one-loop counterterm which occurs in the Hamiltonian density,

$$\frac{\partial}{\partial\psi_{-1}(x)}V_0{}^2[\psi_{-1}(x)]=-F_0{}^2[\psi_{-1}(x)]. \tag{2·27}$$

To the order considered, Eq. (2·20) contains the one-loop counterterms. When the operator products are normal ordered, the contractions combined with the counterterms constitute the one-loop corrections to the soliton solution, the physical particle wavefunction and the physical particle energy.[8)]

From Eq. (2·20) the generalized coordinates are

$$X=x-\left(q+\frac{p}{M}t\right)+\frac{1}{4}[(x-q),\,p^2/M^2]_++\frac{p\tilde{\mathcal{H}}_0}{M^2}t+\cdots\,, \tag{2·28}$$

$$T=t-\frac{1}{2}\left[\frac{p}{M},\,(x-q)\right]_++\frac{p^2}{2M^2}t+\cdots\,. \tag{2·29}$$

The spatial generalized coordinate contains the renormalized Hamiltonian, $\tilde{\mathcal{H}}_0$, of the physical particles and therefore ${}_F\langle 0|X|0\rangle_F$ is a finite renormalized quantity. This result explicitly confirms the general expression for the generalized coordinate given in Ref. 3).

In the next section the collective coordinate will be examined.

§ 3. The collective coordinate

The collective coordinate, $X(t)$, and its conjugate momentum, $p(t)$, are defined by the ansatz

$$\phi(x)=\phi_{-1}(x-X(t))+\chi(x-X(t),\,t), \tag{3·1}$$

$$\begin{aligned}\dot{\phi}(x)=\Pi_0(x)=\pi(x-X(t),\,t)&\\ -\frac{1}{2M}\Big[\phi'_{-1}(x-X(t))\frac{1}{1+\xi/M}&(p(t)+\int dy\chi'(y)\pi(y))\\ +(p(t)+\int dy\pi(y)\chi'(y))&\frac{1}{1+\xi/M}\phi'_{-1}(x-X(t))\Big]\end{aligned} \tag{3·2}$$

with the constraints

$$\int dx\phi'_{-1}(x)\chi(x)=\int dx\phi'_{-1}(x)\pi(x)=0\,, \tag{3·3}$$

where

$$\xi=\int dx\phi'_{-1}(x)\chi'(x) \tag{3·4}$$

and M is defined in Eq. (2·17).

It can be shown[9] that the algebra

$$[\chi(x),\,\pi(y)]_{x^0=y^0}=i\,\mathcal{P}(x,\,y), \tag{3·5}$$

$$[X(t),\,p(t)]=i\,, \tag{3·6}$$

$$[X(t),\,\chi(t)]_{t=x^0}=[X(t),\,\pi(t)]_{t=x^0}=[p(t),\,\chi(x)]_{t=x^0}=[p(t),\,\pi(x)]_{t=x^0}=0\,, \tag{3·7}$$

together with Eq. (3·1)~(3·3) lead to the equal time commutation relation (2·2).

Equations (3·1)~(3·3) can be used, together with Eqs. (2·20) to write the dynamical maps of the operator set $\{X(t), p(t), \chi(x), \pi(x)\}$. A straightforward calculation using Eqs. (3·1)~(3·3) and (2·20) leads to

$$p(t)=p\,, \tag{3·8}$$

$$\begin{aligned}
X(t)=&\,q+\frac{p}{M}t-\frac{p^2a}{2M^2}+\frac{p}{M^2}\int dx\; x\psi'_{-1}(x)\dot{\tilde{\psi}}_0(x)-\alpha_0(t)-\frac{1}{2}\frac{p^3}{M^3}t-\frac{p\tilde{\mathscr{H}}_0}{M^2}t\\
&-\frac{p^2}{2M^3}\int dx\,[(x-a)\psi'_{-1}(x)\tilde{\psi}_0'(x)+x^2\psi'_{-1}(x)\ddot{\tilde{\psi}}_0(x)]\\
&+\frac{p}{M^2}\int dx\; x\psi'_{-1}(x)\dot{\tilde{\psi}}_1(x)\\
&+\frac{1}{2}\frac{p}{M^3}\Big[\int dx\; x\psi'_{-1}(x)\dot{\tilde{\psi}}_0(x),\int dy\psi''_{-1}(y)\tilde{\psi}_0(y)\Big]_+\\
&-\frac{1}{2M}\Big[\alpha_0(t),\int dx\psi''_{-1}(x)\tilde{\psi}_0(x)\Big]_+-\alpha_1(t)+\cdots,
\end{aligned}\tag{3·9}$$

$$\begin{aligned}
\chi(x)=&\,\tilde{\psi}_0(x)+\frac{p^2}{2M^2}(x-a)\psi'_{-1}(x)-\frac{p}{M}\int dy\,\mathscr{P}(x,y)y\dot{\tilde{\psi}}_0(y)+\tilde{\tilde{\psi}}_1(x)\\
&+\frac{p^2}{2M^2}\int dy\,\mathscr{P}(x,y)[(y-a)\tilde{\psi}_0'(y)+y^2\ddot{\tilde{\psi}}_0(y)-t\dot{\tilde{\psi}}_0(y)]\\
&-\frac{p}{M}\int dy\,\mathscr{P}(x,y)y\dot{\tilde{\psi}}_1(y)\\
&+\frac{p}{2M^2}\Big[\int dy\,\mathscr{P}(x,y)\tilde{\psi}_0'(y),\int dz\; z\psi'_{-1}(z)\dot{\tilde{\psi}}_0(z)\Big]_+\\
&-\frac{1}{2}\Big[\int dy\,\mathscr{P}(x,y)\tilde{\psi}_0'(y),\alpha_0(t)\Big]_++\tilde{\tilde{\psi}}_2(x)+\cdots,
\end{aligned}\tag{3·10}$$

$$\begin{aligned}
\pi(x)=&\,\dot{\tilde{\psi}}_0(x)-\frac{p}{M}\int dy\,\mathscr{P}(x,y)[\tilde{\psi}_0'(y)+y\ddot{\tilde{\psi}}_0(y)]+\dot{\tilde{\tilde{\psi}}}_1(y)\\
&-\frac{p^3}{2M^3}\Big[(x-a)\psi''_{-1}(x)+\frac{1}{2}\psi'_{-1}(x)\Big]\\
&+\frac{p^2}{2M^2}\int dy\,\mathscr{P}(x,y)[(3y-a)\dot{\tilde{\psi}}_0'(y)+\dot{\tilde{\psi}}_0(y)+y^2\dddot{\tilde{\psi}}_0(y)-t\ddot{\tilde{\psi}}_0(y)]\\
&-\frac{p^2}{M^3}\psi''_{-1}(x)\int dy\; y\psi'_{-1}(y)\dot{\tilde{\psi}}_0(y)+\frac{p}{M^2}\int dy\; y\psi'_{-1}(y)\dot{\tilde{\psi}}_0(y)\int dz\,\mathscr{P}(x,z)\dot{\tilde{\psi}}_0'(z)\\
&-\frac{p}{M}\int dy\,\mathscr{P}(x,y)[\tilde{\tilde{\psi}}_1'(y)+y\ddot{\tilde{\psi}}_1(y)]\\
&-\frac{1}{2}\int dy\,\mathscr{P}(x,y)[\alpha_0(t),\dot{\tilde{\psi}}_0'(y)]_++\dot{\tilde{\tilde{\psi}}}_2(x)+\cdots,
\end{aligned}\tag{3·11}$$

where

$$a=\frac{1}{M}\int dx\ x\psi'_{-1}(x)\psi'_{-1}(x) \tag{3·12}$$

and

$$\tilde{\tilde{\psi}}_n(x)=\int dy\ \mathscr{P}(x,y)\tilde{\psi}_n(y). \tag{3·13}$$

Equation (3·8) is in fact exact.

It can be verified by direct computation that the commutation relations (2·14) and (2·15) lead, through Eq. (3·8)~(3·11) to the algebra of Eq. (3·5)~(3·7).

An interesting feature of the collective coordinate, $X(t)$, is that it has a complicated time dependence, quite different from the quantum coordinate

$$Q(t)=e^{iHt}qe^{-iHt}\ , \tag{3·14}$$

which satisfies

$$\ddot{Q}(t)=0\ .$$

The Hamiltonian, H, is given by $\sqrt{p^2+(M+H_0)^2}$. The complicated time dependence of $X(t)$ arises from the mixture of quantum mechanical operators and particle-like fields it must contain in order that the condition (3·3) is satisfied.

In fact, $X(t)$ contains parts which are purely particle-like. Even when q and p are disregarded, $X(t)$ has the form

$$\tilde{X}(t)=-\alpha_0(t)-\alpha_1(t)+\cdots\ , \tag{3·15}$$

where $\alpha_0(t)$ and $\alpha_1(t)$ are defined in Eqs. (2·23) and (2·24). These parts are time dependent and therefore cannot be removed by a time-independent canonical transformation.

In light of this property of $X(t)$, great care must be taken in the computation of explicitly space-dependent renormalized quantities in the collective coordinate formalism, as

$$\langle 0_F|X(t)|0_F\rangle \neq X(t) \tag{3·16}$$

and

$$\langle 0_F|X(t)\cdots|0_F\rangle$$

will contain contractions between $X(t)$ and the other operators in the bracket as well as those within $X(t)$.

§ 4. Discussion

The relationship between the quantum coordinate and the collective coordinate has been discussed in detail from the point of view of renormalized perturbation theory. The quantum coordinate is a position operator which is related to the center of mass of a quantum system. The quantum coordinate and the total momentum form a quantum mechanical set of operators which are independent of the physical particle fields. This means that the position of an extended object can be chosen without interference from the quanta at one instant. The time evolution of the position is then given by

$$Q(t)=q+\dot{Q}t\,, \tag{4·1}$$

where

$$\dot{Q}=i[H,q]=pH^{-1} \tag{4·2}$$

and

$$\ddot{Q}=0\,. \tag{4·3}$$

The collective coordinate is, on the other hand, a combination of the quantum coordinate and the physical particle fields as seen in Eq. (3·9). It has a complicated time dependence due to the presence of the physical fields. Even though $X(t)$ and $\chi(x)$ commute at equal times, $X(t)$ does not commute with the physical field operators.

Acknowledgements

This work was supported by the Natural Sciences and Engineering Research Council, Canada and the Faculty of Science, University of Alberta.

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