Positional Analysis in Fuzzy Social Networks

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Abstract— **Social network analysis is a methodology used extensively in social and behavioral sciences, as well as in political science, economics, organization theory, and industrial engineering. Positional analysis of a social network aims to find similarities between actors in the network. One of the the most studied notions in the positional analysis of social networks is *regular equivalence*. According to Borgatti and Everett, two actors are regularly equivalent if they are equally related to equivalent others. In recent years, fuzzy social networks have also received considerable attention because they can represent both the qualitative relationship and the degrees of interaction between actors. In this paper, we generalize the notion of regular equivalence to fuzzy social networks based on two alternative definitions of regular equivalence. While these two definitions are equivalent for social networks, they induce different generalizations for fuzzy social networks. The first generalization, called *regular similarity*, is based on the characterization of regular equivalence as an equivalence relation that commutes with the underlying social relations. The regular similarity is then a fuzzy binary relation that specifies the degree of similarity between actors in the social network. The second generalization, called *generalized regular equivalence*, is based on the definition of role assignment or coloring. A role assignment (resp. coloring) is a mapping from the set of actors to a set of roles (resp. colors). The mapping is regular if actors assigned to the same role have the same roles in their neighborhoods. Consequently, generalized regular equivalence is an equivalence relation that can determine the role partition of the actors in a fuzzy social network.**

I. Introduction

Granular computing (GrC) is a novel problem-solving concept deeply rooted in human thinking. Many objects can be granulated into "sub-objects". For example, the human body can be granulated into the head, the neck, and so forth; while geographic features can be granulated into mountains, plains, etc. Although the notion of granulation is essentially fuzzy, vague, and imprecise, mathematicians have idealized it into partitions (equivalence relations) and developed a fundamental problem-solving methodology based on it. The notion has played a major role in solving many important problems throughout the history of mathematics. In recent years, rough set theory [1], [2] has introduced the idea to computer science, where it has been successfully applied to data analysis and uncertainty management. Nevertheless, the notion of partitions, which does not permit any overlap among granules (equivalence classes), is too restrictive for real-world applications. Even in the natural sciences, classifications permit a small degree of overlap. For example, there are creatures that are the proper subjects of both zoology and botany. A more general theory is thus needed. GrC is a new, rapidly emerging paradigm designed to meet this need [3]–[11].

Currently, there are no widely-accepted formal definitions of GrC. However, informally, any computing theory/technology that processes elements and granules (subsets) of the universe of discourse may be regarded as GrC. Mathematically, rough set theory has two perspectives: algebraically, it is a theory of equivalence relations; and geometrically, it is a theory of topological spaces (approximations).

In rough set theory, objects are partitioned into equivalence classes based on their attribute values, which essentially represent functional information associated with the objects. A natural generalization considers granulation defined by the relational information between objects. Such information is defined by general binary relations, which are extensions of the functional attributes of the objects. Geometrically, such granulation is derived from the neighborhood system of topological spaces [12], where each point/object is assigned at most one neighborhood/granule. This kind of granulation is called *relational granulation*, while granulation based on attribute values only is called *functional granulation* [13], [14].

Interestingly, social scientists have applied the same techniques of relational granulation (albeit by different names) to positional analysis in social networks [15]–[19]. Social network analysis (SNA) is a methodology used extensively in social and behavioral sciences, as well as in political science, economics, organization theory, and industrial engineering [20]–[22]. Positional analysis of a social network tries to find similarities between actors in the network. While many traditional clustering methods are based on the attributes of the individual actors, SNA is more concerned with the structural similarity between the actors. In SNA, a category, called a *social role* or *social position*, is defined in terms of the similarities of the patterns of relations among the actors, rather than the attributes of the actors. For example, one useful way to think about the social role "husband" is to consider it as a set of patterned interactions with a member or members of some other social categories: "wife" and "child" (and probably others) [20]. One of the the most studied notions in the positional analysis of social networks is *regular equivalence* [15], [23]–[25]. According to Borgatti and Everett [15], two actors are regularly equivalent if they are equally related to equivalent others.

In recent years, fuzzy social networks have also received considerable attention because they can represent both the qualitative relationship and the degrees of interaction between actors [26]. In this paper, we generalize the notion of regular equivalence to fuzzy social networks based on two alternative definitions of regular equivalence. While these two definitions are equivalent for social networks, they induce different generalizations for fuzzy social networks. The first generalization, called *regular similarity*, is based on the characterization of regular equivalence as an equivalence relation that commutes with the underlying social relations [16]. The regular similarity is then a fuzzy binary relation that describes the degree of similarity between actors in the social network. The second generalization, called *generalized regular equivalence*, is based on the definition of role assignment or coloring [18]. A role assignment (resp. coloring) is a mapping from the set of actors to a set of roles (resp. colors). The mapping is regular if actors assigned to the same role have the same roles in their neighborhoods. Consequently, generalized regular equivalence is an equivalence relation that can determine the role partition of actors in a fuzzy social network.

The remainder of this paper is organized as follows. In Section II, we review some basic concepts about social networks and fuzzy relations. In Sections III and IV, we present the definitions of regular similarity and generalized regular equivalence respectively. We also discuss the computational process based on the definitions. Finally, in Section V, we present our conclusions and indicate some future research directions.

II. Social Networks and Fuzzy Relations

A. Social networks

Social networks are defined by actors and relations (or nodes and edges in terms of graph theory) [20]. A social network is generally defined as a relational structure $\mathfrak{N} = (A, (\alpha_i)_{i\in I})$, where A is the set of actors in the network, I is an index set, and for each $i \in I$, $\alpha_i \subseteq A^{k_i}$ is a k_i-ary relation on the domain A, where k_i is a positive integer. If $k_i = 1$, then α_i is also called an attribute. In practice, most SNA literature considers a simplified version of social networks with only binary relations. For ease of presentation, we focus on a social network with only one binary relation. Thus, the social network considered in this paper is a structure $\mathfrak{N} = (A, \alpha)$, where A is a *finite set* of actors and α is a binary relation on A. In terms of graph theory, $\mathfrak{N}$ is a directed graph, where A is the set of nodes and α denotes the set of (directed) edges. For each $a \in A$, the out-neighborhood and in-neighborhood of a, denoted respectively by $N^+_\alpha(a)$ and $N^-_\alpha(a)$, are defined as follows:

$$N^+_\alpha(a) = \{b \in A \mid (a, b) \in \alpha\},$$

$$N^-_\alpha(a) = \{b \in A \mid (b, a) \in \alpha\}.$$

A binary relation ρ on A is called an equivalence relation if it satisfies the conditions of reflexivity ($\forall a \in A, (a, a) \in \rho$), symmetry ($\forall a, b \in A, (a, b) \in \rho \Rightarrow (b, a) \in \rho$), and transitivity ($\forall a, b, c \in A, (a, b) \in \rho \wedge (b, c) \in \rho \Rightarrow (a, c) \in \rho$). Given an equivalence relation ρ on A and an actor $a \in A$, the ρ-equivalence class of a is defined as $[a]_\rho = N^+_\rho(a) = N^-_\rho(a)$. Note that the latter equality holds because of the symmetry of ρ. If $(a, b) \in \rho$, then a and b have the same equivalence class. For any $B \subseteq A$, we denote $[B]_\rho$ by the set $\{[a]_\rho \mid a \in B\}$.

Several equivalence relations have been proposed for exploring the role similarity between actors. Among them, regular equivalence has been extensively studied [15]–[19]. There are several alternative definitions of regular equivalence. We consider two of them in this paper. The first is based on the characterization given by Boyd and Everett [16], which states that an equivalence relation ρ is a *regular equivalence* with respect to a binary relation α if it commutes with α, i.e.

$$\alpha\rho = \rho\alpha,$$

where $\alpha\rho = \{(a, b) \mid \exists c \in A, (a, c) \in \alpha \wedge (c, b) \in \rho\}$ is the composition of α and ρ. By this definition, if ρ is a regular equivalence with respect to α and $(a, b) \in \rho$, then for each $c \in N^+_\alpha(a)$(resp. $N^-_\alpha(a)$), there exists $c' \in N^+_\alpha(b)$(resp. $N^-_\alpha(b)$) such that $(c, c') \in \rho$. The property naturally leads to an alternative definition of regular equivalence [18], which states that an equivalence relation ρ is a *regular equivalence* with respect to a binary relation α if for $a, b \in A$,

$$(a, b) \in \rho \Rightarrow [N^+_\alpha(a)]_\rho = [N^+_\alpha(b)]_\rho \text{ and } [N^-_\alpha(a)]_\rho = [N^-_\alpha(b)]_\rho.$$

According to this definition, if a and b are regularly equivalent, then they are connected to equivalent neighborhoods. Obviously, the above definitions are equivalent. However, the situation is quite different when we consider fuzzy social networks. In this paper, a fuzzy social network is defined as a structure $\mathfrak{F} = (A, \alpha)$, where α is a binary fuzzy relation on A, which is defined below.

B. Fuzzy relations

It is well-known that a binary relation α on A can be represented as its characteristic function (adjacency matrix) $\mu_\alpha : A \times A \to \{0, 1\}$. A binary fuzzy relation α on A can thus be characterized by its membership function $\mu_\alpha : A \times A \to [0, 1]$. Obviously, a binary fuzzy relation is a generalization of a binary relation, so the lower-case Greek letters $\alpha, \beta, \rho, \lambda$, etc., are used to denote both fuzzy and crisp relations. Since we only consider binary fuzzy relations in this paper, we call them fuzzy relations hereafter, and the term "binary relation" means crisp relations only. A fuzzy relation α is included in another fuzzy relation β, denoted by $\alpha \subseteq \beta$, if $\mu_\alpha(a, b) \leq \mu_\beta(a, b)$ for all $a, b \in A$. Several basic operations for binary relations [16] can be easily generalized to fuzzy relations.

Definition 1: Given fuzzy relations α and β on A, the following fuzzy relations can be derived:

1) the identity relation ι:

$$\mu_\iota(a, b) = \begin{cases} 1, & \text{if } a = b, \\ 0, & \text{otherwise;} \end{cases}$$

2) the converse of α, α^-:

$$\mu_{\alpha^-}(a, b) = \mu_\alpha(b, a);$$

3) the composition of α and β, $\alpha\beta$:

$$\mu_{\alpha\beta}(a,b) = \sup_{c\in A} \min(\mu_\alpha(a,c), \mu_\beta(c,b));$$

4) the union of α and β, $\alpha \cup \beta$:

$$\mu_{\alpha\cup\beta}(a,b) = \max(\mu_\alpha(a,b), \mu_\beta(a,b));$$

5) the intersection of α and β, $\alpha \cap \beta$:

$$\mu_{\alpha\cap\beta}(a,b) = \min(\mu_\alpha(a,b), \mu_\beta(a,b));$$

6) the right residual of α by β, α/β: the largest fuzzy relation λ such that $\beta\lambda \subseteq \alpha$;
7) the left residual of α by β, $\alpha\backslash\beta$: the largest fuzzy relation λ such that $\lambda\beta \subseteq \alpha$;
8) the symmetric interior of α, α^s: $\alpha^s = \alpha \cap \alpha^-$.

The composition of α with itself k times is denoted by α^k and the transitive closure of α is defined as $\alpha^\infty = \bigcup_{k\geq 1} \alpha^k$. The equivalence relation is generalized to a similarity relation in the fuzzy case.

Definition 2: A fuzzy relation ρ is called a similarity relation if it satisfies:

- reflexivity: $\iota \subseteq \rho$,
- symmetry: $\rho = \rho^-$, and
- (sup-min) transitivity: $\rho^2 \subseteq \rho$.

Intuitively, if ρ is a similarity relation, then $\rho(a,b)$ specifies the degree of similarity between a and b. As in the case of equivalence relations, the set of all similarity relations on a domain A form a lattice. The meet and join of two similarity relations α and β in the lattice are defined as $\alpha \sqcap \beta = \alpha \cap \beta$ and $\alpha \sqcup \beta = (\alpha \cup \beta)^\infty$ respectively.

Given any $S \subseteq A \times A$ and fuzzy relation α on A, the α-*membership image* of S is $\mu_\alpha(S) = \{\mu_\alpha(a,b) \mid (a,b) \in S\}$. Note that $|\mu_\alpha(S)| \leq |S|$, where $|\cdot|$ denotes the cardinality of a set. In particular, $\mu_\alpha(A \times A)$ is a finite subset of [0,1], since A is finite. The following lemma shows that the range of the membership function of a compound fuzzy relation only comprises membership values occurring in the components' fuzzy relations.

Lemma 1: Let α and β be two fuzzy relations on a finite set A, * denote the converse or symmetric interior, $\otimes$ denote composition, union, or intersection, and $|$ denote the right residual or left residual. Then,

1) $\mu_{\alpha^*}(A \times A) \subseteq \mu_\alpha(A \times A)$,
2) $\mu_{\alpha\otimes\beta}(A \times A) \subseteq \mu_\alpha(A \times A) \cup \mu_\beta(A \times A)$,
3) $\mu_{\alpha|\beta}(A \times A) \subseteq \mu_\alpha(A \times A) \cup \{1\}$.

From this lemma, we can derive the following corollary straightforwardly.

Corollary 1: Let φ be any relational expression composed from a set of fuzzy relations $Rel(\varphi)$ by the operations introduced in Definition 1. Then, $\mu_\varphi(A \times A) \subseteq \bigcup_{\alpha\in Rel(\varphi)} \mu_\alpha(A \times A) \cup \{1\}$.

III. Regular Similarity

Just as regular equivalence determines a role partition based on social network data, we can induce a kind of structural similarity between actors from fuzzy social network data. Such similarity is modeled by regular similarity. Formally, a similarity relation ρ is called a *regular similarity* with respect to a fuzzy relation α if it commutes with α, i.e., $\alpha\rho = \rho\alpha$.

As in the case of regular equivalence, regular similarities are closed with respect to the usual join of similarity relations. The closure property makes it possible to define the regular interior of any similarity relation. Let π be any similarity relation. Then, the *(similarity-based) regular interior* of π (with respect to a fuzzy relation α), denoted by π^o, is defined as the join of all regular similarities (with respect to α) included in π, i.e., $\pi^o = \bigsqcup\{\rho \mid \rho \subseteq \pi, \rho$ is a regular similarity (with respect to $\alpha)\}$.

Several basic properties of regular equivalences also hold for regular similarities. These properties, which we summarize in the following lemma, are useful in the computational characterization of the regular interior operator.

Lemma 2:

1) Let α be any fuzzy relation and ρ be a fuzzy relation satisfying reflexivity and transitivity. Then,

$$(\rho\alpha)/\alpha = (\rho\alpha)/(\rho\alpha) \text{ and } (\alpha\rho)\backslash\alpha = (\alpha\rho)\backslash(\alpha\rho).$$

2) A similarity relation ρ is a regular similarity with respect to a fuzzy relation α iff $\rho\alpha\rho \subseteq \alpha\rho$ and $\rho\alpha\rho \subseteq \rho\alpha$.
3) If ρ is a regular similarity with respect to a fuzzy relation α, then $\rho \subseteq ((\alpha\rho)\backslash\alpha) \cap ((\rho\alpha)/\alpha)$.

The next theorem shows that the regular interior can be computed iteratively. It is analogous to Theorem 11 in [16].

Theorem 1: Let α be a fuzzy relation and π be a similarity relation, both on a finite set A. Then, the regular interior of π is equal to

$$\pi^o = \bigcap_{i\geq 0} \pi_i,$$

where $\pi_0 = \pi$ and

$$\pi_{i+1} = [((\alpha\pi_i)\backslash\alpha) \cap ((\pi_i\alpha)/\alpha) \cap \pi_i]^s.$$

Recall that $\cdot^s$ is the symmetric interior of a fuzzy relation.

By this theorem, we have an effective way to obtain regular similarities of a fuzzy social network. Once the regular similarities of a fuzzy social network are obtained, traditional similarity-based clustering methods [27] can be applied to analyze the network data.

IV. Generalized Regular Equivalence

Regular similarity is a fuzzy relation, but we sometimes need a crisp role partition of a fuzzy social network. In such cases, we can use the concept of *generalized regular equivalence* (GRE). To define GRE, we need to consider the neighborhoods in fuzzy social networks. Let $\mathfrak{F} = (A, \alpha)$ be a fuzzy social network. Then, for each $a \in A$, the out-neighborhood and in-neighborhood of a, denoted by $N_\alpha^+(a)$ and $N_\alpha^-(a)$ respectively, are two fuzzy subsets of A with the following membership functions:

$$\mu_{N_\alpha^+(a)}(b) = \mu_\alpha(a,b)$$

$$\mu_{N_\alpha^-(a)}(a) = \mu_\alpha(b,a).$$

Let B be a fuzzy subset of A and ρ be an equivalence relation on A. Then, $[B]_\rho$ is a fuzzy subset of the quotient set $A/\rho = \{[a]_\rho \mid a \in A\}$ with the following membership function:

$$\mu_{[B]_\rho}([a]) = \max_{b \in [a]} \mu_B(b).$$

Thus, an equivalence relation ρ is a GRE with respect to a fuzzy relation α if $(a, b) \in \rho$ implies

$$[N_\alpha^+(a)]_\rho = [N_\alpha^+(b)]_\rho \text{ and } [N_\alpha^-(a)]_\rho = [N_\alpha^-(b)]_\rho.$$

Let us somewhat abuse the notation and write $\mu_\alpha(a, [b]_\rho)$ and $\mu_\alpha([a]_\rho, b)$ to denote $\max_{c\in[b]} \mu_\alpha(a, c)$ and $\max_{c\in[a]} \mu_\alpha(c, b)$ respectively. Then, we have an alternative formulation of GRE.

Lemma 3: An equivalence relation ρ is a GRE with respect to a fuzzy relation α iff for $a, b \in A$, $(a, b) \in \rho$ implies $\mu_\alpha(a, [c]_\rho) = \mu_\alpha(b, [c]_\rho)$ and $\mu_\alpha([c]_\rho, a) = \mu_\alpha([c]_\rho, b)$ for all $c \in A$.

Based on this formulation, we can establish the connection between regular similarity and GRE.

Lemma 4: Let α be a fuzzy relation and ρ be an equivalence relation on a finite set A. Then, ρ is a GRE with respect to α iff $\alpha\rho = \rho\alpha$.

Since an equivalence relation can be seen as a special case of a similarity relation, Lemma 4 shows that regular similarity and GRE are equivalent for equivalence relations.

Like regularity equivalences, GRE is also closed with respect to the usual join of equivalence relations. Thus, given an equivalence relation π, we can define the *generalized regular interior* of π (with respect to a fuzzy relation α), denoted by π^g, as the join of all GRE's (with respect to α) included in π (i.e., the largest GRE included in π). The next theorem shows that the well-known CATREGE algorithm [28] can be used to compute the generalized regular interior of a given equivalence relation. Given an equivalence relation π and a fuzzy relation α on the domain A, the shorthand $ne(\alpha, \pi)$ (meaning the neighborhood equality) is used to denote the equivalence relation $\{(a, b) \mid [N_\alpha^+(a)]_\pi = [N_\alpha^+(b)]_\pi \wedge [N_\alpha^-(a)]_\pi = [N_\alpha^-(b)]_\pi\}$.

Theorem 2: Let α be a fuzzy relation and π be an equivalence relation, both on a finite set A. Then, the generalized regular interior of π with respect to α is equal to

$$\pi^g = \bigcap_{i \geq 0} \pi_i,$$

where $\pi_0 = \pi$ and

$$\pi_{i+1} = \pi_i \cap ne(\alpha, \pi_i).$$

Note that though GRE is equivalent to regular similarity for equivalence relations, the generalized regular interior and similarity-based regular interior are not necessarily the same for a given equivalence relation.

Example 1: Let us consider a trivial two-actor fuzzy social network $(\{1, 2\}, \alpha)$ with α, as shown in Figure 1, where we assume that $r_1 \neq r_2$.

Then, the largest GRE with respect to α is the identity relation, but the largest regular similarity is specified by the following

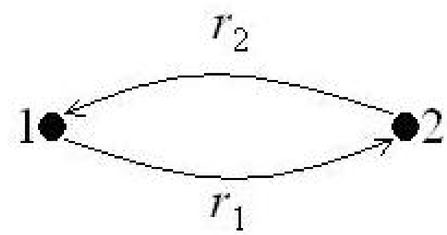

Fig. 1. A fuzzy social relation between two actors

matrix equation

$$\begin{bmatrix} 0 & r_1 \\ r_2 & 0 \end{bmatrix} \begin{bmatrix} 1 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 & x \\ x & 1 \end{bmatrix} \begin{bmatrix} 0 & r_1 \\ r_2 & 0 \end{bmatrix},$$

which can be rewritten into

$$\begin{bmatrix} \min(r_1, x) & r_1 \\ r_2 & \min(r_2, x) \end{bmatrix} = \begin{bmatrix} \min(r_2, x) & r_1 \\ r_2 & \min(r_1, x) \end{bmatrix}.$$

Since the largest solution of $\min(r_1, x) = \min(r_2, x)$ is $x = \min(r_1, r_2)$, the adjacency matrix of the largest regular similarity with respect to α is

$$\begin{bmatrix} 1 & \min(r_1, r_2) \\ \min(r_1, r_2) & 1 \end{bmatrix},$$

which is obviously not equal to the identity matrix if $\min(r_1, r_2) \neq 0$.

V. Conclusion

We have generalized the notion of regular equivalences to fuzzy social networks. There exist different but equivalent definitions of regular equivalences in the literature. However, when generalized to fuzzy social networks, these definitions may result in inequivalent notions of similarity. We consider two kinds of generalizations in this paper. The regular similarity is generalized according to the commutativity between the similarity relation and the underlying fuzzy relation, while the GRE is generalized according to the equality of neighborhoods of equivalent actors. We show that, in some special cases, these two generalizations are still equivalent; however, the regular interiors based on them may be different. We also present effective procedures for computing the regular interiors of a given equivalence relation or similarity relation. Though these procedures are effective, they are not efficient enough for large-scale networks. In the future, we will explore the possibility of adapting the more efficient RCPP (relational coarsest partition problem) algorithm [18] to the fuzzy case. Furthermore, in addition to regular equivalences, we will also consider generalizing other notions of equivalence in SNA [18] to fuzzy social networks.

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