Possible existence of viable models of bi-gravity with detectable graviton oscillations by gravitational wave detectors

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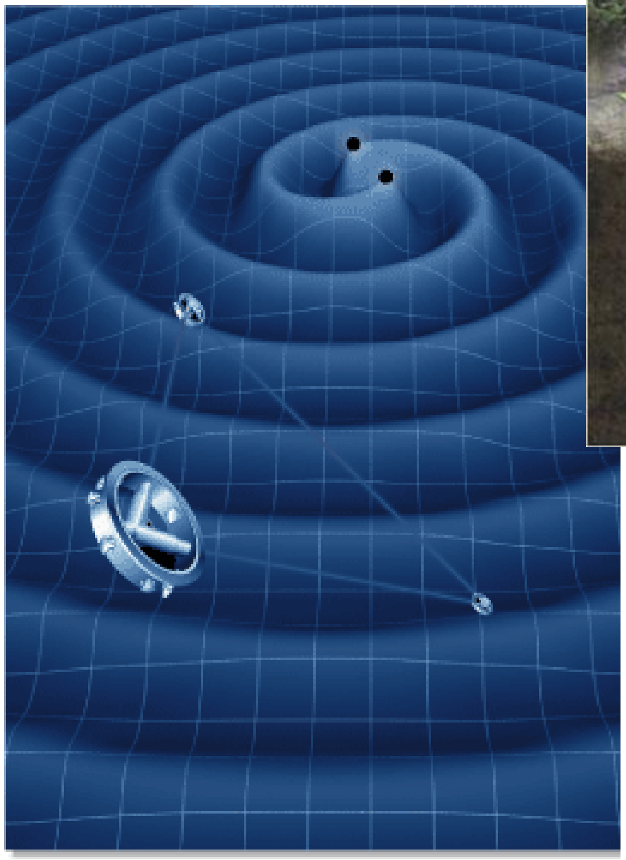
Gravitation wave detectors



eLISA(NGO)
⇒DECIGO/BBO

LIGO⇒adv LIGO

We know that GWs are emitted from binaries.

What is the big surprise?

Is there possibility that graviton disappear during its propagation over cosmological distance?

Braneworld

Induced gravity on the brane

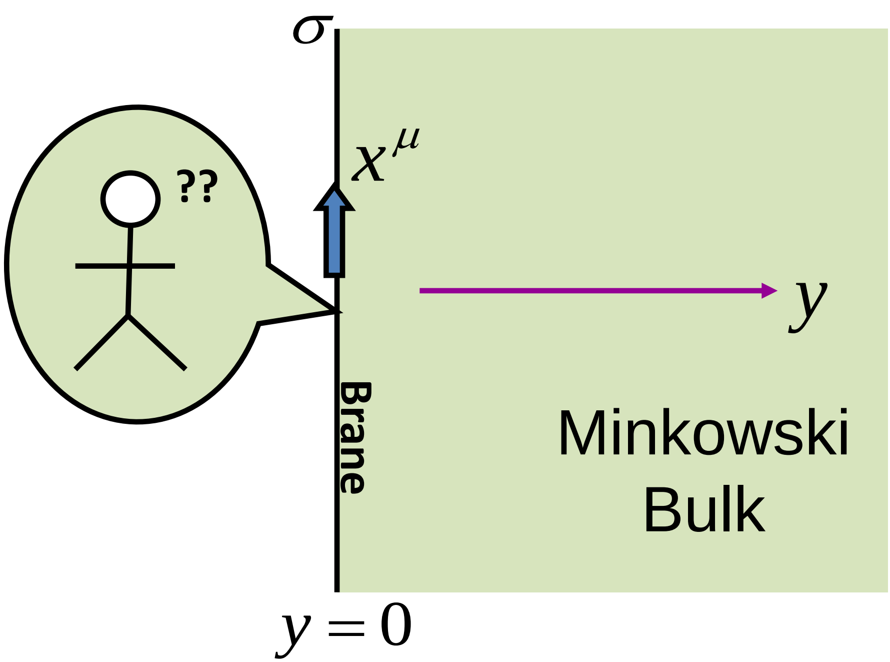
Dvali-Gabadadze-Porrati model (2000)

$$S = \underline{M_5^3} \int d^5x \sqrt{g} R + \int d^4x \sqrt{g^{(4)}} \left(M_4^2 R^{(4)} + L_{matt} \right)$$

$$\boxed{M_5^3 = M_4^2 / 2r_c}$$

Critical length scale

$$\boxed{\frac{M_5^3}{r} : \frac{M_4^2}{r^2} = \frac{1}{rr_c} : \frac{1}{r^2}}$$



- For $r < r_c$, 4-D induced gravity term dominates?
- Extension is infinite, but 4-D GR seems to be recovered for $r < r_c$.

very different from the other braneworld models

Gravitons are trapped to the brane but not completely.

5D scalar toy model:

$$\left[M_5^3\Box+\delta(y)M_4^2\Box^{(4)}\right]\phi=\delta(y)J$$

Source term

$$\boxed{\phi=\tilde{\phi}(y)e^{ip_\mu x^\mu}}$$

$$M_5^3=M_4^2/2r_c$$

$$M_5^3\left(p^2-\partial_y^2\right)\tilde{\phi}+\delta(y)M_4^2p^2\tilde{\phi}=-\delta(y)\tilde{J}$$

$$\boxed{\int_{-\varepsilon}^{\varepsilon}dy\,(\text{equation})}$$

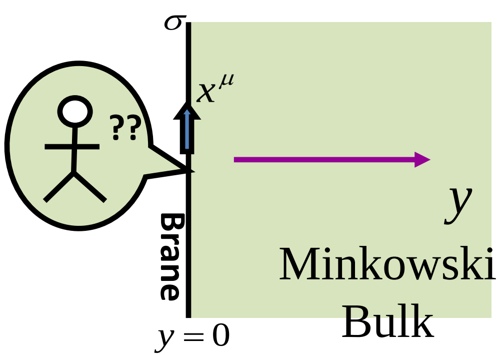
$$-2M_5^3\partial_y\tilde{\phi}+M_4^2p^2\tilde{\phi}=-\tilde{J}$$

Solution in the bulk is given by

$$\tilde{\phi}=\tilde{\phi}_0\,e^{-py}$$

$$\left[2M_5^3p+M_4^2p^2\right]\tilde{\phi}=-\tilde{J}$$

$$\tilde{\phi}\propto\frac{\tilde{J}}{p+r_cp^2}$$



$$\tilde{\phi} \propto \frac{\tilde{J}}{p + r_c p^2}$$

Static pointlike source on the brane

$$\boxed{\tilde{J} \propto \int dt\, e^{-i\omega t} \int d^3x\, e^{i\vec{k}\cdot\vec{x}} \delta(x) \propto \delta(\omega)}$$

large scale (small k) $\quad \phi \propto \int d^3k\, e^{ikr} \frac{1}{k} \propto \frac{1}{r^2}$

five dimensional behavior

small scale (small k) $\quad \phi \propto \int d^3k\, e^{ikr} \frac{1}{k^2} \propto \frac{1}{r}$

four dimensional behavior

After propagation over cosmological distance, GWs may escape into the bulk?

$$\int \frac{d^3k}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}e^{i\mathbf{kx}}$$

$$=\frac{2\pi}{r}\int_{-\infty}^{\infty}\frac{\sin(kr)k\,dk}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}$$

$$=\frac{2\pi}{ir}\int_C\frac{e^{ikr}\;k\,dk}{k^2-\omega^2+r_c^{-1}\sqrt{k^2-\omega^2}}$$

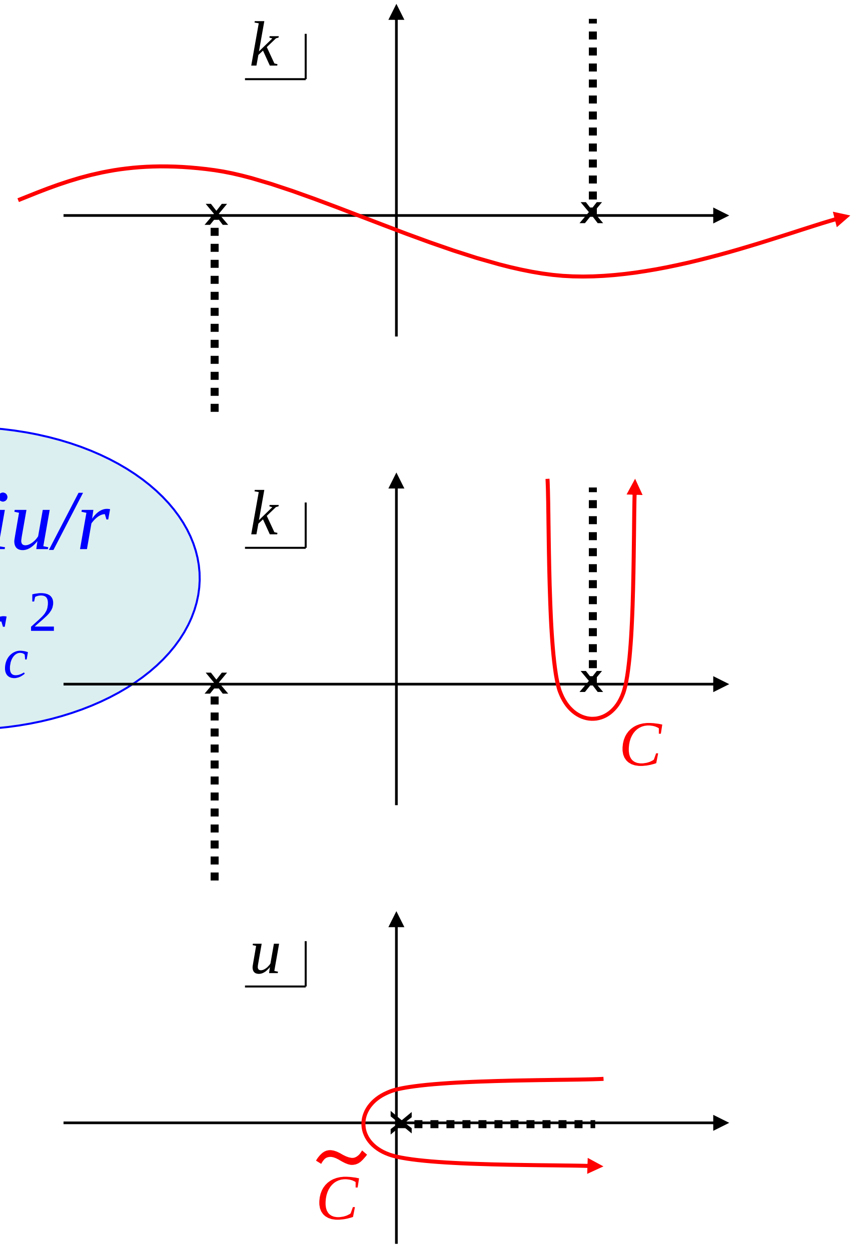
$$\approx\frac{2\pi}{r}\int_{\tilde{C}}\frac{e^{i\omega r}\;e^{-u}\;du}{2iu-\dfrac{e^{\pi i/4}}{\omega r_c}\sqrt{2\omega r u}}$$

$$\approx\frac{2\pi}{r}e^{i\omega r}\int_{\tilde{C}}\frac{e^{-u}\;du}{2iu}\left(1+\frac{e^{-\pi i/4}}{2\omega r_c}\sqrt{\frac{2\omega r}{u}}\right)$$

$$=\frac{2\pi^2}{r}e^{i\omega r}\left(1+\sqrt{\frac{2ir}{\pi\omega r_c^2}}\right)$$

$k=\omega+iu/r$

$r\ll\omega r_c^{\;2}$



Chern-Simons Modified Gravity

$$S \supset \frac{\alpha}{4}\int d^4x\sqrt{-g}\,\theta\,{}^*RR$$

Right handed and left handed gravitational waves are magnified differently during propagation, depending on the frequencies.

$$\mathbf{h}^{(L,R)} = \frac{1}{\sqrt{2}}\left(\mathbf{h}^{(+)} + i\mathbf{h}^{(\times)}\right)$$

$$\boldsymbol{h}_{\mathrm{obs}}^{(L,R)} \approx \boldsymbol{h}^{(L,R)}\sqrt{1 \pm \omega\alpha\dot{\theta}}\,\Big|_{\mathrm{emit}}$$

Current constraint on the evolution of the background scalar field θ :

$$\left|\alpha\dot{\theta}\right| < \left(10^6\,\mathrm{Hz}\right)^{-1}$$: J0737-3039(double pulsar)

(Ali-Haimoud, (2011))

Moreover, $\left|\omega\alpha\dot{\theta}\right| \approx 1$ modes are in the strong coupling regime. outside of the validity of EFT.

bi-gravity

$$L = \frac{\sqrt{-g}R}{16\pi G_N} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi G_N \kappa} + L_{matter}(g, \phi) + \cdots$$

Both massive and massless gravitons exist.

$\rightarrow \nu$ oscillation-like phenomena?

First question is whether or not we can construct a viable cosmological model.

Ghost free massive bi-gravity

When $\tilde{g}$ is fixed, de Rham-Gabadadze-Tolley massive gravity.

$$L = \frac{\sqrt{-g}R}{16\pi G_N} + \sqrt{-g}\sum_{n=0}^{4} c_n V_n + L_{matter}$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots \qquad \tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

No gauge degrees of freedom, but by introducing a field

$$g_{\mu\nu} \to g_{\mu\nu} + 2\pi_{;\mu\nu} + \cdots$$

$$\begin{cases} \pi \to \pi - \Lambda \\ g_{\mu\nu} \to g_{\mu\nu} + 2\Lambda_{;\mu\nu} \end{cases}$$ becomes a gauge symmetry.

Fixing the gauge by $\pi = 0 \Rightarrow$ original theory

Imposing condition on $g_{\mu\nu} \Rightarrow \pi$ becomes dynamical

Setting $\tilde{g}_{\mu\nu} = \eta_{\mu\nu}$, we consider flat metric + π perturbation:

$$g_{\mu\nu} \to \eta_{\mu\nu} + 2\pi_{;\mu\nu} + \pi_{;\mu\rho}\pi^{;\rho}{}_{;\nu}$$

$$\gamma^{\mu}{}_{\nu} = \sqrt{\eta^{\mu\nu} g_{\rho\nu}} = \delta^{\mu}{}_{\nu} + \pi^{,\mu}{}_{;\nu}$$

If $L \supset \pi^{,\mu}{}_{;\nu}\pi^{,\nu}{}_{;\mu}$, its variation gives higher derivative terms.

To avoid higher derivatives of π in the EOM,

$$\varepsilon_{\mu\nu\xi\zeta}\varepsilon^{\alpha\beta\xi\zeta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}, \quad \varepsilon_{\mu\nu\rho\zeta}\varepsilon^{\alpha\beta\gamma\zeta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}\gamma^{\rho}{}_{\gamma}, \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\alpha\beta\gamma\delta}\gamma^{\mu}{}_{\alpha}\gamma^{\nu}{}_{\beta}\gamma^{\rho}{}_{\gamma}\gamma^{\sigma}{}_{\delta},$$

$$\Rightarrow V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2, \ \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

In other words:

10 (metric components) – 4 (constraints) $=6$

Since massive spin 2 filed has 5 components, one scalar remain, which becomes a ghost (kinetic term with wrong sign).

If constraints do not completely fix the Lagrange multipliers, $g_{0\mu}$, their consistency relation gives an additional condition. As a result, the residual scalar degree of freedom disappears.

(Hassan, Rosen (2011))

Ghost free bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{2} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa} + \frac{\sqrt{-g}}{2}\sum_{n=0}^{4} c_n V_n + \frac{L_{matter}}{M_G^2}$$

$$V_0 = 1, \quad V_1 = \tau_1, \quad V_2 = \tau_1^2 - \tau_2, \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \qquad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

Now $\tilde{g}$ is promoted to a dynamical field.

Even in this case, it was shown that the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

$$ds^2 = a^2(t)\left(-dt^2 + dx^2\right)$$

$$T^{(mass)}_{\mu\nu} = 2\frac{\delta S^{(mass)}}{\delta g^{\mu\nu}}$$

$$d\tilde{s}^2 = b^2(t)\left(-c^2(t)dt^2 + dx^2\right)$$

$$\xi \equiv b/a$$

$$\nabla^{\mu} T^{(mass)}_{\mu\nu} = 0 \Rightarrow \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 :

At the linear perturbation, expected scalar and vector perturbations are absent. Strong coupling? Unstable for the homogeneous anisotropic mode.

branch 2 :

All perturbation modes are equipped.

Branch 2 background

$$\underbrace{\left(6c_3\xi^2+4c_2\xi+c_1\right)}_{\text{branch 1}}\left(\underbrace{cba'-ab'}_{\text{branch 2}}\right)=0 \qquad \xi\equiv b/a$$

branch 2 :

$$\rho-\frac{c_1}{\kappa\xi}+\left(c_0-\frac{6c_2}{\kappa}\right)+\left(3c_1-\frac{18c_3}{\kappa}\right)\xi+\left(6c_2-\frac{24c_4}{\kappa}\right)\xi^2+6c_3\xi^3=0$$

ξ becomes a function of ρ. $\xi \to \xi_c$ for $\rho \to 0$.

$$H^2=\frac{\rho+\rho_{mass}}{3M_G^2} \qquad \rho_{mass}:=c_0+3c_1\xi+6c_2\xi^2+6c_3\xi^3$$

effective energy density due to mass term

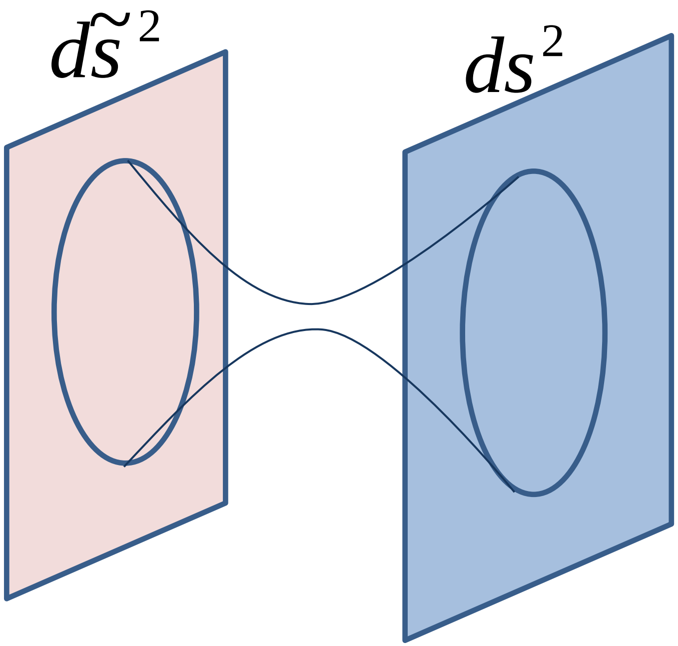
$$\frac{1}{c-1}\frac{\xi'}{\xi}=\frac{a'}{a} \Rightarrow c-1=\frac{3(\rho+P)\kappa\xi_c}{\Gamma_c\left(1+\kappa\xi_c^2\right)M_G^2} \qquad \Gamma=\frac{d\rho_{mass}}{d\xi}$$

Natural Tuning to c=1 for $\rho \to 0$.

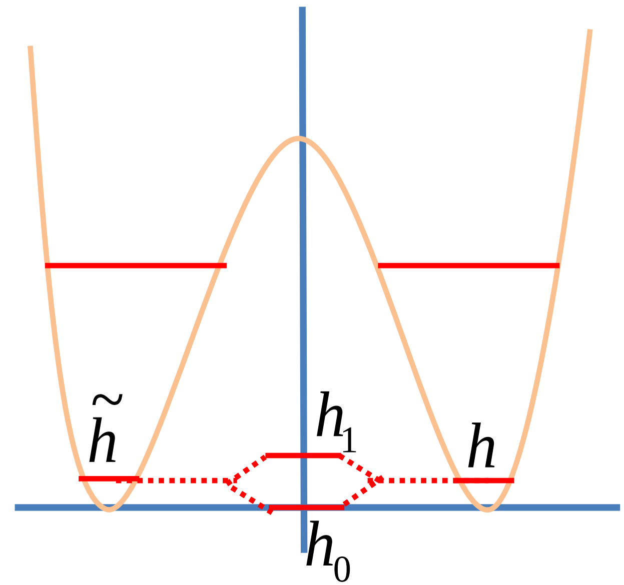
$$H^2=\frac{\rho}{3\left(1+\kappa\xi_c^2\right)M_G^2}$$

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

Why do we have this attractor behavior, $c \to 1$ and $\xi \to \xi_c$, at low energies?



Higher dimensional model



KK graviton spectrum
Only first two modes remain at low energy

Matter on right brane couples to h.

If the internal space is stabilized

⇒ $d\tilde{s}^2 = \xi_c^2 ds^2$ ⇒ $c = 1$

⇒ Only the tensor perturbation should remain as propagating modes at low energies.

EOM of Gravitational waves

$$m_g^2 = \xi\left(c_1 + 2c_2(c+1)\xi + 6cc_3\xi^3\right)$$

$$h'' + 2aHh'' - \Delta h + a^2 m_g^2\left(h - \tilde{h}\right) = 0$$

$$\tilde{h}_2'' + \left(2aH + 2\xi'/\xi - c'/c\right)\tilde{h}' - c^2\Delta\tilde{h} - a^2 m_g^2 \frac{c}{\kappa\xi^2}\left(h - \tilde{h}\right) = 0$$

(Comelli, Crisostomi, Pilo (2012))

Short wavelength approximation: $k >> m_g >> H$

$$\begin{pmatrix} -\omega^2 + k^2 + m_g^2 & -m_g^2 \\ -\frac{1}{\kappa\xi^2} m_g^2 & -\omega^2 + c^2 k^2 + \frac{1}{\kappa\xi^2} m_g^2 \end{pmatrix} \begin{pmatrix} h \\ \tilde{h} \end{pmatrix} = 0$$

Two propagation speeds are not same for $c \neq 1$.
[$\neq \nu-$oscillation]

$\Rightarrow$ Eigen mode decomposition

$$\begin{cases} h_A = \cos\theta_g h + \sin\theta_g \left(\sqrt{\kappa}\xi\tilde{h}\right) \\ h_2 = -\sin\theta_g h + \cos\theta_g \left(\sqrt{\kappa}\xi\tilde{h}\right) \end{cases}$$

$$\mu^2 = m_g^2 \frac{1 + \kappa\xi^2}{\kappa\xi^2}$$

$$k_{1,2}^2 = \omega^2 - \frac{\mu^2}{2}\left(1 + x \mp \sqrt{1 + 2x\frac{1 - \kappa\xi^2}{1 + \kappa\xi^2} + x^2}\right)$$

$$\cot 2\theta_g = \frac{\left(1 + \kappa\xi_c^2\right)x + \left(1 - \kappa\xi_c^2\right)}{2\sqrt{\kappa}\xi_c}$$

$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

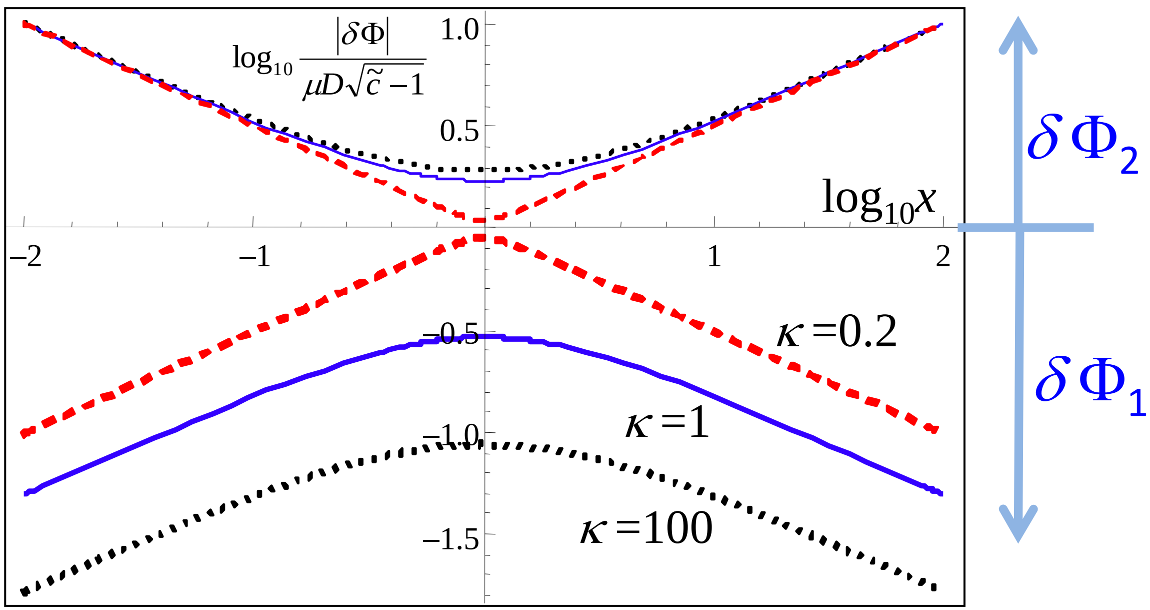
Gravitational wave propagation over a long distance D

Phase shift due to the modified dispersion relation:

$$\delta\Phi_{1,2} \equiv -\frac{\Delta k^2}{2\omega} D = -\frac{\boxed{\mu D\sqrt{c-1}}}{2\sqrt{2x}} \left(1 + x \mp \sqrt{1 + 2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2} + x^2} \right)$$

$$\mu D\sqrt{c-1} \approx HD\sqrt{3\left(1+\kappa\xi_c^2\right)\Omega_0}$$

becomes O(1) after propagation over the horizon distance



$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

Solar system constraint: basics

◆ vDVZ discontinuity

In GR, this coefficient is 1/2

$$\delta g_{\mu\nu} \propto \Box^{-1}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$

current bound <10^{-5}

To cure this discontinuity

- Make the extra helicity 0 mode massive

$$\mu^{-1} \geq 0.1\mathrm{mm} \quad \text{(Chameleon)}$$

- Go beyond the linear perturbation (Vainshtein)

Schematically

$$\cancel{\Delta\delta\Phi} + \mu^{-2}(\partial\partial\delta\Phi)^2 = G_N \rho$$

$$\Rightarrow \delta\Phi/\Phi \approx \mu\sqrt{G_N\rho}\, r^2 \big/ (r_g/r) \approx \mu\sqrt{r^3/r_g}$$

$$10^{-10} \geq \mu\sqrt{(10^{13}cm)^3/(10^5 cm)} \Rightarrow \mu^{-1} \geq 300 Mpc$$

Gravitational potential around a star in the limit $c \to 1$

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$
$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R ,

$$\left(\Delta - \mu^2\right)u - \frac{C}{\mu^2}\left((\Delta u)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$$C \propto \frac{d\left(\log \Gamma\right)}{d\left(\log \xi\right)}$$, which can be tuned to be extremely large.

Then, the Vainshtein radius $r_V \approx \left(\frac{C r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} << 300\mathrm{Mpc}$.

Solar system constraint: $\sqrt{C}\mu^{-1} \geq 300\mathrm{Mpc}$

$$\Delta v \approx \frac{\rho_m}{\tilde{M}_G^2}$$ v is excited as in GR. $$H^2 = \frac{\rho}{3\tilde{M}_G^2}$$

Excitation of the metric perturbation on the hidden sector:

Erasing u, v and R

$$\left(\Delta-\mu^2\right)\tilde{u}-\frac{\tilde{C}}{\mu^2}\left(\left(\Delta\tilde{u}\right)^2-\left(\partial_i\partial_j\tilde{u}\right)^2\right)\approx\frac{\rho_m}{M_G^2}$$

$$\Delta\tilde{v}\approx\frac{\rho_m}{\tilde{M}_G^2}$$

$\tilde{u}$ is also suppressed like u.

$\tilde{v}$ is also excited like v.

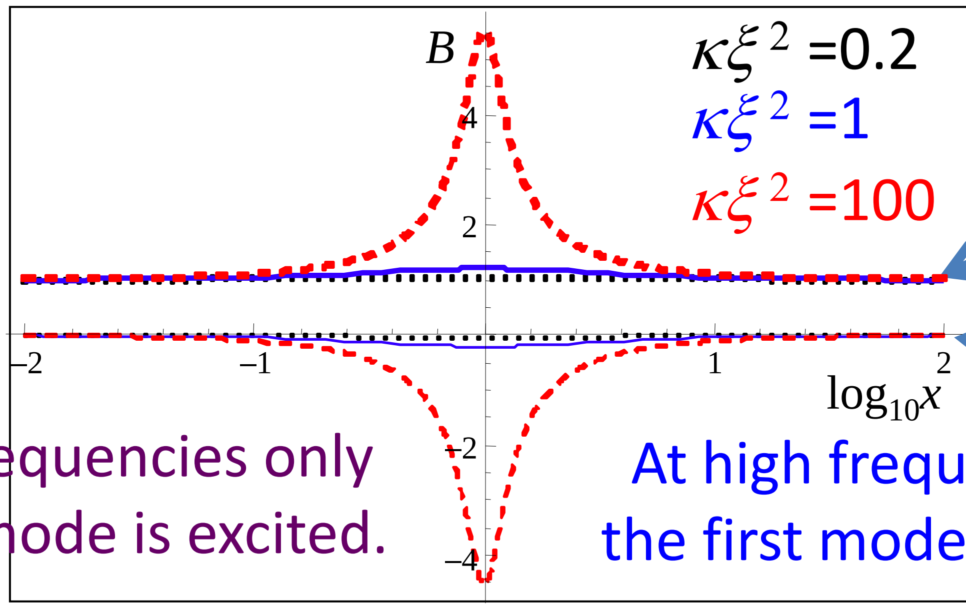
The metric perturbations are almost conformally related with each other: $d\tilde{s}^2\approx\xi_c^2 ds^2$

Non-linear terms of $\tilde{u}$ (or equivalently u) play the role of the source of gravity.

Gravitational wave oscillations

At the time of generation of GWs from coalescing binaries, both h and $\tilde{h}$ are equally excited.

$$\Rightarrow \quad h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$



$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

At low frequencies only the first mode is excited.

At high frequencies only the first mode is observed.

Graviton oscillations occur only around the frequency

$$\omega_{GW} \approx \frac{\mu^2}{\sqrt{6(1+\kappa\xi_c^2)\Omega_0}\,H} = 100\mathrm{Hz}\left(\frac{1+\kappa\xi_c^2}{100}\right)^{-1/2}\left(\frac{\mu}{(0.08\,pc)^{-1}}\right)^4$$

with the phase shift $\sim HD\sqrt{3(1+\kappa\xi_c^2)\Omega_0}$

Summary

Gravitational wave observations give us a new probe to the modified gravity theory.

Even graviton oscillations are not immediately denied, and hence we may find something similar to the case of solar neutrino experiment in near future.

To construct a detectable model, a large $\kappa\xi^2$ is necessary. It is crucial that the effective gravitational coupling for structure formation is identical to the one that determines the background cosmology.